Anomalous transport phenomena on the lattice

Eduardo Garnacho Velasco

egarnacho@physik.uni-bielefeld.de

In collaboration with:
B. B. Brandt, F. Cuteri, G. Endrődi, G. Markó

Journal Club WS Bielefeld, 13-01-2023

Introduction

- Quantum anomalies + $\begin{matrix}\text{EM fields}\\ \text{Vorticity}\end{matrix}$ $\rightarrow$ non-dissipative transport effects:

Anomalous transport phenomena

Introduction

- Quantum anomalies $+ \begin{matrix} \text{EM fields} \\ \text{Vorticity} \end{matrix} \rightarrow$ non-dissipative transport effects:

  Anomalous transport phenomena

- Examples:
  - Chiral Magnetic Effect (CME)
  - Chiral Separation Effect (CSE)
  - Chiral Electric Separation Effect (CESE)
  - Chiral Vortical Effect (CVE)
  - ...

  For a review see Kharzeev, Liao, Voloshin, Wang '16

Introduction

- Quantum anomalies + $\begin{matrix}\text{EM fields}\\ \text{Vorticity}\end{matrix}$ → non-dissipative transport effects:

  Anomalous transport phenomena

- Examples:
  - Chiral Magnetic Effect (CME)
  - Chiral Separation Effect (CSE)
  - Chiral Electric Separation Effect (CESE)
  - Chiral Vortical Effect (CVE)
  - . . .

  For a review see Kharzeev, Liao, Voloshin, Wang '16

- Event-by-event CP-violation → non-trivial topology of QCD vacuum

Quantum anomalies

- Classical symmetry broken at the quantum level → **Anomaly**
  - Global symmetries
  - Gauge symmetries

Quantum anomalies

- Classical symmetry broken at the quantum level $\rightarrow$ **Anomaly**
  - Global symmetries
  - Gauge symmetries

- Formally : (Classical) action symmetric
  PI measure non-symmetric $\rightarrow$ Partition function non-symmetric

Quantum anomalies

- Classical symmetry broken at the quantum level $\rightarrow$ **Anomaly**
  - Global symmetries
  - Gauge symmetries

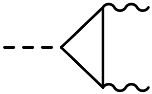
- Formally : (Classical) action symmetric
  PI measure non-symmetric $\rightarrow$ Partition function non-symmetric

- Responsible for physical effects:
  - $\mathrm{U_B}(1)$ anomaly $\rightarrow$ Baryogenesis

Quantum anomalies

- Classical symmetry broken at the quantum level → **Anomaly**
  - Global symmetries
  - Gauge symmetries
- Formally : (Classical) action symmetric
  PI measure non-symmetric → Partition function non-symmetric
- Responsible for physical effects:
  - $\mathrm{U_B}(1)$ anomaly → Baryogenesis
  - $\mathrm{U_A}(1)$ anomaly → $\pi^0 \to \gamma\gamma$, heavy η',...

Adler '69 Bell, Jackiw '69



$\mathsf{U_A}(1)$ anomaly

- Chiral symmetry $\mathsf{U_A}(1)$ (QED or QCD action)

$$\psi \to e^{i\alpha\gamma_5}\psi$$

with Noether current $J_5^\mu = \bar{\psi}\gamma^\mu\gamma^5\psi$.

$\mathrm{U_A}(1)$ anomaly

- Chiral symmetry $\mathrm{U_A}(1)$ (QED or QCD action)

$$\psi \to e^{i\alpha\gamma_5}\psi$$

with Noether current $J_5^\mu = \bar{\psi}\gamma^\mu\gamma^5\psi$.

- Conserved current for massless fermions

$$\partial_\mu J_5^\mu = 2im\bar{\psi}\gamma^5\psi$$

- But in the Path Integral

$$\mathcal{Z} = \int \mathcal{D}\psi\mathcal{D}\bar{\psi}\mathcal{D}A\, e^{iS}$$

the measure $\mathcal{D}\psi\mathcal{D}\bar{\psi}$ is not invariant! Fujikawa '79

$\mathrm{U_A}(1)$ anomaly

- Chiral symmetry $\mathrm{U_A}(1)$ (QED or QCD action)

$$\psi \to e^{i\alpha\gamma_5}\psi$$

with Noether current $J_5^\mu = \bar{\psi}\gamma^\mu\gamma^5\psi$.

- Conserved current for massless fermions

$$\partial_\mu J_5^\mu = 2im\bar{\psi}\gamma^5\psi$$

- But in the Path Integral

$$\mathcal{Z} = \int \mathcal{D}\psi\mathcal{D}\bar{\psi}\mathcal{D}A\, e^{iS}$$

the measure $\mathcal{D}\psi\mathcal{D}\bar{\psi}$ is not invariant! Fujikawa '79

- Current is not conserved even for $m = 0$

$$\text{QED}: \quad \partial_\mu J_5^\mu = 2im\bar{\psi}\gamma^5\psi + \frac{e^2}{16\pi^2}F_{\mu\nu}\tilde{F}^{\mu\nu}\left(= Q_{\text{top}}^{\mathrm{U}(1)}\right)$$

$\mathsf{U_A}(1)$ anomaly

- Chiral symmetry $\mathsf{U_A}(1)$ (QED or QCD action)

$$\psi \to e^{i\alpha\gamma_5}\psi$$

with Noether current $J_5^\mu = \bar{\psi}\gamma^\mu\gamma^5\psi$.

