Exact Random Matrix Spectral Form Factor in Kicked Ising Spin Chain

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Outline

1. Spectral correlations in quantum systems and The Quantum Chaos Conjecture
2. Random-phase model (‘diagonal RMT’): long-range Kicked Ising chain
3. Self-dual Kicked Ising model: Exactly solved model for maximal many body quantum chaos (No small parameter, such as $\hbar$ or inverse local Hilbert space dimension!)
4. Sketch of the derivation/proof

/w Bruno Bertini and Pavel Kos

PRL121, 264101 (2018)
Consider periodically driven systems

$$H(t) = H(t + T)$$

where the set of quasi-energies \(\{\varphi_n \in [0, 2\pi]\}_{n=1,\ldots,N}\) such that \(\{e^{-i\varphi_n}\}\) is the spectrum of the Floquet operator

$$U = \mathcal{T} \exp \left(-i \int_0^T ds \, H(s)\right).$$
The **spectrum** as a **gas** in one dimension

Spectral density:

\[ \rho(\varphi) = \frac{2\pi}{N} \sum_n \delta(\varphi - \varphi_n). \]

Spectral pair correlation function (2-point function):

\[ r(\vartheta) = \frac{1}{2\pi} \int_0^{2\pi} d\varphi \rho(\varphi + \frac{1}{2} \vartheta) \rho(\varphi - \frac{1}{2} \vartheta) - 1. \]

Spectral Form Factor (SFF) (Fourier transform of 2-point function):

\[ K(t) = \frac{N^2}{2\pi} \int_0^{2\pi} d\vartheta r(\vartheta) e^{it\vartheta} = \sum_{m,n} e^{it(\varphi_m - \varphi_n)} - N^2 \delta_{t,0} \]

\[ = |\text{tr} U^t|^2 - N^2 \delta_{t,0}, \quad t \in \mathbb{Z}. \]
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\[ = |\text{tr } U^t|^2 - N^2 \delta_{t,0}, \quad t \in \mathbb{Z}. \]

Caveat: SFF is not self-averaging! Consider instead \( \bar{K}(t) = \mathbb{E}[K(t)] \).
Comparision to RMT spectral form factors

RMT (No time reversal symmetry):

\[ K_{\text{CUE}}(t) = t, \quad t < \mathcal{N}. \]

RMT (With time reversal symmetry):

\[ K_{\text{COE}}(t) = 2t - \log(1 + 2t/\mathcal{N}), \quad t < \mathcal{N}. \]

Random (uncorrelated, Poissonian) spectrum \( \{ \varphi_n \} \):

\[ K_{\text{Poisson}} \equiv \mathcal{N}. \]

Real System:

\[ \mathbb{E}[K(t)] = \mathbb{E} \left[ \sum_{m,n} e^{i(\varphi_m - \varphi_n)} \right]. \]

Saturation \( \bar{K}(t) \sim \mathcal{N} \) beyond Heisenberg time \( t > t_H = \mathcal{N} = 1/\Delta \varphi \).

Non-universal (system-specific) behaviour below Ehrenfest/Thouless time \( t < t_T \).
The Quantum Chaos Conjecture


The spectral fluctuations of quantum systems with chaotic and ergodic classical limit are universal and described by Random Matrix Theory (RMT).

The same holds for periodically-driven systems if one considers the statistics of quasi-energies instead.


Tomaž Prosen | Many body quantum chaos
What about QCC for many-body systems at \( \hbar \sim 1 \)?
(say for interacting spin 1/2 or fermionic systems)
What about QCC for many-body systems at ‘\(\hbar \sim 1\)’? (say for interacting spin 1/2 or fermionic systems)

\[
H = \sum_{j=0}^{L-1} \left( -J c_j^\dagger c_{j+1} - J' c_j^\dagger c_{j+2} + \text{h.c.} + V n_j n_{j+1} + V' n_j n_{j+2} \right), \quad n_j = c_j^\dagger c_j.
\]

From [Rigol and Santos, 2010]
How about the numerical data on SFF?

