Frame independent cosmological perturbations

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Outline of this talk

Part 1: Two problems in quantum cosmology

- Problem 1: Gauge dependence of perturbations
- Problem 2: Frame dependence of perturbations

Part 2: Unified solution to both problems

Part 1: Two problems in quantum cosmology

- Problem 1: Gauge dependence of perturbations

Gauge dependence in general relativity

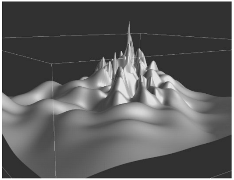
The action

$$S = \int d^4x\sqrt{-g}\left\{-R - \frac{1}{2}g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi - V(\Phi)\right\}$$

is *covariant*: it is invariant under coordinate transformations. Metric and scalar transform as:

$$\tilde{g}_{\mu\nu}(\tilde{x}) = \frac{dx^\alpha}{d\tilde{x}^\mu}\frac{dx^\beta}{d\tilde{x}^\nu}g_{\mu\nu}(x)\,, \qquad \tilde{\Phi}(\tilde{x}) = \Phi(x)$$

Now:



- split the metric and scalar field in a *fixed* background $g^{(0)}_{\mu\nu}$ and $\phi(t)$, and a fluctuation $\delta g_{\mu\nu}$ and φ
- this breaks the general covariance for the fluctuations

$\longrightarrow$ Fluctuations on a fixed background become dependent on coordinate transformations!

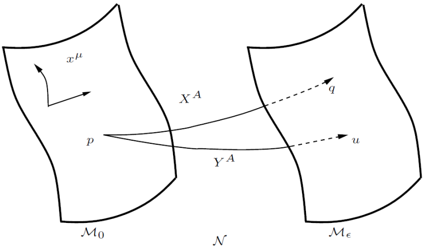
Gauge freedom in general relativity

A general perturbation of a quantity Q is defined as

$$\text{Perturbation } \delta Q = (Q \text{ in perturbed spacetime}) - (Q^{(0)} \text{ in background spacetime})$$

In order to compare Q and $Q^{(0)}$, one has to choose a mapping between the perturbed and background spacetime $\Longrightarrow$ this is a *gauge choice*

The freedom in choosing a mapping is called a *gauge freedom*

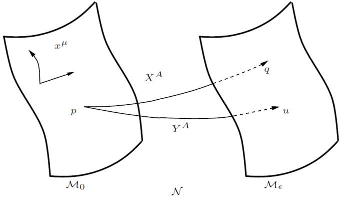


Gauge transformations in general relativity

Under an infinitesimal coordinate transformation

$x^\mu \to \tilde{x}^\mu = x^\mu + \xi^\mu$, gauge transformations of the

fields (at linear order):



$$\delta g_{\mu\nu}(x) \quad \rightarrow \quad \delta g_{\mu\nu}(\tilde{x}) = \delta g_{\mu\nu}(x) - \nabla_\mu \xi_\nu - \nabla_\nu \xi_\mu$$

$$\varphi(x) \quad \rightarrow \quad \varphi(\tilde{x}) = \varphi(x) + \xi^0 \partial_0 \phi(t)$$

(compare to QED: $A_\mu \to A_\mu + \partial_\mu \Lambda$)

Problem with gauge freedom: calculating physical quantities (similar to QED).

The complete action is covariant:

Can the perturbed action be written in a covariant way?

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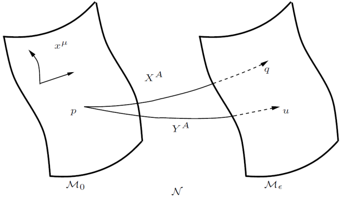

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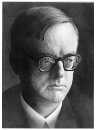
Part 1: Two problems in quantum cosmology

- Problem 2: Frame dependence of perturbations

The Jordan frame and the Einstein frame

Jordan frame action (nonminimal coupling)

$$S_J = \int d^4x\sqrt{-g}\left\{-RF(\Phi) - \frac{1}{2}g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi - V(\Phi)\right\}.$$



Einstein frame action (minimal coupling)

$$S_E = \int d^4x\sqrt{-g_E}\left\{-\frac{1}{2}M_P^2R_E - \frac{1}{2}g_E^{\mu\nu}\partial_\mu\Phi_E\partial_\nu\Phi_E - V_E(\Phi_E)\right\}.$$

