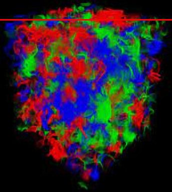
Universität Bielefeld

Magnetic fields in lattice QCD



Dean Valois & Leon Sandbote

December 16, 2022

Department of Physics
Bielefeld University

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Understand QCD thermodynamics in the presence of a background magnetic field.

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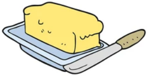

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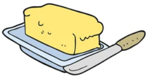
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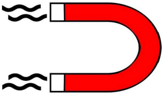
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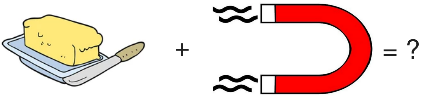

4. Can we go beyond uniform B?

Lattice QCD in one slide

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$$\langle\, \mathcal{O} \,\rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}\bar{\psi}\mathcal{D}\psi\mathcal{D}A \; \mathcal{O} e^{-S[\bar{\psi},\psi,A]}$$

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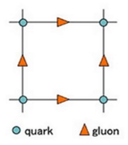

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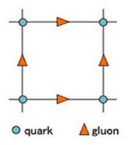

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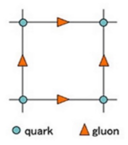

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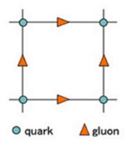

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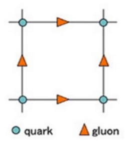

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2. Calculate averages $\langle\, \mathcal{O}\, \rangle = (1/N)\sum_{i=1}^{N} \mathcal{O}_i$

Magnetic fields on the lattice

MAGNETIC FIELD ON THE LATTICE

$$\not{D}\psi(n) = \sum_{\mu=1}^{4} \gamma_\mu \frac{U_\mu(n)\psi(n+\hat{\mu}) - U_\mu(n-\hat{\mu})^\dagger \psi(n-\hat{\mu})}{2a}$$

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- Electromagnetic field: $u_\mu^{\text{em}}(n) = \exp(iaqA_\mu(n))$
- $U_\mu \;\rightarrow\; U_\mu^{\text{gluon}} u_\mu^{\text{em}}$

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Consider a uniform field in the z directions:

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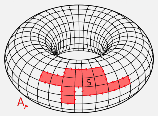
inner area: $\oint A_\mu dx_\mu = SB$



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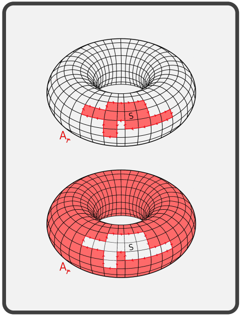

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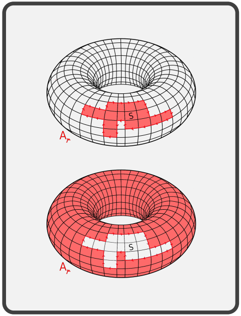

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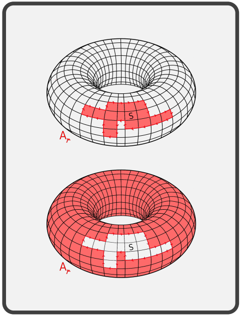

$$\boxed{qB = \frac{2\pi N_b}{L_x L_y}, \quad N_b \in \mathbb{Z}}$$

The magnetic flux is quantized inside a box!

Defining U(1) links

$$\vec{B} = \nabla \times \vec{A}$$

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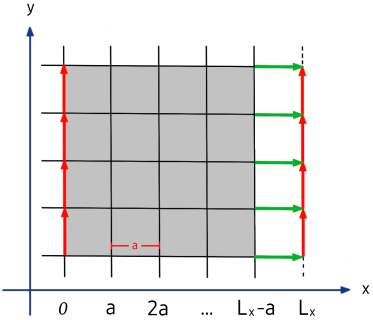

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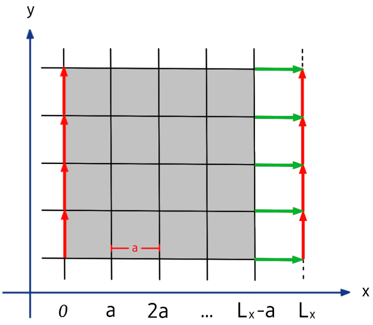

We can perform gauge transformations on the links

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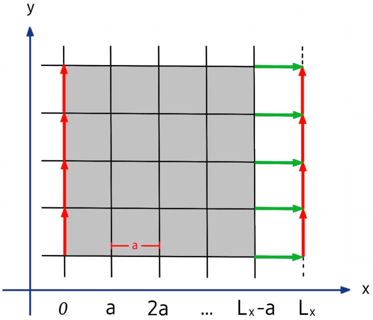


