Bielefeld 2012

On the Unlikeliness of Multi-Field Infation: Bounded Random Potentials and our Vacuum

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What is the inflatons' most likely resting place, if inflation is driven by multiple fields in a random potential that is softly bounded from above and below, with equal likelihood for positive and negative values?

Some Known Results based on Random Matrix theory:

Super exponential suppression for non-diagonal Hessians with zero mean:
For large D, almost all critical points are saddle points.

Aazami, Easther 05;
Dean, Majumdar 06; Vivo, Majumdar, Bohigas 07;
Marsh, McAllister, Wrase 11;
Chen, Shiu, Sumitomo, Tye 11

If Inflation takes place, it will most likely occur at a single saddle point.

Aazami, Easther 05

If Inflation takes place, it is more likely to be of short duration.

Freivogel, Kleban, Rodriguez Martinez, Susskind 05
Agarwal, Bean, McAllister, Xu 11

Tegmark 04;
Frazer, Liddle 11;

Our model for a bounded random potential

Truncated Fourier Series (usually n=5)

$$V = \sum_{J_1,\ldots,J_D=1}^{n} \left(a_{J_1\ldots J_D} \cos \sum_{i=1}^{D} J_i x_i + b_{J_1\ldots J_D} \sin \sum_{i=1}^{D} J_i x_i \right)$$

$$\equiv \sum_{J_1,\ldots,J_D=1}^{n} V_{J_1,\ldots,J_D} \,,$$

$$\mathbf{J} \equiv J_1, \ldots, J_D \,,$$

$$x_i = \varphi_i / M_p, \; M_p = (8\pi G)^{-1/2} \equiv 1 \qquad \mathbf{x} \equiv x_1, \ldots, x_D \,, \;\; \text{etc.} \,,$$

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Choose coefficients according to

$$P_{a_{J_1\ldots J_D}} = \frac{1}{\sqrt{2\pi}\sigma_{J_1\ldots J_D}} \exp\left(-\frac{a^2_{J_1\ldots J_D}}{2\sigma^2_{J_1\ldots J_D}} \right)$$

$$\sigma_{J_1\ldots J_D} = \exp\left(-\sum_{i=1}^{D} \frac{J_i^2}{Dn} \right)$$

Boundedness is crucial for viability of low energy EFT.

Coefficients chosen s.t. potential resembles beginners ski-slope (no sharp cliffs etc.) to support inflation.

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Variance of V

$$\sigma_V^2 = \sum_{\mathbf{J}} \sigma_{\mathbf{J}}^2 \approx \left(\frac{Dn\pi}{8} \right)^{D/2}$$

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The Hessian:

$$H_{kl} \equiv -\left(\bar{V} + h_{kl}\right)$$

Height of potential at critical point $V(\bar{\mathrm{x}}) = \bar{V}$

h_kl are Gaussian-distributed, have zero mean and variance

$$\sigma_{kl}^2 \approx \begin{cases} \left(\frac{Dn\pi}{8}\right)^{D/2} \frac{3}{16} D^2 n^2 \equiv \sigma^2 \text{ for } k = l \\ \sigma^2/3 \text{ for } k \neq l \,, \end{cases}$$

Crucial features:

1. Matrix elements of the Hessian have non-zero mean set by the height of the potential at the critical point! This is a generic feature of bounded potentials.

2. Hessian is not diagonal (the different variance for diagonal and off-diagonal elements is model dependent, as well as the value of the variance; that the matrix is not diagonal is again generic).

Probability that a Critical Point is a Minimum:

