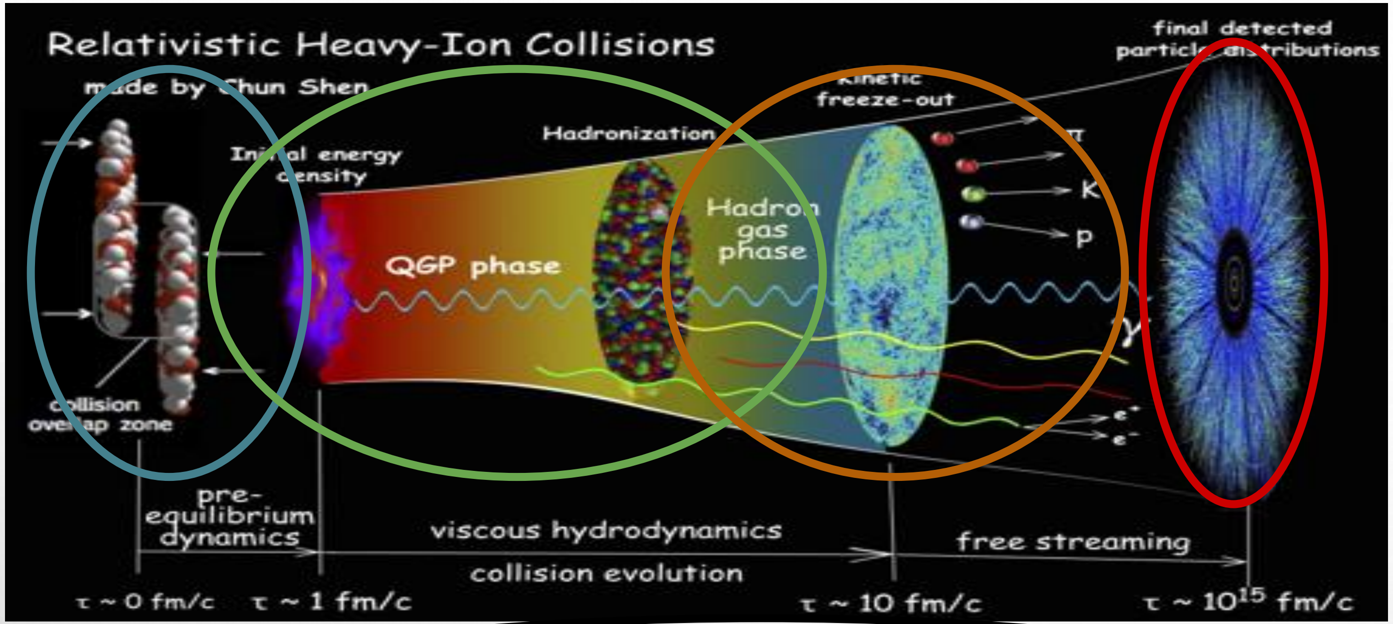
THE FAR-FROM-EQUILIBRIUM SEARCH FOR THE QCD CRITICAL POINT

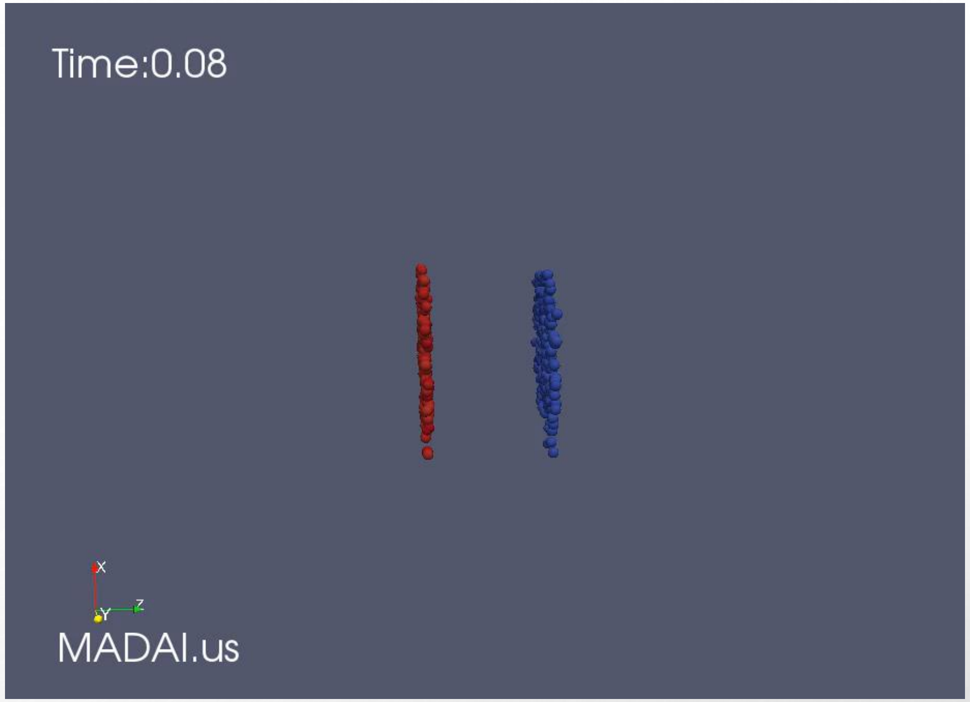
TRAVIS DORE

TD, ET AL., ARXIV: 2207.04086 [NUCL-TH].

TD, ET AL., PHYS. REV. D, VOL. 102, NO. 7, P. 074 017, 2020

HEAVY-ION COLLISIONS IN A NUTSHELL





Initial conditions and pre-equilibrium

Viscous hydrodynamic evolution, Quark Gluon Plasma

Hadronization, confinement transition, and freeze-out

Measurement of particle distributions

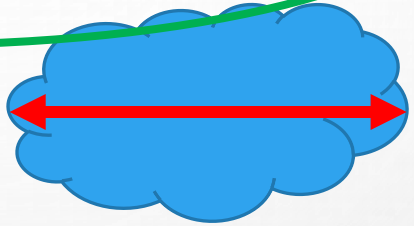
This talk will focus on the connection between the initial conditions and the following hydrodynamic evolution

WHAT IS HYDRODYNAMICS?

Fundamentally based on a hierarchy of scales:

$$\lambda_{micro} \ll \ell_{hydro} \leq L_{global}$$

$\lambda_{micro} \sim 1/E_{micro}$ $\ell_{hydro} \sim D^{\mu}$



Thermodynamics **Local Equilibrium**

Microscopics encoded into *transport coefficients* (e.g. shear viscosity)

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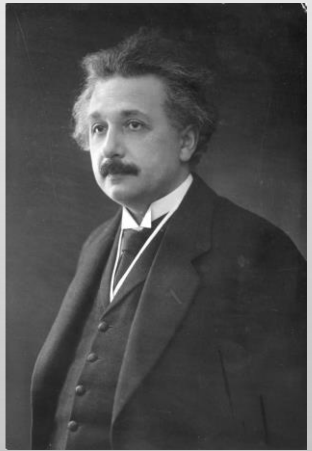
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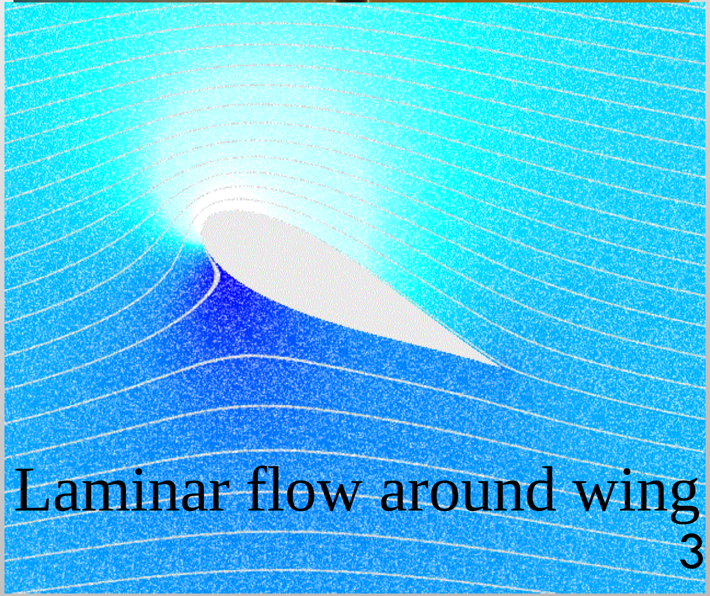


FUNDAMENTALS OF RELATIVISTIC HYDRODYNAMICS

1) Energy-Momentum Conservation:

(also charge conservation)

$$\partial_\mu J^\mu = 0$$

$$\partial_\mu T^{\mu\nu} = 0$$

Energy-Momentum tensor

2) Coarse Graining:

(thermodynamic)

$$J^\mu = J^\mu_{(0)} + N^\mu$$

$$T^{\mu\nu} = T^{\mu\nu}_{(0)} + \Pi^{\mu\nu}$$

Local Thermodynamic Equilibrium

Out-of-equilibrium Effects

$$T^{\mu\nu}_{(0)} = f\left(\epsilon, p, u^\mu, x^\mu\right)$$

$$\epsilon = \epsilon(T, \mu_B) \quad p = p(T, \mu_B)$$

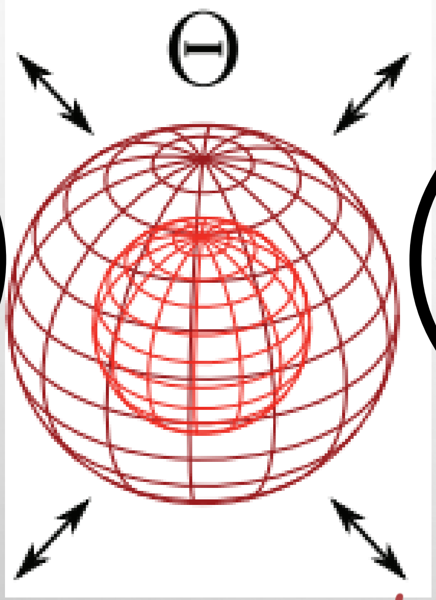
$\Pi^{\mu\nu}$ Local shear and bulk viscous effects

WHAT DO WE MEAN BY OUT OF EQUILIBRIUM?

From a hydro perspective, traditionally related to inhomogeneities and gradients

Part of $\Pi^{\mu\nu}$

Π



$\pi^{\mu\nu}$

Part of charge current

n^{μ}

Bulk ζ

Resistance to expansion/compression

Shear η

Resistance to deformation

Diffusion κ

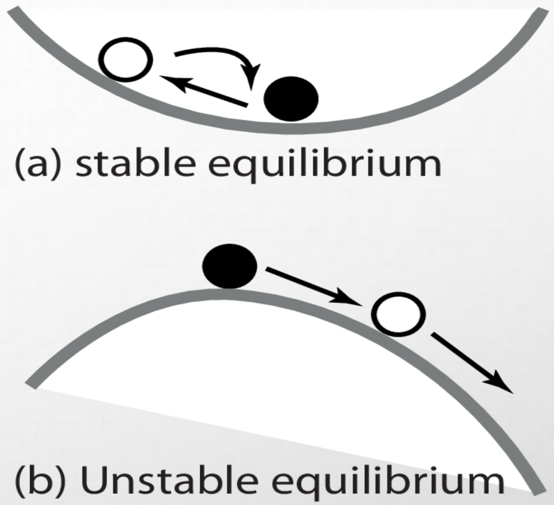
Spreading from high to low density

OUT-OF-EQUILIBRIUM HYDRODYNAMICS

Upgrading traditional Navier-Stokes equations to be relativistic…

$$\Pi = -\zeta \partial_\mu u^\mu$$

Leads to acausal (super-luminal) mode propagation and thermodynamic instabilities



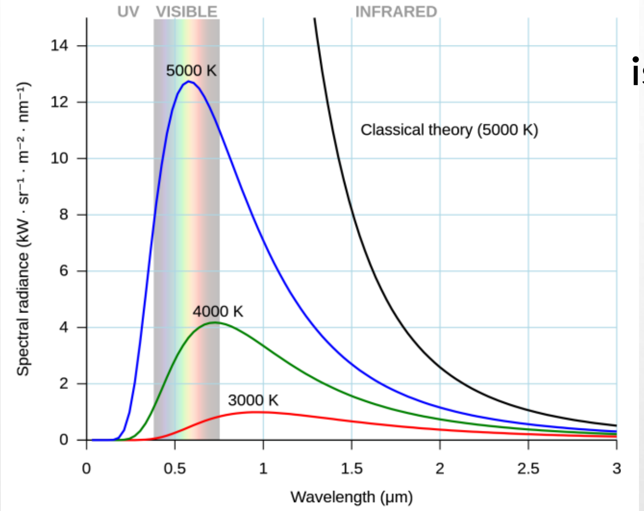
One way to ensure linear stability and causality in your system:
dynamic relaxation of viscous components

Must be initialized independently

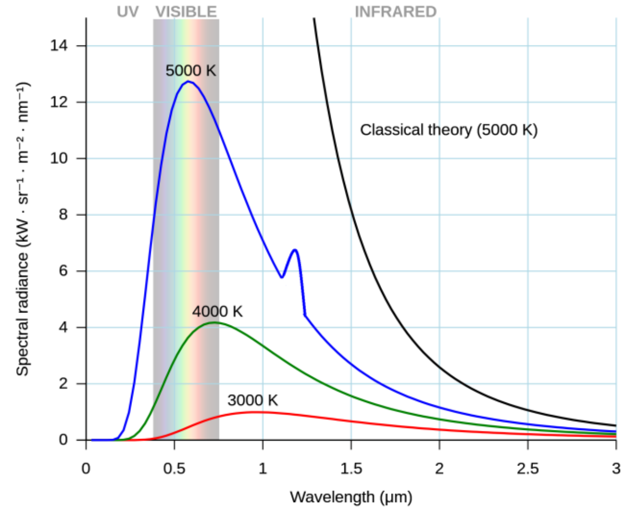
$$\tau_\Pi \dot{\Pi} + \Pi = -\zeta \partial_\mu u^\mu + \ldots$$

DIGRESSION: PHYSICALITY OF INDEPENDENT VISCOUS FIELDS

Consider the following thought experiment:



isotropic



Small deviation from eq

THERMALIZATION

What is expected to be in this black box?

