Chiral Magnetic Effect: Discussion on the Lattice Journal

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26/06/2020

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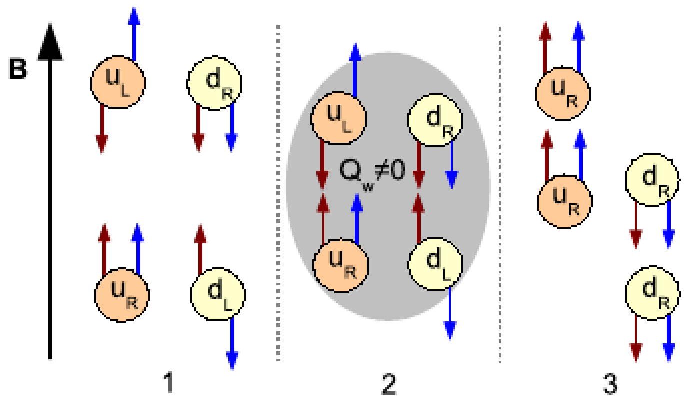

Figure 1: Qualitative illustration of the Chiral Magnetic Effect, from Ref. [2]. The red arrows denote the direction of momentum, the blue arrows the spin of the quarks. (1) Initial conditions of the system. (2) Chirality imbalance, induced for example by the interaction between quarks and topological non trivial gluonic configurations. (3) The right-handed up quarks will move upwards, the right-handed down quarks will move downwards. A charge difference of $Q = 2e$ will be created between two sides of a plane perpendicular to the magnetic field.

1 Introduction

1.1 What is the Chiral Magnetic Effect?

What is the Chiral Magnetic Effect ([1])? In a system composed of free or quasi-free fermions, initially chirally symmetric, it is possible to show that a strong magnetic field, in the presence of an imbalance of chirality, induces an electric dipole moment and then a current along the magnetic field. This is called the "Chiral Magnetic Effect" (CME from now on).

1.2 A Qualitative Description of the CME

Before speaking about the reasons that make the CME an interesting research topic, let me first explain what the CME actually is in a sketchy and intuitive way, inspired by Ref.[2].
Let us consider a generic system composed of free massless fermions. It is well known that the energy of a particle with magnetic moment μ in the presence of a background magnetic field B follows the Pauli coupling, that is:

$$E = -q\vec{\mu} \cdot \vec{B} \tag{1}$$

So, in a strong background field, the spin tends to be aligned along the magnetic field: for the positively charged fermions, in order to minimize the energy above, the spin is parallel to $\vec{B}$. For the negatively charged particle, instead, the spin is antiparallel to $\vec{B}$.

Now we assume that our system is chirally symmetric, thus there are as many left-handed as right-handed fermions. If fermions are massless chirality coincides with helicity, then the momentum of right-handed fermions is parallel to their spin and antiparallel for left-handed fermions. The situation is illustrated in

part (1) of Fig. 1, where we call positively charged fermions u (like for example quark up) and negatively charged ones d (like down quarks).
Now, if for any reason, it occurs an imbalance of chirality, some of the fermions will change their helicity. In the presence of a strong background magnetic field we assume that $eB \gg p^2$. Hence, they can only change helicity by reversing their momenta, since spin flip is energetically suppressed. If, for example, we assume that an imbalance of chirality changes a positively and negatively charged left-handed fermion in a right-handed one, then the situation becomes the one in part (3) of Fig. 1
Therefore, we can see that the imbalance of chirality in the presence of a large background magnetic induces a charge separation in the system. As a result an electromagnetic current is generated along the direction of B.
Summarizing, in order to produce the CME we need the following ingredients, that we will use as a general guideline during all the discussion:

1. A chirally symmetric system composed of free massless fermions
2. A large background magnetic field
3. Some process that induces a chirality imbalance in the system

If we have such a system with all the ingredients above, as a result we can see a charge separation and a current along B. This is the Chiral Magnetic Effect.

