Exact Random Matrix Spectral Form Factor in Kicked Ising Spin Chain

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Outline

1. Spectral correlations in quantum systems and
   The Quantum Chaos Conjecture
2. Random-phase model ('diagonal RMT'): long-range Kicked Ising chain
3. Self-dual Kicked Ising model:
   Exactly solved model for maximal many body quantum chaos
   (*No small parameter, such as $\hbar$ or inverse local Hilbert space dimension!*)
4. Sketch of the derivation/proof

/w Bruno Bertini and Pavel kos

`PRL`**`121`**`, 264101 (2018)`

Consider periodically driven systems

$$H(t) = H(t+T)$$

where the set of **quasi-energies** $\{\varphi_n \in [0, 2\pi]\}_{n=1,\ldots,\mathcal{N}}$ such that $\{e^{-i\varphi_n}\}$ is the spectrum of the Floquet operator

$$U = \hat{\mathcal{T}} \exp\left(-i \int_0^T \mathrm{d}s\, H(s)\right).$$

The **spectrum** as a **gas** in one dimension

Spectral density:

$$\rho(\varphi) = \frac{2\pi}{\mathcal{N}} \sum_n \delta(\varphi - \varphi_n).$$

Spectral pair correlation function (2-point function):

$$r(\vartheta) = \frac{1}{2\pi} \int_0^{2\pi} \mathrm{d}\varphi \rho(\varphi + \tfrac{1}{2}\vartheta)\rho(\varphi - \tfrac{1}{2}\vartheta) - 1.$$

Spectral Form Factor (SFF) (Fourier transform of 2-point function):

$$\begin{aligned} K(t) &= \frac{\mathcal{N}^2}{2\pi} \int_0^{2\pi} \mathrm{d}\vartheta r(\vartheta) e^{it\vartheta} = \sum_{m,n} e^{it(\varphi_m - \varphi_n)} - \mathcal{N}^2 \delta_{t,0} \\ &= \left|\mathrm{tr}\, U^t\right|^2 - \mathcal{N}^2 \delta_{t,0}, \quad t \in \mathbb{Z}. \end{aligned}$$

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Caveat: SFF is not self-averaging! Consider instead $\bar{K}(t) = \mathbb{E}[K(t)]$.

RMT (No time reversal symmetry):

$$K_{\text{CUE}}(t) = t, \quad t < \mathcal{N}.$$

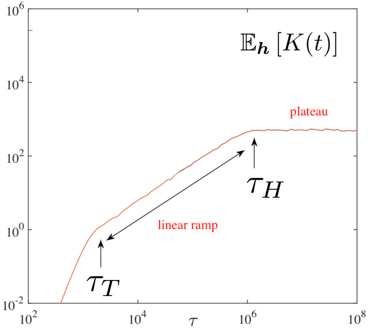
RMT (With time teversal symmetry):

$$K_{\text{COE}}(t) = 2t - \log(1 + 2t/\mathcal{N}), \quad t < \mathcal{N}.$$

Random (uncorrelated, Poissonian) spectrum $\{\varphi_n\}$:

$$K_{\text{Poisson}} \equiv \mathcal{N}.$$

Real System:



Review: Chen and Ludwig 2017

$$\mathbb{E}[K(t)] = \mathbb{E}\left[\sum_{m,n} e^{i(\varphi_m - \varphi_n)}\right].$$

Saturation $\bar{K}(t) \sim \mathcal{N}$ beyond Heisenberg time $t > t_{\text{H}} = \mathcal{N} = 1/\Delta\varphi$.

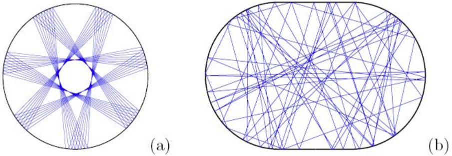
Non-universal (system-specific) behaviour below Ehrenfest/Thouless time $t < t_{\text{T}}$.

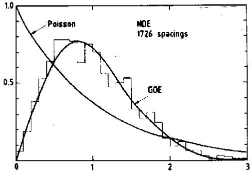
The Quantum Chaos Conjecture

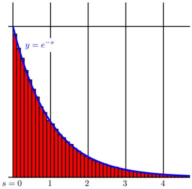
Casati,Guarnerri,Valz-Gris1980, Berry1981, Bohigas,Giannoni,Schmidt1984

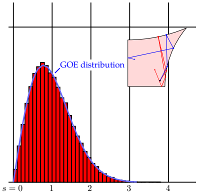
The spectral fluctuations of quantum systems with chaotic and ergodic classical limit are *universal* and described by Random Matrix Theory (RMT).