Frames related through conformal transformation of $g_{\mu\nu}$ + redefinition of Φ

$$g_{\mu\nu,E} = \omega^2 g_{\mu\nu}\,, \qquad \frac{d\Phi_E}{d\Phi} = \sqrt{\frac{1}{F} + 3\frac{F'^2}{F^2}}\,, \qquad V_E(\Phi_E) = \frac{1}{\omega^4}V(\Phi)\,, \qquad \omega^2 = \frac{F(\Phi)}{\frac{1}{2}M_P^2}$$

Just field redefinitions, therefore *frames are physically equivalent*

Higgs inflation

Higgs inflation - nonminimally coupled Higgs field

Salopek, Bond, Bardeen 1989; Bezrukov, Shaposhnikov 2008

Jordan frame action with $\xi \gg 1$:

$$S = \int d^4x\sqrt{-g}\left\{-\frac{1}{2}(M_P^2 + \xi\Phi^2)R - \frac{1}{2}g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi - \frac{\lambda}{4}(\Phi^2 - v^2)^2\right\}.$$

Einstein frame

$$S_E = \int d^4x\sqrt{-g_E}\left\{-\frac{1}{2}M_P^2 R_E - \frac{1}{2}g_E^{\mu\nu}\partial_\mu\Phi_E\partial_\nu\Phi_E - \frac{\lambda(\Phi[\Phi_E]^2 - v^2)^2}{4(1+\xi\frac{\Phi[\Phi_E]^2}{M_P^2})^2}\right\}.$$

If we consider the background/classical level only:

- Jordan frame: modified G_N, gravity weaker $\longrightarrow$ inflation
- Einstein frame: modified potential, flat at large field values $\longrightarrow$ inflation

At the background level, both frames give inflationary expansion!

Quantum corrections to Higgs inflation

Quantum corrections to Higgs inflation have been calculated:

- Einstein frame Bezrukov, Magnin, Shaposhnikov 2009
- Jordan frame Barvinsky, Kamenshchik, Starobinsky 2008, De Simone, Hertzberg, Wilczek 2009; Barvinsky et al. 2009,2010
- Both frames: Bezrukov, Shaposhnikov 2009

The results in Einstein and Jordan frame seem to differ!

See also Steinwachs, Kamenshchik 2011 and Christian Steinwachs' talk @ 6. Kosmologietag

If quantum results are different, which frame is the physical one?

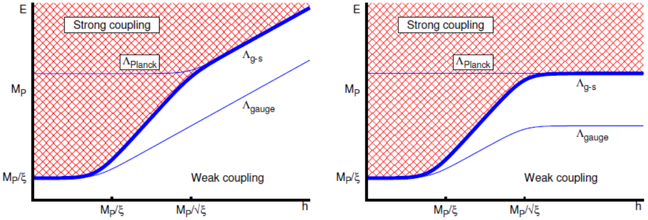
How can we rhyme this with the observation that the frames are related by field redefinitions?

Naturalness problem in Higgs inflation

The naturalness problem appears through power-counting arguments

Higher dimensional operators are suppressed by some energy scale - the cut-off

- appears in Einstein frame due to non-polynomial potential
- appears in Jordan frame due to nonminimal coupling $\xi\Phi^2 R$



Bezrukov et al. 2011

Left: Jordam frame

Right: Einstein frame

Can we consistently describe perturbations/quantum corrections in both frames?

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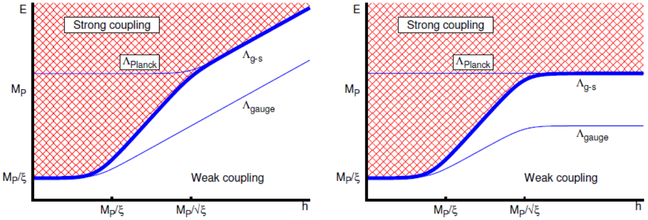

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Part 2: Unified solution to both problems

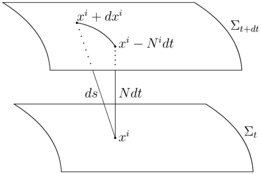
Solution to problem 1 - Gauge dependence

How to solve the problem of gauge (coordinate) dependence of perturbations?