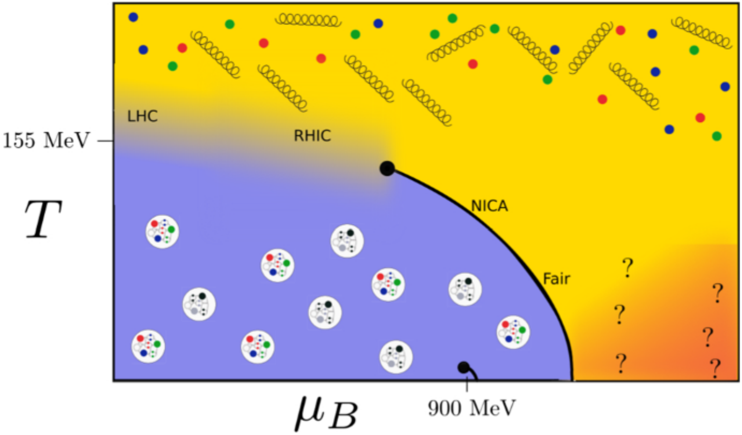
$$u_x = \begin{cases} e^{-iqBL_x y} & \text{if } x = L_x - a \\ 1 & \text{if } x \neq L_x - a \end{cases}$$

$$u_y = e^{iaqBx}$$

$$u_z = 1$$

$$u_t = 1$$

WHAT DO WE KNOW AT $B = 0$?



Approximate order parameters: $\bar{\psi}\psi$, P

QCD Physics at $B \neq 0$

Analytical results for the quark condensate

At $T = 0$ chiral perturbation theory predicts Shushpanov and Smilga 1997

$$\Delta\langle\overline{\psi}\psi\rangle(B) = \langle\overline{\psi}\psi\rangle(0)\frac{|qB|\ln(2)}{16\pi^2 F_\pi^2}$$

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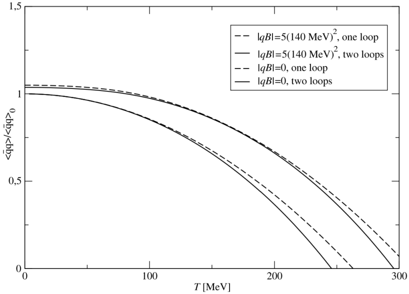

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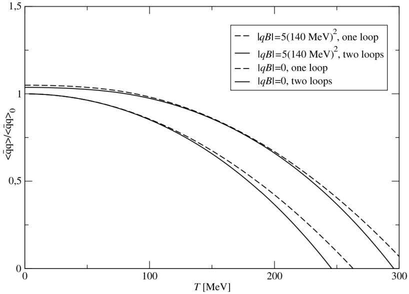
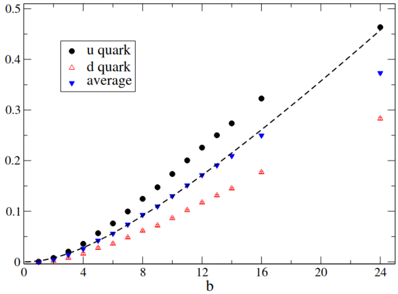


Andersen 2012 condensate dependent on T for $qB = 0$ and $qB = 0.1\text{GeV}^2$

- increases linear with qB
- decreases with T
- chiral perturbation theory will break down at high temperature

Impact of B in full QCD

IMPACT OF B IN FULL QCD



Magnetic catalysis D'Elia and Negro 2011

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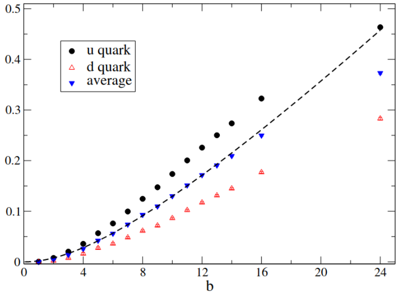

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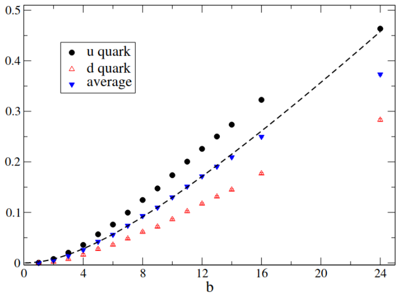

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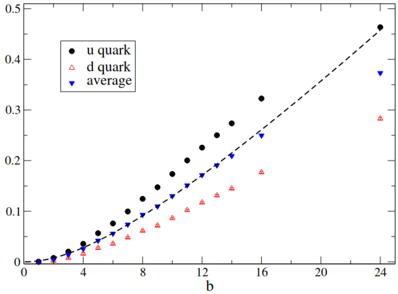