The same holds for periodically-driven systems if one considers the statistics of quasi-energies instead.









[Berry 1985, Sieber&Richter 2001, Müller, Heusler, Braun, Haake,Altland 2004...]

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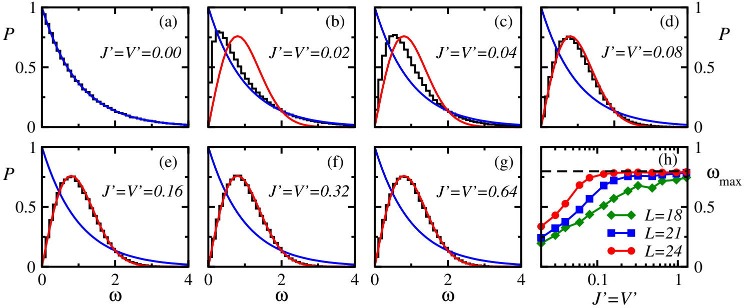
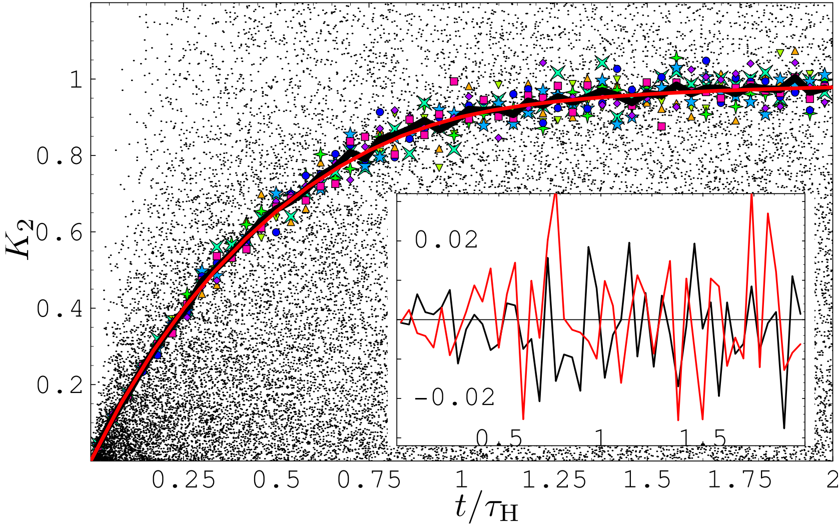
$$H = \sum_{j=0}^{L-1} (-J c_j^\dagger c_{j+1} - J' c_j^\dagger c_{j+2} + \text{h.c.} + V n_j n_{j+1} + V' n_j n_{j+2}), \quad n_j = c_j^\dagger c_j.$$



From [Rigol and Santos, 2010]

Clean non-integrable Kicked Ising Chain [Pineda and TP, PRE 2007]



Only very recently first analytic results arrived..

Floquet long-ranged (non-integrable/non-mean field) spin 1/2 chains `[arXiv:1712.02665]`

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Many-Body Quantum Chaos: Analytic Connection to Random Matrix Theory

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(Received 5 February 2018; revised manuscript received 12 April 2018; published 8 June 2018)

Floqeut local quantum circuits with random unitary gates in the limit of large local Hilbert space dimension $q \to \infty$ `[PRL` **121**`, 060601 (2018); PRX` **8**`, 041019 (2018)]`

Solution of a minimal model for many-body quantum chaos

Amos Chan, Andrea De Luca and J. T. Chalker
Theoretical Physics, Oxford University, 1 Keble Road, Oxford OX1 3NP, United Kingdom
(Dated: December 20, 2017)

Spectral statistics in spatially extended chaotic quantum many-body systems

Amos Chan, Andrea De Luca and J. T. Chalker
Theoretical Physics, Oxford University, 1 Keble Road, Oxford OX1 3NP, United Kingdom
(Dated: April 4, 2018)

/w Marko Ljubotina and Pavel Kos

Setup: Kicked Ising models with (non-mean field!) long range interactions

$$H(t) = H_0 + H_1 \sum_{m \in \mathbb{Z}} \delta(t-m)$$

where

$$H_0 = \sum_{j=1}^{L} J_j^1 \sigma_j^z + \sum_{j<j'} J_{j,j'}^2 \sigma_j^z \sigma_{j'}^z + \cdots, \quad H_1 = h \sum_{j=1}^{L} \sigma_j^x .$$