Non-diagonal Hessian: (well motivated) Ansatz

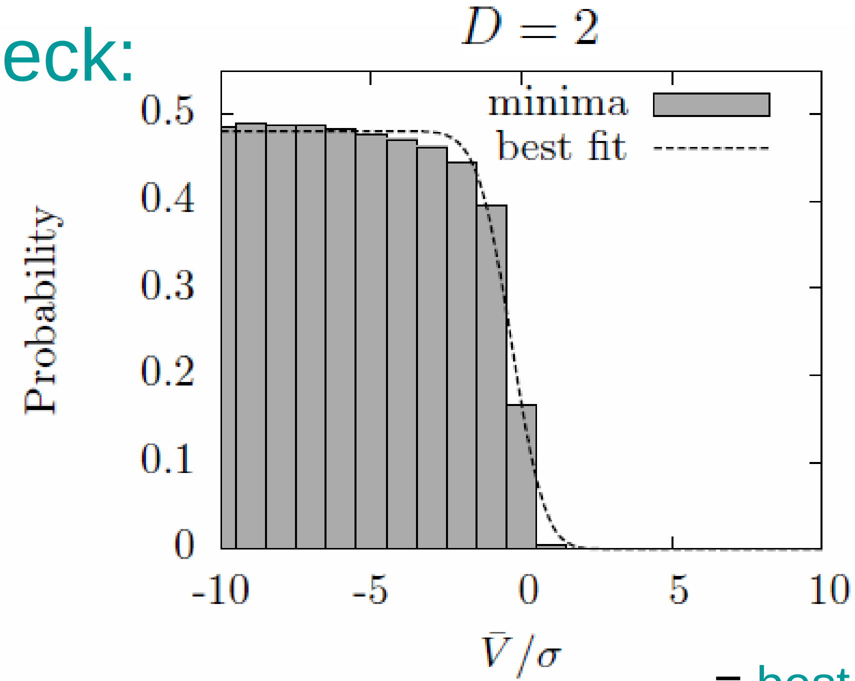
$$P_{min} \sim \left(e^{-c} \left(1 - \text{erf} \left(\frac{\bar{V}}{\sqrt{2}\sigma} \right) \right) \right)^{D^p}$$

Upper bound results by setting

$$c \equiv \ln(2) \left(1 + \frac{D-1}{D^p} \right)$$

such that $P_{min}^{\text{limit}}(V \to -\infty) \equiv 2^{-D+1}$

Check:



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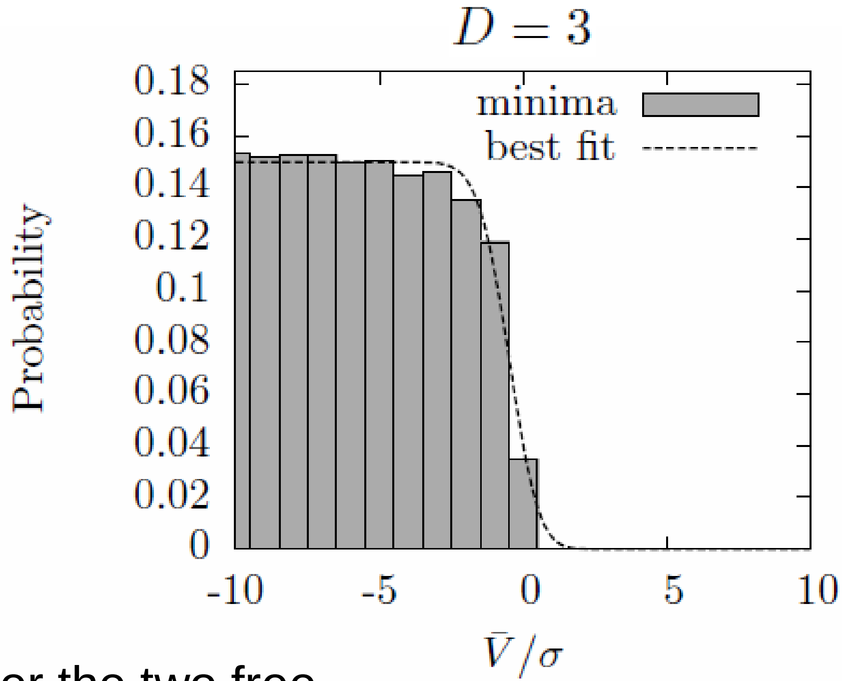