1.3 Motivation

Why is this effect so interesting? There are many reasons to study the so called CME, since it can be theoretically found in many fields of modern physics. One of the most interesting applications, from the lattice QCD point of view, is that the CME can be used, in principle, as a way of probing the generation of the Quark Gluon Plasma (QGP) in non central Heavy Ion Collision (HIC), where a large magnetic field can be produced. The current induced by the CME could be a signature that the system is entered in a deconfined, chirally symmetric phase, that is indeed the QGP.

The CME is also a very interesting feature of many other systems. It can be found in condensed matter physics, it is linked to interesting transport phenomena like chiral superconductivity, and it can have even some intriguing applications in Cosmology and the physics of the primordial magnetic fields in the Early Universe.

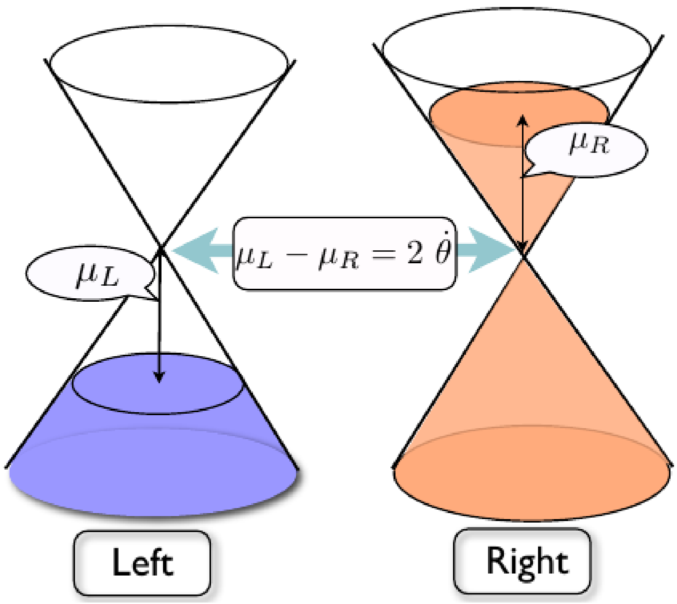

Figure 2: Dirac cones of the left and right fermions (image taken from Ref. [3]). In the presence of a chirality imbalance there is an asymmetry between the Fermi energies of left and right fermions, i.e. μ_L and μ_R.

2 A More Quantitative Description of the CME

Now that we have an intuitive comprehension of the CME, in this section we will use a simple formalism to show the mathematical derivation of the electric dipole moment and the current induced by this effect.

2.1 Description of the System

We consider the most simple system of the type described in the previous section: a "Dirac sea" of massless chiral fermions ([3]). I recall that at zero temperature, in absence of any external fields, fermions will occupy the lowest levels of energy from the vacuum state to the highest one that allows them to obey the Pauli exclusion principle, that has an energy equal to the Fermi Energy $E_F \equiv \mu$.

Let us start considering that, in the absence of external fields and in the case of massless fermions we are in the chiral limit, so the chirality is conserved. Therefore there are two disconnected Fermi surfaces of left- and right-handed fermions. We can see a representation of the Dirac cones of our double Dirac sea of quarks in Fig.2.

Now, we have to introduce a chiral imbalance: then, let us turn on adiabatically the external classical electric and magnetic fields $\vec{E}$ and $\vec{B}$, both aligned along the z direction. I say that this two fields will be able to create a chirality imbalance. Indeed, as I said in the first section, the presence of magnetic field B aligns the spins of the positive (negative) fermions in the direction parallel (anti-parallel) to B. The situation will be, then, exactly as part (1) of Fig.(1), both for the right and left surface. Now, the right positive (negative) fermions will have their momentum parallel (anti-parallel) to the electric field, while the left