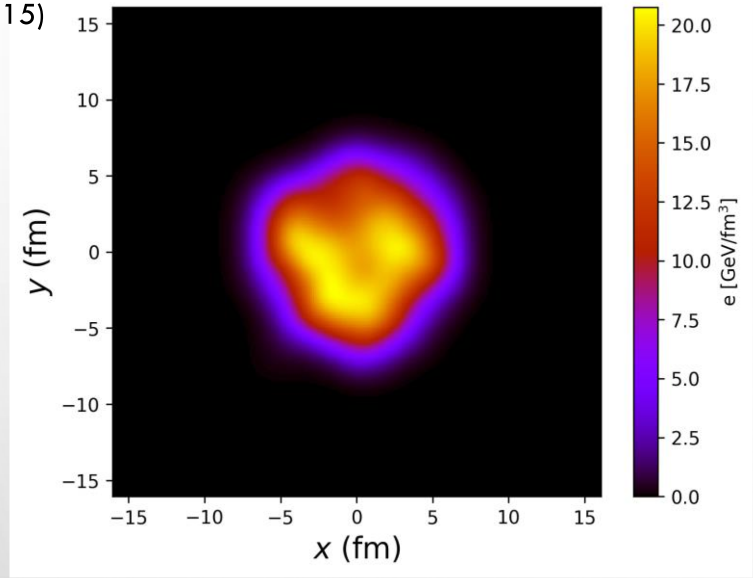
A kinetic theory perspective tells us that the viscous fields in relaxation hydro are given by moments of the distribution function

$$\Pi \sim \int dK \, (\Delta_{\mu\nu} k^\mu k^\nu) \, \delta f$$

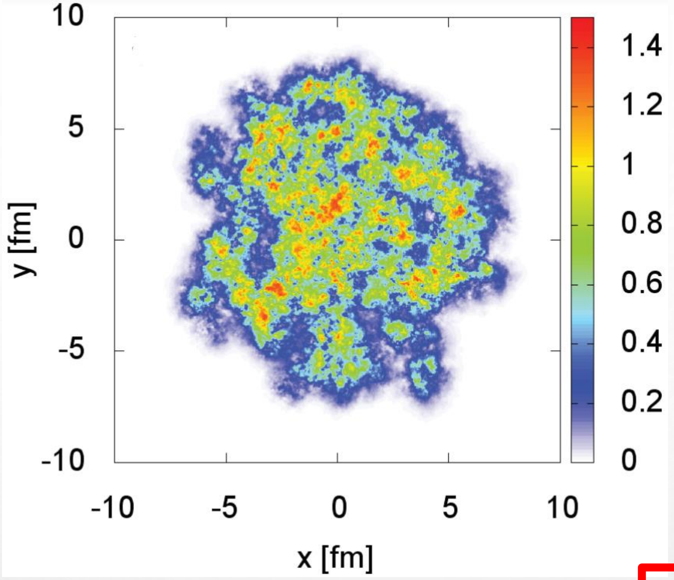
This is more information than only spatial gradients

WHAT DOES THE INITIAL STATE OF HIC LOOK LIKE?

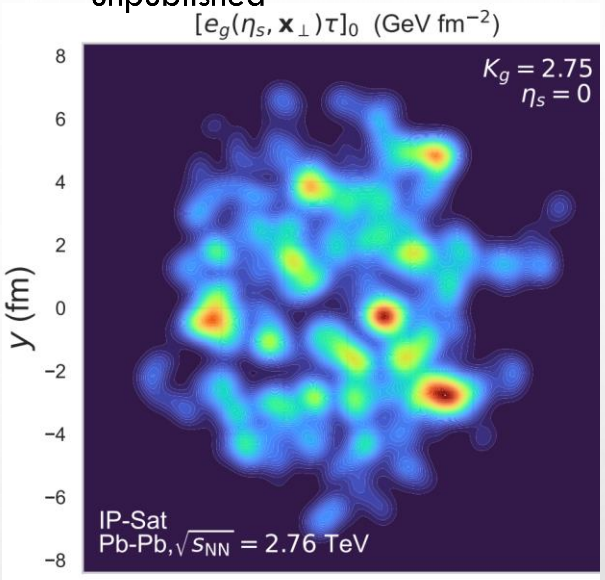
Courtesy Christopher Plumberg using **Trento** J. S. Moreland, J. E. Bernhard, and S. A. Bass, Phys. Rev. C92, 011901 (2015)



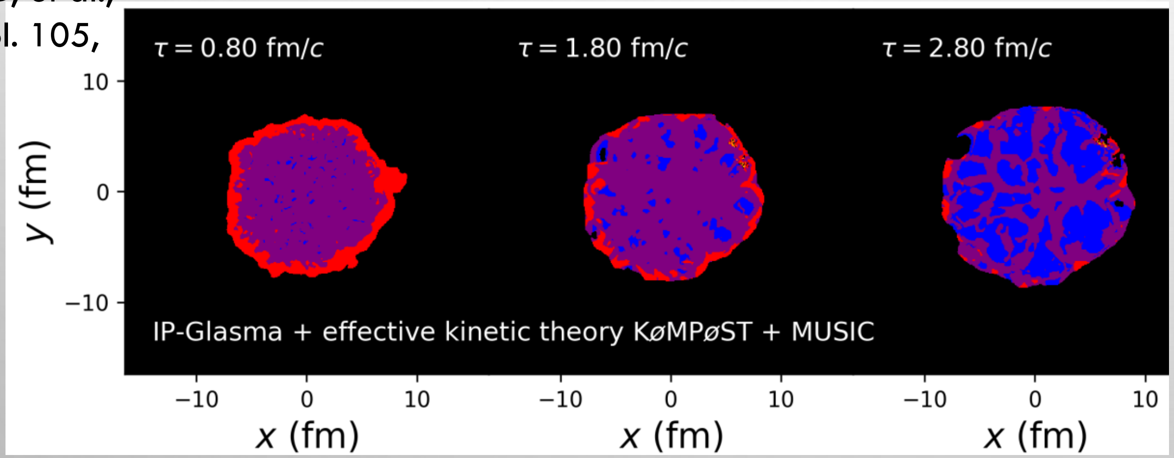
IP-Glasma S. Schliching, B. Schenke
Phys.Rev.C 94 (2016) 4, 044907



IP-Sat courtesy Oscar Garcia-Montero, unpublished



C. Plumberg, TD, et al., Phys. Rev. C, vol. 105, p. L061901

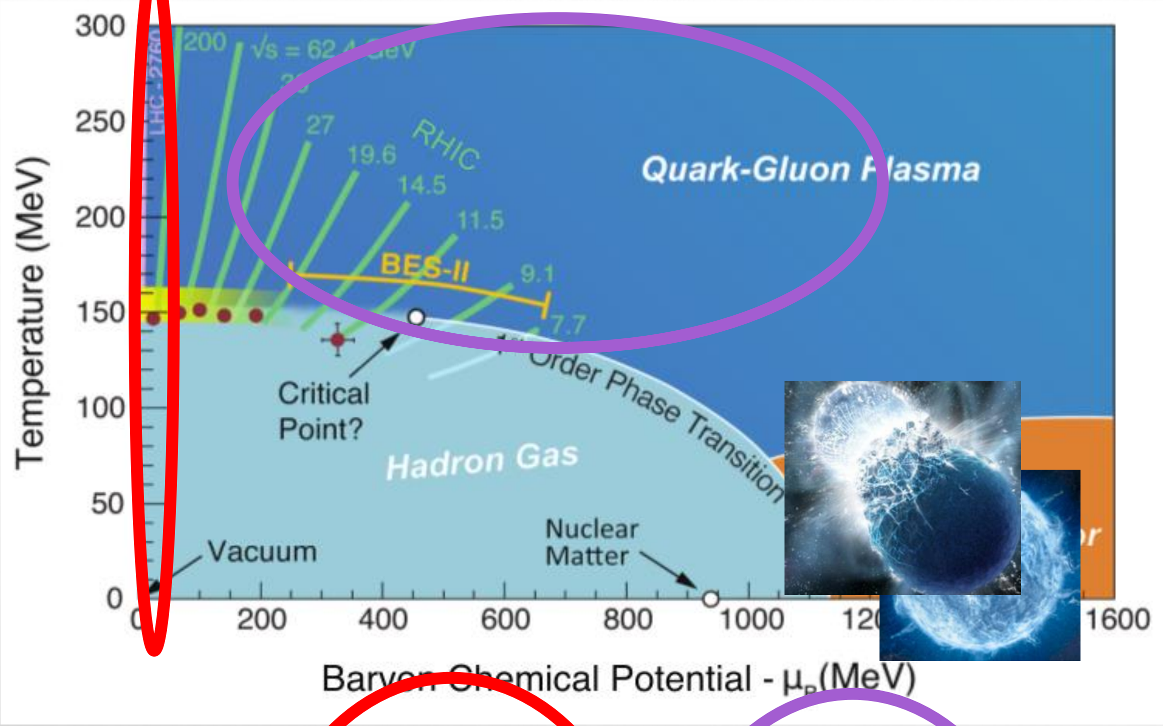


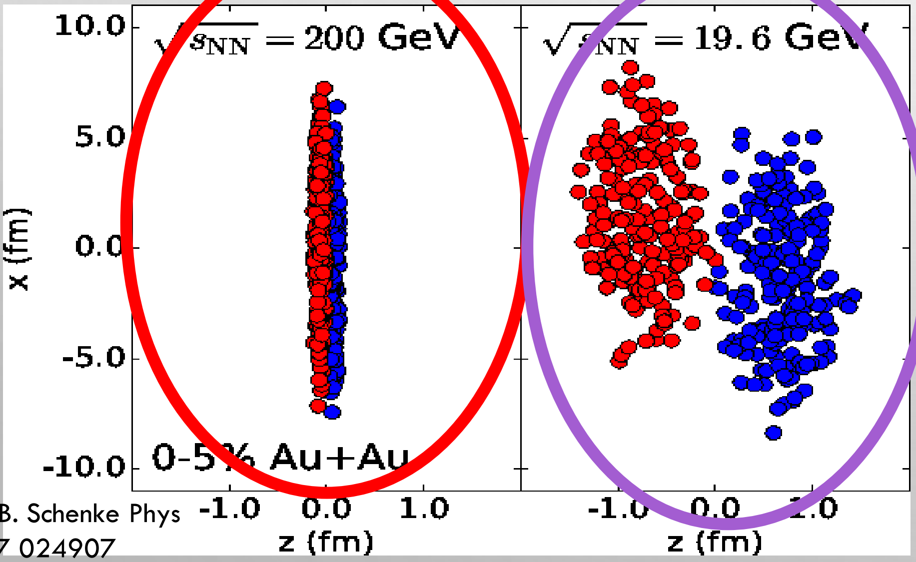
Red and Purple: Far from equilibrium

$$\Pi^{\mu\nu}\big|_{t_0} \neq 0$$

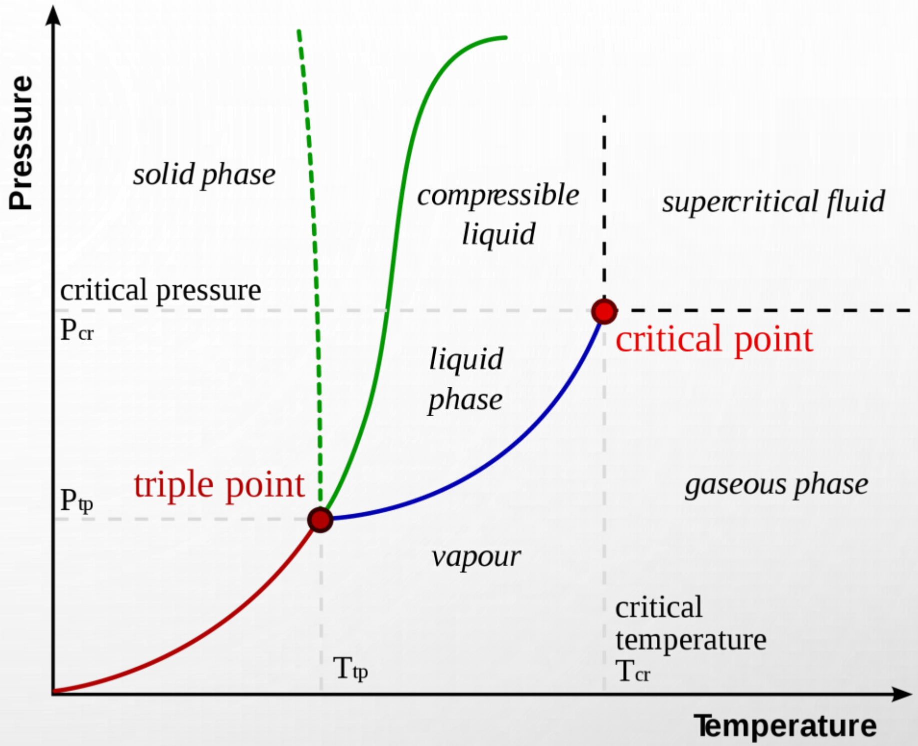
There is no reason to believe this fact changes at lower energies

