Axion-photon coupling from Lattice QCD

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Outline

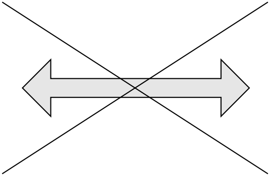
- Topology in QCD with EM fields
- Axion mechanism and observables
- Electric fields on the lattice
- Computation of $g_{a\gamma\gamma}$ on the lattice
- Preliminary results: $g_{a\gamma\gamma}^{QCD}$ and χ_{top}
- Conclusions

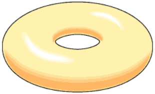
Topology in QCD with EM fields

Topology: the general idea

- Study of continuous transformations.
- Can be characterised by integers: # holes.
- Under continuous transformations one remains in the same homotopy group (same # holes).

Wikimedia





Pinterest

Topology in QCD

- We usually expect that our gluon fields vanish at the boundary $|x| \to \infty$,

$$A_\mu(x) = 0.$$

- But we need to consider all possible gauge transformations:

$$A_\mu = i\Omega\partial_\mu\Omega^\dagger, \;\; \Omega \in SU(3).$$

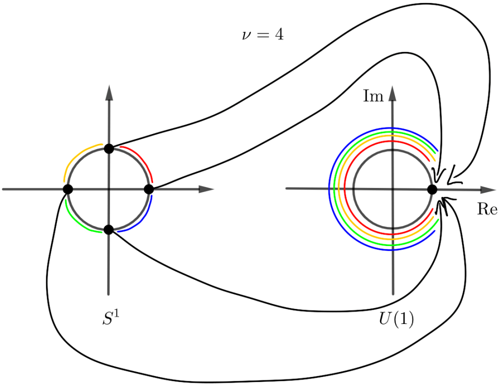
Hence we have an infinite set of solutions.

- They can be classified by an integer label, the "winding number".

$$Q_{top} = \int d^4x \, q_{top}(x), \;\; q_{top} = \frac{1}{32\pi^2}\epsilon^{\mu\nu\rho\sigma}\mathrm{Tr}\, G_{\mu\nu} G_{\rho\sigma}.$$

In general, $Q \propto \int F\widetilde{F}$ for any gauge group.

Winding numbers in $U(1)$



Adding EM fields I

- Adding electric or magnetic fields *separately*: no changes in topology.

$$\langle Q_{top} \rangle = 0.$$

- If $F_{\mu\nu} \neq 0$ such that $\vec{E} \cdot \vec{B} \neq 0$ it can be interpreted as an effective θ-therm D'Elia et al., 2012.
- Hence non-orthogonal EM fields $\Longleftrightarrow$ non-trivial topology.

$$\langle Q_{top} \rangle \neq 0.$$

Adding EM fields II

- EM fields can induce topologies in the gluon sector. But how? $\longrightarrow$ Index theorem.
- The index theorem says (for QCD) Atiyah, Singer '71:

$$\text{Index}(D) \equiv n_- - n_+ = Q_{top}$$

Since in QCD $\langle Q_{top} \rangle = 0$, we don't see imbalances in chirality.

- But after including electromagnetic fields the situation is different:

$$Q_{top} \longrightarrow Q_{top} + Q_{U(1)}.$$

We have two different topological contributions to the zero modes.

- Path integral favours as little zero modes as possible: $\det M \uparrow\uparrow$.
- Hence, it selects gluon field configurations such that:

$$Q_{U(1)} \uparrow \Longleftrightarrow \; Q_{top} \downarrow .$$

Axion mechanism and observables

Strong CP problem

- In principle, the QCD Lagrangian could include an extra term:

$$\mathcal{L}_{\mathrm{QCD}+\theta} = \mathcal{L}_{\mathrm{QCD}} + \theta\, q_{top}.$$

  - This term is CP and T odd.
  - Induces an electric dipole moment in n: $|\theta| < 10^{-10}$ Abel et al 2020.
- Why don't we see it 😞? $\longrightarrow$ Axions 🤔? Peccei, Quinn '77

$$\mathcal{L}_a = \frac{1}{2}\partial_\mu a \partial^\mu a + \frac{a}{f_a} q_{top} + \mathcal{L}_{\mathrm{int}}.$$

- Essence of the idea: new pseudoscalar a whose minimum is $\langle a \rangle = -\theta f_a$.