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Floquet propagator:

$$\begin{aligned} U &= \mathcal{T}\text{-}\exp\left(-\mathrm{i}\int_0^1 \mathrm{d}t\, H(t)\right) = VW, \\ W &= e^{-\mathrm{i}H_0}, \quad V = e^{-\mathrm{i}H_1} = v^{\otimes L}, \quad v = \begin{pmatrix} \cos h & \mathrm{i}\sin h \\ \mathrm{i}\sin h & \cos h \end{pmatrix}. \end{aligned}$$

- Averaging over $J^{(r)}_{j,j',\ldots}$ implies twisted 1D Ising model representation:

$$K(t) = \sum_{\pi\in S_t} Z_\pi^L, \qquad \text{where} \qquad Z_\pi = \sum_{s_1,\ldots,s_t} \prod_{\tau=1}^{t} v_{s_\tau,s_{\tau+1}} v^*_{s_{\pi(\tau)},s_{\pi(\tau+1)}}.$$

- The leading order (in the limit $L\to\infty$) contributions come from t cyclic permutations and t anti-cyclic permutations, i.e. all $2t$ permutations π which do not change any neigbour the sequence $\underline{s}$:

$$\begin{aligned} K(t) &\simeq 2t(\mathrm{tr}T^t)^L = 2t(1+(\cos 2h)^t)^L \\ &\simeq 2t \quad \text{for} \quad t \gg t^* = -\frac{\ln L}{\ln\cos 2h} \end{aligned}$$

where $T = \begin{pmatrix} \cos^2 h & \sin^2 h \\ \sin^2 h & \cos^2 h \end{pmatrix}$ is 1D Ising model transfer matrix.

- This is exactly the leading term of the Random-Matrix-Theory result!

$$K_{\mathrm{OE}}(t) = 2t - t\ln(1+2t/\mathcal{N}) = 2t - 2t^2/\mathcal{N} + \cdots,$$

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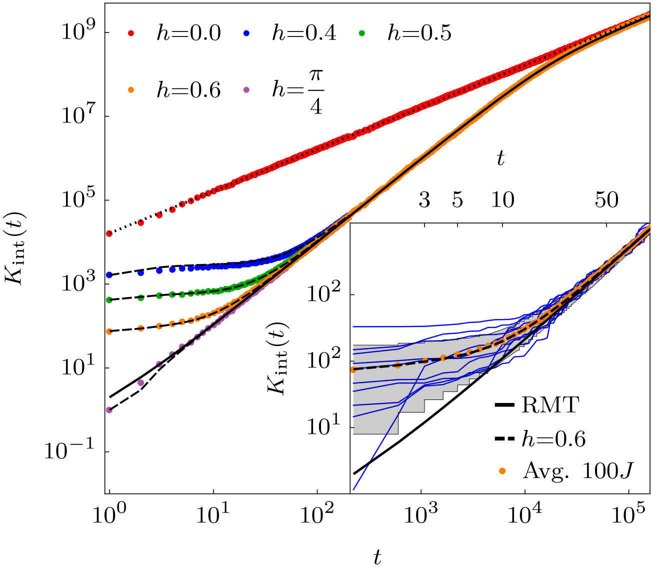
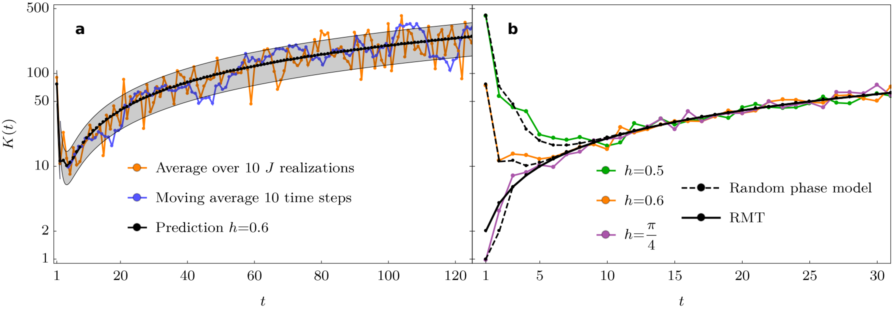
$$J_j^1 = a + b/x^\alpha, \quad J_{j,j'}^2 = J/(j'-j)^\alpha, \quad \alpha \in [1,2]$$





a
b
K(t)
500
100
50
10
2
1
Average over 10 J realizations
Moving average 10 time steps
Prediction h=0.6
1
20
40
60
80
100
120
t
h=0.5
h=0.6
h=π/4
Random phase model
RMT
1
5
10
15
20
25
30
t