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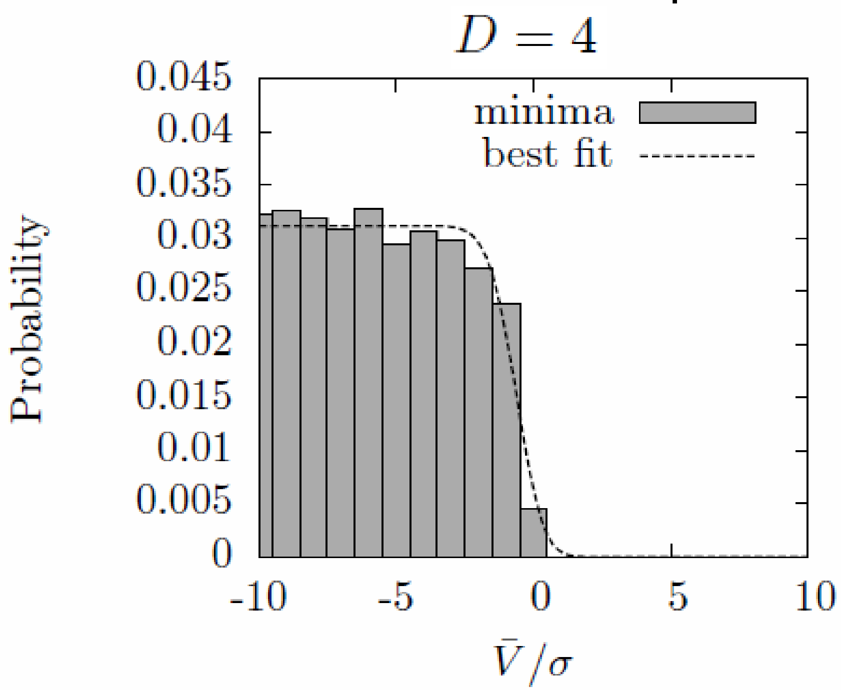
---- = best fit over the two free parameters in our ansatz.



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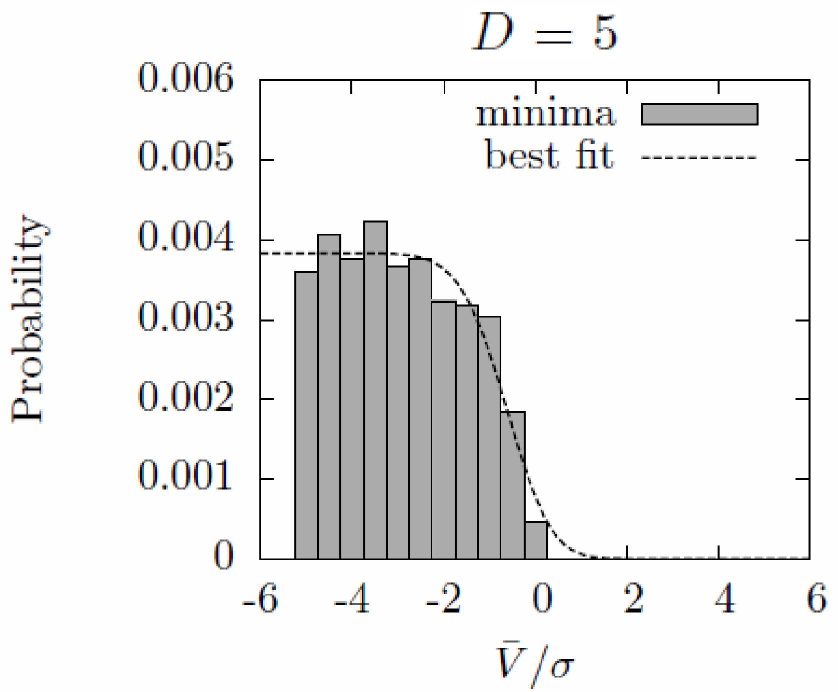

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Where are the Minima?