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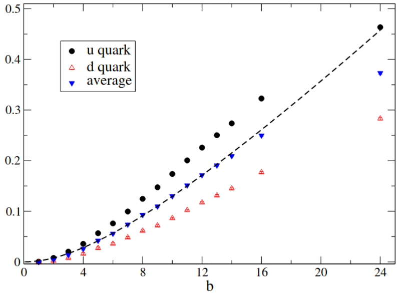

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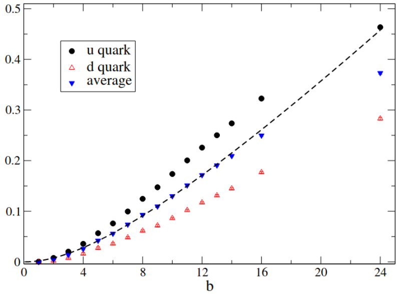

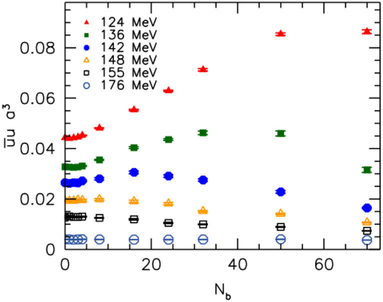

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Inverse catalysis G. Bali et al. 2012

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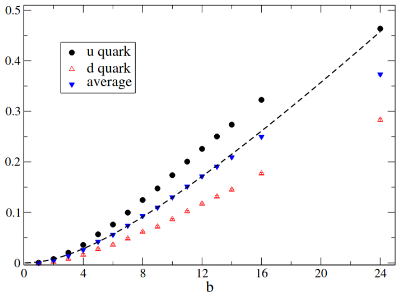

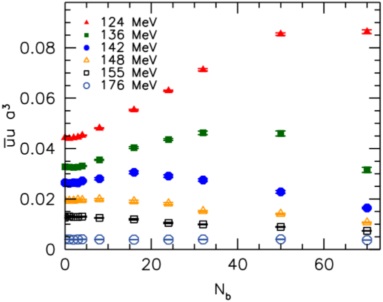

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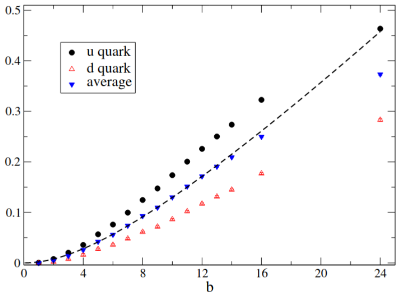

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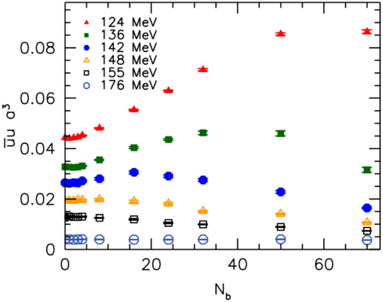

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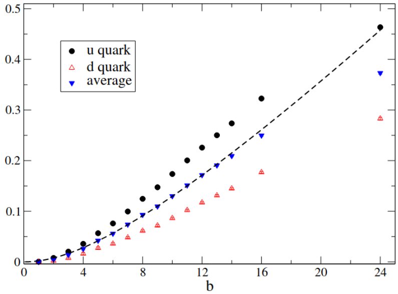

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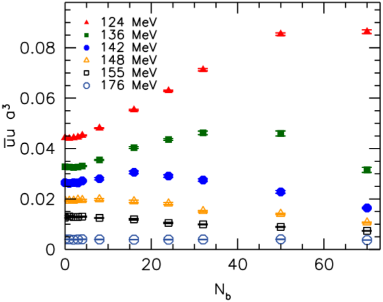

Inverse catalysis

G. Bali et al. 2012

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- No IMC in quenched QCD $(\det(\not{D}+m)=1)$

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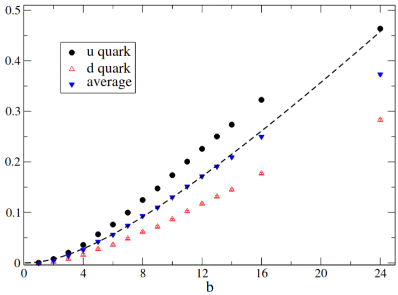

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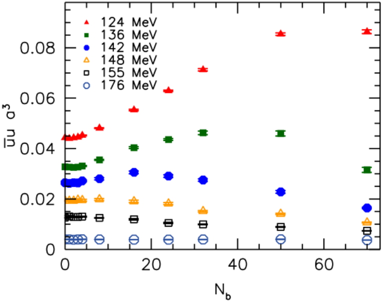