- Conserved current for massless fermions

$$\partial_\mu J_5^\mu = 2im\bar{\psi}\gamma^5\psi$$

- But in the Path Integral

$$\mathcal{Z} = \int \mathcal{D}\psi\mathcal{D}\bar{\psi}\mathcal{D}A\, e^{iS}$$

the measure $\mathcal{D}\psi\mathcal{D}\bar{\psi}$ is not invariant! Fujikawa '79

- Current is not conserved even for $m = 0$

$$\text{QED}: \quad \partial_\mu J_5^\mu = 2im\bar{\psi}\gamma^5\psi + \frac{e^2}{16\pi^2}F_{\mu\nu}\tilde{F}^{\mu\nu}\left(= Q_{\mathsf{top}}^{\mathsf{U}(1)}\right)$$

$$\text{QCD}: \quad \partial_\mu J_5^\mu = 2im\bar{\psi}\gamma^5\psi + \frac{e^2}{32\pi^2}G^a_{\mu\nu}\tilde{G}_a^{\mu\nu}\left(\equiv Q_{\mathsf{top}}^{\mathsf{SU}(3)}\right)$$

Anomalous transport

- Macroscopic manifestations of quantum anomalies
- $U_A(1)$ anomaly origin $\sim Q_{top} \propto G^a_{\mu\nu}\tilde{G}^{\mu\nu}_a \rightarrow$ CP-odd phenomena

Anomalous transport

- Macroscopic manifestations of quantum anomalies
- $U_A(1)$ anomaly origin $\sim Q_{top} \propto G^a_{\mu\nu}\tilde{G}^{\mu\nu}_a \rightarrow$ CP-odd phenomena
- Event-by-event CP-violation:

$$\langle j_{\text{anom.}} \rangle = 0$$
$$\langle j_{\text{anom.}} \rangle_{\text{event}} \neq 0$$

Anomalous transport

- Macroscopic manifestations of quantum anomalies
- $U_A(1)$ anomaly origin $\sim Q_{top} \propto G^a_{\mu\nu}\tilde{G}^{\mu\nu}_a \rightarrow$ CP-odd phenomena
- Event-by-event CP-violation:

$$\langle j_{\text{anom.}} \rangle = 0$$
$$\langle j_{\text{anom.}} \rangle_{\text{event}} \neq 0$$

- Experimental detection:

Anomalous transport

- Macroscopic manifestations of quantum anomalies
- $U_A(1)$ anomaly origin $\sim Q_{top} \propto G^a_{\mu\nu}\tilde{G}^{\mu\nu}_a \to$ CP-odd phenomena
- Event-by-event CP-violation:

$$\langle j_{\text{anom.}} \rangle = 0$$
$$\langle j_{\text{anom.}} \rangle_{\text{event}} \neq 0$$

- Experimental detection:
  - Condensed matter systems Li, Kharzeev, Zhan et al '14

Anomalous transport

- Macroscopic manifestations of quantum anomalies
- $U_A(1)$ anomaly origin $\sim Q_{top} \propto G^a_{\mu\nu}\tilde{G}^{\mu\nu}_a \rightarrow$ CP-odd phenomena
- Event-by-event CP-violation:

$$\langle j_{\text{anom.}} \rangle = 0$$
$$\langle j_{\text{anom.}} \rangle_{\text{event}} \neq 0$$

- Experimental detection:
  - Condensed matter systems 🔗 Li, Kharzeev, Zhan et al '14
  - Heavy-ion collisions 🔗 STAR collaboration '21

Anomalous transport

- Macroscopic manifestations of quantum anomalies
- $U_A(1)$ anomaly origin $\sim Q_{top} \propto G^a_{\mu\nu}\tilde{G}^{\mu\nu}_a \rightarrow$ CP-odd phenomena
- Event-by-event CP-violation:

$$\langle j_{\text{anom.}} \rangle = 0$$
$$\langle j_{\text{anom.}} \rangle_{\text{event}} \neq 0$$

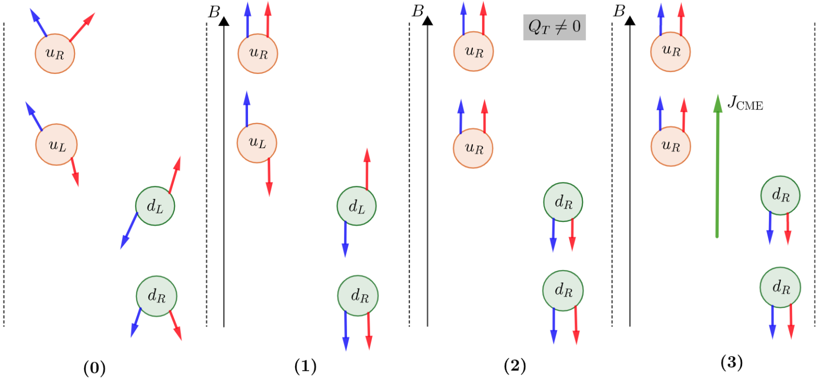
- Experimental detection:
  - Condensed matter systems Li, Kharzeev, Zhan et al '14
  - Heavy-ion collisions STAR collaboration '21

    Latest signals suggest suppression of CME:
    Can we understand this from theory?