Gauge freedom: certain degrees of freedom are unphysical

In gravity, this becomes nicely visible using the ADM metric

$$ds^2 = -N^2 dt^2 + g_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$$



The N and N_i appear as constraint fields in the action

Solving for the constraint fields removes unphysical degrees of freedom

The remaining, physical, d.o.f.'s are *gauge invariant* Bardeen 1980

Result: Gauge-invariant action for perturbations (linear order)

EF action $S_E = \int d^4x\sqrt{-g_E}\left\{-R_E - \frac{1}{2}g_E^{\mu\nu}\partial_\mu\Phi_E\partial_\nu\Phi_E - V_E(\Phi)\right\}$ up to second order is

$$\boxed{S_E^{(2)} = \int d^3x dt \bar{N}_E a_E^3 \left\{ \frac{\dot{\phi}_E{}^2}{36H_E^2}\left[\frac{1}{2}\dot{\mathcal{R}}_E{}^2 - \frac{1}{2}\left(\frac{\partial_i\mathcal{R}_E}{a_E}\right)^2\right] + \frac{1}{4}\left[(\dot{h}_{ij,E}^{TT})^2 - \left(\frac{\partial h_{ij,E}^{TT}}{a_E}\right)^2\right]\right\}}$$

Mukhanov 1981

- 1 dynamical scalar: comoving curvature perturbation:

$$\mathcal{R}_E = (h_E - \nabla^2\tilde{h}_E) - 6\frac{H_E}{\dot{\phi}_E}\varphi_E$$

  Combination of scalar metric and inflaton fluctations

  Together with inflation forms the primordial power spectrum of the CMB

- 1 dynamical transverse-traceless tensor: graviton $h_{ij,E}^{TT}$
- All fields are gauge invariant!

$\mathcal{R}_E$ and $h_{ij,E}^{TT}$ are *gauge invariant cosmological perturbations*

Solution to problem 2: Frame dependence

What about our second problem - perturbations in different frames?

The perturbations can be written in the Jordan frame

$$\mathcal{R}_E = (h_E - \nabla^2 \tilde{h}_E) - 6\frac{H_E}{\dot{\phi}_E}\varphi_E \xrightarrow{\text{FrameTransform}} (h - \nabla^2 \tilde{h}) - 6\frac{H}{\dot{\phi}}\varphi = \mathcal{R}$$

$$h^{TT}_{ij,E} \xrightarrow{\text{FrameTransform}} h^{TT}_{ij}$$

$\mathcal{R}$ and h^{TT}_{ij} are *frame independent cosmological perturbations*!

Makino, Sasaki 1991; Fakir, Habib, Unruh 1992

Jordan frame action at second order via conformal transformation Hwang 1996

$$S^{(2)} = \int d^3x dt \bar{N} a^3 \left\{ \frac{(1+3\frac{F'^2}{F})\dot{\phi}^2}{36(H+\frac{1}{2}\frac{\dot{F}}{F})^2} \left[\frac{1}{2}\dot{\mathcal{R}}^2 - \frac{1}{2}\left(\frac{\partial_i \mathcal{R}}{a}\right)^2 \right] + \frac{F(\phi)}{4}\left[(\dot{h}^{TT}_{ij})^2 - \left(\frac{\partial h^{TT}_{ij}}{a}\right)^2 \right] \right\}$$

Also found directly from Jordan frame action JW, Prokopec 2010

Gauge invariance at non-linear order

Showed: the second order gauge invariant action is also frame independent

$\longrightarrow$ Consistent description of perturbations of nonminimally coupled field

What about higher order perturbations?

The third order EF action (second order perturbations) has been found by Maldacena

Maldacena 2003

Here two different gauge fixing were used to obtain third order action

Maldacena showed that by nonlinear field redefinition both gauges lead to same result

$\longrightarrow$ gauge invariant action at third order

Project: find third order gauge invariant action without gauge fixing

Collaboration with Prokopec, Rigopoulos

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Frame independence at non-linear order

At higher order in perturbations, the gauge invariant variables become non-linear

Malik, Wands 2004,2008; Bartolo et al. 2004; Noh, Hwang 2004; and many more

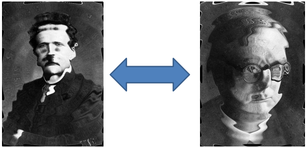
What about frame independence at higher order?

Proofs of frame independence of curvature perturbation:

- explicitly at second order Sugiyama, Futamase 2010
- general proof at all orders Koh 2005; Chiba, Yamaguchi 2008; Gong et al. 2011
- application of frame transformation to cubic action Kubota et al. 2012

This suggests that the Jordan and Einstein frame are physically equivalent, at every order in perturbation theory

Quantum equivalence of frames!



Summary and outlook

General conclusion:

In order to consistently describe cosmological quantum corrections in an arbitrary frame, one should use those perturbations which are both gauge invariant and frame independent

Outlook

- Find the gauge invariant action at third order in fluctuations in Jordan frame
- Show quantum equivalence of frames at third order
- Revisit the naturalness problem in Higgs inflation

Thank you for your attention!

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