Clean non-integrable Kicked Ising Chain [Pineda and TP, PRE 2007]

![Graph showing numerical data on the Clean non-integrable Kicked Ising Chain with key points highlighted.](image-url)
Floquet local quantum circuits with random unitary gates in the limit of large local Hilbert space dimension $q \rightarrow \infty$ [PRL 121, 060601 (2018); PRX 8, 041019 (2018)]

Solution of a minimal model for many-body quantum chaos

Amos Chan, Andrea De Luca and J. T. Chalker

Theoretical Physics, Oxford University, 1 Keble Road, Oxford OX1 3NP, United Kingdom

(Dated: December 20, 2017)

Spectral statistics in spatially extended chaotic quantum many-body systems

Amos Chan, Andrea De Luca and J. T. Chalker

Theoretical Physics, Oxford University, 1 Keble Road, Oxford OX1 3NP, United Kingdom

(Dated: April 4, 2018)
Random phase model [PRX 8, 021062 (2018)]

/w Marko Ljubotina and Pavel Kos

Setup: Kicked Ising models with (non-mean field!) long range interactions

\[ H(t) = H_0 + H_1 \sum_{m \in \mathbb{Z}} \delta(t - m) \]

where

\[ H_0 = \sum_{j=1}^{L} J_j^{1} \sigma_j^z + \sum_{j<j'} J_j^{2} \sigma_j^z \sigma_j'^z + \cdots, \quad H_1 = \hbar \sum_{j=1}^{L} \sigma_j^x. \]
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where

\[ H_0 = \sum_{j=1}^{L} J^1_j \sigma^z_j + \sum_{j<j'} J^2_{j,j'} \sigma^z_j \sigma^z_{j'} + \cdots, \quad H_1 = \hbar \sum_{j=1}^{L} \sigma^x_j. \]

Floquet propagator:

\[ U = \mathcal{T} \exp \left( -i \int_0^1 dt \, H(t) \right) = VW, \]

\[ W = e^{-iH_0}, \quad V = e^{-iH_1} = v \otimes L, \quad v = \begin{pmatrix} \cos \hbar & i \sin \hbar \\ i \sin \hbar & \cos \hbar \end{pmatrix}. \]
Averaging over $J_{j,j'}^{(r)}$ implies twisted 1D Ising model representation:

$$K(t) = \sum_{\pi \in S_t} Z^L_{\pi}, \quad \text{where} \quad Z_{\pi} = \sum_{s_1,\ldots,s_t} \prod_{\tau=1}^{t} v_{s_{\tau},s_{\tau+1}} v_{s_{\pi(\tau)},s_{\pi(\tau+1)}}.$$

The leading order (in the limit $L \to \infty$) contributions come from $t$ cyclic permutations and $t$ anti-cyclic permutations, i.e. all $2t$ permutations $\pi$ which do not change any neigbour the sequence $s$:

$$K(t) \simeq 2t(\text{tr} T^t)^L = 2t(1 + (\cos 2h)^t)^L \simeq 2t \quad \text{for} \quad t \gg t^* = -\frac{\ln L}{\ln \cos 2h}$$

where $T = \begin{pmatrix} \cos^2 h & \sin^2 h \\ \sin^2 h & \cos^2 h \end{pmatrix}$ is 1D Ising model transfer matrix.

This is exactly the leading term of the Random-Matrix-Theory result!

$$K_{\text{OE}}(t) = 2t - t \ln(1 + 2t/N) = 2t - 2t^2/N + \cdots,$$
Averaging over $J^{(r)}_{j,j'}...$ implies twisted 1D Ising model representation:

$$K(t) = \sum_{\pi \in S_t} Z^L_\pi, \quad \text{where} \quad Z_\pi = \sum_{s_1,...,s_t} \prod_{\tau=1}^t v_{s_\tau,s_{\tau+1}}v^*_{s_\pi(\tau),s_\pi(\tau+1)}.$$

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$$K(t) = \sum_{\pi \in S_t} Z_{\pi}^L,$$

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$$Z_{\pi} = \sum_{s_1,\ldots,s_t} \prod_{\tau=1}^{t} v_{s_{\pi}(\tau),s_{\pi}(\tau+1)} v_{s_\tau,s_{\tau+1}}.$$  