Analytic upper bound based on above ansatz:

$$P_{no\,min}(\bar{V}) \approx (1 - P_{min}(-\infty))^{-(\bar{V}-\bar{V}_c)n_c/\sigma}$$

$$\bar{V}_c \equiv \sigma\sqrt{2}\,\mathrm{erf}^{-1}\left(1 - e^c\left(2^{-D/D^p}\right)\right)$$

Thus, the peak of the distribution of minima encountered dynamically is bounded from above by

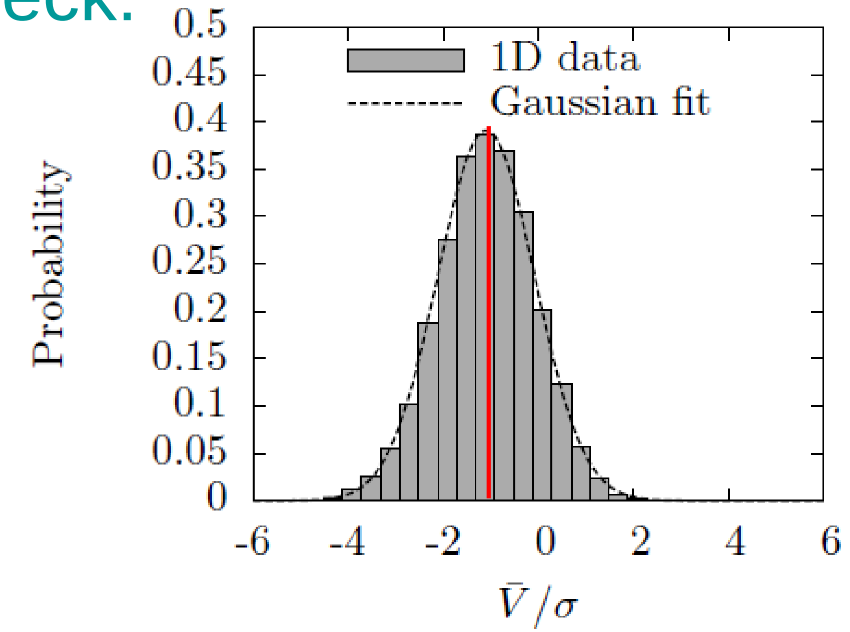
$$\bar{V}_p = \bar{V}_c + \frac{\sigma}{n_c}\frac{\ln(2)}{\ln(1-2^{-D+1})}$$

here the number of critical points encountered while traversing $\Delta V \sim \sigma$ is $n_c \sim 1$

Large D limit:

$$\boxed{\bar{V}_p \approx -\sigma 2^{D-1}\ln(2)/n_c}$$

Check:



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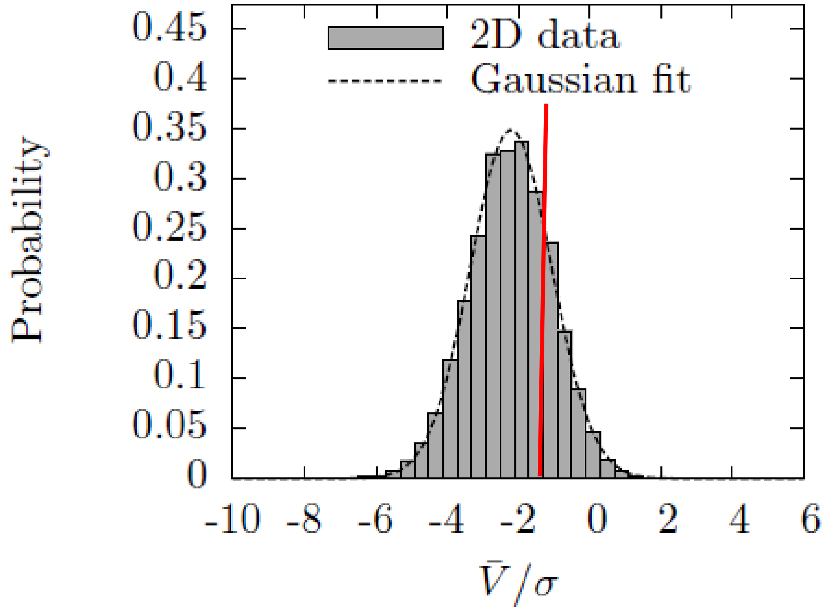