Axion models

- The interaction lagrangians depend on the specific model.
- Examples:
  - KSVZ Kim, '79 : introduces a new fermion, $\mathcal{Q} \sim (3,1,0)$ and a complex scalar $\phi \sim (1,1,0)$.
  - DFSZ Dine et al., '81 : introduces two Higgs doublets $H_u \sim (1,2,-\frac{1}{2})$ and $H_d \sim (1,2,+\frac{1}{2})$, and a complex scalar $\phi \sim (1,1,0)$.

Topological susceptibility χ_{top}

- Is the second moment of Q_{top}: $\chi_{top} = \frac{T}{V}\langle Q_{top}^2 \rangle$
- It is also the mass of the axion:

$$m_a^2 = \frac{T}{V}\frac{\delta^2}{\delta a^2}\log \mathcal{Z}\left(\frac{a}{f_a}\right)\Bigg|_{a=0} = \frac{1}{f_a^2}\frac{T}{V}\frac{\partial^2}{\partial\theta^2}\log \mathcal{Z}(\theta)\Bigg|_{\theta=0} = \frac{\chi_t}{f_a^2}.$$

- Hence, an analysis of χ_t gives information on m_a.
- Current estimates from ChPT. : $m_a = 5.70(6)(4)(10^{12}\mathrm{GeV}/f_a)$ μeV Cortona et al 2016.
  - Lattice calculations give almost the same central value but with a bigger error Borsanyi et al 2016.
- It also gives us cosmological information about a.

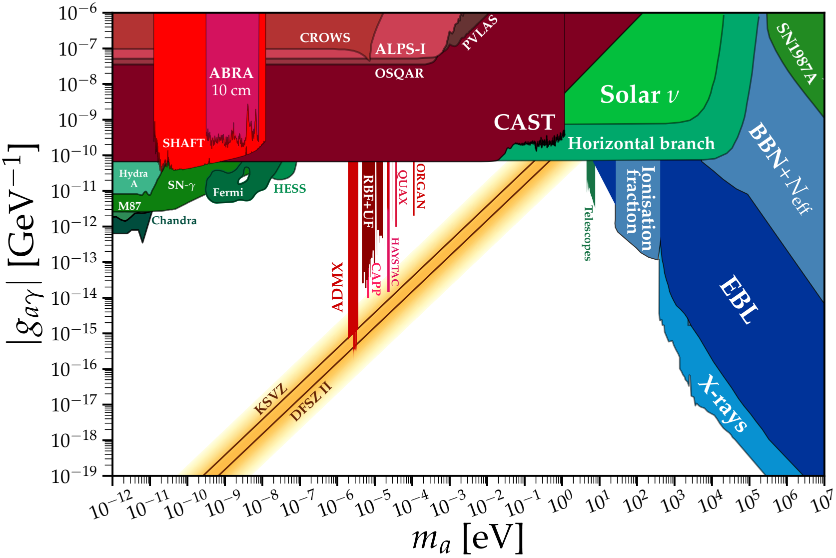
The photon-axion coupling $g_{a\gamma\gamma}^{QCD}$

- The axion couples directly and indirectly to photons.
- ChPT calculations show:

$$\mathcal{L}_{a\gamma\gamma} = \frac{1}{4} g_{a\gamma\gamma} a F_{\mu\nu} \widetilde{F}^{\mu\nu} = g_{a\gamma\gamma} a \mathbf{E} \cdot \mathbf{B}.$$

- Current estimates from ChPT.: $g_{a\gamma\gamma} = g_{a\gamma\gamma}^{0} + g_{a\gamma\gamma}^{QCD} = \frac{\alpha_{em}}{2\pi f_a} \left(\frac{E}{N} - 1.92(4) \right)$ Cortona et al 2016.