Example 1: CME

1. Magnetic field induces polarization
2. Finite chiral density: $N_L - N_R \propto Q_{\text{top}}$ (Index theorem)
3. **Chiral Magnetic Effect** (CME):

Magnetic field + Finite chiral density → Vector current

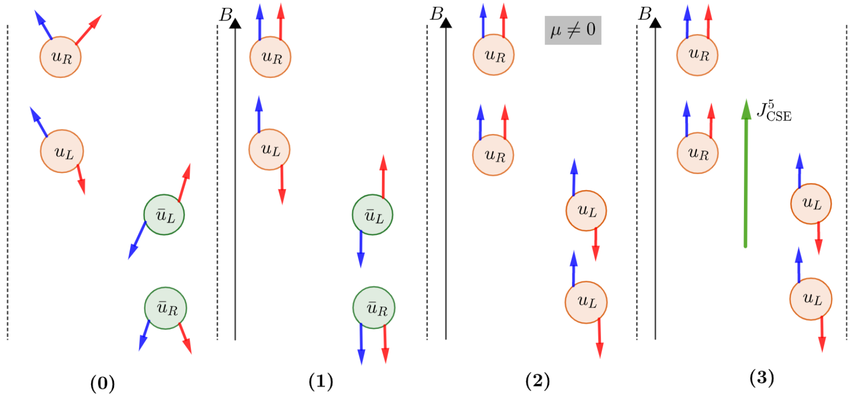


spin, momentum

Example 2: CSE

1. Magnetic field induces polarization
2. Finite density: $N_q - N_{\bar{q}} \propto \mu$
3. **Chiral Separation Effect** (CSE):

Magnetic field + Finite density → Axial current



spin, momentum

Conductivities

- Quark chemical potential μ induces imbalance between n_q and $n_{\bar{q}}$:

$$\mu\bar{\psi}\gamma_4\psi$$

Conductivities

- Quark chemical potential μ induces imbalance between n_q and $n_{\bar{q}}$:

$$\mu\bar{\psi}\gamma_4\psi$$

- Chiral chemical potential μ_5 induces imbalance between n_L and n_R:

$$\mu_5\bar{\psi}\gamma_4\gamma_5\psi$$

Conductivities

- Quark chemical potential μ induces imbalance between n_q and $n_{\bar{q}}$:

$$\mu\bar{\psi}\gamma_4\psi$$

- Chiral chemical potential μ_5 induces imbalance between n_L and n_R:

$$\mu_5\bar{\psi}\gamma_4\gamma_5\psi$$

- Currents linear in B and μ/μ_5 to first order:

$$J^V_{\mathsf{CME}} = C_{\mathsf{CME}}\, eB\mu_5 + \mathcal{O}(\mu_5^3)$$
$$J^A_{\mathsf{CSE}} = C_{\mathsf{CSE}}\, eB\mu + \mathcal{O}(\mu^3)$$

Conductivities

- Analytical predictions for **free fermions**.
- CME Fukushima, Kharzeev, Warringa '08:

$$C_{\mathsf{CME}} = \frac{1}{2\pi^2}$$

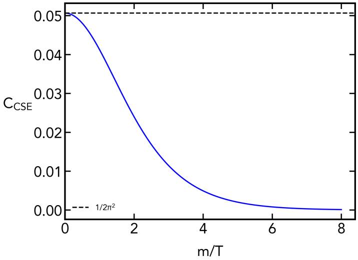
Conductivities

- Analytical predictions for **free fermions**.
- CME Fukushima, Kharzeev, Warringa '08:

$$C_{\mathsf{CME}} = \frac{1}{2\pi^2}$$

- CSE Son, Zhitnitsky '04 Metlitski, Zhitnitsky '05:

$$C_{\mathsf{CSE}} = C_{\mathsf{CSE}}(m/T) \xrightarrow{m\to 0} \frac{1}{2\pi^2}$$



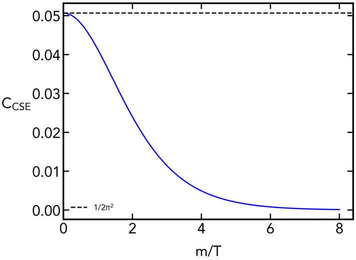
Conductivities

- Analytical predictions for **free fermions**.
- CME Fukushima, Kharzeev, Warringa '08:

$$C_{\mathsf{CME}} = \frac{1}{2\pi^2}$$

- CSE Son, Zhitnitsky '04 Metlitski, Zhitnitsky '05:

$$C_{\mathsf{CSE}} = C_{\mathsf{CSE}}(m/T) \xrightarrow{m \to 0} \frac{1}{2\pi^2}$$



- Problem to solve: Corrections in QCD?

Chemical potential on the lattice

- Chemical potential in the continuum

$$S_F = \int \mathrm{d}^4x \, \bar{\psi}(x) \left(\not{D} + m + \mu\gamma_4 \right) \psi(x)$$

Chemical potential on the lattice

- Chemical potential in the continuum

$$S_F = \int \mathrm{d}^4x \, \bar{\psi}(x) \left(\not{D} + m + \mu\gamma_4 \right) \psi(x)$$

- Then on the lattice (remember $U_\mu(n) = \exp\big(iaA_\mu\big)$)

$$S_F = a^4 \sum_{n\in\Lambda} \bar{\psi}(n) \Bigg(\sum_{\mu=1}^{4} \gamma_\mu \frac{U_\mu(n)\psi(n+\hat{\mu}) - U_\mu^\dagger(n-\hat{\mu})\psi(n-\hat{\mu})}{2a} + m\psi(n) + \mu\gamma_4\psi(n) \Bigg)$$

Chemical potential on the lattice

- Chemical potential in the continuum

$$S_F = \int \mathrm{d}^4x \, \bar{\psi}(x) \left(\not{D} + m + \mu\gamma_4 \right) \psi(x)$$

- Then on the lattice (remember $U_\mu(n) = \exp\big(iaA_\mu\big)$)

$$S_F = a^4 \sum_{n\in\Lambda} \bar{\psi}(n) \Bigg(\sum_{\mu=1}^{4} \gamma_\mu \frac{U_\mu(n)\psi(n+\hat{\mu}) - U_\mu^\dagger(n-\hat{\mu})\psi(n-\hat{\mu})}{2a} + m\psi(n) + \mu\gamma_4\psi(n) \Bigg)$$

- Divergence appears!

$$\epsilon(\mu) - \epsilon(0) \propto \left(\frac{\mu}{a}\right)^2 \xrightarrow{a\to 0} \infty$$

Chemical potential on the lattice

- The divergence is a lattice artifact $\rightarrow$ What is wrong?