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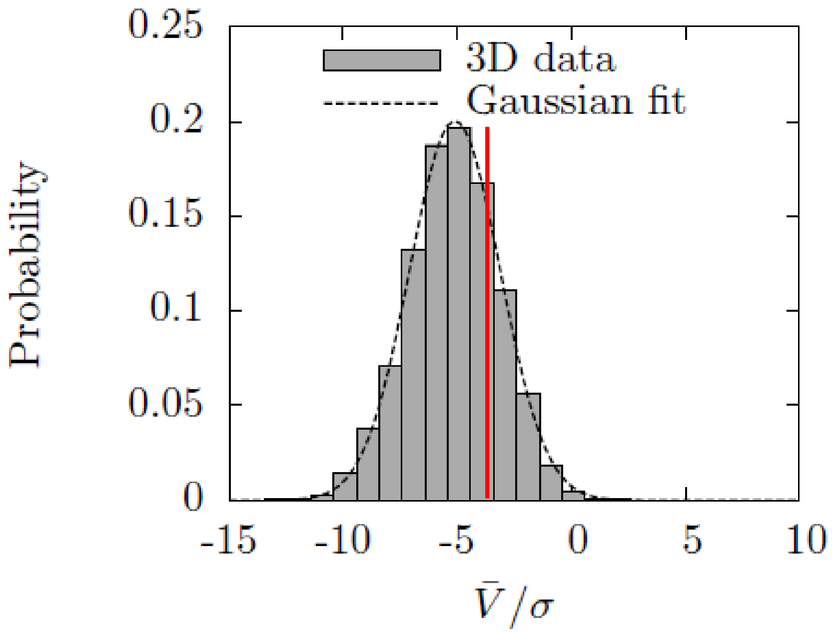


(c)

Red line:

the analytic upper bound to the peaks position.

Inflation in a Random Potential:

Equations of motion:

$$3H^2 = V + \sum_{i=1}^{D} \frac{\dot{\varphi}_i^2}{2}$$

$$\ddot{\varphi}_i + 3H\dot{\varphi}_i = -\frac{\partial V}{\partial \varphi_i}$$

e-folds

$$N = \int H(t)\, dt$$

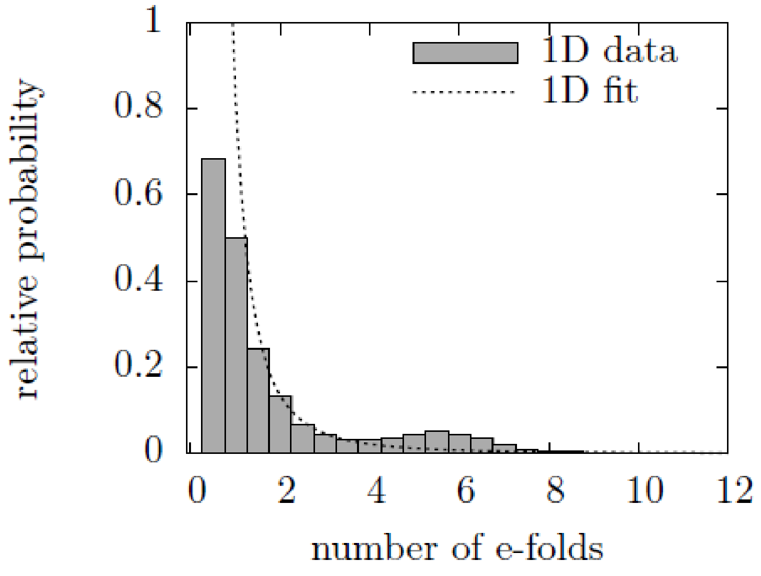
Steps:

1. Crate random potential.
2. Start field at origin (if V<0 reject).
3. Evolve until either a minimum is reached or V<0.
4. Note number of e-folds.
5. Repeat.

To speed up computations, we set terms in V to zero if coefficients are below a prescribed cutoff (we checked that results are independent of the chosen cutoff).