What about fermionic or spin $1/2$ systems with strictly local interactions?

$$H_{\mathrm{KI}}[\boldsymbol{h};t] = H_{\mathrm{I}}[\boldsymbol{h}]+\delta_p(t)H_{\mathrm{K}}, \quad H_{\mathrm{I}}[\boldsymbol{h}] \equiv \sum_{j=1}^{L}\left\{J\sigma_j^z\sigma_{j+1}^z + h_j\sigma_j^z\right\}, \;\; H_{\mathrm{K}} \equiv b\sum_{j=1}^{L}\sigma_j^x,$$

with Floquet propagator

$$U_{\mathrm{KI}} = e^{-iH_{\mathrm{K}}}e^{-iH_{\mathrm{I}}}.$$

J, b: homogeneous spin-coupling and transverse field
h_j position dependent longitudinal field

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Remarks:

- KI model is integrable if $b = 0$ or $h_j \equiv 0$.
- For generic h_j and $b \neq 0$, the model has no symmetries.
- The clean model $h_j \equiv h$, for $J \sim b \sim h \sim 1$ appears to be ergodic and its spectral statistics well described by RMT
- The clean model appears to display non-trivial non-ergodicity – ergodicity transition when h is varied [`TP PRE 2002, TP JPA 2002, TP JPA 2007, see also Vajna, Klobas, TP, Polkovnikov, PRL 120, 200607 (2018)`]

The disorder averaging

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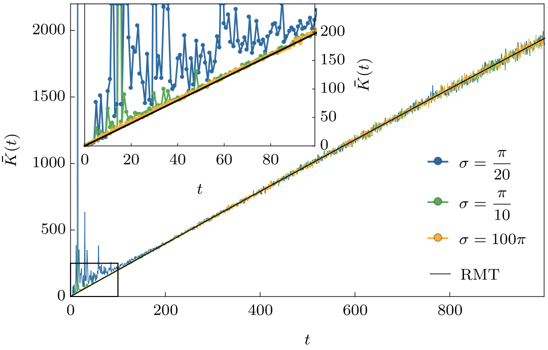
Consider longitudinal magnetic field h_j to be i.i.d. (Gaussian) variable with mean $\bar{h}$ and standard deviation σ

$$\bar{K}(t) = \mathbb{E}_{\boldsymbol{h}}[K(t)] = \int_{-\infty}^{\infty} \left(\prod_{j=1}^{L} \frac{\mathrm{d}h_j}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(h_j - \bar{h})^2}{2\sigma^2} \right) \right) K(t).$$



For $|J| = |b| = \pi/4$ and σ large enough the behaviour seems immediately RMT-like ($t_{\mathrm{T}} \sim 1$)

Interpreting $\bar{K}(t)$ in terms of a partition function of $2d$ classical statistical model, we can study SFF analytically in thermodynamic limit!

Key result: Exact SFF in thermodynamic limit

Theorem: For *odd* t:

$$\lim_{L\to\infty} \bar{K}(t) = \begin{cases} 2t-1\,, & t \le 5 \\ 2t\,, & t \ge 7 \end{cases}.$$

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Conjecture: For *even* t:

$$\bar{K}(2) = 2,\ \bar{K}(4) = 7,\ \bar{K}(6) = 13,\ \bar{K}(8) = 18,\ \bar{K}(10) = 22,$$
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Remarks:

- Results independent of $\sigma>0$: The model is ergodic for any disorder strength (**no Floquet-MBL!**). In particular, we can take the limit of a clean system at the end $\sigma \searrow 0$.
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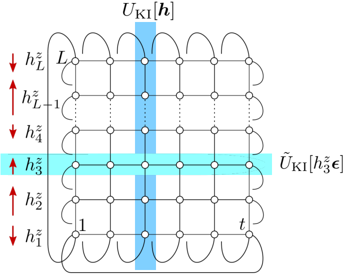
- Results independent of $\bar{h}$: We can set $\bar{h} = 0$ which corresponds to a limiting integrable system.