Chemical potential on the lattice

- The divergence is a lattice artifact $\rightarrow$ What is wrong?
- Photon polarization:

Chemical potential on the lattice

- The divergence is a lattice artifact $\rightarrow$ What is wrong?
- Photon polarization:

- Cutoff calculation gives quadratic divergence instead of logarithmic

Chemical potential on the lattice

- The divergence is a lattice artifact $\rightarrow$ What is wrong?
- Photon polarization:

- Cutoff calculation gives quadratic divergence instead of logarithmic
- Not gauge invariant $\rightarrow$ Violation of the Ward identity
- Can something similar happen in this case?

Chemical potential on the lattice

- Chemical potential $\mu \sim$ (Euclidean) imaginary 4th component photon
- It should enter like a gauge field to preserve gauge invariance!

  Hasenfratz, Karsch '83

Chemical potential on the lattice

- Chemical potential $\mu \sim$ (Euclidean) imaginary 4th component photon
- It should enter like a gauge field to preserve gauge invariance!

  Hasenfratz, Karsch '83
- It enters exponentially

$$S_F = a^4 \sum_{n\in\Lambda} \bar{\psi}(n) \Bigg(\sum_{j=1}^{3} \gamma_j \frac{U_j(n)\psi(n+\hat{j}) - U_j^\dagger(n-\hat{j})\psi(n-\hat{j})}{2a} + m\psi(n) + \gamma_4 \frac{e^{a\mu}U_4(n)\psi(n+\hat{4}) - e^{-a\mu}U_4^\dagger(n-\hat{4})\psi(n-\hat{4})}{2a} \Bigg)$$

- No longer divergent in the continuum limit $a \to 0$

Sign problem

- Discretized Dirac operator is (usually) γ_5-hermitian

$$\gamma_5 D \gamma_5 = D^\dagger$$

- That implies $\det D \in \mathbb{R} \Rightarrow$ Required for Monte Carlo simulations ($e^{-S_g} \det D$ probability distribution)

Sign problem

- Discretized Dirac operator is (usually) γ_5-hermitian

$$\gamma_5 D \gamma_5 = D^\dagger$$

- That implies $\det D \in \mathbb{R} \Rightarrow$ Required for Monte Carlo simulations ($e^{-S_g} \det D$ probability distribution)
- Chemical potential breaks this!

$$\gamma_5 D(\mu) \gamma_5 = D(-\mu)^\dagger$$

Sign problem

- Discretized Dirac operator is (usually) γ_5-hermitian

$$\gamma_5 D \gamma_5 = D^\dagger$$

- That implies $\det D \in \mathbb{R} \Rightarrow$ Required for Monte Carlo simulations ($e^{-S_g} \det D$ probability distribution)
- Chemical potential breaks this!

$$\gamma_5 D(\mu) \gamma_5 = D(-\mu)^\dagger$$

- Then $\det D \in \mathbb{C} \Rightarrow$ **Sign problem**

Sign problem

- Discretized Dirac operator is (usually) γ_5-hermitian

$$\gamma_5 D \gamma_5 = D^\dagger$$

- That implies $\det D \in \mathbb{R} \Rightarrow$ Required for Monte Carlo simulations ($e^{-S_g} \det D$ probability distribution)
- Chemical potential breaks this!

$$\gamma_5 D(\mu) \gamma_5 = D(-\mu)^\dagger$$

- Then $\det D \in \mathbb{C} \Rightarrow$ **Sign problem**
- Chiral chemical potential μ_5 has no sign problem

CSE on the lattice

- Some previous results:
  - Overlap: Quenched QCD Puhr, Buividovich '17
    No significant corrections found
  - Wilson/Domain Wall: SU(2) Buividovich, Smith, von Smekal '21
    CSE suppressed at low T

CSE on the lattice

- Some previous results:
  - Overlap: Quenched QCD Puhr, Buividovich '17
    No significant corrections found
  - Wilson/Domain Wall: SU(2) Buividovich, Smith, von Smekal '21
    CSE suppressed at low T

  No results for full QCD with physical masses!

CSE on the lattice

- Some previous results:
  - Overlap: Quenched QCD Puhr, Buividovich '17
    No significant corrections found
  - Wilson/Domain Wall: SU(2) Buividovich, Smith, von Smekal '21
    CSE suppressed at low T

  No results for full QCD with physical masses!

- Our setup:
  - Improved staggered fermions, $2+1$ flavors, physical quark masses
  - Background B field (z direction)

CSE on the lattice

- Some previous results:
  - Overlap: Quenched QCD Puhr, Buividovich '17
    No significant corrections found
  - Wilson/Domain Wall: SU(2) Buividovich, Smith, von Smekal '21
    CSE suppressed at low T

  No results for full QCD with physical masses!