QCD PHASE DIAGRAM: EQUILIBRIUM DYNAMICS





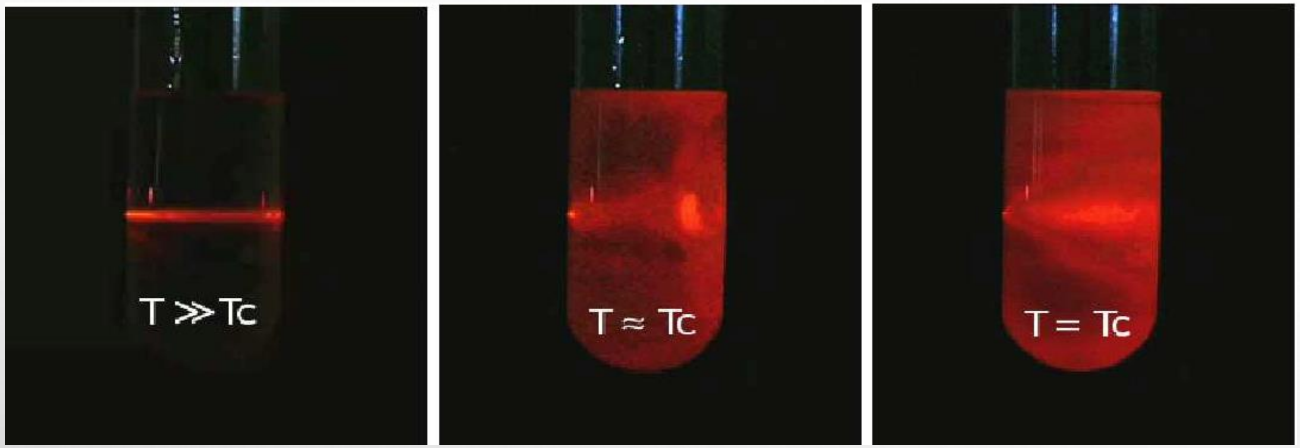
C. Shen, B. Schenke Phys Rev C. 97 024907



QCD has a conjectured critical point similar to that of water

SOME PHYSICS OF CRITICALITY

Critical Opalescence



Important point:

Static system in a well-defined equilibrium state, measure at any point in time

How does this compare to a heavy-ion collision?

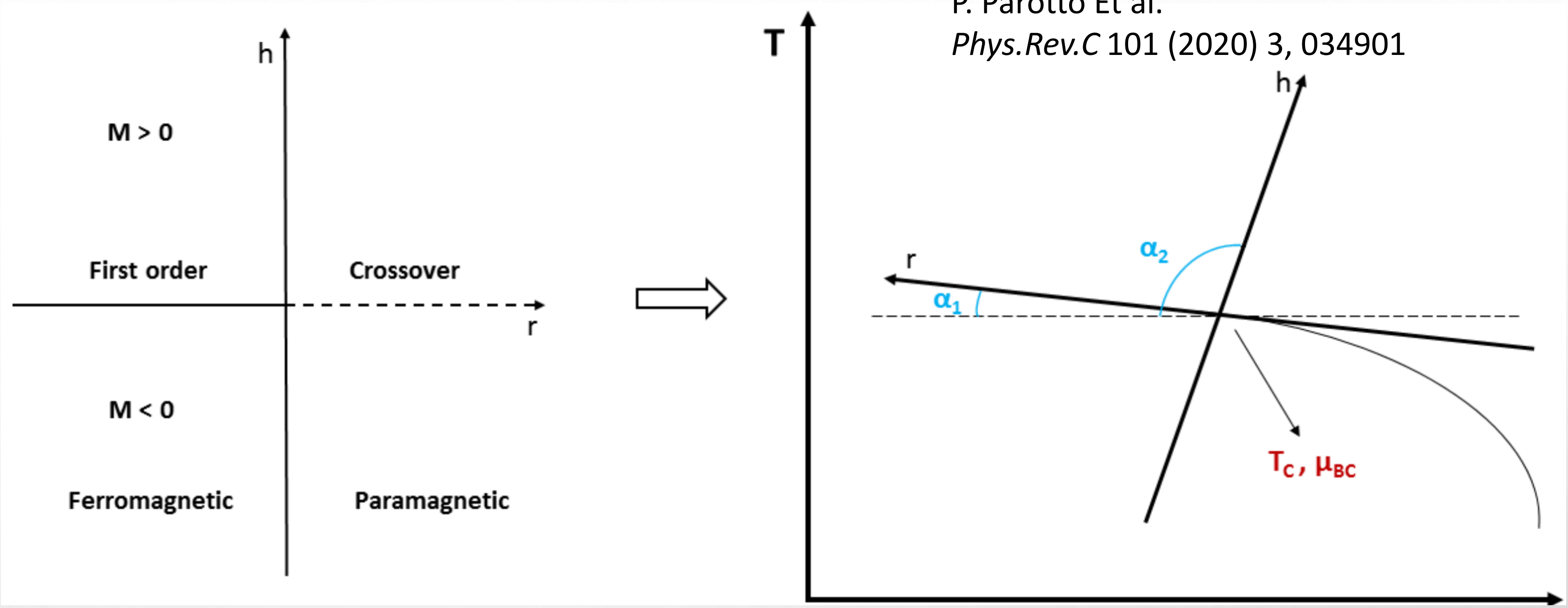
Divergence of correlation length ξ

- Characteristic fluctuations on all length scales of the system
- Light scatters when correlations on the scale of its wavelength develop

MAPPING THE 3D ISING MODEL TO QCD

P. Parotto Et al.
Phys.Rev.C 101 (2020) 3, 034901

Due to its symmetries, QCD is expected to be in the 3D Ising universality class



3D Ising

$$\xi \sim \left|\frac{T - T_c}{T_c}\right|^{-\nu} \quad \chi \sim \left|\frac{T - T_c}{T_c}\right|^{-\gamma}$$

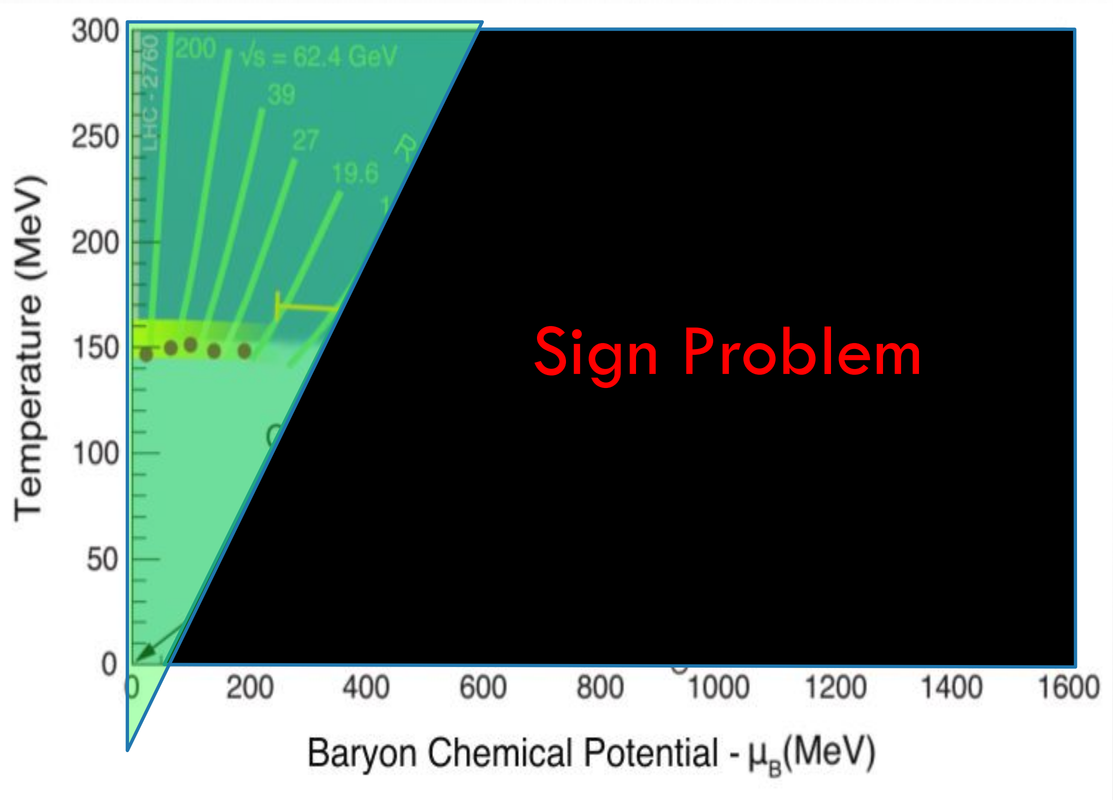
QCD

$$\chi_2^B \sim \xi^2 \quad \chi_4^B \sim \xi^{11}$$

M. A. Stephanov, Phys. Rev. Lett. 107, 052301 (2011)

SEARCHING FOR THE QCD CRITICAL POINT

Lattice QCD

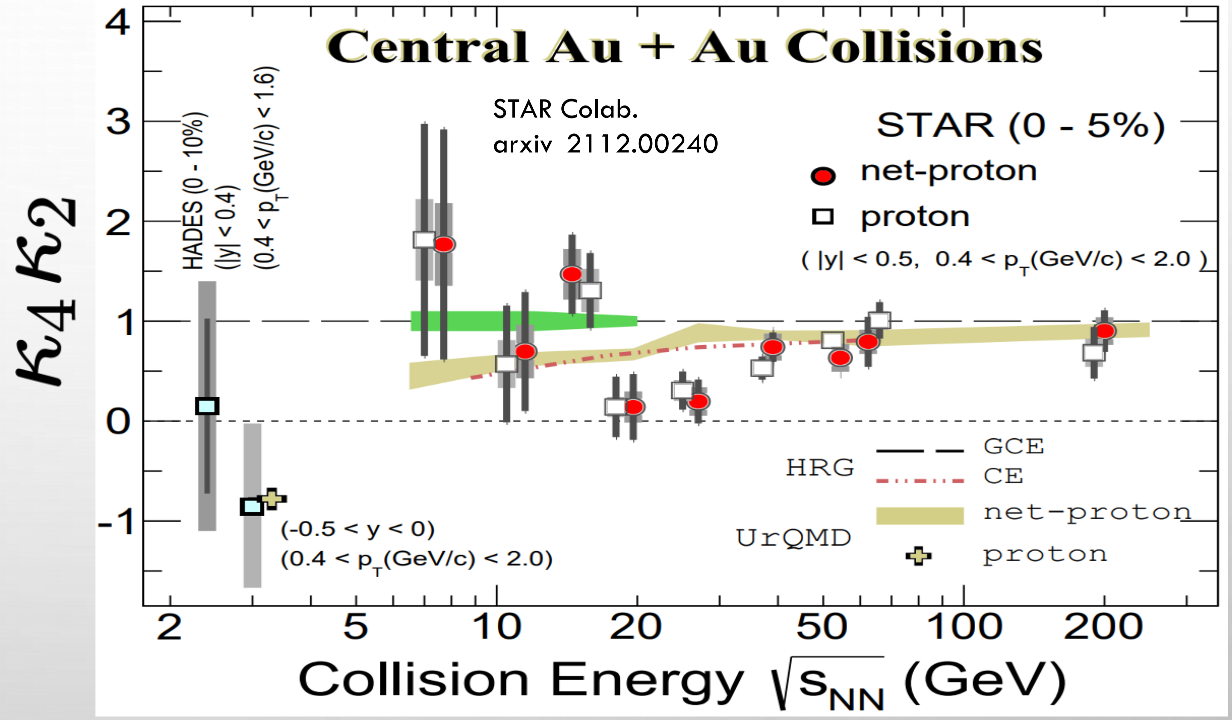


Crucial Experimental Observable:

M. A. Stephanov, Phys. Rev. Lett. 107, 052301 (2011)

$$\kappa_4 \kappa_2 = \frac{\chi_4}{\chi_2} \sim \xi^9$$

In the critical region



Final State Measurements of Fluctuations

How does the *structure* of the critical region affect observables?