positive (negative) fermions will have their momentum anti-parallel (parallel) to the electric field. All of the fermions will experience a force $F = qE$ along the z direction, but looking at Fig.(1) we can easily convince ourselves that all the right fermions will increase their momentum, while all the left ones will decrease their momentum. That means that also the Fermi momentum of the right distribution will increase to $p_R^F = E_R^F/c = eEt$. The left fermions, instead, will have their Fermi momentum decreased of the same quantity. Increasing and decreasing the Fermi energy of the two Dirac surfaces will also change the number of fermions in each cone, creating an imbalance of chirality.
Now, we recall that a quantum particle in a constant, transverse magnetic field can only occupy orbits with discrete energy values, called Landau levels. So, in the transverse direction, the motion of fermions is quantized and the density of Landau levels is $\mathrm{d}^2N_R/\mathrm{d}x\mathrm{d}y = eB/2\pi$. The one-dimensional density of states along the axis z (the axis parallel to the direction both of $\vec{B}$ and $\vec{E}$), instead, comes from the Dirac distribution, thus is given by $\mathrm{d}N_R/\mathrm{d}z = p_R^F/2\pi$. Therefore the density of right fermions increases per unit time as:

$$\frac{\mathrm{d}^4N_R}{\mathrm{d}t\mathrm{d}V} = \frac{e^2}{(2\pi)^2}\vec{E}\cdot\vec{B} \tag{2}$$

Then, the density of left fermions decreases with the same rate, $\mathrm{d}^4N_L/\mathrm{d}t\mathrm{d}V = -\mathrm{d}^4N_R/\mathrm{d}t\mathrm{d}V$. Now, let us define the local rate of chirality $Q_5 = N_R - N_L$. Using equation above, and this definition of Q_5, the local rate of chirality generation is thus:

$$\frac{\mathrm{d}^4Q_5}{\mathrm{d}t\mathrm{d}V} = \frac{e^2}{2\pi^2}\vec{E}\cdot\vec{B} \tag{3}$$

Summarizing, with an external electric and magnetic field it is possible to create a difference between the number of right-handed and left-handed fermions. Coming back to our system, after the chirality imbalance in the Dirac sea, induced by E and B fields, the situation is illustrated in Fig.2.
Finally, we stress that the quantity (3) is CP-odd, since both B and E are odd under C transformations, but E is P-odd while B is P-even. So we can already see that even in a simple model like this one, the creation of a chirality imbalance is a CP-odd phenomenon. We will find the same results in QCD, that will lead to the famous strong-CP problem.
In addition, we notice that if we perform the integration of the right hand side of Eq. (3) over four-dimensional space we get:

$$q[A] = \frac{e^2}{8\pi^2}\int \mathrm{d}^4x F^{\mu\nu}\tilde{F}_{\mu\nu} \tag{4}$$

Where $F_{\mu\nu}$ is the usual Abelian force tensor, and $\tilde{F}_{\mu\nu}$ is its dual, i.e. $\tilde{F}_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\alpha\beta}F^{\alpha\beta}$. We stress that the equation above is strictly connected to the quantity that in QCD we call *topological charge*, that we will briefly introduce in section 3.1. From this we can already see a qualitative connection between the chirality imbalance and the topological properties of the gauge configurations of our theory. Also, we linked the chirality generation rate to a quantity that will naturally arise in the framework of the discussion of the anomaly of $U(1)_A$ symmetry in QCD, linking these two phenomena.

2.2 The Chern-Simons Model

Now let us introduce a model in which we can calculate the mathematical form of the current induced by the CME. Consider the Lagrangian of QED with flavored fermions, in the presence of a so called θ-term that violates CP, as we said at the end of the previous section.

$$\mathscr{L}_{MCS} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - A_\mu J^\mu - \frac{c}{4}\theta\tilde{F}^{\mu\nu}F_{\mu\nu} \tag{5}$$