Inverse catalysis G. Bali et al. 2012

- Decreases $\bar{\psi}\psi$
- m_π-dependent
- No IMC in quenched QCD $(\det(\not{D} + m) = 1)$
- Only effective around T_c

QCD TRANSITION AT $B \neq 0$

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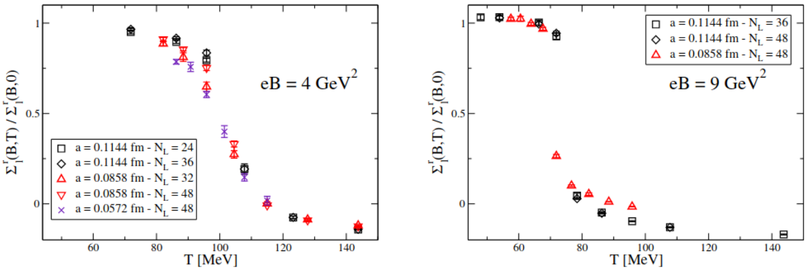

Figure 1: Renormalized chiral condensate at $eB = 4$ GeV^2 (left) and $eB = 9$ GeV^2 (right) D'Elia et al. 2022

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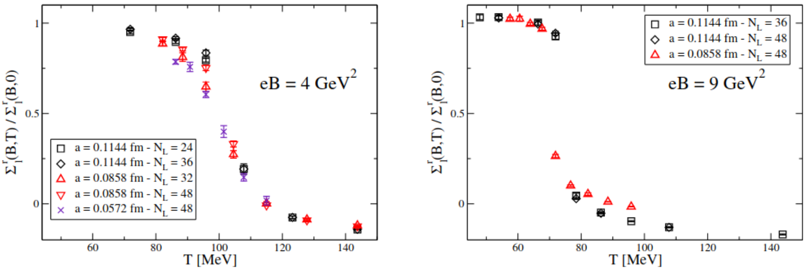

Figure 1: Renormalized chiral condensate at $eB = 4$ GeV^2 (left) and $eB = 9$ GeV^2 (right) D'Elia et al. 2022

There must be a critical point somewhere in the range
4 GeV$^2 < eB < 9$ GeV^2.

Beyond uniform B

WHY NON-UNIFORM B?

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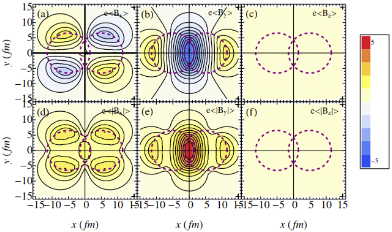
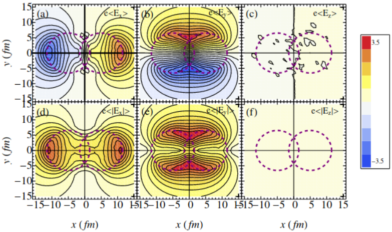
Peripheral heavy-ion collisions





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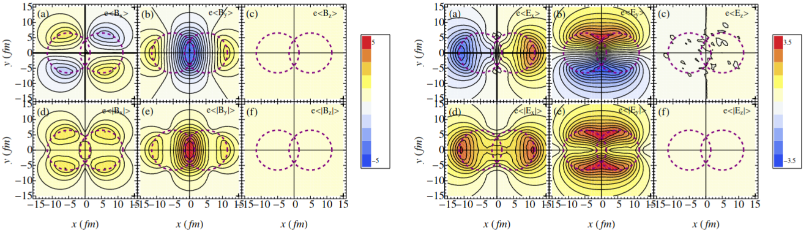

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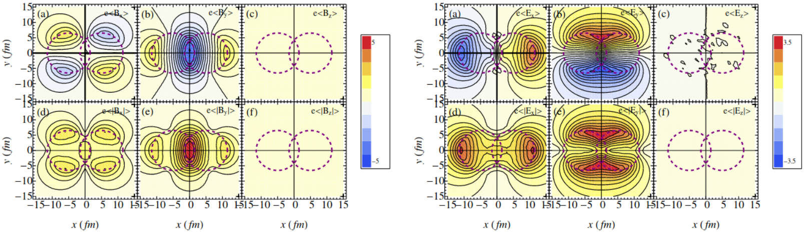

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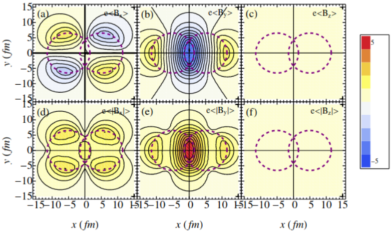