- Our setup:
  - Improved staggered fermions, $2+1$ flavors, physical quark masses
  - Background B field (z direction)

- Simulations at finite real μ suffer from sign problem

CSE on the lattice

- Measure derivatives of the current:

$$\frac{\mathrm{d}\langle J_z^A\rangle}{\mathrm{d}\mu}\Big|_{\mu=0} = \frac{T}{V}\big[\langle \mathsf{Tr}(\gamma_4 M^{-1})\mathsf{Tr}(\gamma_3\gamma_5 M^{-1})\rangle_{\mu=0} - \langle \mathsf{Tr}(\gamma_4 M^{-1}\gamma_3\gamma_5 M^{-1})\rangle_{\mu=0}\big] = C_{\mathsf{cse}} eB_z$$

CSE on the lattice

- Measure derivatives of the current:

$$\frac{\mathrm{d}\langle J_z^A \rangle}{\mathrm{d}\mu}\bigg|_{\mu=0} = \frac{T}{V}\big[\langle \mathsf{Tr}(\gamma_4 M^{-1})\mathsf{Tr}(\gamma_3\gamma_5 M^{-1})\rangle_{\mu=0} - \langle \mathsf{Tr}(\gamma_4 M^{-1}\gamma_3\gamma_5 M^{-1})\rangle_{\mu=0}\big] = C_{\mathsf{cse}} e B_z$$

- Simulations at $\mu = 0 \rightarrow$ no sign problem

CSE on the lattice

- Measure derivatives of the current:

$$\frac{\mathrm{d}\langle J_z^A\rangle}{\mathrm{d}\mu}\Big|_{\mu=0} = \frac{T}{V}[\langle \mathsf{Tr}(\gamma_4 M^{-1})\mathsf{Tr}(\gamma_3\gamma_5 M^{-1})\rangle_{\mu=0} - \langle \mathsf{Tr}(\gamma_4 M^{-1}\gamma_3\gamma_5 M^{-1})\rangle_{\mu=0}] = C_{\mathsf{cse}} eB_z$$

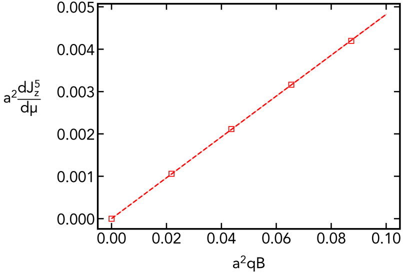
- Simulations at $\mu = 0 \to$ no sign problem
- Numerical derivative (linear fit) w.r.t. B to obtain C_{cse}:

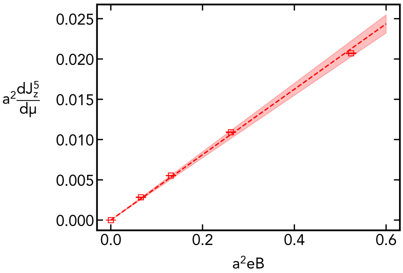
CSE on the lattice

- Measure derivatives of the current:

$$\frac{\mathrm{d}\langle J_z^A\rangle}{\mathrm{d}\mu}\Big|_{\mu=0} = \frac{T}{V}\big[\langle \mathsf{Tr}(\gamma_4 M^{-1})\mathsf{Tr}(\gamma_3\gamma_5 M^{-1})\rangle_{\mu=0}$$
$$- \langle \mathsf{Tr}(\gamma_4 M^{-1}\gamma_3\gamma_5 M^{-1})\rangle_{\mu=0}\big]$$
$$= C_{\mathsf{cse}} e B_z$$

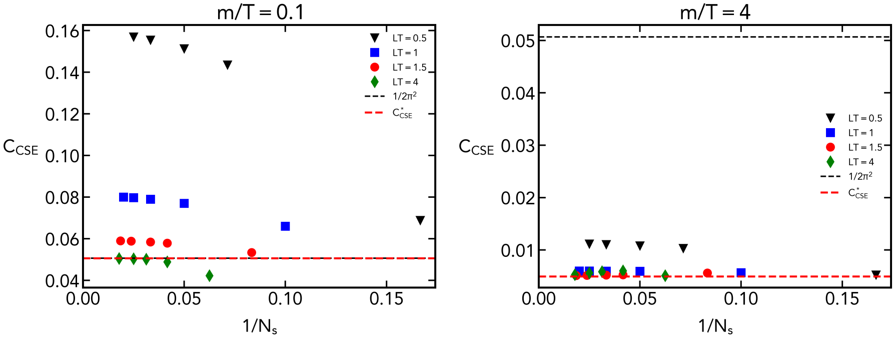
- Simulations at $\mu = 0 \rightarrow$ no sign problem
- Numerical derivative (linear fit) w.r.t. B to obtain C_{cse}:





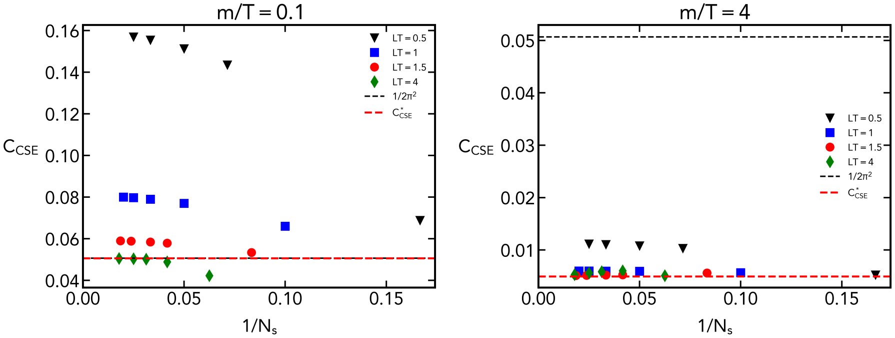
Results for CSE: free case

- Consistency check in the free case ($A_\mu = 0 \iff U_\mu = 1$)



