Scale setting in Lattice QCD

References: arXiv:1401.3270 , arXiv:1111.1710 , arXiv:1407.6387

“Tell me and I forget. Teach me and I remember. Involve me and I learn.”
Benjamin Franklin

Lattice action and its parameters

- Lattice action,

$$Z(T,\mu) = \int [\mathscr{D}U] det[M_{ud}(\mu_f)]^{1/2} det[M_s(\mu_f)]^{1/4} \exp[-S_G]$$

$$M_f = D_{HISQ}(\mu_f) + m_f \ ,$$

m_f, μ_f **are in dimensional units i.e. in lattice units.**

$$m_f \rightarrow am_f \, , \mu_f \rightarrow a\mu_f$$

Temperature?? , $T = \dfrac{1}{N_\tau a}$**,** S_G **contains the bare coupling** g_0

$$a(g_0) = M_p(g_0)/m_p$$

$$m_i^{lat}(g_0) = M_i(g_0)/a_p(g_0)$$

$a = ??$ fm

Pseudo-scalar meson,

$$m_\pi(f = u, d) \sim 135 \text{ MeV}$$

Vector meson,

Experimental measured values!!

$$m_\phi(f = s) \sim 1020 \text{ MeV}$$

What is a good scale(LQCD)??

arXiv:1401.03270

- Calculations can be done with low numerical effort
- Has a good statistical precision
- Has small systematic uncertainties
- Has a weak quark mass dependence.