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$$K_{OE}(t) = 2t - t \ln(1 + 2t/N) = 2t - 2t^2/N + \cdots,$$
\begin{align*}
J_j^1 &= a + b/x^\alpha, \quad J_{j,j'}^2 = J/(j' - j)^\alpha, \quad \alpha \in [1, 2] 
\end{align*}
What about fermionic or spin 1/2 systems with strictly local interactions?
Kicked Ising model \([TP, JPA\ 1998;\ PTPS\ 2000;\ PRE\ 2002]\)

\[
H_{KI}[\hbar; t] = H_1[\hbar] + \delta_p(t)H_K,
\]

\[
H_1[\hbar] \equiv \sum_{j=1}^{L} \left\{ J \sigma_j^z \sigma_{j+1}^z + h_j \sigma_j^z \right\},
\]

\[
H_K \equiv b \sum_{j=1}^{L} \sigma_j^x,
\]
with Floquet propagator

\[
U_{KI} = e^{-iH_K} e^{-iH_1}.
\]

\(J, b\): homogeneous spin-coupling and transverse field

\(h_j\): position dependent longitudinal field
Kicked Ising model [TP, JPA 1998; PTPS 2000; PRE 2002]

\[ H_{KI}[h; t] = H_1[h] + \delta_p(t)H_K, \quad H_1[h] \equiv \sum_{j=1}^{L} \{ J \sigma^z_j \sigma^z_{j+1} + h_j \sigma^z_j \}, \quad H_K \equiv b \sum_{j=1}^{L} \sigma^x_j, \]

with Floquet propagator

\[ U_{KI} = e^{-iH_K} e^{-iH_1}. \]

\( J, b \): homogeneous spin-coupling and transverse field
\( h_j \): position dependent longitudinal field

Remarks:

- KI model is integrable if \( b = 0 \) or \( h_j \equiv 0 \).
- For generic \( h_j \) and \( b \neq 0 \), the model has no symmetries.
- The clean model \( h_j \equiv h \), for \( J \sim b \sim h \sim 1 \) appears to be ergodic and its spectral statistics well described by RMT.
- The clean model appears to display non-trivial non-ergodicity – ergodicity transition when \( h \) is varied [TP PRE 2002, TP JPA 2002, TP JPA 2007, see also Vajna, Klobas, TP, Polkovnikov, PRL 120, 200607 (2018)].
The disorder averaging

\[ H_{KI}[\mathbf{h}; t] = H_1[\mathbf{h}] + \delta_p(t) H_K, \quad H_1[\mathbf{h}] = \sum_{j=1}^{L} \left\{ J \sigma_j^z \sigma_{j+1}^z + h_j \sigma_j^x \right\}, \quad H_K \equiv b \sum_{j=1}^{L} \sigma_j^x, \]
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Consider longitudinal magnetic field \( h_j \) to be i.i.d. (Gaussian) variable with mean \( \bar{h} \) and standard deviation \( \sigma \)

\[ \bar{K}(t) = \mathbb{E}_h[K(t)] = \int_{-\infty}^{\infty} \left( \prod_{j=1}^{L} \frac{dh_j}{\sqrt{2\pi}\sigma} \exp \left( -\frac{(h_j - \bar{h})^2}{2\sigma^2} \right) \right) K(t). \]

For \( |J| = |b| = \pi/4 \) and \( \sigma \) large enough the behaviour seems immediately RMT-like \( (t_T \sim 1) \)

Interpreting \( \bar{K}(t) \) in terms of a partition function of 2d classical statistical model, we can study SFF analytically in thermodynamic limit!
Key result: Exact SFF in thermodynamic limit

**Theorem:** For odd $t$:

\[
\lim_{L \to \infty} K(t) = \begin{cases} 
2t - 1, & t \leq 5 \\
2t, & t \geq 7 
\end{cases}
\]
Key result: Exact SFF in thermodynamic limit

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Conjecture: For even \( t \):

\[
\bar{K}(2) = 2, \quad \bar{K}(4) = 7, \quad \bar{K}(6) = 13, \quad \bar{K}(8) = 18, \quad \bar{K}(10) = 22,
\]

\[
\bar{K}(t) = 2(t + 1), \quad t \geq 12.
\]
Theorem: For odd $t$:

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Remarks:

- Results independent of $\sigma > 0$: The model is ergodic for any disorder strength (no Floquet-MBL!). In particular, we can take the limit of a clean system at the end $\sigma \searrow 0$.