Astrophysical bounds



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Q_{top} and $g_{a\gamma\gamma}^{QCD}$

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- If we include both electric and magnetic background fields, the only CP odd operators in our theory are:

$$\mathrm{Tr}\, G_{\mu\nu} \widetilde{G}^{\mu\nu} \quad \& \quad \mathbf{E} \cdot \mathbf{B}.$$

So by symmetry arguments, Q_{top} can only be (for weak fields):

$$Q_{top} \propto \mathbf{E} \cdot \mathbf{B} + \mathcal{O}\left([\mathbf{E} \cdot \mathbf{B}]^3 \right).$$

- By looking at $\mathcal{Z}$:

$$\left. \frac{\delta \log \mathcal{Z}(a)}{\delta a} \right|_{a=0} = \frac{\langle Q_{top} \rangle_{E,B}}{f_a} \longrightarrow g_{a\gamma\gamma}^{QCD} f_a = \frac{T}{V} \frac{\partial}{\partial (\mathbf{E} \cdot \mathbf{B})} \langle Q_{top} \rangle_{E,B} \Big|_{\mathbf{E},\mathbf{B}=0}$$

- So for homogeneous, static and weak EM fields

$$\frac{T}{V} \langle Q_{top} \rangle_{E,B} \approx \frac{g_{a\gamma\gamma}^{QCD} \cdot f_a}{e^2} e^2 \mathbf{E} \cdot \mathbf{B} \quad \textit{and } g_{a\gamma\gamma}^{QCD} < 0.$$

Electric fields on the lattice

Introducing $\vec{E}$ on the lattice

- We already saw how to introduce magnetic fields $\longrightarrow$ D. Valois L. Sandbote's talk.
- For electric fields we follow the same approach:
  - Considering an electric field in the z direction + Stoke's theorem:

$$qE = \frac{2\pi N_e}{L_z L_t} \quad \text{with } N_e \in \mathbb{Z}.$$

  - And the prescription for the links is:

$$u_t = e^{iaqEz}, \quad u_z\Big|_{z+L_z-a} = e^{-iqEL_z t},$$

$$u_x = u_y = u_z\Big|_{\text{rest}} = 1.$$

- Caveat $\longrightarrow$

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- Caveat $\longrightarrow$ is an imaginary electric field!

Computation of $g_{a\gamma\gamma}^{QCD}$ on the lattice

Approaches to the computation of $g_{a\gamma\gamma}^{QCD}$

- We have explored two ways fro computing $g_{a\gamma\gamma}^{QCD}$:
  - Correlator method: computing $\frac{T}{V}\frac{\partial^2 \log Z}{\partial E \partial a}\Big|_{a,E=0}$ and fitting to $\mathbf{B}$.

$$\text{Im}\left[\int d^4x \langle q_{top}(0) j_4(x)\rangle\right] = g_{a\gamma\gamma} f_a B.$$

  - Electric method: measuring $\langle Q_{top}\rangle_{E,B}$ and fitting to $\mathbf{E}\cdot\mathbf{B}$

$$\frac{T}{V}\langle Q_{top}\rangle_{E,B} = \frac{g_{a\gamma\gamma}^{QCD}\cdot f_a}{e^2} e^2 \mathbf{E}\cdot\mathbf{B}.$$

To take into account...

- We are going to simulate gluons with background (homogeneous) EM fields. Thus, we have to deal with two issues:
    1. The sign problem. We can't simulate real electric fields, so $\mathbf{E} \longrightarrow i\mathbf{E}$, *already discussed*.
    2. UV fluctuations of the gluon fields $\longrightarrow$ *Wilson flow*.

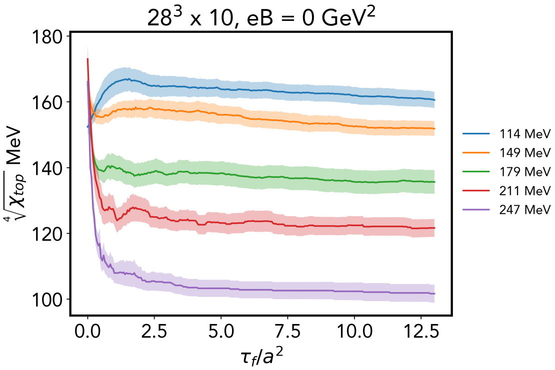
Wilson flow

- The gluon fields need to be smeared in order to reduce UV fluctuations.
- One method $\longrightarrow$ Wilson flow Lüscher 2010.
- General idea: $U_\mu \equiv U_\mu(\tau_f)$ and links evolved through a diff. eq..
- For increasing τ_f, the fields evolve towards stationary points of the action S.
- Goal? $\longrightarrow$ we obtain renormalised gluon fields and Q_{top} closer to integers.
- Related talk in previous JC JH,2022.