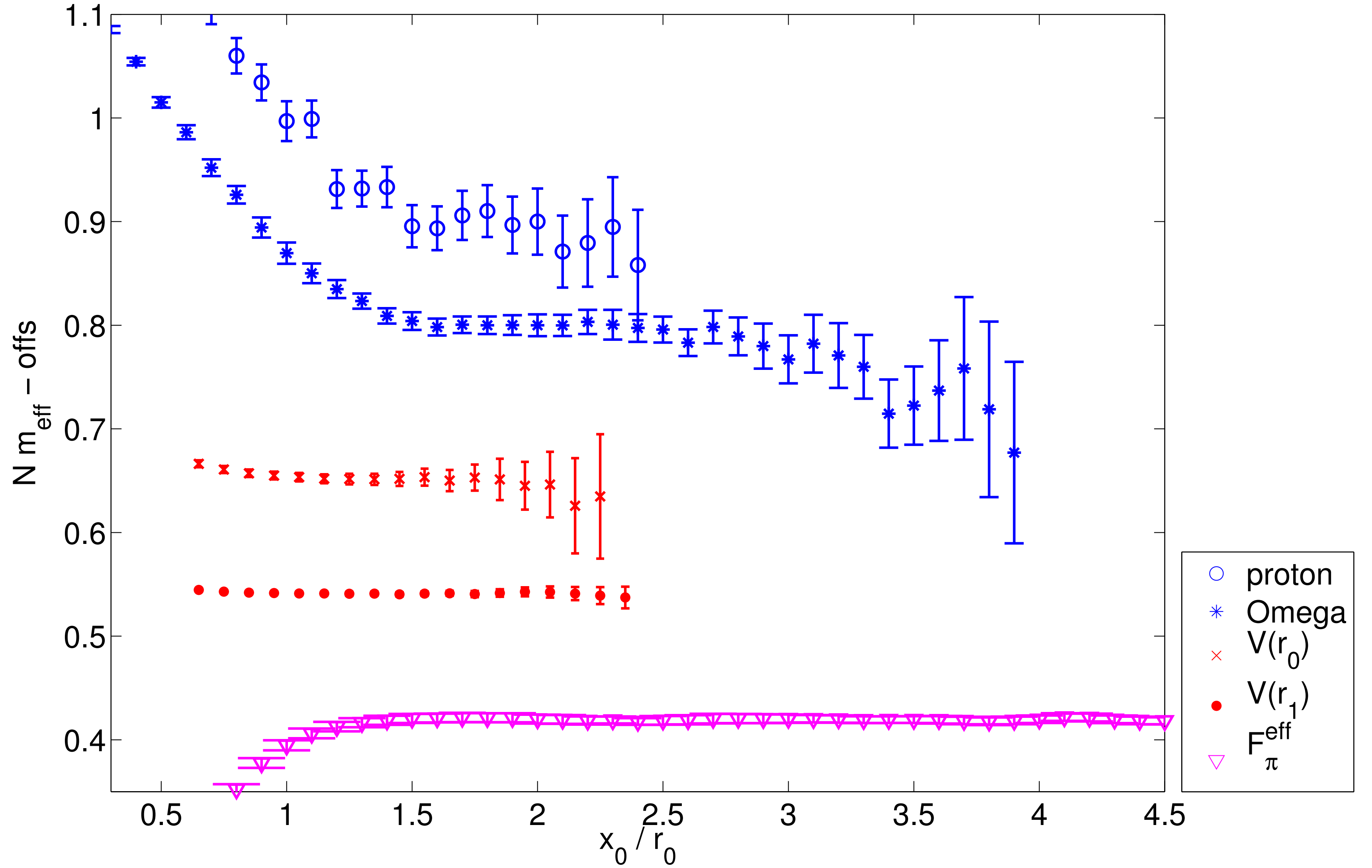


Figure 1: Effective masses for m_p [12], m_Ω [13], $V(\approx r_0)$, $V(\approx r_1)$ [14] and f_π [9] on CLS ensemble N6 (see [9]). All effective "masses" have been scaled such that the errors in the graph reflect directly the errors of the determined scales. They have been shifted vertically.

Scale setting with static quark potential

- Lattice simulation parameters, gauge coupling β, quark masses m_f and $N_\sigma^3 \times N_\tau$.

- Wilson line, $W(r, T)$ is a purely gluonic observable, whose expectation value at large T gives static potential in the lattice.

- Two infinitely heavy quarks at fixed positions,

- $W(r, T) \sim \exp[-V(r)\Delta T + \ldots.]$

we can calculate the static quark–antiquark potential from the large-Δ behavior of the Wilson loop

$$V(r) \sim \lim_{\Delta T \to \infty} \frac{1}{T} \ln \frac{W(r, T)}{W(r, T + \Delta T)}$$

Scale setting with static quark potential

- The potential ansatz, $V(r) = C + \frac{B}{r} + \sigma r - \frac{\lambda}{r}$

Coulomb term **Linear term** **Corrections for lattice artefacts**

Phenomenological value, [quarkonium potential models]

$$-r^2 \frac{\partial V(r)}{\partial r}\Bigg|_{r=r_0} = 1.65 \text{ where, } \mathrm{r}_0 \sim 0.5 \text{ fm}$$