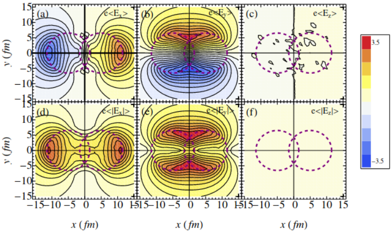

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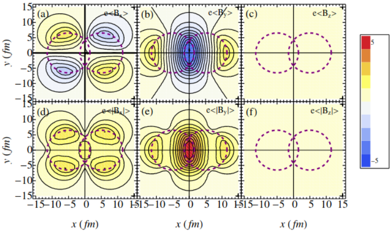

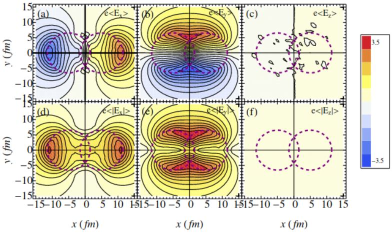

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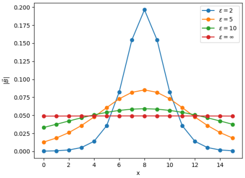

$$\vec{B}(x) = B \cosh\left(\frac{x - L_x/2}{\epsilon}\right)^{-2} \hat{z}$$

Cao 2018

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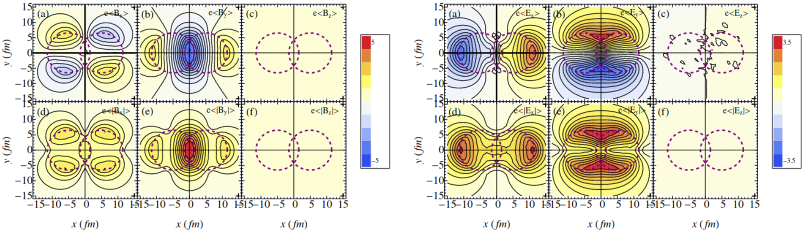

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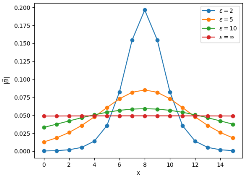

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Cao 2018

We need to change the background (links).

U(1) LINKS

- Flux quantization

$$qB = \frac{\pi N_b}{L_y \epsilon \tanh\left(\frac{L_x}{2\epsilon}\right)}, \quad N_b \in \mathbb{Z}$$

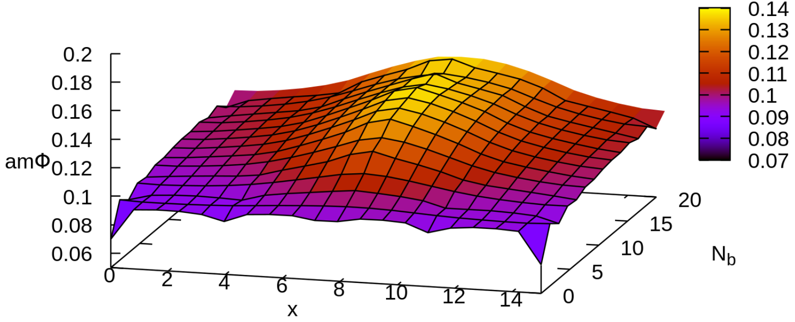
- Periodic U(1) links

$$u_y(n) = \exp\left(i\pi \frac{N_b}{N_y} \frac{\tanh\left(a\frac{n_x - N_x/2}{\epsilon}\right)}{\tanh\left(a\frac{N_x}{2\epsilon}\right)} \right) \quad 0 \leq n_x \leq N_x - 1$$

$$u_x(n) = \exp\left(-i2\pi \frac{N_b}{N_y} n_y \delta_{n_x, N_x - 1} \right) \quad u_z(n) = u_t(n) = 1$$

The quark condensate

FREE QUARK CONDENSATE



Quark condensate for $\epsilon = 2.0$ and $am = 0.2$ dependent on x and N_b.

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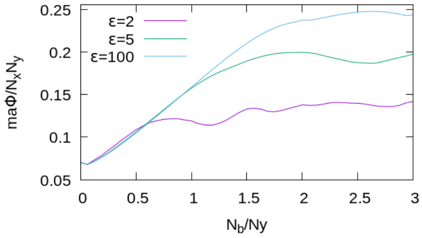

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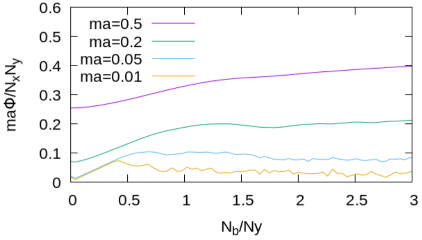

Condensate for $\epsilon = 5a$ dependent on N_b.