Preliminary results

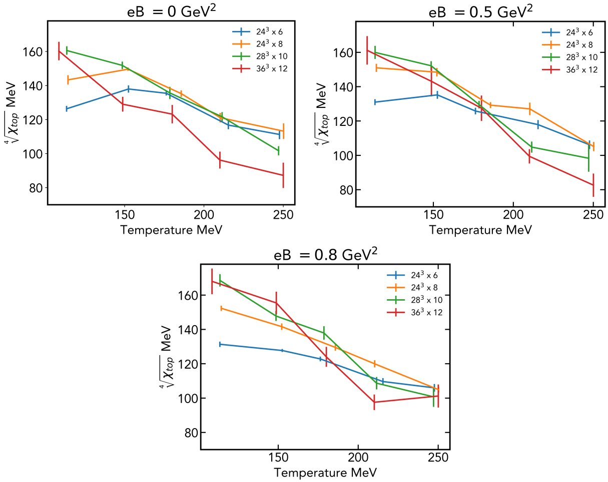
χ_{top} flow: preliminary results

Wilson evolution of χ_{top}. Note the plateaus.



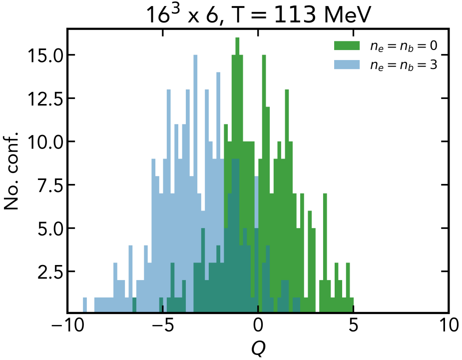
$\chi_t(B)$ vs T: preliminary results

$\chi_t(B)$ as a function of T. Note that $\sqrt{\chi_t} = m_a f_a$.

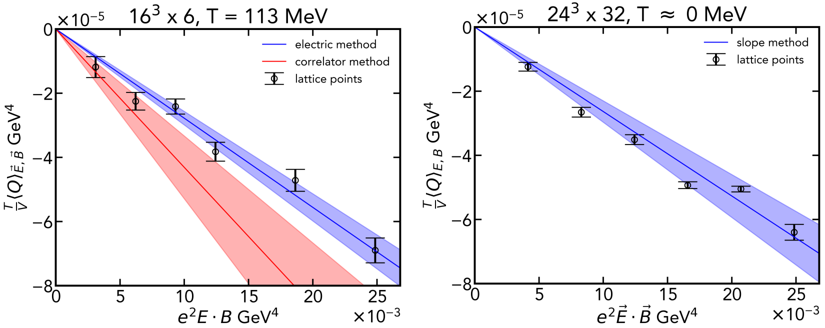


Shift of Q_{top}: preliminary results

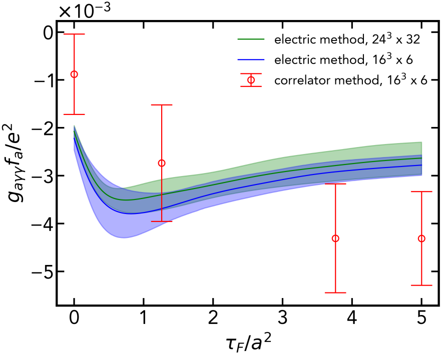
Shift of Q_{top} at non-zero $\mathbf{E}_I \cdot \mathbf{B}$. Effect also shown in D'Elia et al 2016.



$\langle Q_{top} \rangle_{E_I,B}$ as a function of $\mathbf{E}_I \cdot \mathbf{B}$.



$g_{a\gamma\gamma}^{QCD}$ as a function of flow time for the two different methods.
$g_{a\gamma\gamma}f_a/e^2 = -0.00243(5)$ from ChPT.



Conclusions and further work

Conclusions and further work

- We have shown:
  - how there is an interplay between EM fields and $SU(3)$ topology.
  - that there is a linear response of Q_{top} with $\mathbf{E}_I \cdot \mathbf{B}$ for weak fields.
  - preliminary results for χ_{top} as a function of $\mathbf{B}$, T as well as for $g_{a\gamma\gamma}^{QCD}$.
- Further work:
  - Mimic the effect of zero modes by rewiwghting the determinant for χ_{top}.
  - Generate more statistics and perform continuum limit for both observables.
  - Eventually $\longrightarrow$ Use experimental bounds and lattice results to constrain axion models.

Thank you for your attention!