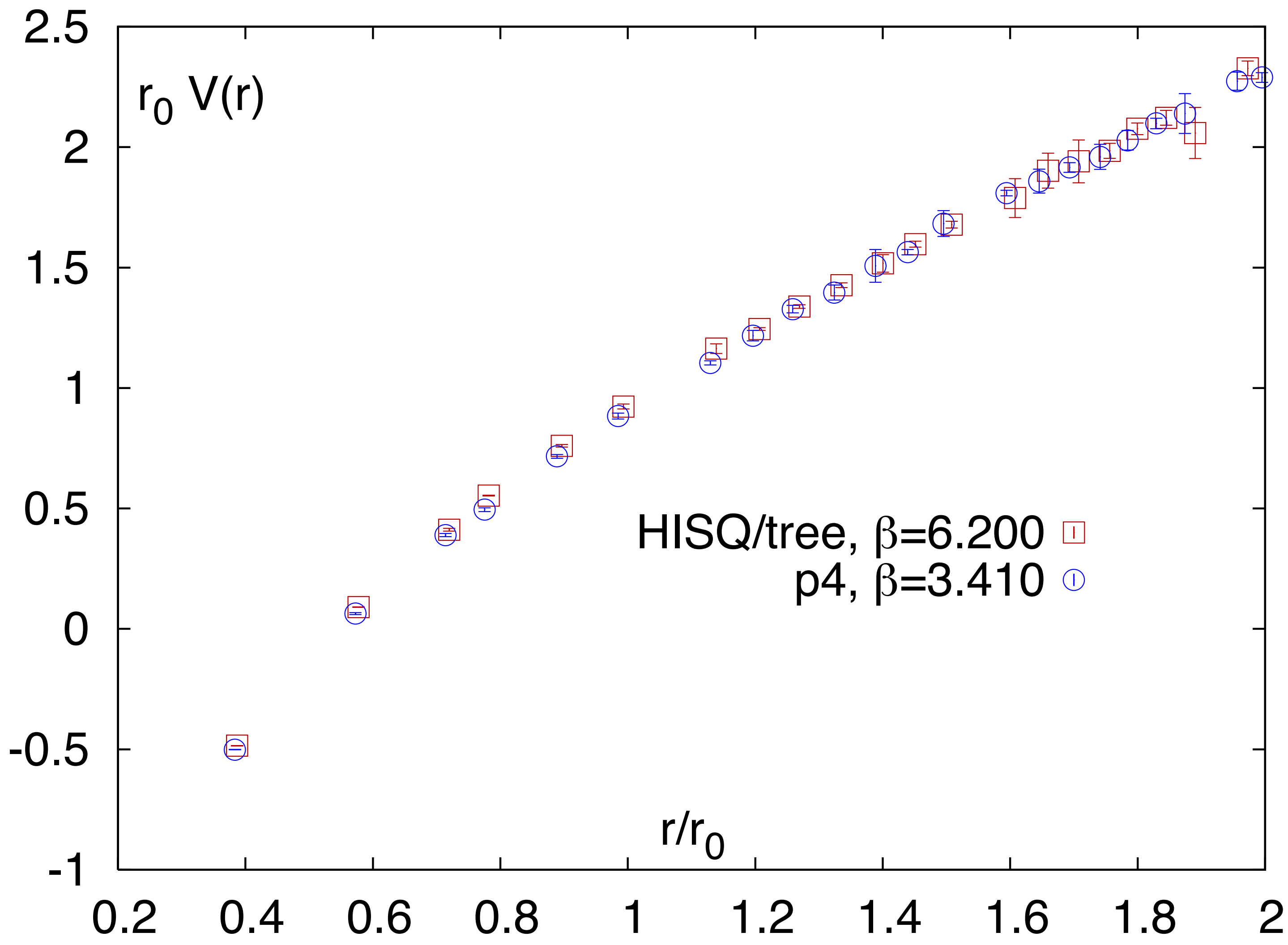
Sommer parameter

$$-r^2 \frac{\partial V(r)}{\partial r}\Bigg|_{r=r_1} = 1.00$$

We need a reference value of r_1 in physical units

$r_1 = 0.3106$ fm **From MILC**

HPQCD, $r_1 = 0.3091$ fm, from bottonium splitting



$\beta = 6.200$

$r_0/a = 2.501,\ r_1/a = 1.722$

$r_1 = 0.3106\ \mathrm{fm}$

$a \Rightarrow 0.3106/1.722\ \mathrm{fm} = 0.184\ \mathrm{fm}$

$r_0 \Rightarrow 0.46\ \mathrm{fm}$

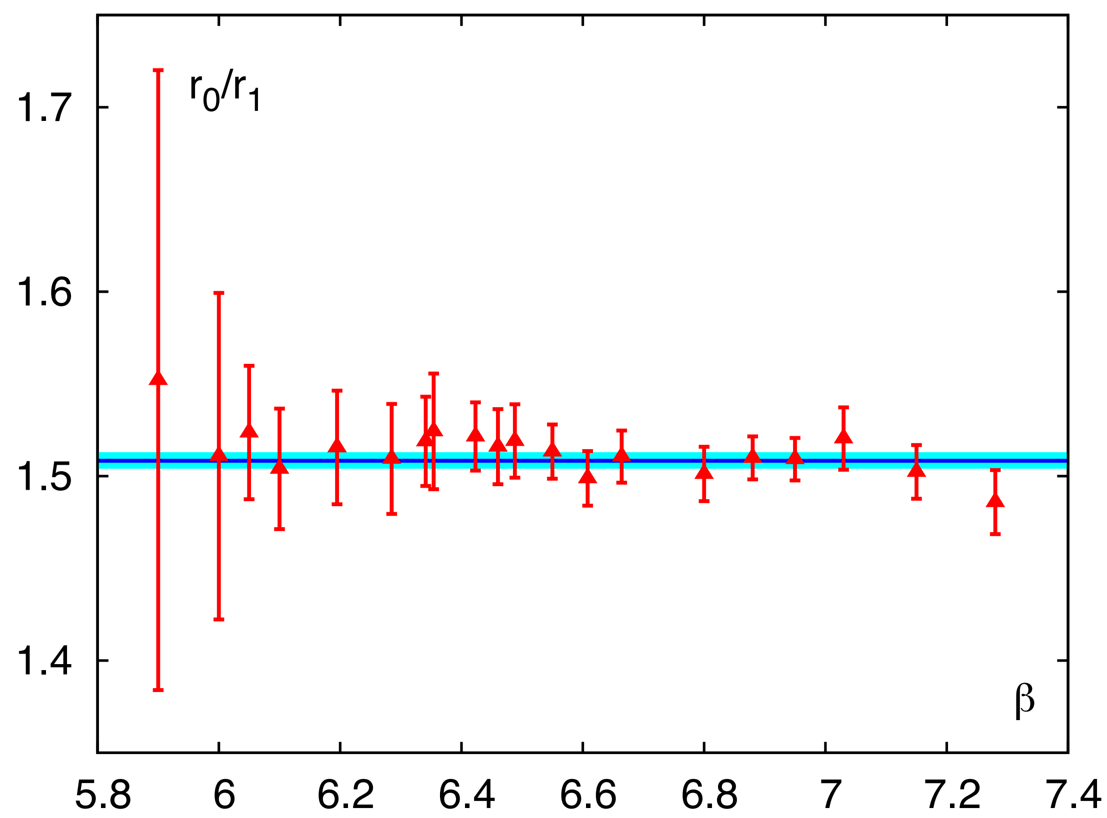
$r_0 = 0.50\ \mathrm{fm}$

$a \Rightarrow 0.50/2.501\ \mathrm{fm} = 0.19\ \mathrm{fm}$

$r_1 \Rightarrow 0.3271\ \mathrm{fm}$

$$r_0 = \sqrt{\frac{1.65 + B - \lambda}{\sigma}} \quad r_1 = \sqrt{\frac{1 + B - \lambda}{\sigma}}$$

