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Mathematical aspects of random matrices and applications in statistical physics and field theory

The area of random matrix theory (RMT) is a branch of mathematical physics, where many different tools from mathematics enter, like analysis, probability theory or combinatorics, and where results apply to many different areas in physics, such as quantum field theory, quantum chaotic systems, condensed matter or models of statistical mechanics. This is perhaps the reason why RMT continues to be such a lively and interdisciplinary area of research.

One of its goals is to develop a statistical theory of spectra, e.g. of Hamilton or Dirac operators in physics, much like the concept of the scalar quantities as temperature or pressure in standard thermodynamics. Naively, such an operator is replaced by a finite matrix with random variables as entries, depending only on global symmetries like time reversal or charge conjugation. We then seek the laws of how the corresponding matrix eigenvalues are correlated in the limit of large matrix size, in particular if they are universal, that is independent of the details of the distribution of matrix elements. Clearly such a statistical theory of spectra will only describe certain aspects of the underlying physical system. While a comparison with RMT statistics often remains heuristic, several examples exist where we well understand the approximations to be made, that allow for a RMT description to apply, e.g. in lattice gauge theory or quantum chaos.

Many mathematical aspects of RMT that are studied here in Bielefeld are inspired by integrable systems. Advances on a technical level have led us to study and solve RMTs consisting of sums or products of several random matrices. Introducing such an additional structure allows to describe more details of the underlying physical system, as for example the effect the effect of chemical potential or finite lattice spacing in lattice gauge theory, the effect of topological zero modes in quantum Hamiltonians, or properties of the Lyapunov exponents in dynamical systems. Ideally, such multi-matrix models can be mapped to a determinantal or Pfaffian point process of its eigenvalues, and the asymptotic properties of the underlying kernel can be studied, including its universality. Among the most recent examples are non-Hermitian random matrices, including their eigenvector statistics, where universality is still not fully understood.

Part of this research is conducted in the framework of the following programs:

SFB1283 https://www.sfb1283.uni-bielefeld.de/

IRTG2235 https://irtg.math.uni-bielefeld.de/

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