Results for CSE: free case

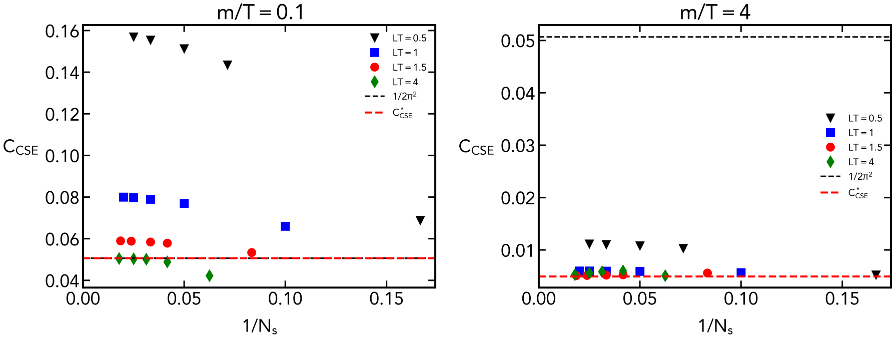
- Consistency check in the free case $(A_\mu = 0 \iff U_\mu = 1)$



- Finite size effects at $LT \to 0$ sizeable if m/T not large enough

Results for CSE: free case

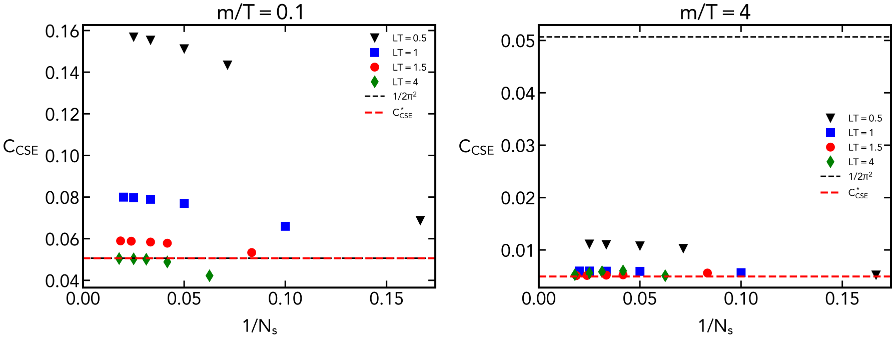
- Consistency check in the free case ($A_\mu = 0 \iff U_\mu = 1$)



- Finite size effects at $LT \to 0$ sizeable if m/T not large enough
- Importance of continuum limit

Results for CSE: free case

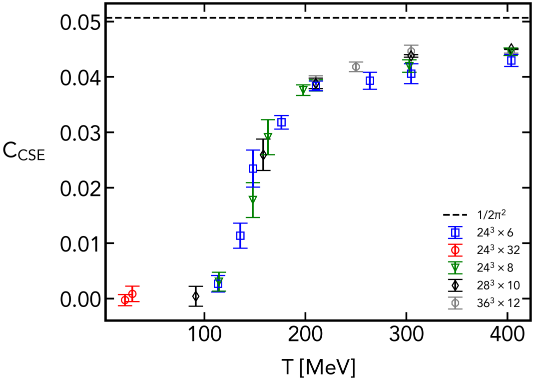
- Consistency check in the free case $(A_\mu = 0 \iff U_\mu = 1)$



- Finite size effects at $LT \to 0$ sizeable if m/T not large enough
- Importance of continuum limit
- C_{cse} approaches analytical prediction when $LT \to \infty$

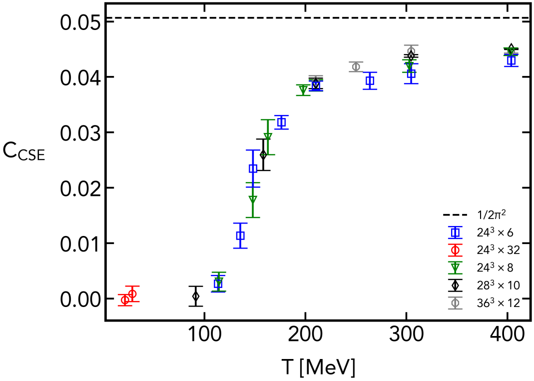
Results for CSE: interacting case

- 2+1 flavors, physical masses



Results for CSE: interacting case

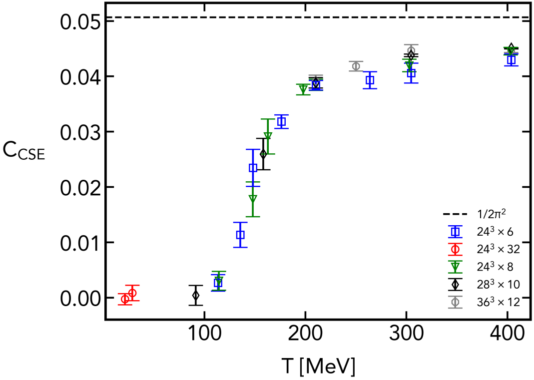
- 2+1 flavors, physical masses



- High T $(T > T_c)$: approaches free case value

Results for CSE: interacting case

- 2+1 flavors, physical masses



- High T $(T > T_c)$: approaches free case value
- Low T $(T < T_c)$: CSE suppressed Buividovich, Smith, von Smekal '21

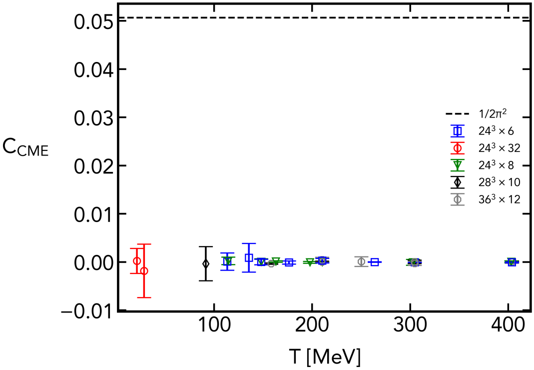
  Chiral effective theories Avdoshkin, Sadofyev, Zakharov '18

And CME?