β	r_0/a	r_1/a	r_0/r_1
5.900	1.909(11)	1.230(133)	1.552(168)
6.000	2.094(21)	1.386(80)	1.511(89)
6.050	2.194(22)	1.440(31)	1.524(36)
6.100	2.289(21)	1.522(30)	1.504(33)
6.195	2.531(24)	1.670(30)	1.516(31)
6.285	2.750(30)	1.822(30)	1.509(30)
6.341	2.939(11)	1.935(30)	1.519(24)
6.354	2.986(41)	1.959(30)	1.524(31)
6.423	3.189(22)	2.096(21)	1.522(18)
6.460	3.282(32)	2.165(20)	1.516(20)
6.488	3.395(31)	2.235(21)	1.519(20)
6.550	3.585(14)	2.369(21)	1.513(15)
6.608	3.774(20)	2.518(21)	1.499(15)
6.664	3.994(14)	2.644(23)	1.511(14)
6.740	4.293(32)	2.856(11)	1.503(13)



$$\frac{a}{r_1} = \frac{c_0 f(\beta) + c_2 (10/\beta) f^3(\beta)}{1 + d_2 (10/\beta) f^2(\beta)}$$

Renormalization group inspired ansatz,

$$f(\beta) = \left(\frac{10 b_0}{\beta}\right)^{-b_1/(2b_0^2)} \exp(-\beta/(20 b_0))$$