Where J_μ is the electromagnetic current, i.e. $J_\mu = \sum_f q_f\overline{\psi}_f\gamma_\mu\psi_f$, and coefficient c is:

$$c = \sum_f \frac{q_f^2 e^2}{(2\pi)^2} \tag{6}$$

This is the so-called Lagrangian of Maxwell-Chern-Simons. Basically we added to the QED Lagrangian a term that is directly related, as we said at the end of the previous section, to the local rate of chirality generation (3), allowing the CP-violating process that creates an imbalance of chirality.
We notice that the last term in the equation can be written as a full divergence:

$$\tilde{F}^{\mu\nu}F_{\mu\nu} = \partial_\mu K^\mu \tag{7}$$

Where K^μ is the Abelian version of the so-called Chern-Simons current:

$$K^\mu = 4\varepsilon^{\mu\nu\rho\sigma}\mathrm{Tr}[A_\nu\partial_\rho A_\sigma + \tfrac{2}{3}A_\nu A_\rho A_\sigma] \tag{8}$$

As we know, in an Abelian gauge theory this term does not affect the equations of motion. The situation, though, is different if the field $\theta \equiv \theta(\vec{x}, t)$ varies in space-time. Indeed, in this case we have:

$$\theta\tilde{F}^{\mu\nu}F_{\mu\nu} = \partial_\mu[\theta K^\mu] - \partial_\mu\theta K^\mu \tag{9}$$

The first term on right hand side is again a full derivative and, for the considerations above, can be omitted. Now let us introduce the new notation:

$$P_\mu \equiv \partial_\mu\theta \equiv (M, \vec{P}) \tag{10}$$

Using equation above we can rewrite the Lagrangian (5) in the following form:

$$\mathscr{L}_{MCS} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - A_\mu J^\mu + \frac{c}{4}P_\mu K^\mu \tag{11}$$

Now that we have the Lagrangian let us write down the Euler-Lagrange equations of motion. In analogy with the covariant form of the Maxwell equations we found:

$$\partial_\mu F^{\mu\nu} = J^\nu - P_\mu\tilde{F}^{\mu\nu} \tag{12}$$

If we write down these equations in terms of the electric $\vec{E}$ and magnetic $\vec{B}$ fields we get:

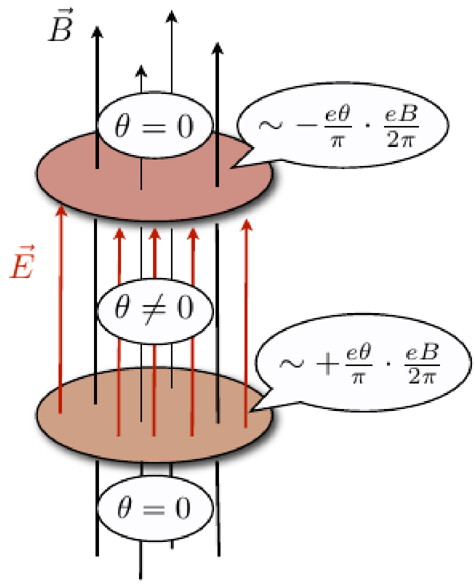

Figure 3: Charge separation effect, from Ref. [3]: domain walls that separate region of different θ become charged in the presence of an external magnetic field. This induces an electric dipole.

$$\vec{\nabla} \times \vec{B} = \frac{\partial E}{\partial t} + \vec{J} + c(M\vec{B} - \vec{P} \times \vec{E}) \tag{13}$$

$$\vec{\nabla} \cdot \vec{E} = \rho + c\vec{P} \cdot \vec{B} \tag{14}$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial B}{\partial t} \tag{15}$$

$$\vec{\nabla} \cdot \vec{B} = 0 \tag{16}$$

Where $(\rho, \vec{J})$ are the electric charge and current densities. One can see that, as expected, the presence of θ-term leads to essential modifications of the Maxwell theory.

2.3 Separation of Charges

We start with the simple case, where we assume that θ is a static field, that means $\dot{\theta} = 0$.