- What about CME?
- Wilson: Quenched and phys. masses Yamamoto '11
  $C_{\mathsf{cme}} = 0.02 - 0.03$ at high T

And CME?

- What about CME?
- Wilson: Quenched and phys. masses Yamamoto '11
  $C_{\text{cme}} = 0.02 - 0.03$ at high T
- Our result for physical masses



CME

- Our result for CME is 0 for free fermions and in QCD for physical and higher than physical pion masses

CME

- Our result for CME is 0 for free fermions and in QCD for physical and higher than physical pion masses
- What is going on?

CME

- Our result for CME is 0 for free fermions and in QCD for physical and higher than physical pion masses
- What is going on?
  - CME is zero in equilibrium? Buividovich '14 Landsteiner '16

CME

- Our result for CME is 0 for free fermions and in QCD for physical and higher than physical pion masses
- What is going on?
  - CME is zero in equilibrium? Buividovich '14 Landsteiner '16
  - Problems with some lattice discretizations for CME?

CME

- Our result for CME is 0 for free fermions and in QCD for physical and higher than physical pion masses
- What is going on?
  - CME is zero in equilibrium? Buividovich '14 Landsteiner '16
  - Problems with some lattice discretizations for CME?
  - Something else?

Summary & Outlook

Summary

- Study of CSE with staggered fermions, 2+1 flavors, physical masses
- Free case consistent with analytical prediction
- Full QCD: suppression at low T, approach free case value at high T
- Zero CME

Summary & Outlook

Summary

- Study of CSE with staggered fermions, 2+1 flavors, physical masses
- Free case consistent with analytical prediction
- Full QCD: suppression at low T, approach free case value at high T
- Zero CME

Outlook

- Perform continuum limit for C_{CSE}
- Clarify CME

Backup slides

- Transport effects:

$$\begin{pmatrix} \vec{J} \\ \vec{J}_5 \end{pmatrix} = \begin{pmatrix} \sigma_{\text{Ohm}} & \sigma_{\text{CME}} \\ \sigma_{\text{CESE}} & \sigma_{\text{CSE}} \end{pmatrix} \begin{pmatrix} \vec{E} \\ \vec{B} \end{pmatrix}$$

- Chiral Vortical Effect: vector/axial current generated by rotation + $\mu + \mu_5$:

$$\vec{J} = \frac{1}{\pi^2}\mu_5\mu\vec{\omega}$$
$$\vec{J}_5 = \left[\frac{1}{6}T^2 + \frac{1}{2\pi^2}(\mu_5^2 + \mu^2)\right]\vec{\omega}$$

Free case continuum limit

- Continuum limit: $a \to 0$, $V = L^3 = a^3 N_s^3 =$ fixed (L fixed)
- Also keep $T = 1/(N_t a)$, $m = \tilde{m}/a$ and B fixed
- Then $LT =$ fixed so
$$\frac{N_s}{N_t} = \text{fixed}$$
- Also $mL =$ fixed, $m/T =$ fixed so
$$\tilde{m} N_s = \text{fixed}, \quad \tilde{m} N_t = \text{fixed}$$
- And
$$B = \frac{2\pi N_b}{L_y L_x} \Rightarrow N_b = \text{fixed}$$

Staggered gammas

- Staggered gammas $\sim$ Taste singlets $(\gamma_\mu \otimes \not{1})$, $(\gamma_5 \otimes \not{1})$:

$$\Gamma_\mu(x,y) = \frac{1}{2}\eta_\mu(x)[U(x)u(x)e^{a\mu_q}\delta_{x+\hat{\mu},y} + U^\dagger(y)u^\dagger(y)e^{-a\mu_q}\delta_{x-\hat{\mu},y}]$$

$$\Gamma_5(x,y) = \frac{1}{4!}\sum_{\text{perm}} \epsilon_{\text{perm}}\Gamma_1\Gamma_2\Gamma_3\Gamma_4$$

$$(\Gamma_5\Gamma_4)(x,y) = \frac{1}{3!}\sum_{\text{perm}} \epsilon_{\text{perm}}\Gamma_1\Gamma_2\Gamma_3$$

with

$$\eta_\mu(x) = (-1)^{\sum_{\nu<\mu} x_\nu}, \quad \eta_1(x) = 1$$

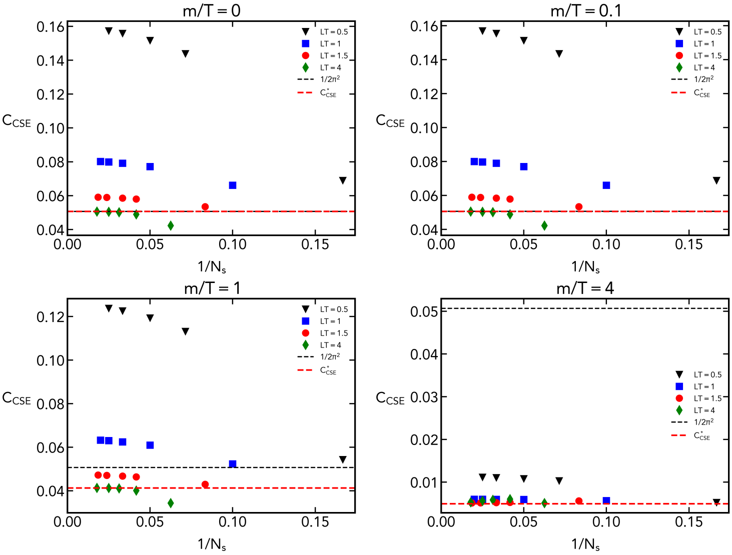
and $U \in \mathsf{SU}(3), u \in \mathsf{U}(1)$