- Increases linear and quadratic for $N_b \approx 0$
- Independent of ϵ for $N_b \approx 0$
- Calculation breaks down for high N_b

FULL QCD - CHIRAL CONDENSATE

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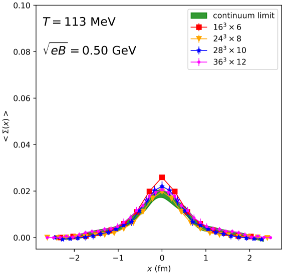

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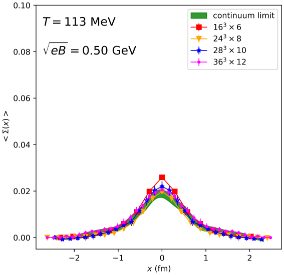

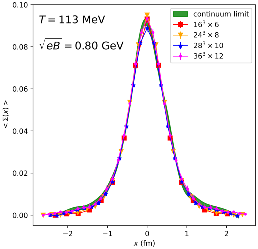

$T < T_c$

Full QCD - chiral condensate

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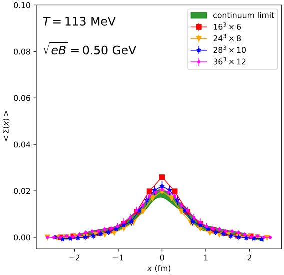

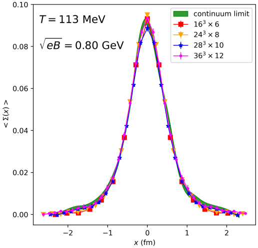

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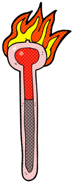

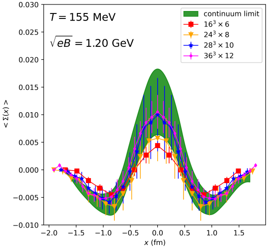

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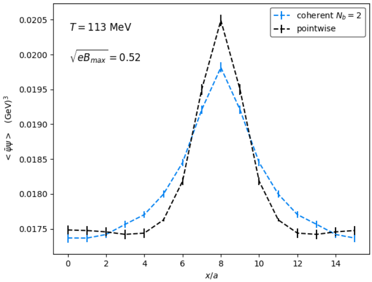

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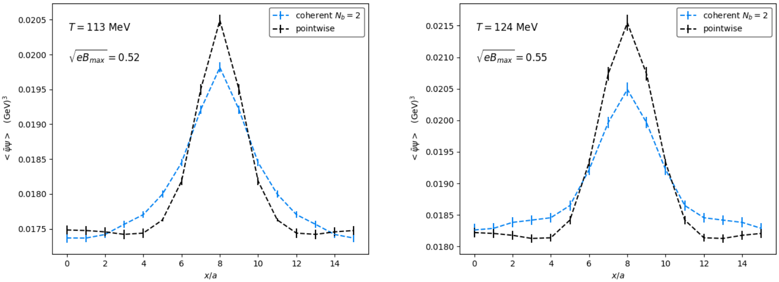

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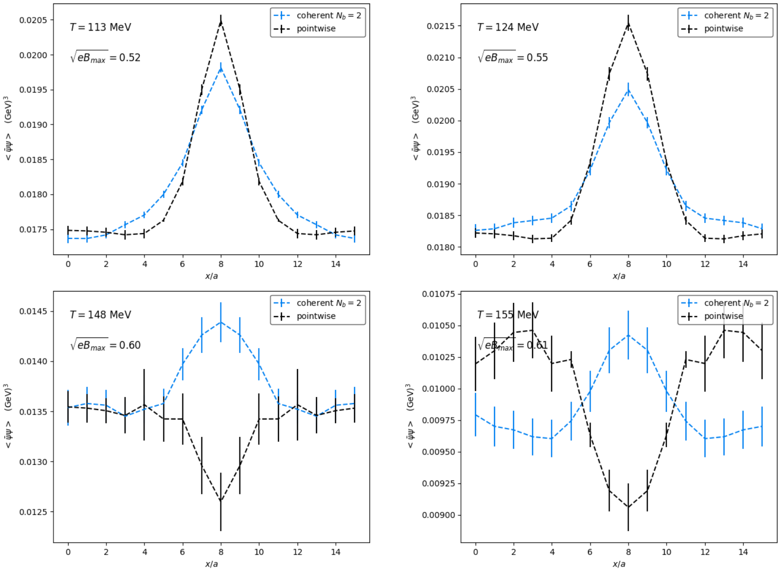

Full QCD - Polyakov loop

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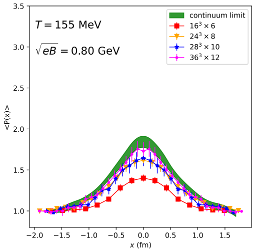