$$\beta = 10/g^2$$

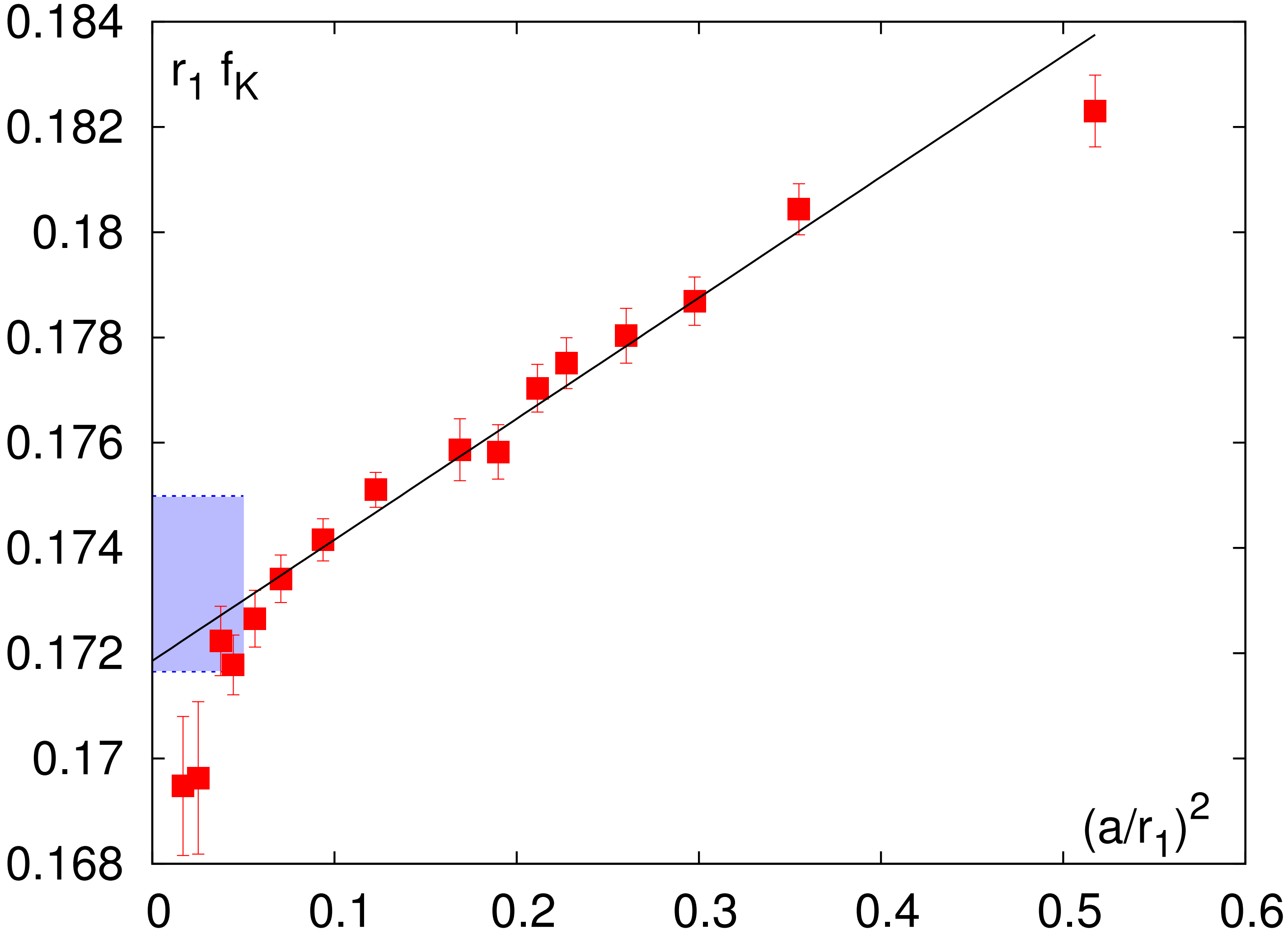
$$c_0 = 43.1 \pm 0.3\ ,$$

$$c_2 = 343236 \pm 41191\ ,$$

$$d_2 = 5514 \pm 755\ .$$

$T_{pc} = 156.5(1.5)$ MeV**,** $T_c \sim 132$ MeV **??**

$$(r_1 f_K)^{cont} = \frac{0.3106\ \textbf{fm} \cdot 156.1/\sqrt{2}\ \textbf{MeV}}{197.3\ \textbf{fm} \cdot \textbf{MeV}} \simeq 0.1738.$$



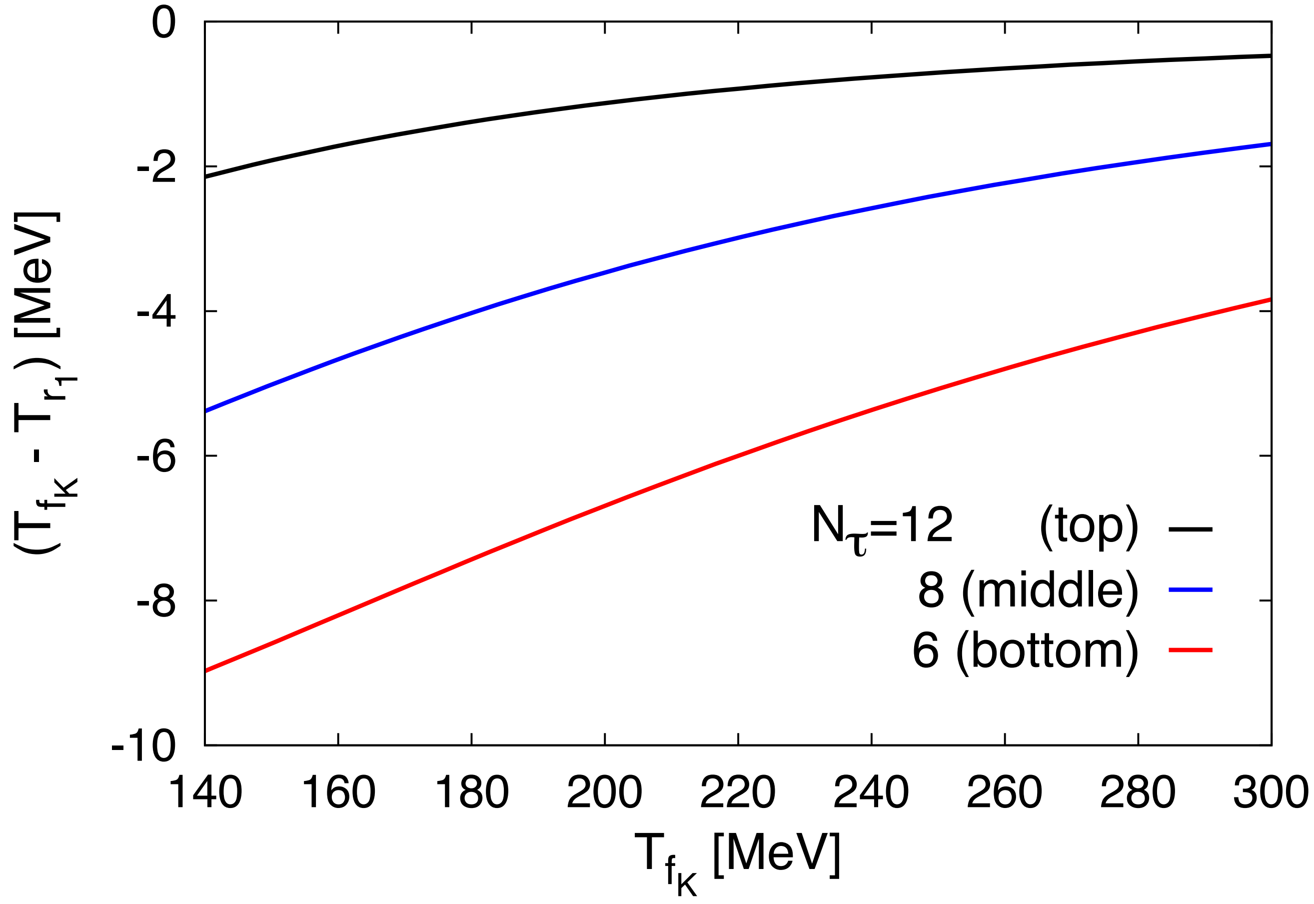
$$f_k r_1 = (f_k r_1)^{cont} + e_k (a/r_1)^2$$

