The field of thermodynamics is one of the crown jewels of classical physics. However, only comparatively recently, due to the advent of experiments in cold atomic systems with long coherence times, has our detailed understanding of its connection to quantum statistical mechanics seen remarkable progress. Extending these ideas and concepts to the non-equilibrium setting is a challenging topic, in itself of perennial interest. Here, we study perhaps the simplest non-equilibrium class of quantum problems, namely Floquet systems, i.e. systems whose Hamiltonians depend on time periodically, $H(t + T) = H(t)$. For these, there is no energy conservation, and hence not even a natural concept of temperature. We find that it is nonetheless possible to identify several fundamentally distinct thermodynamic ensembles. We also ask if there exists a sharp notion of a phase in such driven, interacting quantum systems. Disorder turns out to play a crucial role, enabling the existence of states which are straightforward analogues of equilibrium states with broken symmetries and topological order, while others – genuinely new to the Floquet problem – are characterized by a combination of order and non-trivial periodic dynamics. This work was done in collaboration with Arnab Das, Vedika Khemani, Achilleas Lazarides and Shivaji Sondhi.