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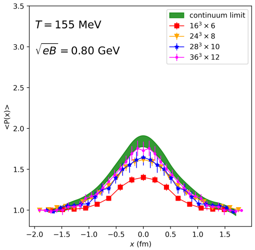

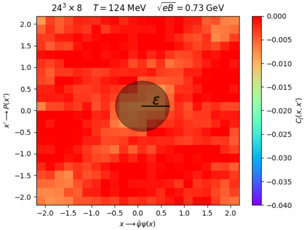

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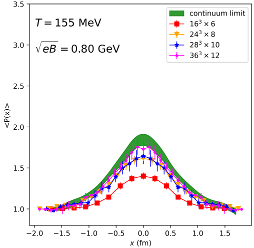

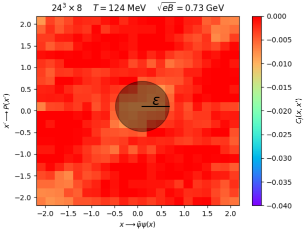

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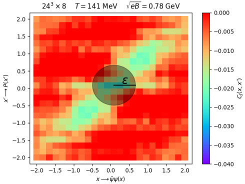

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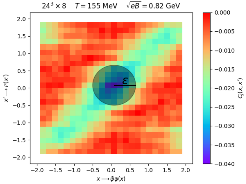

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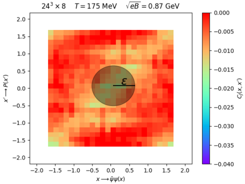

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FULL QCD - ELECTRIC CURRENT

Full QCD - Electric current

Mawell's equations in a medium:

$$\nabla \times \vec{B} = \mu_0 \vec{J}$$

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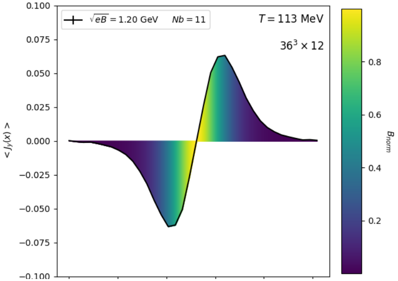

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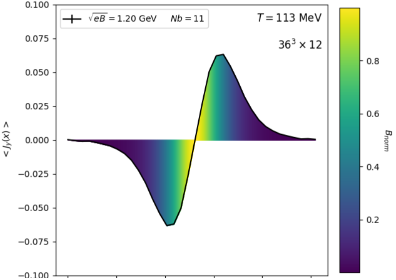

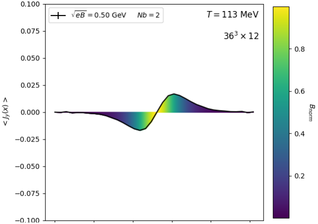

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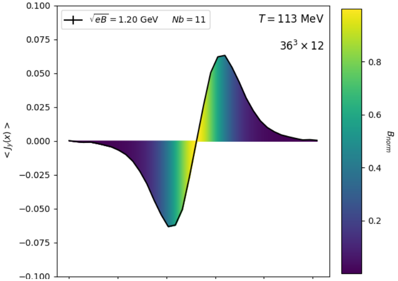

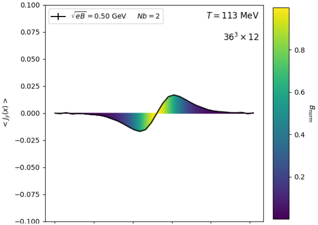

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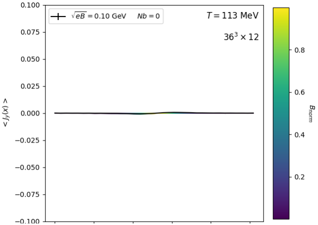

$\sqrt{eB} = 0.1$ GeV (RHIC-like)

Summary

What have we learned?

- What happens to B in a periodic box?

- Impact of B?

- What happens to the nature of the QCD transition?

- Beyond uniform B?

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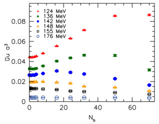

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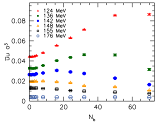

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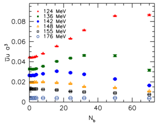

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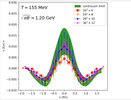

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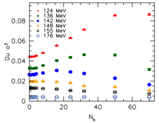

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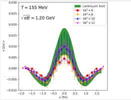

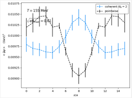

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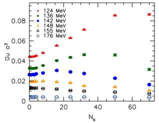
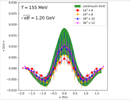
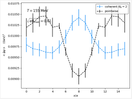
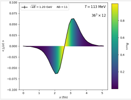


- What happens to the nature of the QCD transition? crossover $\rightarrow$ 1$^{\text{st}}$ order at $4\ \text{GeV}^2 < eB < 9\ \text{GeV}^2$

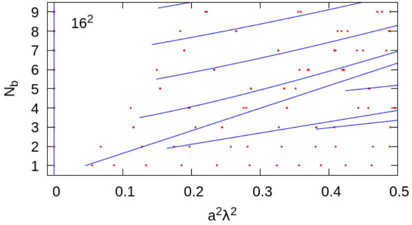
- Beyond uniform B? YES!