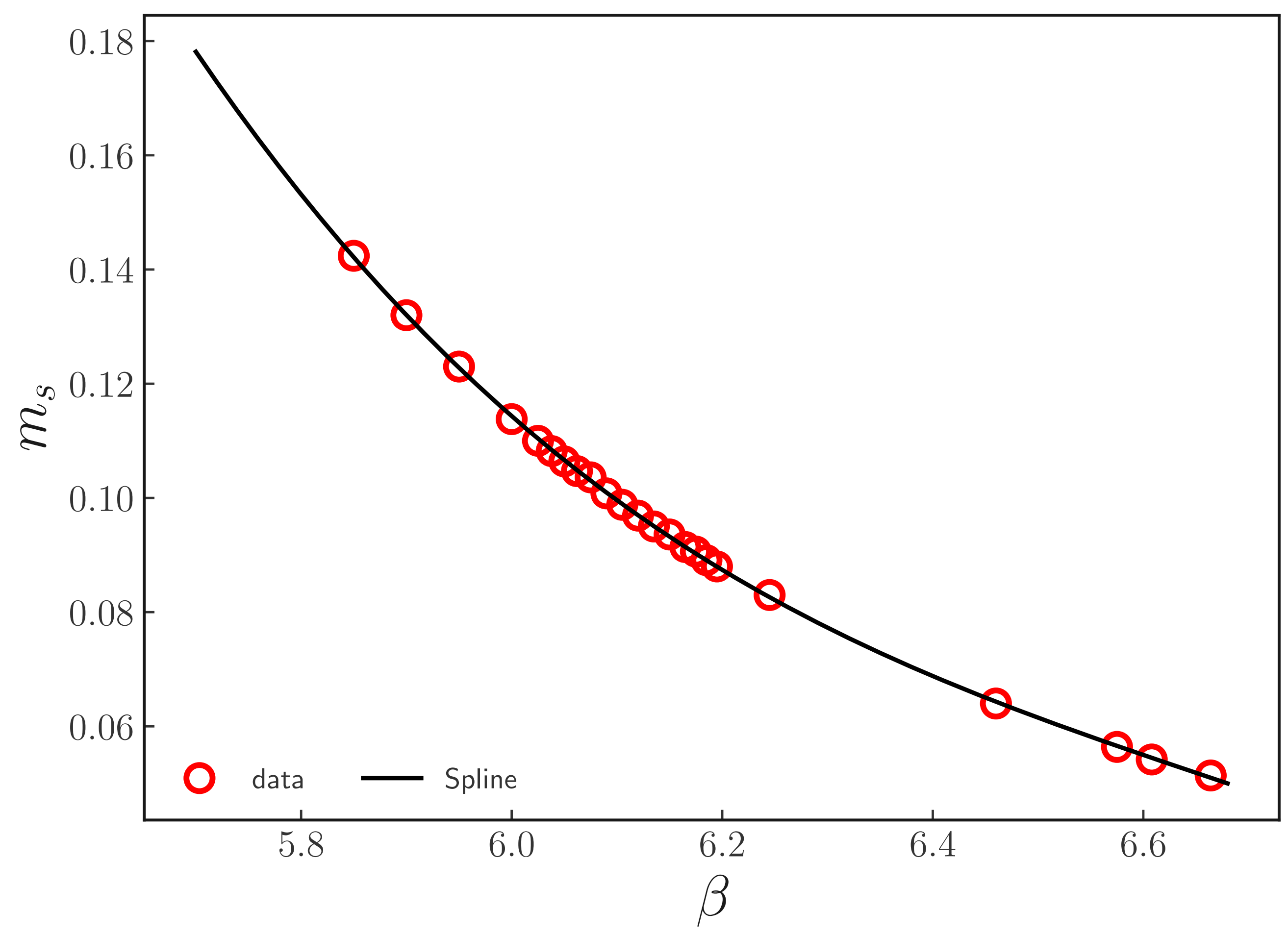
$$(r_1 f_K)^{cont} = 0.17186(24)$$

$$e_K = 0.0230(11)$$

For a given β, which scale will generate a larger temperature??