Let us examine a situation like the one shown on Fig. 3, where an external magnetic field pierces a domain with $\theta \neq 0$ inside and $\theta = 0$ outside. In this case we have a gradient of the pseudo-scalar field θ, thus, for the reasoning of the previous section, the θ-term in $\mathscr{L}_{MCS}$ can not be neglected, because $\partial_\mu \theta \neq 0$. From the Maxwell-Chern-Simons equation (14) we can see that the magnetic field in the presence of $\theta \neq 0$ generates an electric field $E \sim \theta \cdot B$.
In fact using the equivalent of the Gauss Theorem from the Chern-Simons model, that could be derived from Eq. (14), we find that the upper domain wall acquires the charge density per unit area S:

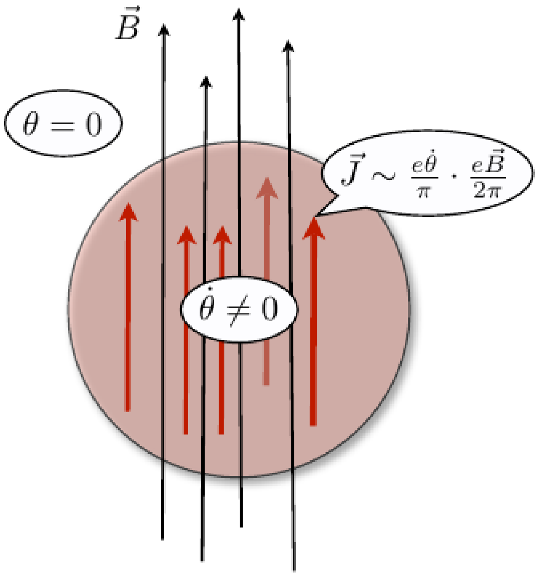

Figure 4: Inside a domain with $\dot{\theta} \neq 0$ an external magnetic field induces an electric current $\vec{J}$. Image taken from Ref. [3].

$$\left(\frac{Q}{S}\right)_{up} = +c\theta B \tag{17}$$

Meanwhile, the lower domain wall acquires the same in magnitude but opposite in sign charge density, i.e.:

$$\left(\frac{Q}{S}\right)_{down} = -c\theta B \tag{18}$$

Then, assuming that the domain walls are thin compared to the distance L between them, we find that the system possesses an electric dipole moment:

$$d = c\theta(B \cdot S)L = \sum_f q_f^2 \frac{e^2}{2\pi^2}(B \cdot S)L \tag{19}$$

Then, we showed as a separation of charge, parametrized with an electric dipole moment, naturally arises in a system described by a Chern-Simons Lagrangian, that was one of the aspect of the CME.

2.4 Induced Electromagnetic Current

This time, in order to show the induced electric current, let us consider instead the case of a dynamical θ, where $|\vec{P}| = 0$ but $M = \dot{\theta} \neq 0$. Then we introduce again an external irrotational magnetic field B (thus $\vec{\nabla} \times \vec{B} = 0$) and assume that no external electric field is present (as, for example, for the system in Fig. 1). Such a system is depicted in Fig.(4).
In this case we immediately get from Eq. (13) that there is an induced current:

$$\vec{J} = -cM\vec{B} = -\frac{e^2}{2\pi^2}\dot{\theta}\vec{B} \tag{20}$$

Finally, we demonstrated that the presence of a magnetic field in a system with an imbalance of chirality (thus with a non negligible θ-term) can generate an electric current (20). The situation is summarized in Fig. 4.

In conclusion, we have seen a very simple model in which, using all ingredients discussed in section (1.2), it is possible to see the consequences of the Chiral Magnetic Effect:

1. We proposed a system composed of massless chiral fermions: a double Dirac sea of right and left-handed fermions.

2. We considered a chirality imbalance, parametrized by a chiral chemical potential: in this case it is induced by an external electric field.

3. We considered the presence of an external magnetic field.

As a result, a separation of charge is induced in the system, and then an electric dipole (19). Finally we have seen how the chirality imbalance and the external magnetic field generate an electric current (20).

3 Chiral Magnetic Effect in QCD

In this section I will briefly hint at the applications of the CME in QCD. First I will introduce the basic ideas that will be necessary to understand the generation of this effect in QCD. Then, in the following section, I will show how the CME could be found, in principle, in a system like the Quark Gluon Plasma.