LATTICE QCD EOS WITH PARAMETERIZED CP

EoS = Non-critical + Parameterized Critical

P. Parotto Et al.
Phys.Rev.C 101 (2020) 3, 034901

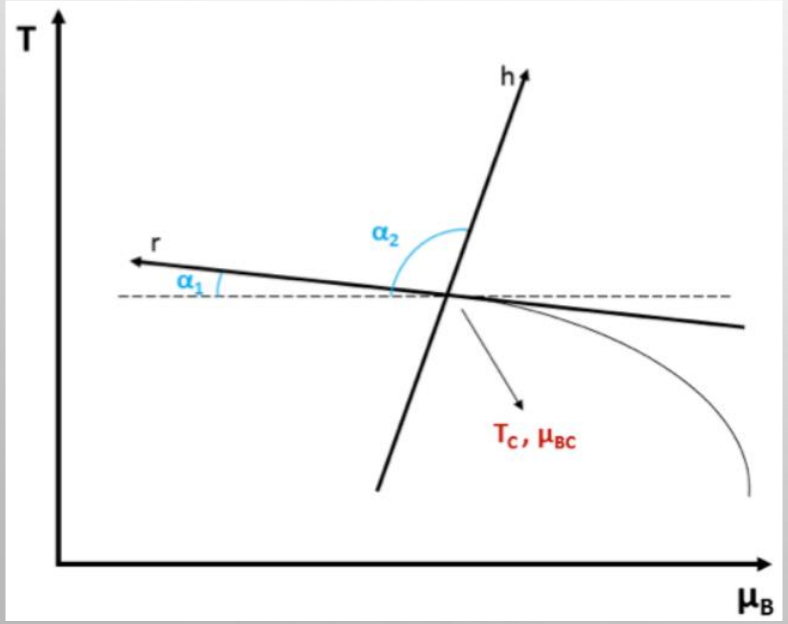
$$p(T,\mu_B) = T^4 \sum_n \mathbf{c_n^{non-crit}}(T)\left(\frac{\mu_B}{T}\right)^n + \mathbf{p^{crit}(T,\mu_B)}$$

$$c_n^{non-crit}(T) = c_n^{LAT}(T) - c_n^{crit}(T; w, \rho, \Delta\alpha, \cdots)$$

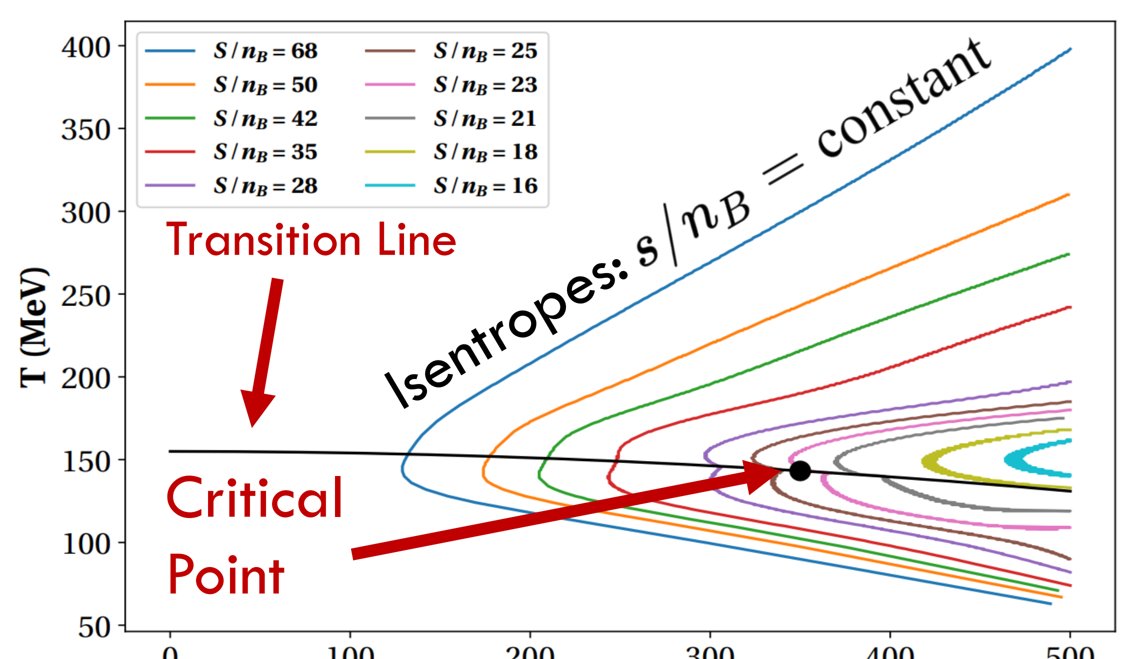
$$p^{crit} = p^{crit}(T, \mu_B; w, \rho, \Delta\alpha, \cdots)$$

Multiple parameters control size, shape, and strength of critical region

Ideal hydrodynamics evolves along isentropes: one initial (T, μ_B), unique evolutions



Even in equilibrium, evolution is dynamic

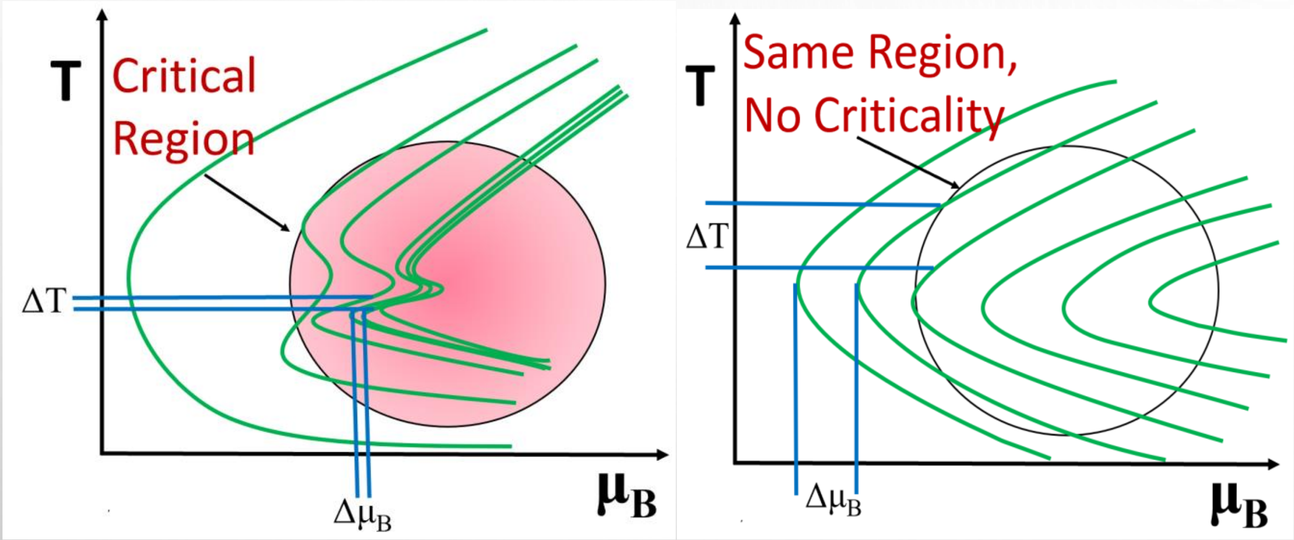


J. Karthein, Et al. Eur. Phys. J. Plus 136, 621 (2021),

THERMODYNAMICS AND EQUILIBRIUM LENSING

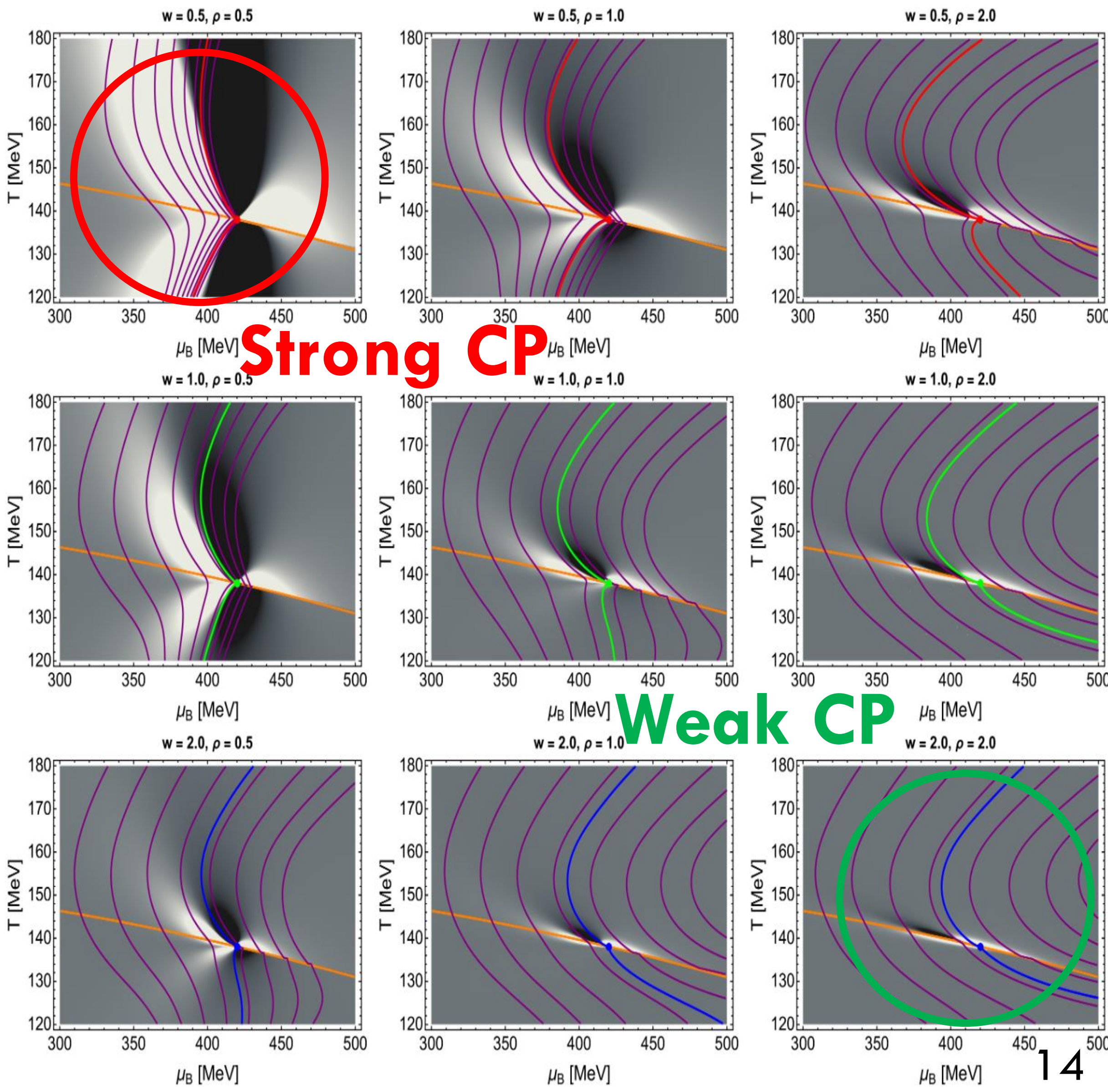
TD, et al., arXiv: 2207.04086 [nucl-th].