We found a simple locally interacting model with finite dimensional local Hilbert space with proven RMT spectral correlations at all time-scales!

The trace of U_{KI}^t is equivalent to a partition sum of a classical 2d Ising model with **row-homogeneous field** h_j:

$\mathrm{tr}\,(U_{\mathrm{KI}}^t[\boldsymbol{h}]) \sim$



Duality relation

$$tr\,(U_{\mathrm{KI}}[\boldsymbol{h}])^t = tr\left(\prod_{j=1}^{L} \tilde{U}_{\mathrm{KI}}[h_j\boldsymbol{\epsilon}]\right)$$

where $\boldsymbol{\epsilon} = (1, 1 \ldots, 1)$ and $\tilde{U}_{\mathrm{KI}}$ is a KI model on a ring of size t with twisted parameters $\tilde{J}(J, b)$, $\tilde{b}(J, b)$.

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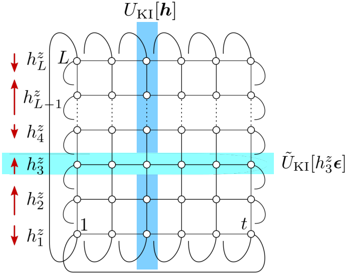

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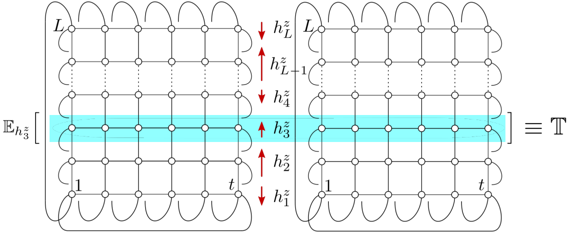
Remarkably: $\tilde{U}_{\mathrm{KI}}$ is **unitary** for $|J| = |b| = \pi/4$ (Self-dual, $J = \pm\tilde{J}, b = \pm\tilde{b}$)

Space-time duality allows to simply express the disorder averaging:

$$\mathbb{E}_{\boldsymbol{h}}\left[K(t)\right]$$

$$\equiv$$



$$=$$

$$\mathrm{tr}\left(\mathbb{T}^L\right)$$

Unitary

$$\mathbb{T} \equiv \mathbb{E}_h\left[\tilde{U}_{\mathrm{KI}}[h\boldsymbol{\epsilon}] \otimes \tilde{U}_{\mathrm{KI}}[h\boldsymbol{\epsilon}]^*\right] = (\tilde{U}_{\mathrm{KI}} \otimes \tilde{U}^*_{\mathrm{KI}}) \cdot \mathbb{O}_\sigma$$

Non-Unitary

Contraction

$$\mathbb{O}_\sigma = \exp\left[-\tfrac{1}{2}\sigma^2\left(M_z \otimes I - I \otimes M_z\right)^2\right]$$

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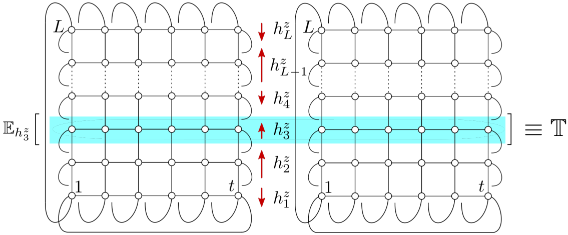

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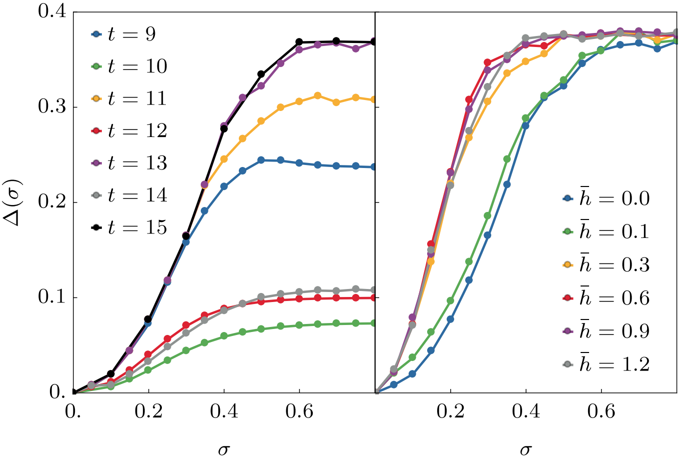
Computation of thermodynamic SFF $\lim_{L\to\infty} \mathbb{T}^L$ thus amounts to determining the multiplicity of eigenvalue 1 of $\mathbb{T}$ and proving positive spectral gap.