Setting the Quark masses

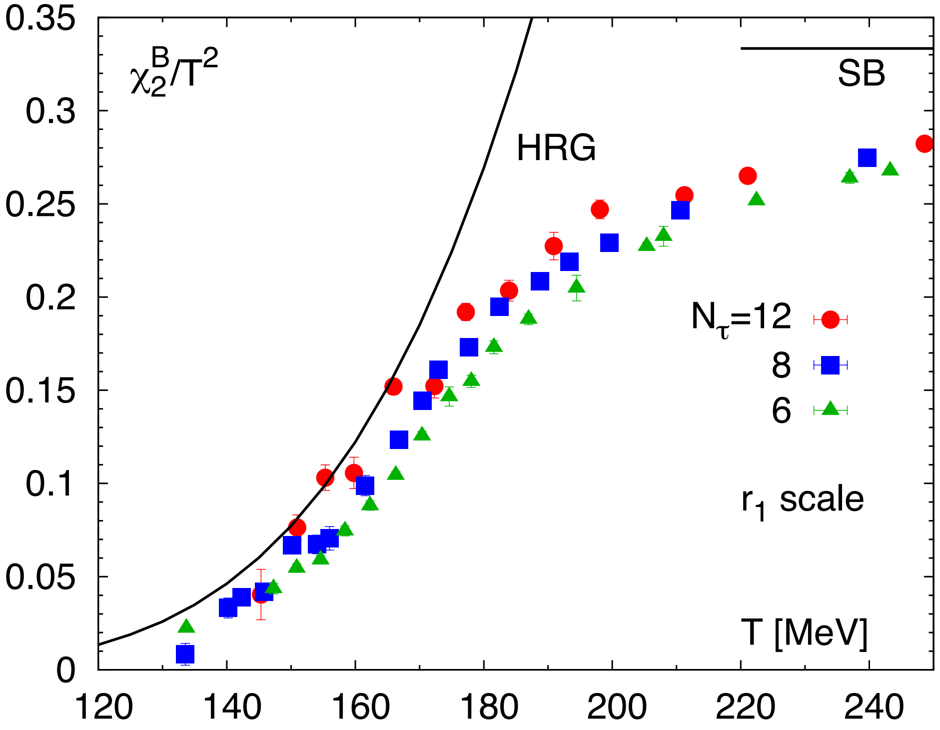


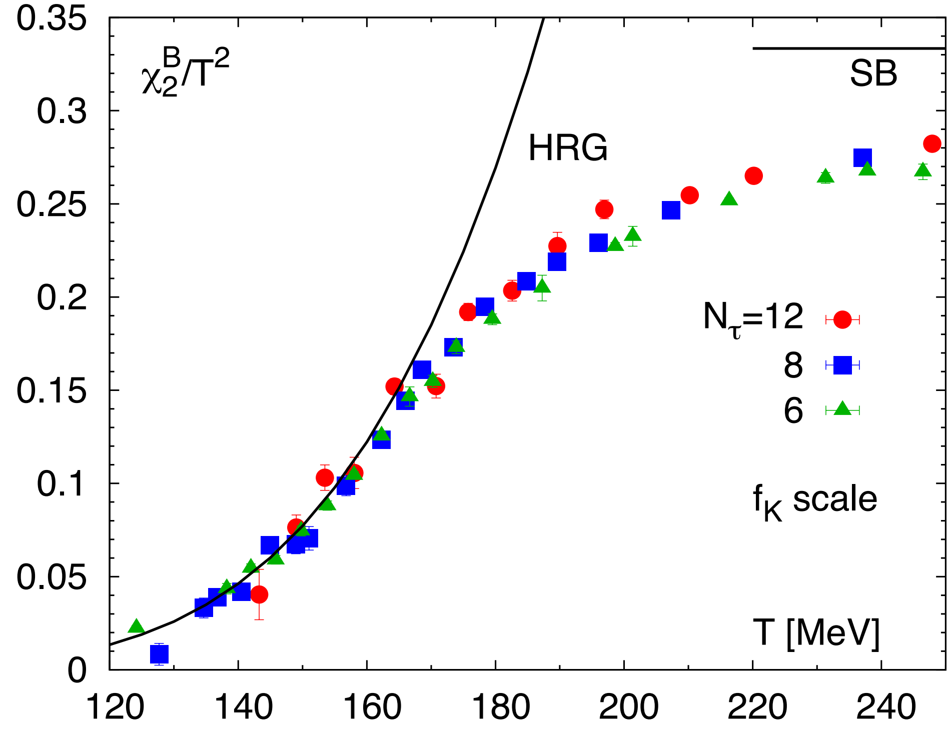


$$M_{\eta_{s\bar{s}}} = \sqrt{2m_K^2 - m_\pi^2} = 686 \text{ MeV}$$

$m_l = m_s/27$**, physical point**

Lattice QCD and HRG





$$\sim e^{-m_H/T}$$

$$P = \sum_H \frac{g}{2\pi^2} T^2 m_H^2 \sum_{n=1}^{\infty} \frac{(-\eta)^{n+1}}{n^2} K_2\left(\frac{n m_H}{T}\right) \exp[n\vec{\mu}\cdot\vec{c}/T]$$

$$\mu = (\mu_B, \mu_Q, \mu_S), c = (B, Q, S)$$

m_H is from PDG while T is from Lattice!! Although, in the continuum limit they should be in same scale!!