$$\Pi^{\mu\nu} = 0$$

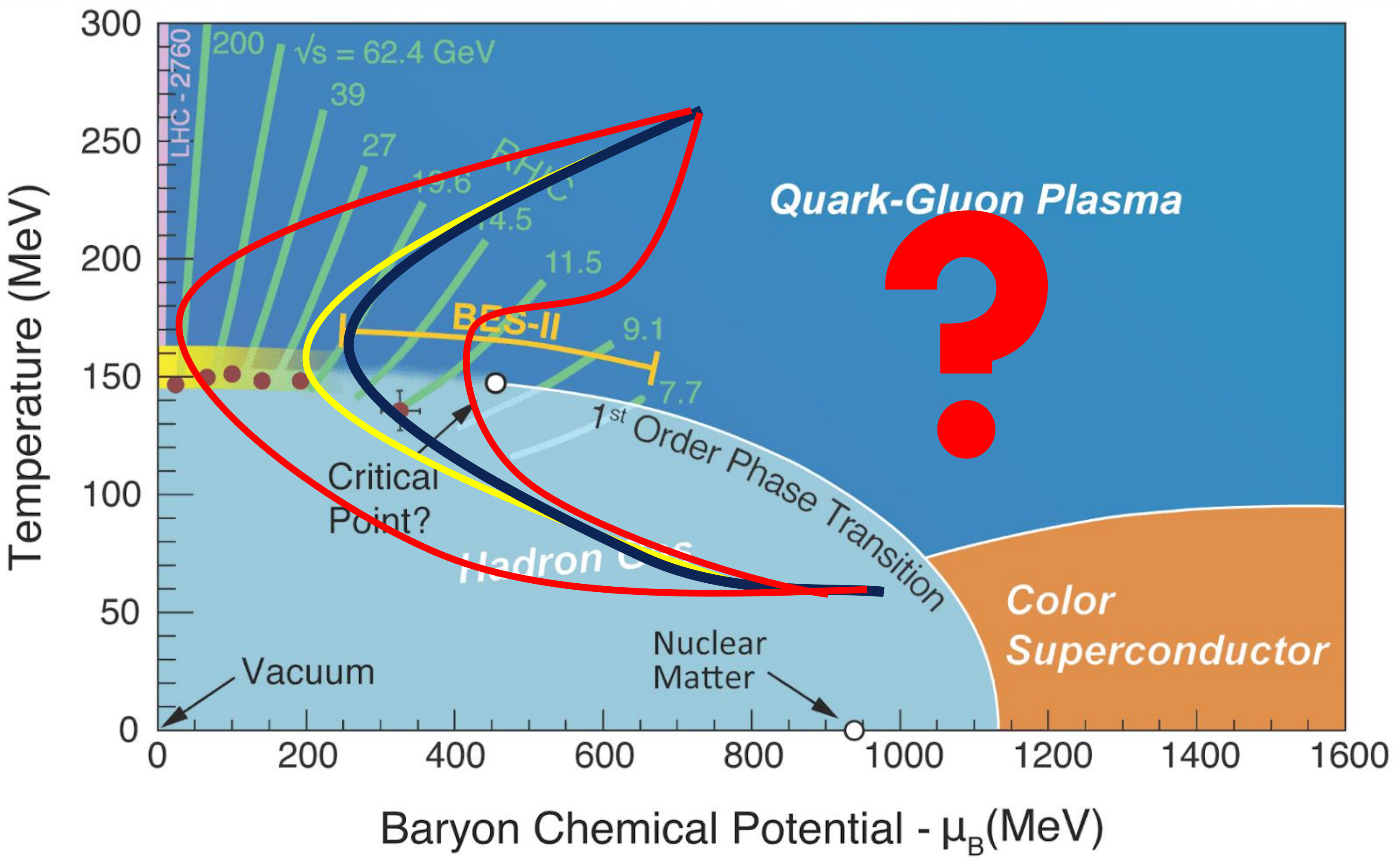


$$dT|_{\mu_B} = \frac{d\varepsilon|_{\mu_B} c^2_{\mu_B}}{s} \quad d\mu_B|_T = \frac{dn|_T}{\chi_2}$$

$$c^2_{\mu_B}|_{crit} \to 0 \quad \chi_2|_{crit} \to \infty$$



THE BEHAVIOR OF T-μ_B TRAJECTORIES



SIMPLE MODEL, QUALITATIVE INVESTIGATION

Toy model: Bjorken Symmetric Flow

Highly symmetric scenario, functions of space and time become *only* functions of time

$$\text{e.g. } \epsilon(\tau, \vec{x}) = \epsilon(\tau)$$

Coupled PDE'S become coupled ODE's

$$\dot{\epsilon} = -\frac{1}{\tau}\left[\epsilon + p + \Pi - \pi_\eta^\eta\right]$$

Energy Conservation

$$\rho(\tau) = \frac{\rho_0 \tau_0}{\tau}$$

Charge Conservation

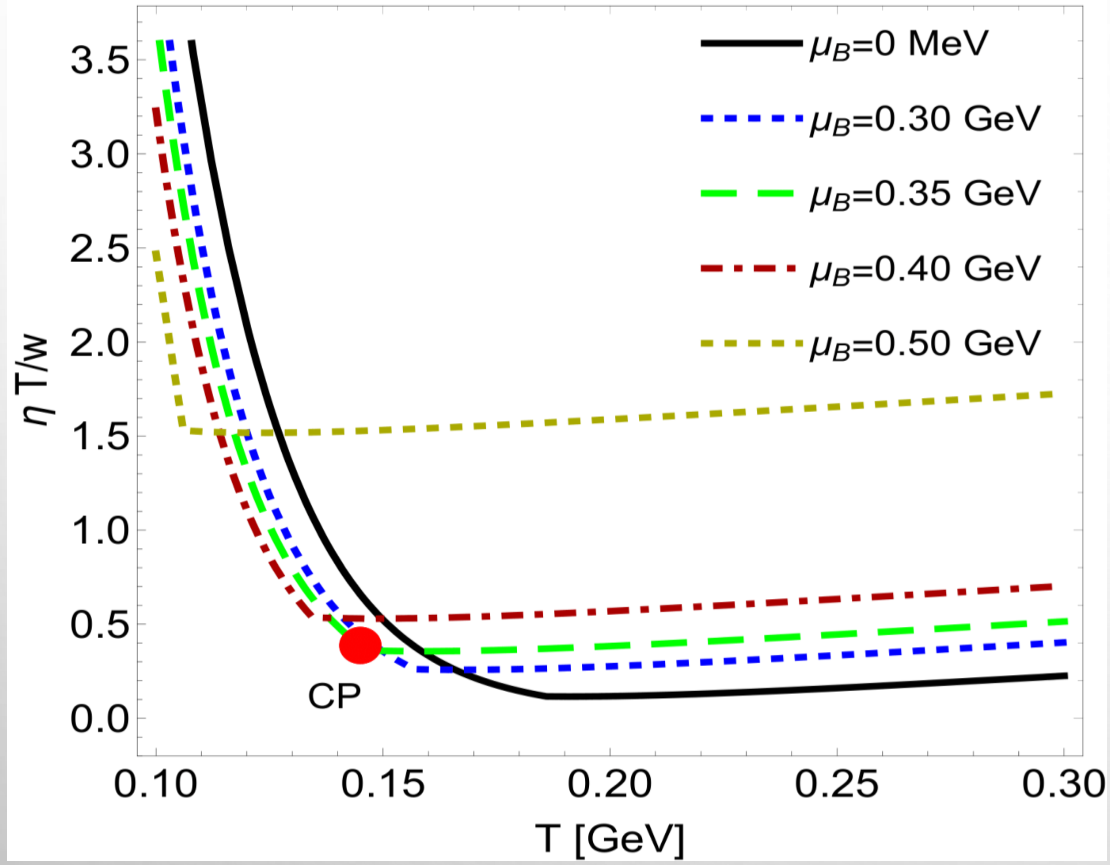
Viscous Relaxation type equations

Denicol et al. *Phys. Rev. D* 85 (2012 11407

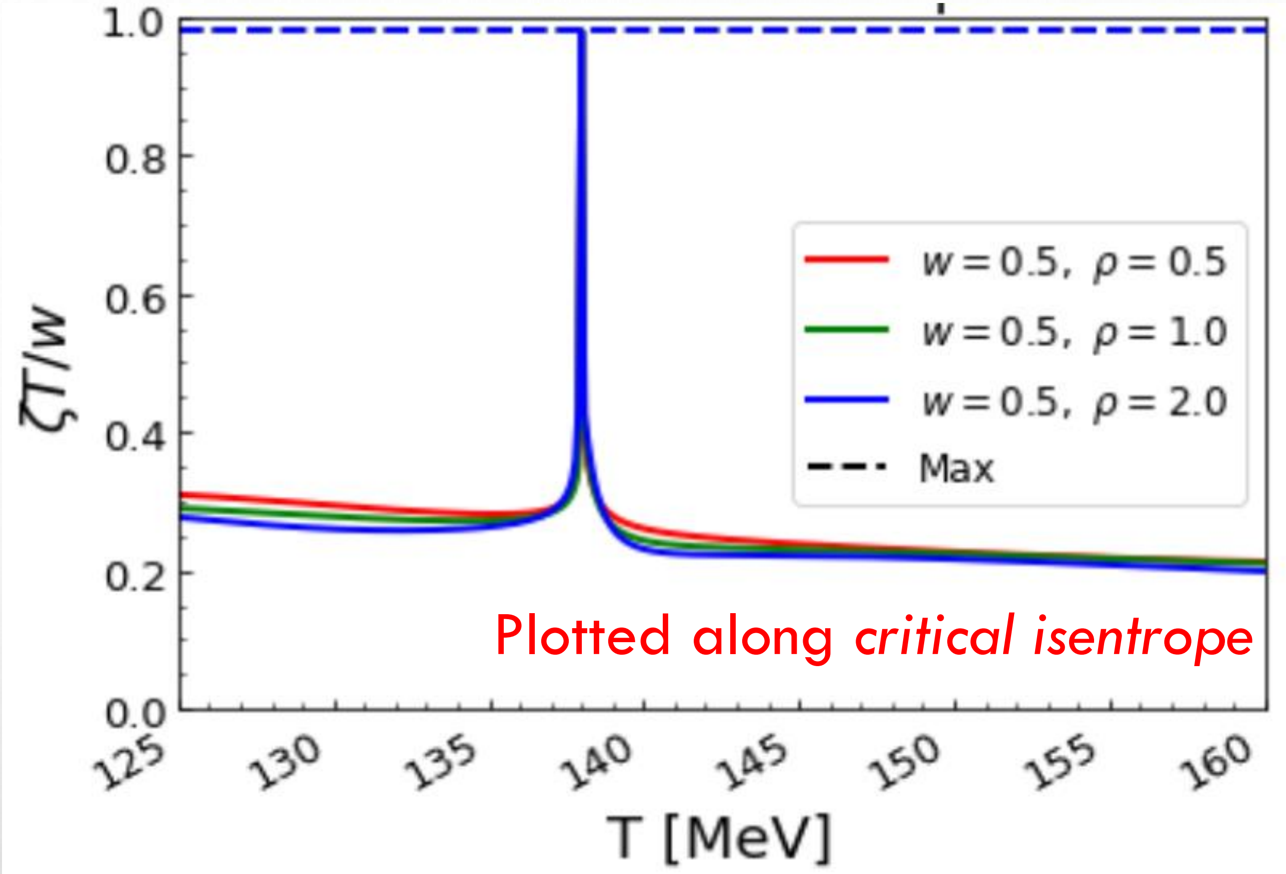
$$\tau_\pi \dot{\pi}_\eta^\eta + \pi_\eta^\eta = \frac{1}{\tau}\left[\frac{4\eta}{3} - \pi_\eta^\eta\left(\delta_{\pi\pi} + \tau_{\pi\pi}\right) + \lambda_{\pi\Pi}\Pi\right]$$

$$\tau_\Pi \dot{\Pi} + \Pi = -\frac{1}{\tau}\left(\zeta + \delta_{\Pi\Pi}\Pi + \frac{2}{3}\lambda_{\Pi\pi}\pi_\eta^\eta\right)$$

Transport Coefficients



TD,E. McLaughlin, J. Noronha-Hostler, *Phys.Rev.*D 102 (2020) 7



Shear viscosity not sensitive to criticality explicitly

Critically Scaled Bulk:

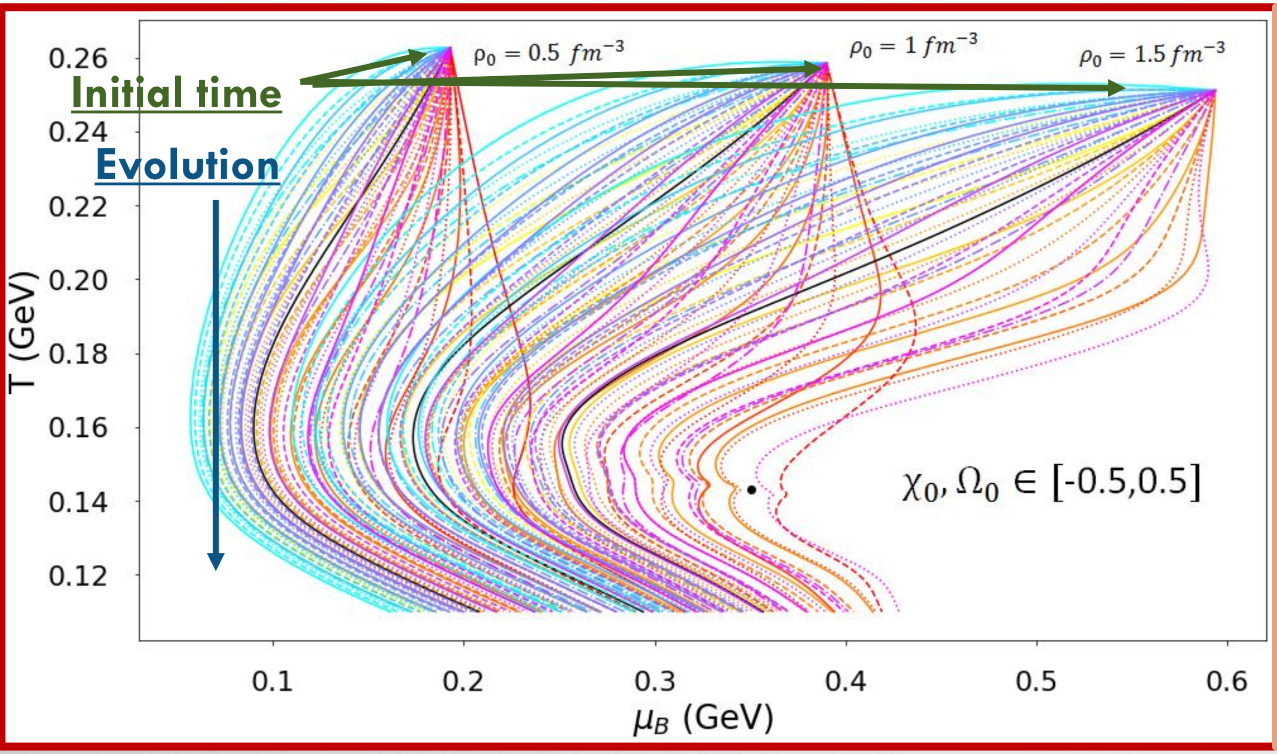
$$\left(\frac{\zeta T}{w}\right)_{CS} = \frac{\zeta T}{w}\left[1 + \left(\frac{\xi}{\xi_0}\right)^3\right]$$

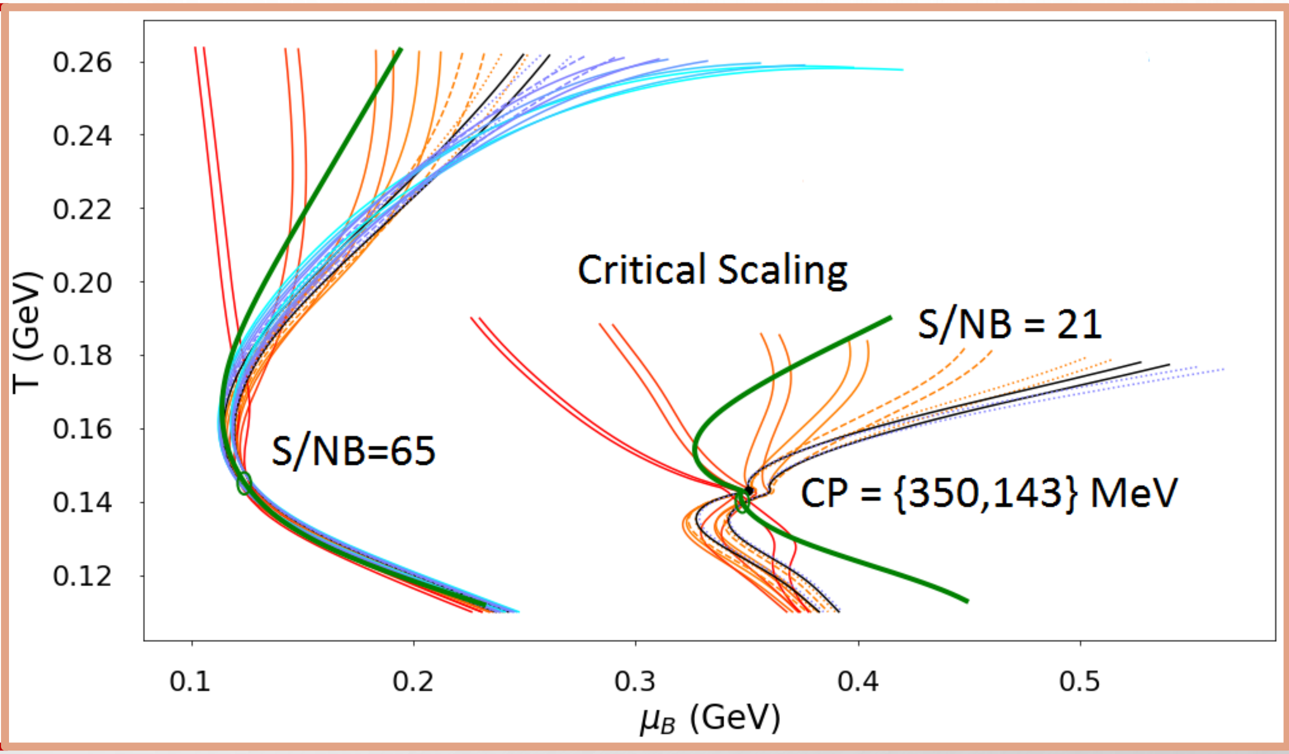
Monnai, Akihiko et al, Nucl. Phys. ,A967,2017

$$\chi_0 = \frac{\pi}{\epsilon+p}, \Omega_0 = \frac{\Pi}{\epsilon+p}$$

$$\{T(\tau), \mu_B(\tau)\}$$

Green Line: Equilibrium Hydro trajectory





TD,E. McLaughlin, J. Noronha-Hostler, *Phys.Rev.*D 102 (2020) 7

Takeaways:

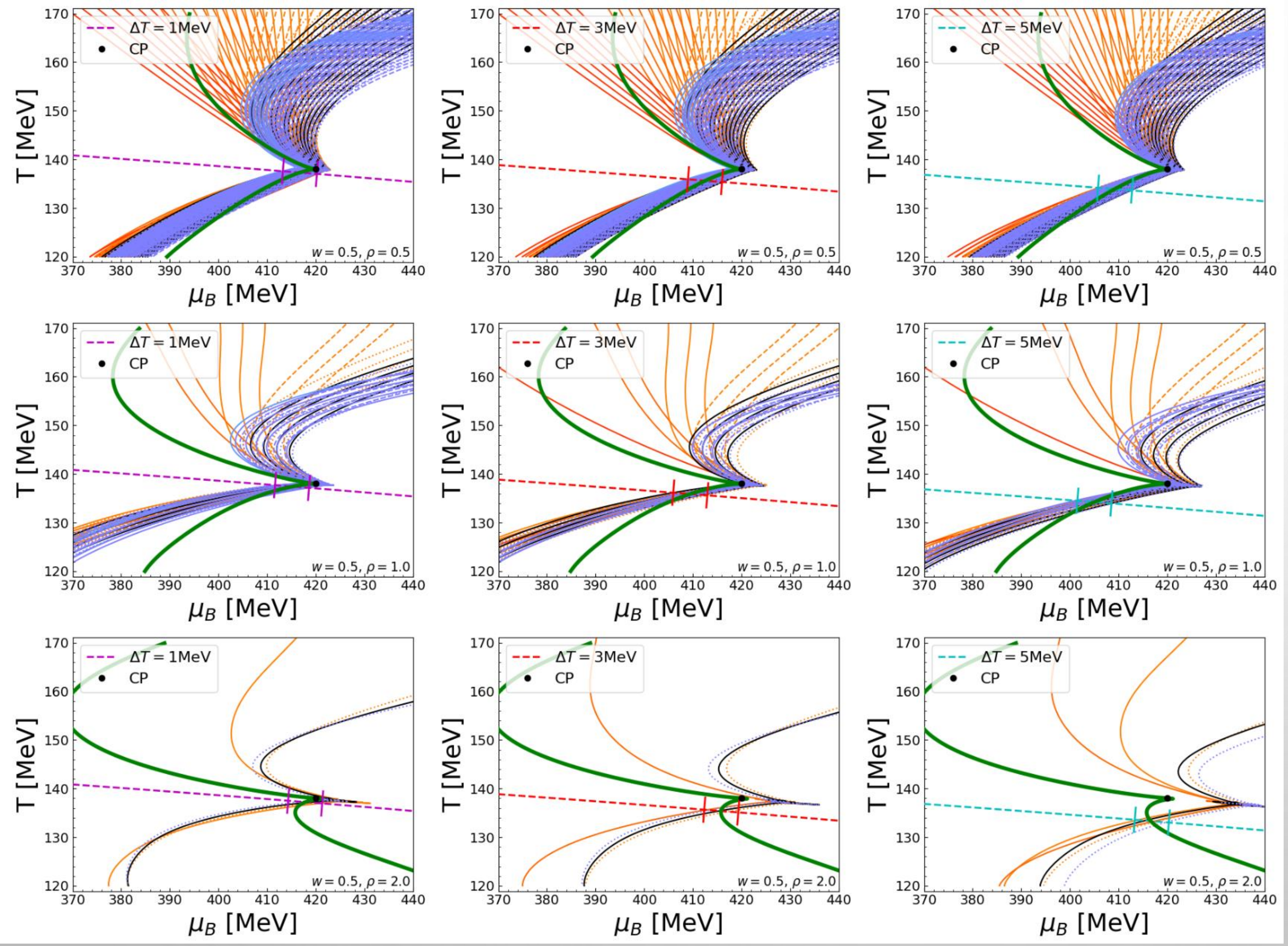
1. Pushed to or away from CP on event-by-event basis
2. Degeneracy of final state mapping to initial state

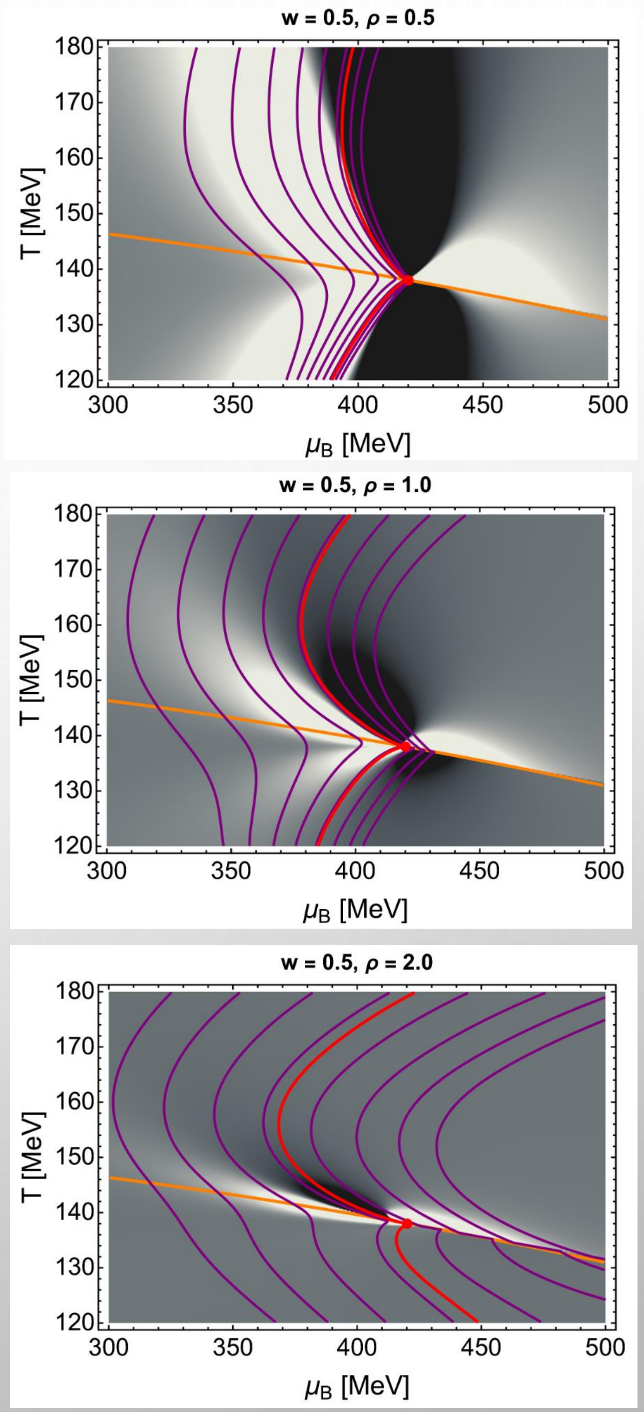
How does the kurtosis behave at freeze-out in the critical region?