3.1 Anomaly of the Chiral Symmetry

Before discussing where the CME can be found in QCD, we have to introduce the idea of the anomaly of the chiral symmetry.
Why is this interesting in the framework of the CME? The third elements of the list of ingredients of the CME is indeed some process that is able to induce a chirality imbalance in our system. In the Chern-Simons model that was the presence of an external electric field. In QCD, this is linked to the chiral anomaly.
Then, let us first remind the basic mathematical ideas behind the anomaly of the $U(1)_A$ group.

During this journal we already discussed a lot about Chiral Symmetry, but since we are going through a mathematical discussion of the anomaly, let me just give a quick reminder of the basic definitions of the Chiral Group.

Let us start from defining the spinor ψ_f as a fundamental representation of the group SU(l) in flavor space. Then, the most general transformation of this field belongs to the flavor group:

$$\mathrm{U}(l) = \mathrm{U}(1) \otimes \mathrm{SU}(l) \tag{21}$$

If we define the chiral components of the field:

$$\psi_L(x) = \left(\frac{1-\gamma_5}{2}\right)\psi(x)$$
$$\psi_R(x) = \left(\frac{1+\gamma_5}{2}\right)\psi(x)$$

We can then transform separately the two components of the spinor fields and then define the Chiral Group:

$$\mathcal{G} = \mathrm{U}(l)_R \otimes \mathrm{U}(l)_L \tag{22}$$

Or, alternatively, if we define axial and vectorial components:

$$\mathcal{G} = \mathrm{U}(l)_V \otimes \mathrm{U}(l)_A \tag{23}$$

Now, at classical level, when the Lagrangian of a system is invariant under a certain transformation, you can define a Noether current and a conserved charge. When you try to quantize the symmetry, there are a few scenarios that can happen.
You can have an exact symmetry, a spontaneously broken one, or an anomaly. The first case happens when you are able to define a Noether current and a conserved charged operator. The symmetry is spontaneously broken when you

are not able to define a conserved charge operator, because it does not annihilate the vacuum state. Finally, the symmetry is called anomalous when the Lagrangian is still invariant under the group transformations, but you can not even define a conserved Noether current.
During this journal we already shown that when we quantize the chiral symmetry, $U(l)_V$ is an exact symmetry in the chiral limit, that is when the mass tends to zero, while $SU(l)_A$ is spontaneously broken and $U(1)_A$ is an anomalous symmetry.

Let us focus on this last case: an easy way to quantize a symmetry is via the functional formalism. The anomaly of a symmetry can occur if the functional measure of a generic field ϕ is not invariant under the group transformations. If I exponentiate the variation of the measure I get:

$$\mathcal{D}\phi' = e^{i\mathcal{A}[\alpha]}\mathcal{D}\phi \tag{24}$$

Where α is the parameter of the transformations that we are considering. Now, if I parametrize the variation in the following way:

$$\mathcal{A}[\alpha] = \int \mathrm{d}^4x \alpha_a(x)\mathcal{A}_a(x) \tag{25}$$

We found that the non-invariance of the functional measure added a term $\mathcal{A}_a(x)$ to the Lagrangian. Since the general result of the Noether Theorem is that the derivative of the Noether Current is $\partial_\mu J^\mu_a = -\delta_a \mathscr{L}$, where $\delta_a \mathscr{L}$ is the variation of the Lagrangian under the group transformations, we can easily see that the anomalous term $\mathcal{A}_a(x)$ modifies the derivative of the Noether current in the following way:

$$\partial_\mu J^\mu_a = -\mathcal{A}_a \tag{26}$$

That means that, in presence of an anomalous symmetry, the quantized version of the Noether Current is not conserved anymore. Using the functional formalism it is possible to calculate the anomalous term for the case of a non-Abelian gauge theory like QCD, in presence of an axial $U(1)_A$ flavor symmetry. The result is:

$$\mathcal{A}(x) = -\frac{g^2}{32\pi^2} N_f \varepsilon_{\mu\nu\rho\sigma} F^{\mu\nu}_a F^{\rho\sigma}_a \tag{27}$$