One Inflaton:

Bump is most likely due to prescribed scale in our potential (not seen in Frazer, Liddle 11)



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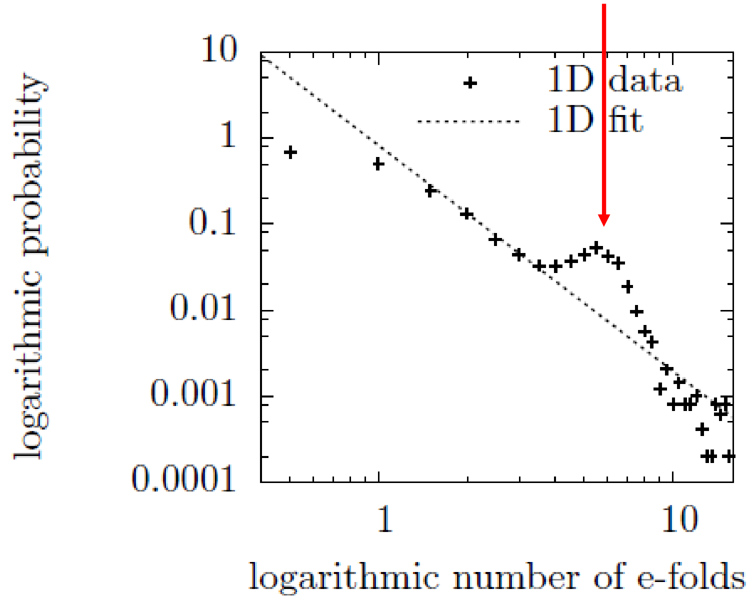

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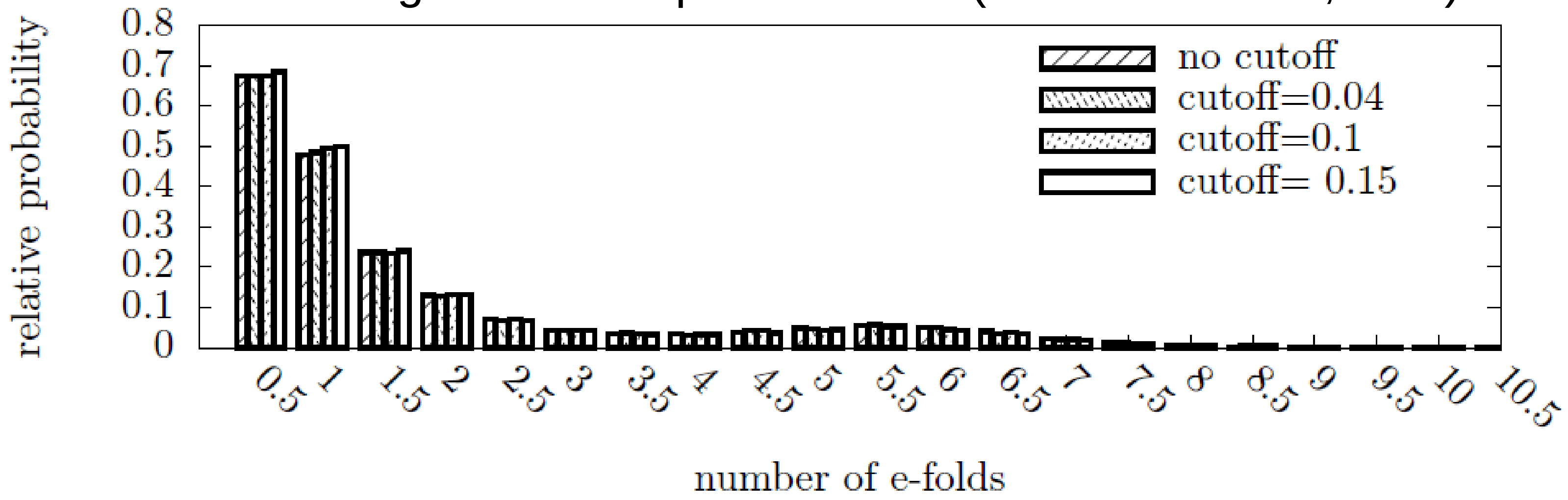
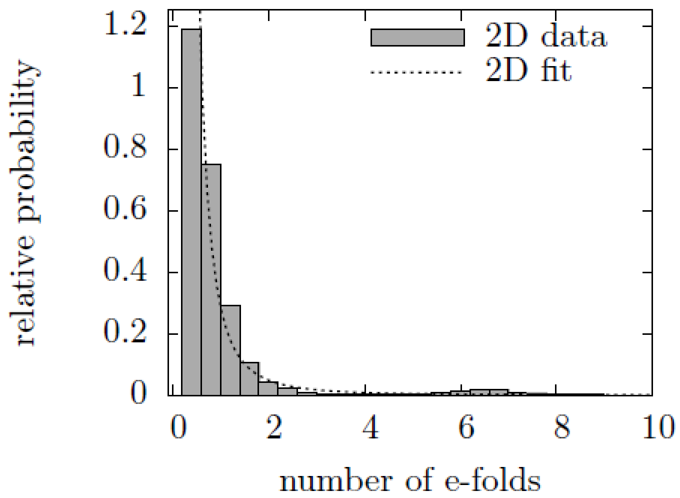
No strong effect of imposed cut-off (checked in D=2,3 too):