PROCEDURE

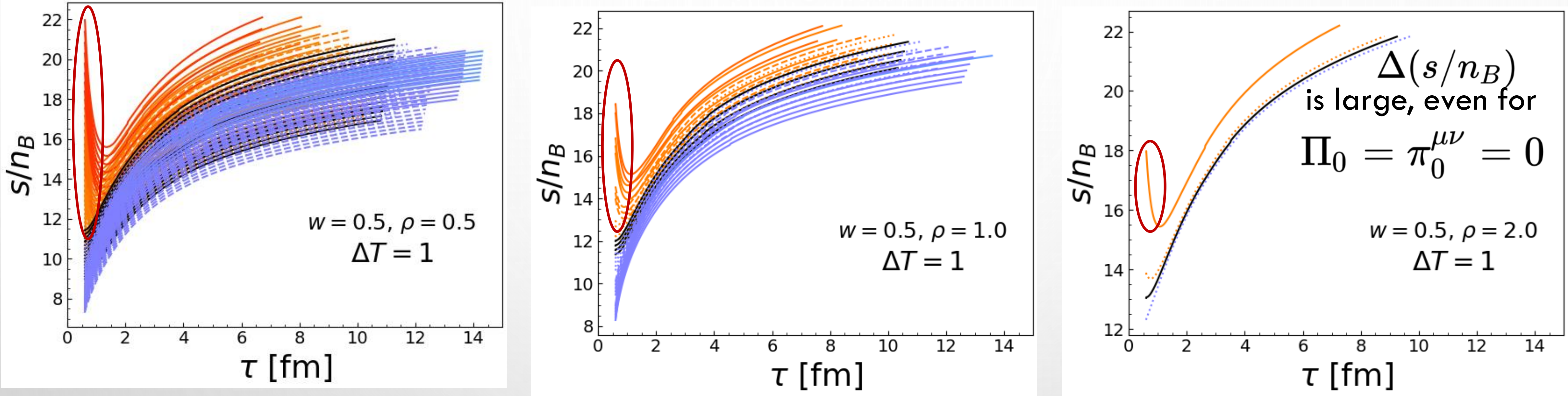
- INTIALIZE MANY DIFFERENT HYDRODYANMIC TRAJECTORIES SYSTEMATICALLY FROM A LIST OF n_{B_0} , Π_0, $\pi_0^{\mu\nu}$ (SAME ENERGY DENSITY)
- SELECT ON TRAJECTORIES THAT PASS THROUGH FREEZE-OUT WINDOW, CENTERED ON THE ISENTROPE THAT GOES THROUGH THE CRITICAL POINT, AND ALONG SHIFTED TRANSITION PARABOLAS
- REPEAT PROCEDURE FOR MANY DIFFERENT REALIZATIONS OF THE EQUATION OF STATE

SIZE AND SHAPE OF REGION IS IMPORTANT





IS IDEAL HYDRODYNAMICS A 'GOOD ENOUGH' APPROXIMATION?



"Thermal" entropy production, violation of second law?

$$\partial_\mu (s u^\mu) = \frac{1}{T}(\pi^{\mu\nu}\sigma_{\mu\nu} - \Pi\theta) < 0\ ?$$

Clearly negative for

$$\pi^{\mu\nu}\sigma_{\mu\nu} < 0,\ \Pi\theta > 0$$

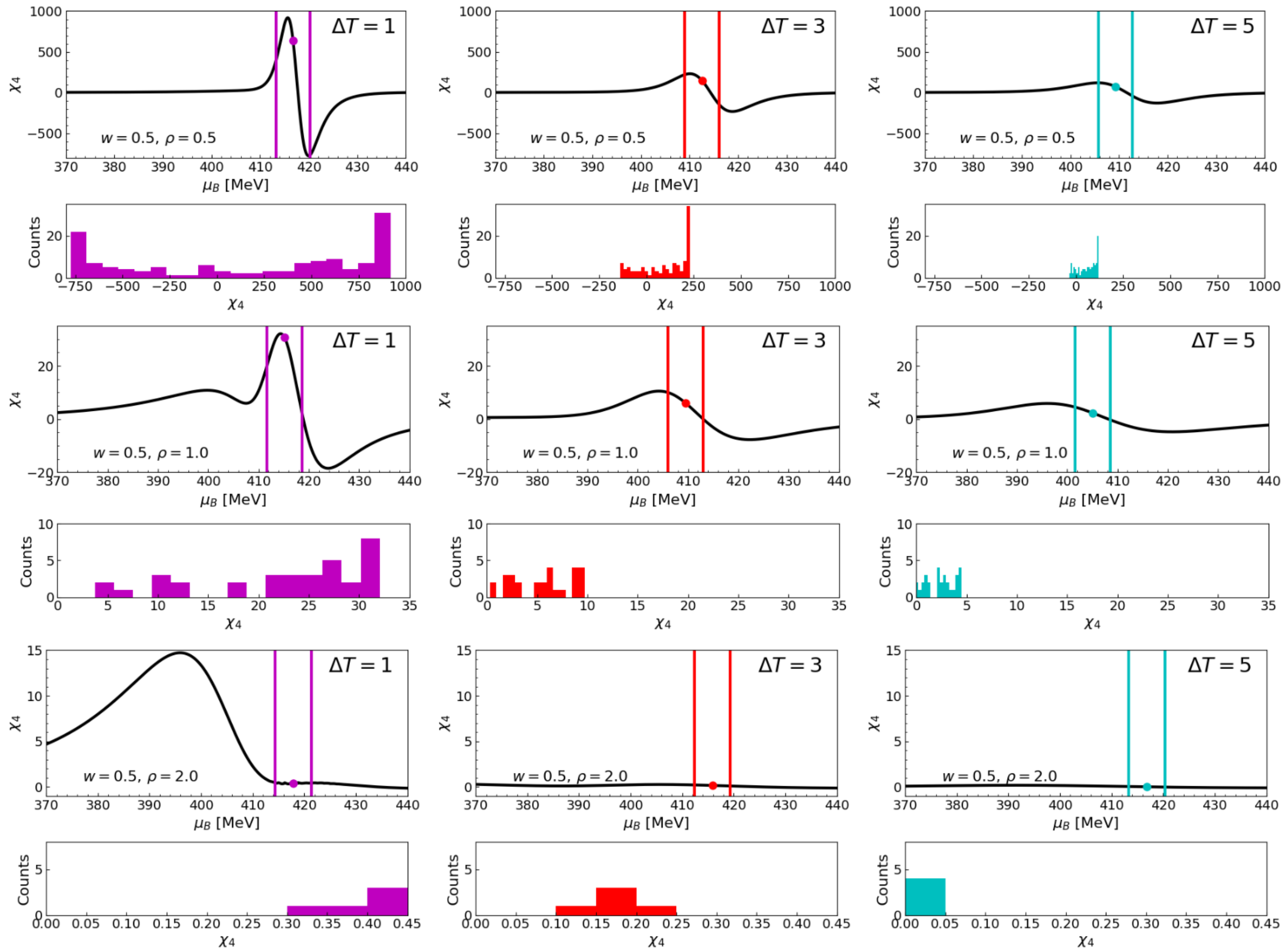
Real entropy production:

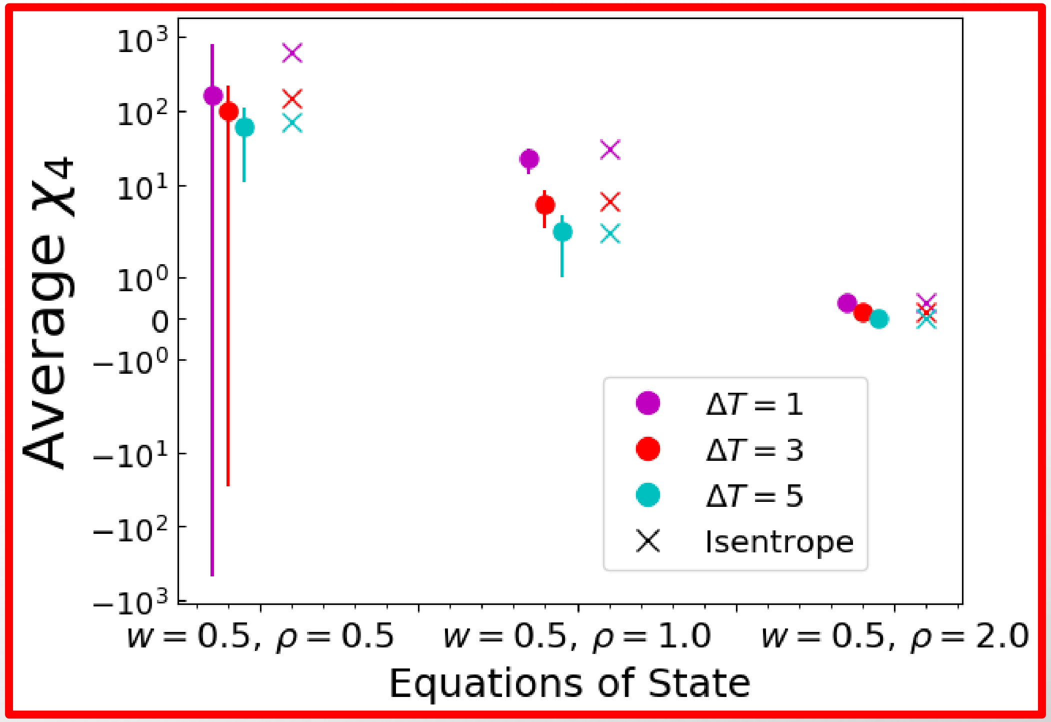
$$\partial_\mu S^\mu \approx s u^\mu - \beta_\Pi \Pi\dot{\Pi} - \beta_\pi \pi^{\mu\nu}\dot{\pi}_{\mu\nu} > 0\ !$$

Recent work has confirmed this conjecture

C. Chattopadhyay, U. Heinz, T, Schaefer, arXiv: 2209.10483 [hep-ph].

NON-TRIVIAL DISTRIBUTIONS OF FOURTH MOMENT

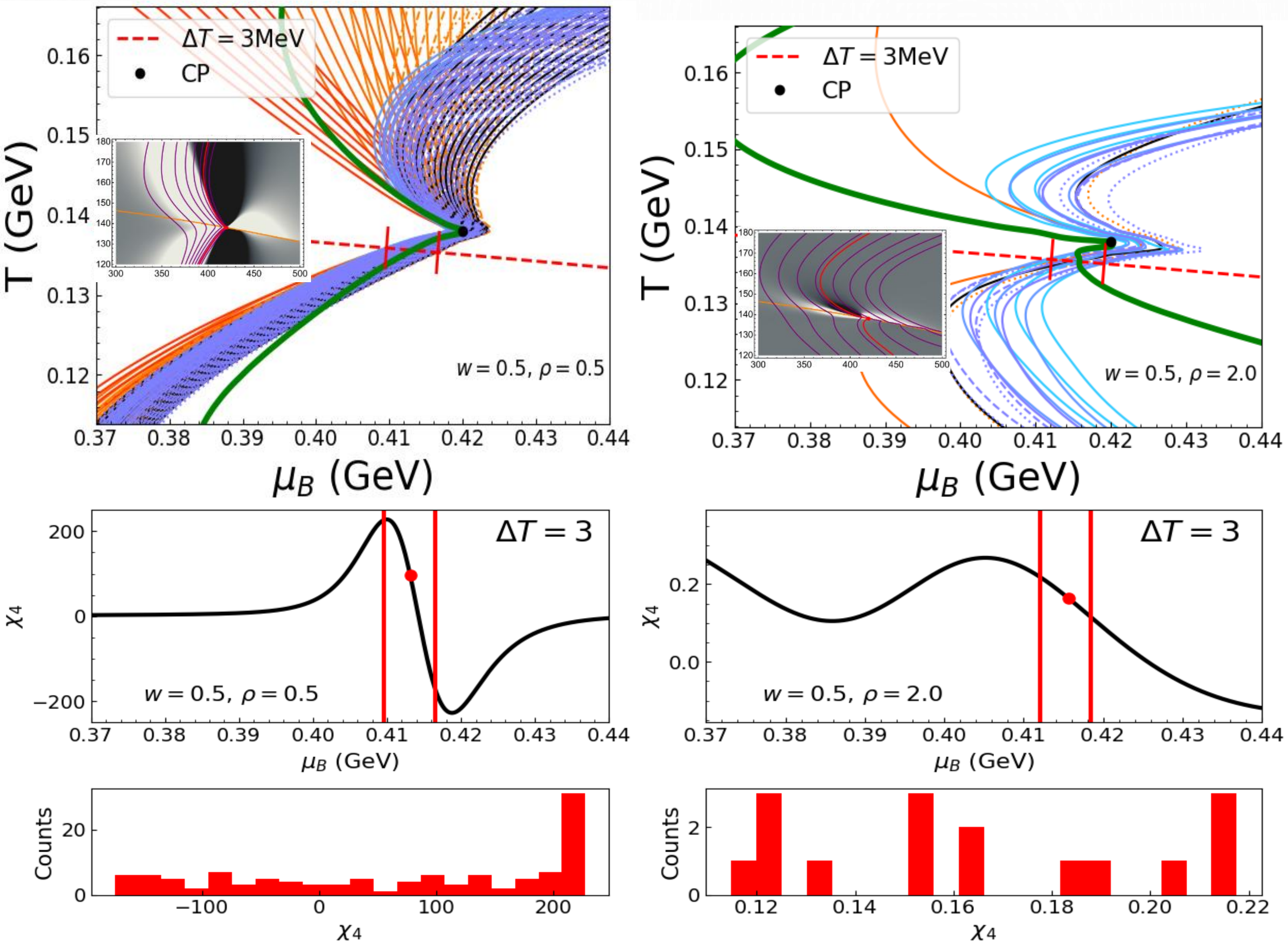




While the spread may be large when there are many trajectories, average values remain close to the central isentropic value.