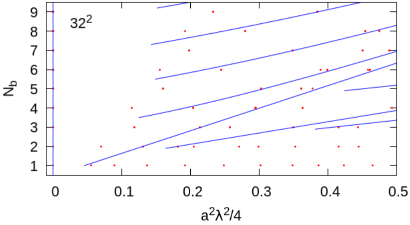






Backup

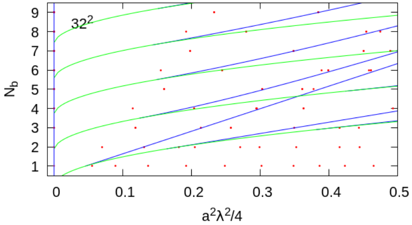




Spectrum in red of the Dirac operator for $\epsilon = 5a$ on a 16^2 lattice and 32^2 and exact eigenvalues in blue by Cangemi, D'Hoker, and Dunne n.d.

- $\lambda = 0$ shows a $2N_b$-fold degeneracy
- $\lambda \neq 0$ shows a 2-fold degeneracy
- For $\epsilon \to \infty$, N_b $\lambda \neq 0$ reform $2N_b$-folded degeneracy of Hofstadter's Butterfly
- Why are so many red dots lonely?

Backup



Spectrum of the Dirac operator with green lines representing the starting point of a continuous spectrum

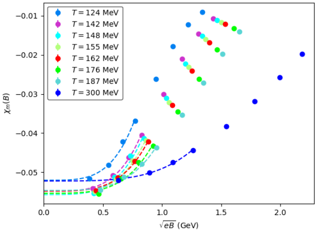
What did Cangemi, D'Hoker, and Dunne n.d. calculate?

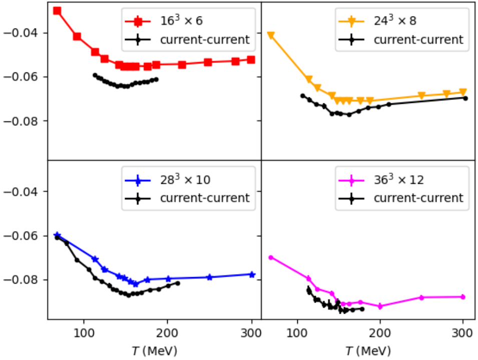
- $\not{D}^2 \to -\partial_x^2 + V_k(x)$
- Calculated $-\partial_x^2 + V_k(x)$'s spectrum
- Found bound state solution's eigenvalues

- Lonely red dots belong to eigenfunctions with $\lim_{x\to\infty} \psi(x,y) \neq 0$
- Smooth transition between discrete and continuous spectrum

(BARE) MAGNETIC SUSCEPTIBILITY

- Linear response term: $\vec{M} \approx \chi_m \vec{H}$
- $\frac{\chi_m}{1+\chi_m} \nabla \times \vec{B} = \vec{J}_m$

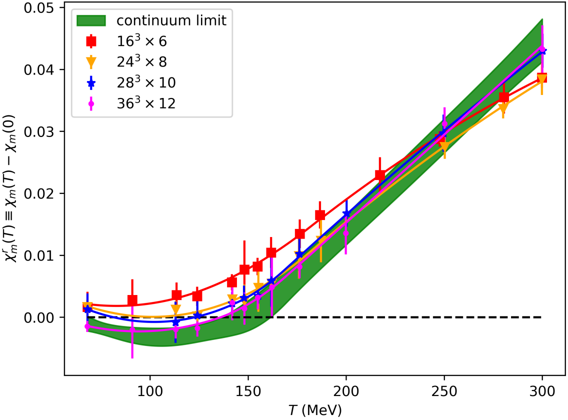




The susceptibility contains an additive divergence $\chi_m \sim \log(a)$

(Renormalized) magnetic susceptibility

The divergence is independent of T: $\chi_m^r(T) \equiv \chi_m(T) - \chi_m(0)$



- $\chi_m^r < 0$: diamagnetism
- $\chi_m^r > 0$: paramagnetism

Material	χ_m
Al	$+2.2\times10^{-5}$
Glass	-1.13×10^{-5}

Great agreement with the current-current method! 🔗 Gunnar S Bali, Endrődi, and Piemonte 2020

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