Empirical convergence of the spectral gap



The following is straightforward to show:

Property 1

1. The eigenvalues of $\mathbb{T}$ of maximal (unit) magnitude are either $+1$ or -1.
2. Each eigenvector associated to the eigenvalue ± 1 is uniquely paramertrized by an operator $A \in \mathrm{End}((\mathbb{C}^2)^{\otimes t})$ satisfying

$$UAU^\dagger = \pm A, \quad [A, M_\alpha] = 0, \quad \alpha \in \{x, y, z\}. \tag{1}$$

   where we have defined $M_\alpha = \sum_{\tau=1}^{t} \sigma_\tau^\alpha$,
   $U = \exp\left[i\frac{\pi}{4}\sum_{\tau=1}^{t}(\sigma_\tau^z\sigma_{\tau+1}^z - 1)\right]$.

U is the parity of half-number of domain walls in the spin configuration, $U^2 = \mathbb{1}$.

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Observation: The operators U, M_α are translationally invariant and reflection symmetric $\Rightarrow$ All elements of
$\mathcal{D}_t = \{\Pi^n R^m, n \in \{0, 1, \ldots, t-1\}, m \in \{0, 1\}\}$ fullfil (1) with $+1$, where

$$\Pi = \prod_{\tau=1}^{t-1} P_{\tau,\tau+1}, \quad R = \prod_{\tau=1}^{\lfloor t/2 \rfloor} P_{\tau, t+1-\tau}$$

are *translation* and *reflection* on a spin ring of length t.

Property 2

The number of linearly independent elements of $\mathcal{D}_t$ is $2t$ for $t \geq 6$, $2t-1$ for $t \in \{1,3,4,5\}$, and 2 for $t=2$.

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For odd t, Eq. (1) can be fulfilled only for eigenvalue $+1$.

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For odd t, all A satisfying (1) are given by linear combination of elements of $\mathcal{D}_t$.

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Theorem

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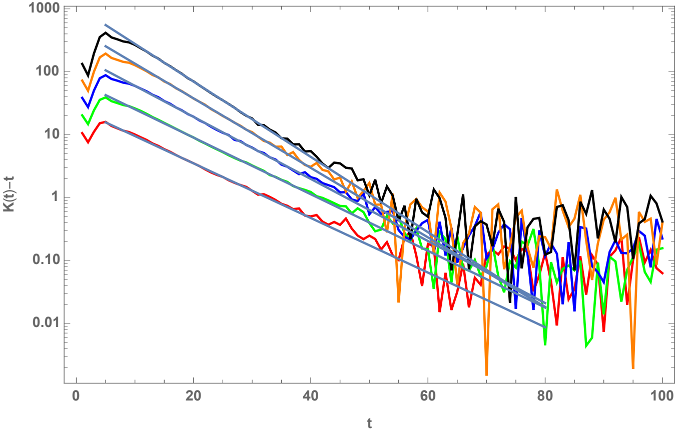
Observation: For even t, we find generically exactly one additional operator A satisfying Eq. (1). For special values of $t \leq 10$ we find an extra additional operator, and also solutions of Eq. (1) for eigenvalue -1.

t	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
$\#_{+1}$	2	5	7	9	13	14	18	18	22	22	25	26	29	30	33	34
$\#_{-1}$	0	0	0	0	2	0	0	0	2	0	0	0	0	0	0	0

Model with broken time reversal:
Data for $L = 8, 10, 12, 14, 16$ suggest that

$$|K(t) - t| \le C \exp(\alpha L - \beta t), \qquad C, \alpha, \beta > 0.$$



More numerical analysis in Hamiltonian disordered spin chains in:
`Šuntajs, Bonča, TP, Vidmar, arXiv:1905.06345`

Conclusions

- The first (?) exact result on ergodicity in terms of spectral correlations for an interacting quantum many-body problem
- Self-dual instances of Kicked Ising chain provide a minimal model of quantum many-body chaos with no intrinsic time scales

Open problems and promising future directions:

1. Complete the picture by rigorous analysis of the even t case.
2. Structural stability of the self-dual point: Perturbation theory may have a finite radius of convergence?
3. Potential to characterise ergodicity - MBL transition, approaching from ergodic side.
4. Path to a rigorous approach to ETH?

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