Staggered observable

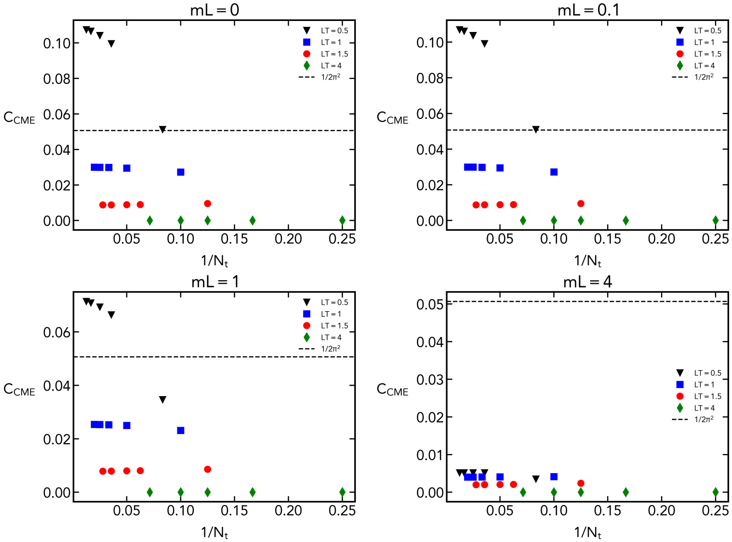
- Staggered observable has an extra term

$$\left.\frac{\mathrm{d}\left\langle J_z^5\right\rangle}{\mathrm{d}\mu}\right|_{\mu=0} = \frac{T}{V}\left[\frac{1}{4}\left\langle \mathsf{Tr}\left(\Gamma_4 M^{-1}\right)\mathsf{Tr}\left(\Gamma_3\Gamma_5 M^{-1}\right)\right\rangle_{\mu=0} - \frac{1}{16}\left\langle \mathsf{Tr}\left(\Gamma_4 M^{-1}\Gamma_3\Gamma_5 M^{-1}\right)\right\rangle_{\mu=0} + \frac{1}{4}\left\langle \mathsf{Tr}\left(\frac{\partial(\Gamma_3\Gamma_5)}{\partial\mu}M^{-1}\right)\right\rangle_{\mu=0}\right] \tag{1}$$

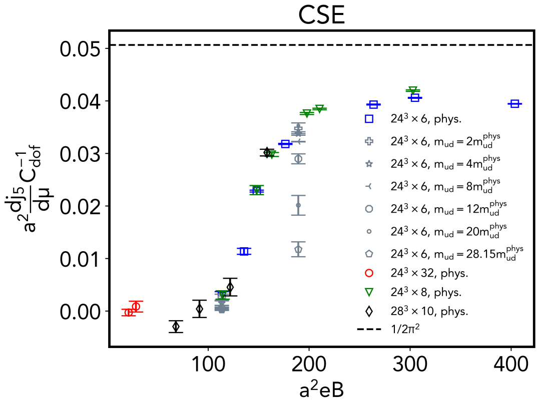
CSE free case



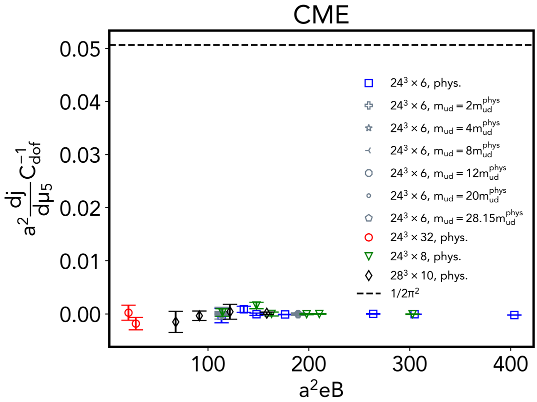
CME free case



CSE masses


CSE
0.05
0.04
0.03
0.02
0.01
0.00
100
200
300
400
a^2eB
$a^2 \frac{dj_5}{d\mu} C_{dof}^{-1}$
$24^3 \times 6$, phys.
$24^3 \times 6$, $m_{ud} = 2m_{ud}^{phys}$
$24^3 \times 6$, $m_{ud} = 4m_{ud}^{phys}$
$24^3 \times 6$, $m_{ud} = 8m_{ud}^{phys}$
$24^3 \times 6$, $m_{ud} = 12m_{ud}^{phys}$
$24^3 \times 6$, $m_{ud} = 20m_{ud}^{phys}$
$24^3 \times 6$, $m_{ud} = 28.15m_{ud}^{phys}$
$24^3 \times 32$, phys.
$24^3 \times 8$, phys.
$28^3 \times 10$, phys.
$1/2\pi^2$

CME masses



CME
0.05
0.04
0.03
0.02
0.01
0.00
100
200
300
400
a^2eB
$a^2\frac{dj}{d\mu_5}C_{dof}^{-1}$
$24^3 \times 6$, phys.
$24^3 \times 6$, $m_{ud} = 2m_{ud}^{phys}$
$24^3 \times 6$, $m_{ud} = 4m_{ud}^{phys}$
$24^3 \times 6$, $m_{ud} = 8m_{ud}^{phys}$
$24^3 \times 6$, $m_{ud} = 12m_{ud}^{phys}$
$24^3 \times 6$, $m_{ud} = 20m_{ud}^{phys}$
$24^3 \times 6$, $m_{ud} = 28.15m_{ud}^{phys}$
$24^3 \times 32$, phys.
$24^3 \times 8$, phys.
$28^3 \times 10$, phys.
$1/2\pi^2$