Static Potential discussions

Lattice QCD text books : H. Rothe ; J. Smit ; Gattringer and Lang

Cornell type potential,

$$V(r) \sim V_0 - K/r$$

When, r small

This is related to asymptotic freedom!!

$$V(r) \sim V_0' + \sigma r$$

When, r large

This is related to Confinement!!

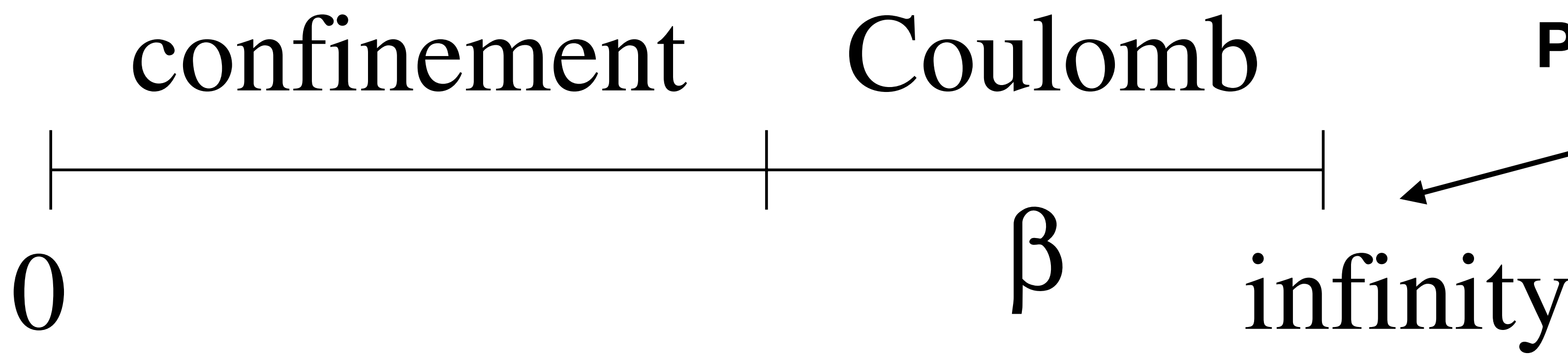
At large distances, measurement of $V(r)$ is effectively measure of σ i.e. the string tension, $\sigma \sim 400\ \mathrm{MeV}^2$.

In lattice gauge theory weak coupling expansion, i.e. expansion in bare coupling g_0 , generates the Coulomb type potential from the PT and a two loop beta function,

$$f(\beta) = \left(\frac{10b_0}{\beta}\right)^{-b_1/(2b_0^2)} \exp(-\beta/(20b_0))$$

In strong coupling expansion $\sim 1/g_0^2$, potential is linearly confining ,

$$V(r) \sim \sigma r$$



Phase diagram of Lattice, $U(1)$

Phase diagram of Lattice, $SU(N)$

$$V(r) \sim V_0 - K/r + \sigma r$$

Based on numerical studies, in $U(1)$ theory it is found that at some critical coupling , $1/g_c$, string tension, $\sigma \sim 0$.

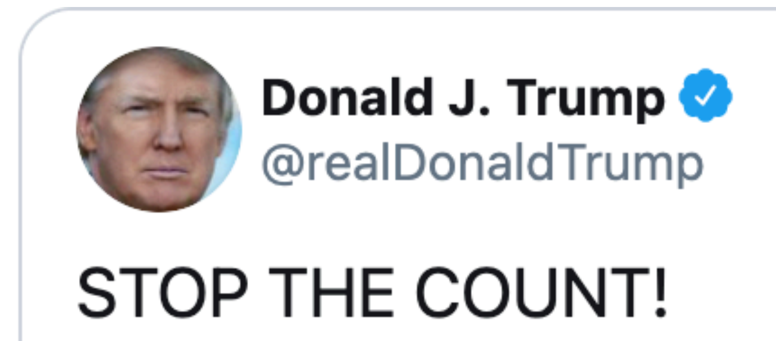
No such thing for $SU(N)$ gauge theories.

Confinement for all energies?? Asymptotic freedom at very high energies ??

Further reading : Erhard Seiler, hep-th/0312015, "The Case against asymptotic freedom"

r_0, r_1 and f_k **We discussed three scales!!**

There are other scales from Wilson flow (t_0, w_0) and hadron masses as well!!



Thank You