- Results independent of $\bar{h}$: We can set $\bar{h} = 0$ which corresponds to a limiting integrable system.
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- Results independent of $\sigma > 0$: The model is ergodic for any disorder strength (**no Floquet-MBL!**). In particular, we can take the limit of a clean system at the end $\sigma \downarrow 0$.
- Results independent of $\bar{h}$: We can set $\bar{h} = 0$ which corresponds to a limiting integrable system.

We found a simple locally interacting model with finite dimensional local Hilbert space with proven RMT spectral correlations at all time-scales!
The trace of $U_{KI}^t$ is equivalent to a partition sum of a classical 2d Ising model with **row-homogeneous field** $h_j$:

\[
\text{tr} \left( U_{KI}^t[h] \right) \sim \text{tr} \left( \prod_{j=1}^{L} \tilde{U}_{KI}[h_j \epsilon] \right)
\]

Duality relation

\[
tr \left( U_{KI}[h] \right)^t = tr \left( \prod_{j=1}^{L} \tilde{U}_{KI}[h_j \epsilon] \right)
\]

where $\epsilon = (1, 1, \ldots, 1)$ and $\tilde{U}_{KI}$ is a KI model on a ring of size $t$ with twisted parameters $\tilde{J}(J, b)$, $\tilde{b}(J, b)$. 
Sketch of the proof. Space-time duality

The trace of $U_{KI}^t$ is equivalent to a partition sum of a classical 2d Ising model with **row-homogeneous field** $h_j$:

$$\text{tr} \left( U_{KI}^t[h] \right) \sim \text{tr} \left( \tilde{U}_{KI}[h_3 \epsilon] \right)$$

Duality relation

$$tr \left( U_{KI}[h] \right)^t = tr \left( \prod_{j=1}^{L} \tilde{U}_{KI}[h_j \epsilon] \right)$$

where $\epsilon = (1, 1 \ldots, 1)$ and $\tilde{U}_{KI}$ is a KI model on a ring of size $t$ with twisted parameters $\tilde{J}(J, b), \tilde{b}(J, b)$.

Remarkably: $\tilde{U}_{KI}$ is **unitary** for $|J| = |b| = \pi/4$ (Self-dual, $J = \pm \tilde{J}, b = \pm \tilde{b}$)
Space-time duality allows to simply express the disorder averaging:

$$
\mathbb{E}_h \left[ K(t) \right] = \text{tr} \left( \mathbb{T}_L \right)
$$

Where

$$
\mathbb{T} = \mathbb{E}_h \left[ \tilde{U}_{KI}[h\epsilon] \otimes \tilde{U}_{KI}[h\epsilon]^* \right] = (\tilde{U}_{KI} \otimes \tilde{U}_{KI}^*) \cdot \mathcal{O}_\sigma
$$

Contraction

$$
\mathcal{O}_\sigma = \exp \left[ -\frac{1}{2} \sigma^2 (M_z \otimes I - I \otimes M_z)^2 \right]
$$

Non-Unitary

Unitary

Tomaž Prosen

Many body quantum chaos
Space-time duality allows to simply express the disorder averaging:

\[ \mathbb{E}_h [K(t)] \equiv \text{tr} \left( \mathbb{T}^L \right) \]

\[ \mathbb{T} \equiv \mathbb{E}_h \left[ \tilde{U}_{KI}[h\epsilon] \otimes \tilde{U}_{KI}[h\epsilon]^* \right] = (\tilde{U}_{KI} \otimes \tilde{U}_{KI}^*) \cdot \mathbb{O}_\sigma \]

\[ \mathbb{O}_\sigma = \exp \left[ -\frac{1}{2} \sigma^2 \left( M_z \otimes I - I \otimes M_z \right)^2 \right] \]