Power-law fit:

$P(N) = \alpha N^{-\beta}$

Dimension	# of runs	cut-off σ_{min}	α	β
1	20000	–	0.8	2.6
2	20000	0.14	0.3	2.3
3	20000	0.14	0.1	2.9



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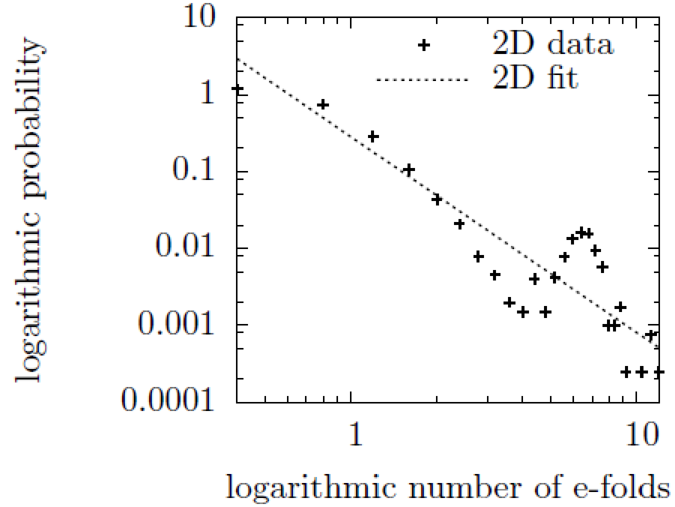

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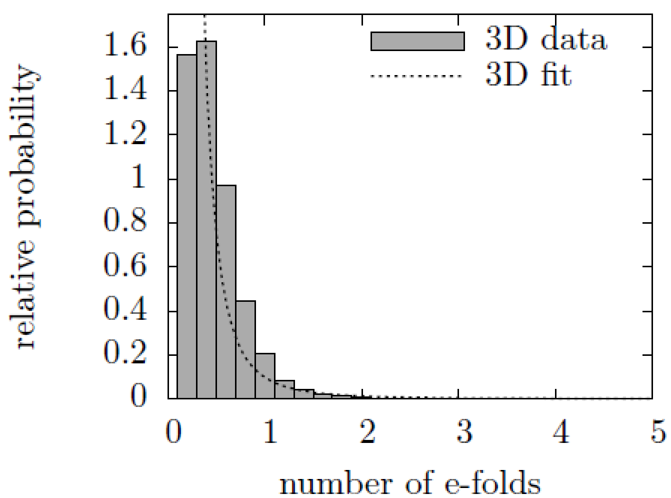

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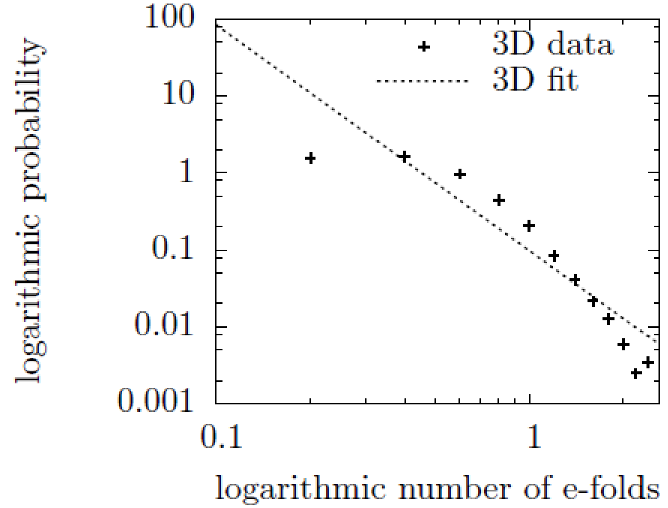

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Results (in agreement with our analytic predictions):

- If inflation takes place, it is near a saddle (small field model).