Where g is the gauge coupling, N_f the number of flavors, $F^a_{\mu\nu}$ is the force tensor of the non-Abelian gauge theory, so QCD in our case. If we substitute the equation above in Eq. (26) we obtain:

$$\partial_\mu J^\mu_5 = -2N_f q(x) \tag{28}$$

That will be an useful result later. Here $q(x)$ is the well known *topological charge density*, i.e.:

$$q(x) = \frac{g^2}{64\pi^2} \varepsilon_{\mu\nu\rho\sigma} F^{\mu\nu}_a F^{\rho\sigma}_a \tag{29}$$

I remind then that from the topological charge density it is possible to define the *topological charge* of a gauge configuration, through the analytic continuation of the density:

$$Q_E = \int \mathrm{d}^4 x_E q_E(x) \tag{30}$$

Where the index E means that we shifted to Euclidean space.
I would like to conclude this section with some theoretical hints that will be important in the discussion of the CME in QGP. The purpose of these last words is not explaining these complex theoretical topics, but just to hint at them, and maybe use them as a starting point for some interesting discussions.
I would like then to briefly remind that, as we said for the easier Chern-Simons model, also the anomalous term (27) can be written as a total derivative. One can think, then, that it could be neglected. T'Hooft, though, demonstrated that with the correct boundary conditions for the gauge field the contribution of this term to the action is different from zero, hence one has to put it in the QCD Lagrangian.
In addition, one can find that such configurations with a non-trivial topology (i.e. with $Q \neq 0$) exist, and they are indeed a minimum of the action. These are the so-called instantonic configurations. There are, then, an infinite number of configurations that are all minima of the action, but they are characterized by a different topological charge. That means that the vacuum of QCD has a complex topological structure.
It can be shown that these instantonic configurations represent a path in the Euclidean space that connects vacuum states with different topological charges. These vacua belongs to different homotopy classes, so they can not be continuously deformed into each other: you need energy to pass from one minima to another. That means that there is a potential barrier that separates them. These instantonic configurations represent then the tunneling between these different vacuum states.
I also would like to stress that such a term as (27) breaks CP-symmetry, as the equivalent one proportional to $\vec{E} \cdot \vec{B}$ in the Chern-Simons model. The problem is that, experimentally, CP symmetry seems to be preserved in QCD. At the same time, because of the previous discussion, from a theoretical point of view this term should be there. That is the so called strong CP problem, that is still one of the greatest mystery of modern physics. This puzzle inspired some really interesting solution like the QCD axion, that could solve both this problem and act as a candidate for the constituent of dark matter.
Again, starting from the anomaly of the chiral symmetry, that is a basic ingredient of the CME in QCD, we can easily jump from experimental high energy physics, to cosmology and dark matter.
This shows again the beauty and the multi-disciplinary aspects of this interesting effect.

3.2 CME in Quark Gluon Plasma

We already hinted that in HIC the CME could be in principle be detected. In order to have a more detailed discussion, let us write down again the main ingredients of the CME:

1. A chirally symmetric system composed of free massless fermions
2. A large background magnetic field

3. Some process that induces a chirality imbalance in the system

From what we know about the QCD Phase Diagram, that we already discussed in this journal, we can say that the ordinary quark matter goes through a phase transition at very high temperature, entering in a new phase of matter called Quark Gluon Plasma (QGP).
Since QCD enjoys the property of asymptotic freedom, at very high temperature the running coupling constant goes to zero. That means that this new phase is actually composed of semi-free quarks. In addition, at the very high temperatures of the QGP, the light quarks can be approximated as nearly massless. At last, following the QCD Phase Diagram we know that while we go through the deconfinement of the quarks, the spontaneously broken chiral symmetry is restored. So, that means that the QGP could be indeed a nice example of a chirally symmetric medium, composed of free massless fermions, that is the requirement (1) of the list above.
In addition, it can be shown that non-central collisions create a very strong magnetic field that is aligned, on the average, perpendicular to the reaction plane. A quantitative derivation of this effect is beyond the aim of this discussion, but qualitatively one can see that if two beam of particles collide with a non zero impact parameter, indeed, we have a non zero total current in the pipe: this current induces a very strong magnetic field. It is possible to theoretically calculate the magnitude of this induced magnetic field, that is $eB \sim 10m_\pi^2$ (or $B \sim 10^{18}G$). Nevertheless, we stress that the magnetic field is indeed very strong at the early moments of the collision, but it rapidly decreases as a function of time. So, in the QGP produced by a non central HIC it is possible also to find the second requirements of our list of ingredients of the CME.
Finally, we need some process to create an imbalance of chirality in the system. We already hinted, though, that the chiral anomaly does the trick for us.
In section (3.1) we said that, in the case of the anomaly of the U$(1)_A$ symmetry, the associated Noether current is Eq. (28):

$$\partial_\mu J_5^\mu = -2N_f q(x)$$

Now, from the equation above we can see an important property of gauge configurations with non-trivial topology: these configurations can, depending on the sign of their winding number, transform left- into right-handed quarks or vice versa. Indeed, the quark number associated with the axial current is what we called Q_5, the difference between the net number of right and left fermions. Then, if we perform the spatial integration of Eq. (28) we find an exact relation for the rate of the chirality change induced by topological configurations ([1]), i.e.:

$$\frac{\mathrm{d}(N_R - N_L)}{\mathrm{d}t} = -\frac{g^2 N_f}{16\pi^2} \int \mathrm{d}^3x F_a^{\mu\nu} \tilde{F}_{\mu\nu}^a \tag{31}$$

I stress again that instantonic configurations have a non-trivial topology, (i.e. $Q \neq 0$), that means that, for these gauge fields, the right hand side of equation above is different from zero. Then, as a result of the axial anomaly, the interactions of the quarks through instantonic configurations create an imbalance of chirality, similarly to the simple case discussed in section (2).
Now it remains the question of what is the rate of production of these instantonic

configurations. As I hinted at the end of section 3.1, they are the solution of the gauge actions that allows the tunneling between different QCD vacua, each of them characterized by a different topological charge, and then belonging to different homotopy classes. If one calculates the tunneling rate then he will find that is proportional to $e^{-2|Q|/\alpha_s}$. That means that at very high temperature this probability is extremely low. Still, instantons are solutions of the euclidean equation of motion, while the creation of the QGP is a real-time phenomenon. In a real time dynamic, at non zero temperature, the temperature itself can act as a catalyst, that allows to jump over the potential barrier between different vacua, instead of tunneling through it. Then these transitions are called *sphalerons* and they can be produced with relatively high probability.
That means that a chiral imbalance can be actually induced in the QGP via the axial anomaly.

Then, also the third ingredients of the CME can in principle be found in the QGP, that becomes a very promising system in which this effect can actually occurs. Also, the CME can be used as a signature to the production of the QGP. Experimentally speaking it is possible to search for the CME looking at the event-by-event fluctuations of the azimuthal charge distribution.

In conclusion, as I mentioned in the motivation section, the CME has a quite broad spectrum of applications, that makes it a very interesting multidisciplinary subject, from condensed matter to cosmology. To have a look of some of these applications see references therein Ref.[4].

4 Conclusion

Summarizing this discussion, we started introducing the CME in a qualitative and intuitive way.
Once we understood the basic idea behind the CME and why it could be interesting, we went through a more quantitative comprehension of the mechanism. We used the Chern-Simons model to introduce the basic properties of the CME like the separation of charges and the induced electromagnetic current. In such a simple system, compared to the QGP in a non-Abelian gauge theory like QCD, we were able to derive all the main features of the effect.
Finally, in the last section, we showed how the CME can occur in the framework of QCD, and how the main ingredients of the CME could be find in QGP, making the CME a promising tool to search for experimental signature of the generation of the QGP in HIC.

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