Computation of thermodynamic SFF \( \lim_{L \to \infty} \mathbb{T}^L \) thus amounts to determining the multiplicity of eigenvalue 1 of \( \mathbb{T} \) and proving positive spectral gap.
Empirical convergence of the spectral gap
The following is straightforward to show:

Property 1

1. The eigenvalues of $\mathbb{T}$ of maximal (unit) magnitude are either $+1$ or $-1$.

2. Each eigenvector associated to the eigenvalue $\pm 1$ is uniquely parametrized by an operator $A \in \text{End}((\mathbb{C}^2)^\otimes t)$ satisfying

$$UAU^\dagger = \pm A, \quad [A, M_\alpha] = 0, \quad \alpha \in \{x, y, z\}.$$  \hspace{1cm} (1)

where we have defined $M_\alpha = \sum_{\tau=1}^{t} \sigma^\alpha_{\tau}$, $U = \exp \left[ i \frac{\pi}{4} \sum_{\tau=1}^{t} (\sigma^z_{\tau}\sigma^z_{\tau+1} - 1) \right]$.

$U$ is the parity of half-number of domain walls in the spin configuration, $U^2 = 1$. 

Tomaž Prosen  Many body quantum chaos
Unimodular eigenvalues of $\mathbb{T}$

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$$UAU^\dagger = \pm A, \quad [A, M_\alpha] = 0, \quad \alpha \in \{x, y, z\}. \quad (1)$$

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$U$ is the parity of half-number of domain walls in the spin configuration, $U^2 = 1$.

Observation: The operators $U, M_\alpha$ are translationally invariant and reflection symmetric $\Rightarrow$ All elements of $\mathcal{D}_t = \{ \Pi^n R^m, n \in \{0, 1, \ldots, t-1\}, m \in \{0, 1\} \}$ fulfill (1) with $+1$, where

$$\Pi = \prod_{\tau=1}^{t-1} P_{\tau,\tau+1}, \quad R = \prod_{\tau=1}^{\lfloor t/2 \rfloor} P_{\tau, t+1-\tau},$$

are translation and reflection on a spin ring of length $t$. 

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Property 2

The number of linearly independent elements of $D_t$ is $2t$ for $t \geq 6$, $2t - 1$ for $t \in \{1, 3, 4, 5\}$, and 2 for $t = 2$. 
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Property 3

For odd $t$, Eq. (1) can be fulfilled only for eigenvalue +1.
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For odd $t$, Eq. (1) can be fulfilled only for eigenvalue $+1$.

Theorem

For odd $t$, all $A$ satisfying (1) are given by linear combination of elements of $D_t$. 
Property 2
The number of linearly independent elements of $D_t$ is $2t$ for $t \geq 6$, $2t - 1$ for $t \in \{1, 3, 4, 5\}$, and 2 for $t = 2$.

Property 3
For odd $t$, Eq. (1) can be fulfilled only for eigenvalue +1.

Theorem
For odd $t$, all $A$ satisfying (1) are given by linear combination of elements of $D_t$.

*Observation:* For even $t$, we find generically exactly one additional operator $A$ satisfying Eq. (1). For special values of $t \leq 10$ we find an extra additional operator, and also solutions of Eq. (1) for eigenvalue $-1$.

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Generic (non-self-dual) Floquet chain with maximal field disorder

Model with broken time reversal:
Data for $L = 8, 10, 12, 14, 16$ suggest that

$$|K(t) - t| \leq C \exp(\alpha L - \beta t), \quad C, \alpha, \beta > 0.$$

More numerical analysis in Hamiltonian disordered spin chains in:
Conclusions

- The first (?) exact result on ergodicity in terms of spectral correlations for an interacting quantum many-body problem
- Self-dual instances of Kicked Ising chain provide a minimal model of quantum many-body chaos with no intrinsic time scales

Open problems and promising future directions:
1. Complete the picture by rigorous analysis of the even $t$ case.
2. Structural stability of the self-dual point: Perturbation theory may have a finite radius of convergence?
3. Potential to characterise ergodicity - MBL transition, approaching from ergodic side.
4. Path to a rigorous approach to ETH?

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[Logo of the European Research Council]