- As D is increased, it becomes exceedingly unlikely to come close to a saddle (fields follow steepest descent): inflation becomes unlikely.

- Brief spurts of inflation are more likely than longer ones: power-law with exponent close to -3.

- The trajectory during inflation is usually not strongly bend.

- The final resting place after inflation shifts to negative values of the potential (ADS): only ~0.25 % of runs have V>0 for D=3 (analytics overestimate this percentage as ~1.9%, i.e. they are too conservative)

Opposite to our naive expectation, the likelihood of finding a positive cosmological constant decreases rapidly as D increases (the huge growth of the absolute number of critical points is irrelevant.)

Is Multi-Field Inflation unlikely?

We do not live in ADS, but in a universe with a positive CC.

Without invoking anthropics, the most likely resting place after MFI in a random potential is at V<0, yielding a large negative contribution to the CC.

Two options:

1. Effective dimensionality directly after inflation is low (requires decoupIding of almost all (massive) moduli, Chen, Shiu, Sumitomo, Tye 11)

2. If the effective dimensionality remains large, we either need tremendous luck, or a strongly but precisely uplifted potential (not natural at all) or a sharp bound at V=0 (not generic; ADS vacua are the norm)

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If the effective dimensionality of field-space directly after inflation is low,
Why should it be large during/before inflation?

Is Multi-Field Inflation unlikely?

Thus, this type of multi-field inflation is ruled out by the observation of a positive CC! This conclusion applies to `realistic' potentials in which we encode unkown physics via random parameters, in the absence of any anthropic reasoning.

The conclusion does not apply to assisted inflation models (several copies of the same field, i.e. axions in N-flation, etc.) or potentials which are very simple. These models are the result of deliberate choices of designer: even though they are concrete and predictive, they tell us little about models and predictions that are likely in string theory.

Note: current data does not need or constrain more than one field anyhow (see i.e. Norena, Wagner,Verde, Peiris, Easther 12.) Apply Okkhams razor!

The effective-field-theory of single-field inflation in the presence of some heavier fields could not only be entirely sufficient to tell us everything we will ever know about inflation, but appears to be favored over more complex multi-field models in light of a positive cosmological constant.

For recent work on EFT, see

Cheung, Creminelli, Fitzpatrick, Kaplan, Senatore 07;
Weinberg 08;
Achucarro, Gong, Hardeman, Palma, Patil 12;
Jackson, Schalm 10,11,12;

A way out via anthropics?

Anthropic bound on a positive (Weinberg 87) and negative (Bousso, Leichenauer 08) CC exist and work well (merely requiring gravitationally bound objects to form).

To use them, we need to invoke eternal inflation to sample all possible trajectories. Our universes' history in a high dimensional random inflaton potential thus requires:

- eternal inflation (i.e. in a rare long lived minimum high up in the potential)
- the unlikely encounter with a saddle point
- an unlikely long phase of inflation (N ~ 60)
- the extremely unlikely encounter with a metastable, low deSitter vacuum directly after inflation terminates.

Our Universe is all but generic.

Problem: statistical predictions in eternal inflation are hampered by our inability to define a unique measure on infinite sets (Measure Problem: i.e. Olum 12).

Possible way out: inflation is not past eternal (Borde/Vilenkin); introduces a cutoff (a measure becomes justified) that is however inaccessible to direct tests.

Questions/Work in Progress

- Can we tell such a peculiar, anthropic multi-field model apart from much simpler ones? (e.g. via non-Gaussianities? D.Battefeld, T.Battefeld, D.Langlois in prep.)

- Is eternal inflation as generic as it appears to be even if D is large? (added mobility? T.Battefeld, A.Eggermeier in prep.)

- Should we better focus on single/few fields and EFT (Achucarro, Gong, Hardeman, Palma, Patil 12, …)?

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