A tight freeze-out window makes this possible

DYNAMIC LENSING AND KURTOSIS



On the left, trajectories *pulled* to larger values of χ_4

Lensing effect may persist for strong viscous corrections

Very sensitive to EoS parameters

SUMMARY AND OUTLOOK

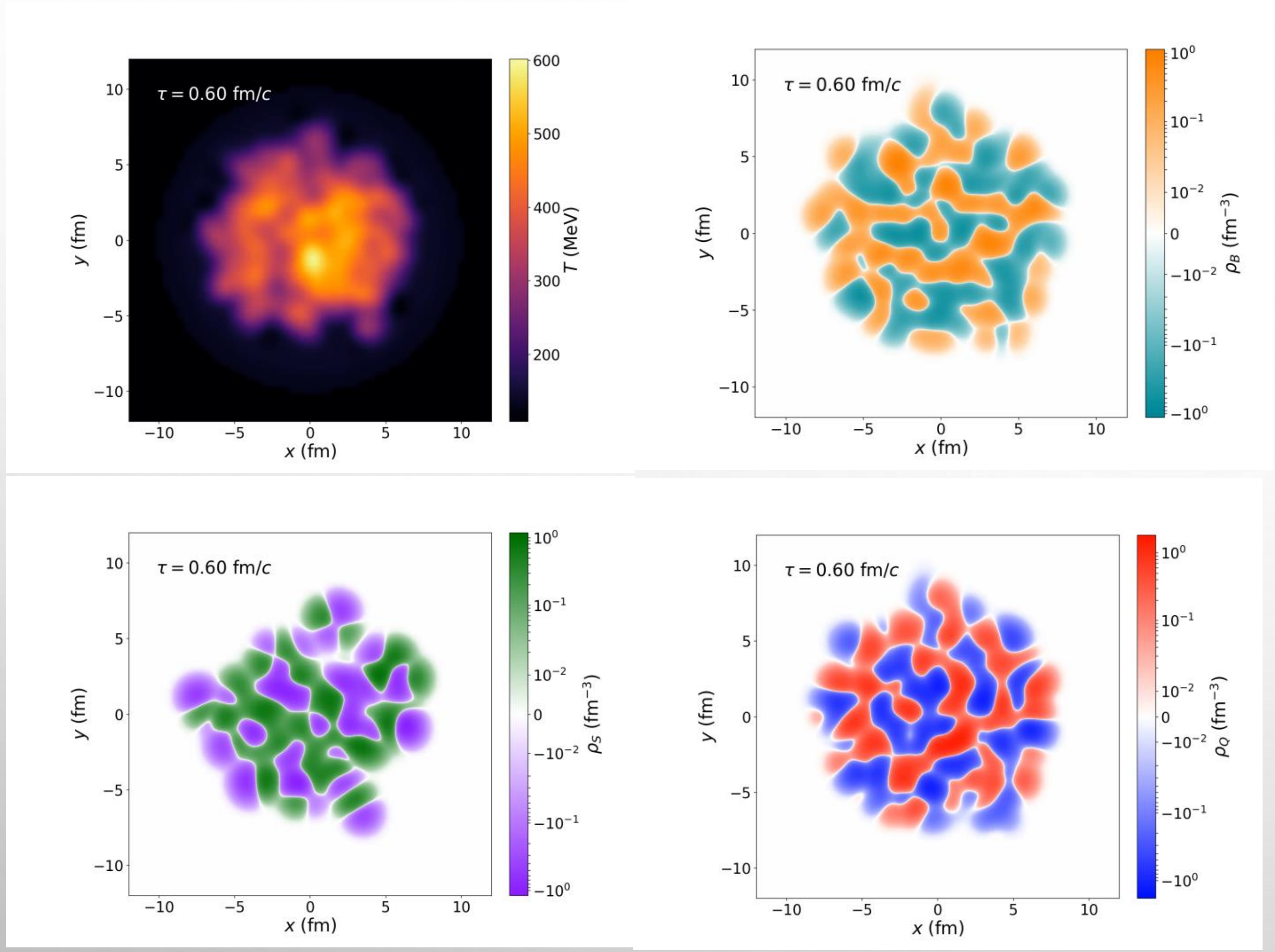
- Out of equilibrium effects will be very important to take into account in our search for the QCD critical point

- Work is ongoing to begin modelling charge dynamics in more realistic hydrodynamic models

- Models which include the initialization of out-of-equilibrium components will be a crucial part of our ability to unambiguously find critical behavior if it is there

BACKUP

WORK IN PROGRESS: ICCING+HYDRO



First code to implement this with realistic EoS

To-Do List:

$(0+1)\mathrm{D}$ ✔

$(2+1)\mathrm{D}$ ...

Can we study finite charge effects at the highest energies?

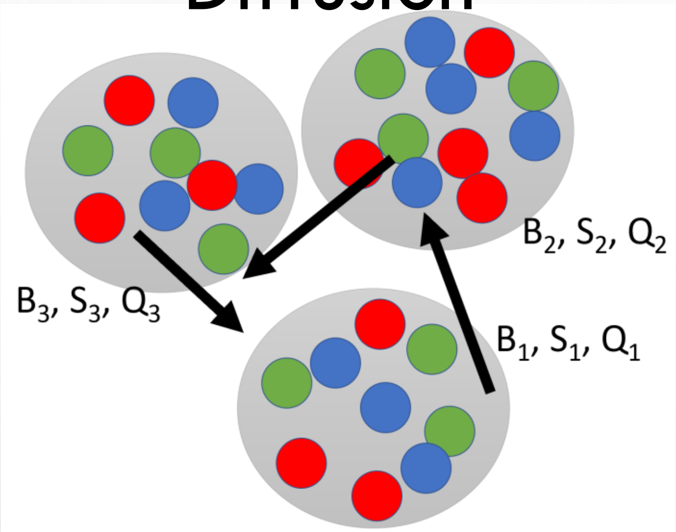
ICCING

Initial Conditions of Conserved Charges in Nuclear Geometry

P. Carzon, et al., *Phys.Rev.*C 105 (2022) 3, 034908
M. Martinez, et al., arXiv:1911.10272

WORK IN PROGRESS

Coupled Diffusion



4D Equation of State

$$\{T, \mu_B, \mu_S, \mu_Q\} \to \{\epsilon, \rho_B, \rho_S, \rho_Q\}$$

Complicated equations of motion

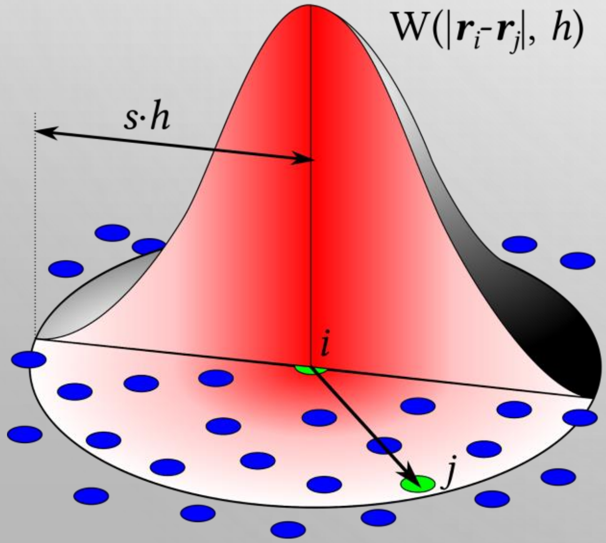
$$\tau_\Pi \dot{\Pi} + \Pi = -\zeta\theta - \frac{\zeta}{2\beta}(\Pi\dot{\beta}_\Pi + \beta_\Pi \Pi\theta + n^l_\mu \nabla^\mu \gamma^l_0) - \frac{\zeta\gamma^l_0}{\beta}\partial_\mu n^\mu_l$$

$$\tau_{lm}\dot{n}^\mu_m + n^\mu_l = -\kappa_{lm}\nabla^\mu \alpha_m - \frac{\tau_{lm}}{2\beta} n^\mu_m \theta - \frac{\kappa_{lm}}{2\beta}\dot{\beta}_{mn} n^\mu_n$$

$$- \frac{\kappa_{lm}}{\beta}\left(\gamma^m_0 \nabla^\mu \Pi + \frac{\Pi}{2}\nabla^\mu \gamma^m_0 + \gamma^m_1 \partial_\nu \pi^{\mu\nu} + \frac{\pi^{\mu\nu}}{2}\nabla_\nu \gamma^m_1\right)$$

$$\tau_\pi \dot{\pi}^{\mu\nu} + \pi^{\mu\nu} = 2\eta\sigma^{\mu\nu} - \frac{\eta}{\beta}(\pi^{\mu\nu}\dot{\beta}_\pi$$

$$+ \beta_\pi \pi^{\mu\nu}\theta + n^{\langle\mu}\nabla^{\nu\rangle}\gamma^l_1) - \frac{2\eta\,\gamma^l_1}{\beta}\nabla^{\langle\mu} n^{\nu\rangle}\gamma^l_1$$



Smoothed Particle Hydrodynamic Formalism

TOWARDS A COMPLETE MODEL

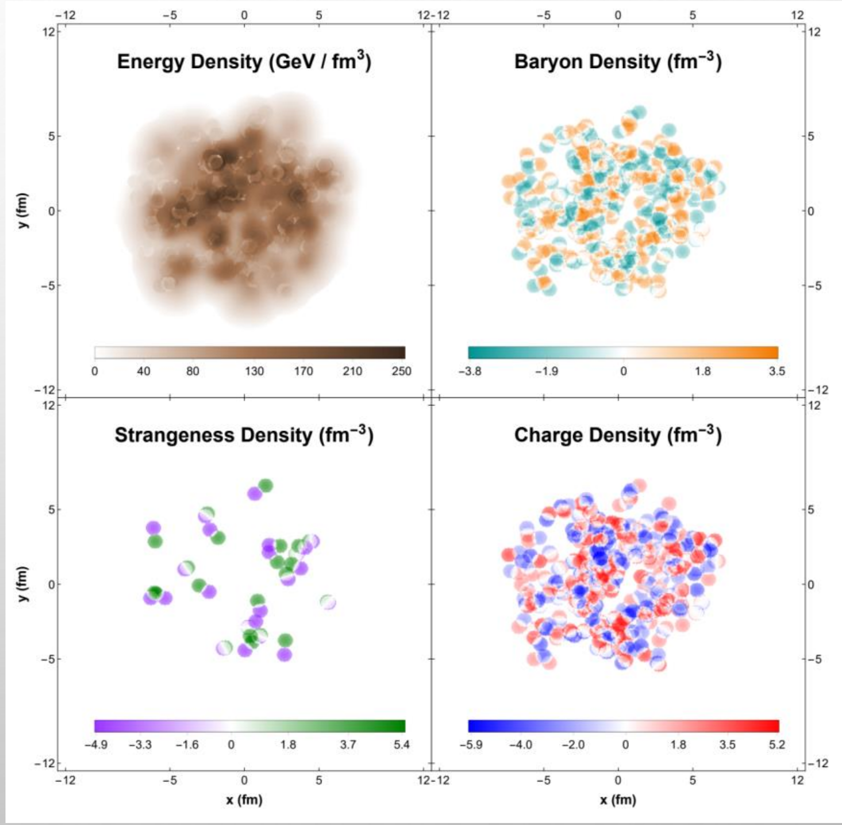
Start with what we know and build our way up

$(0+1)$D ☑ $(2+1)$D [...] $(3+1)$D ☐

Highest beam energies — Lower beam energies

Can we study finite charge effects at the highest energies?

<u>ICCING</u>: Initial Conditions of Conserved Charges in Nuclear Geometry

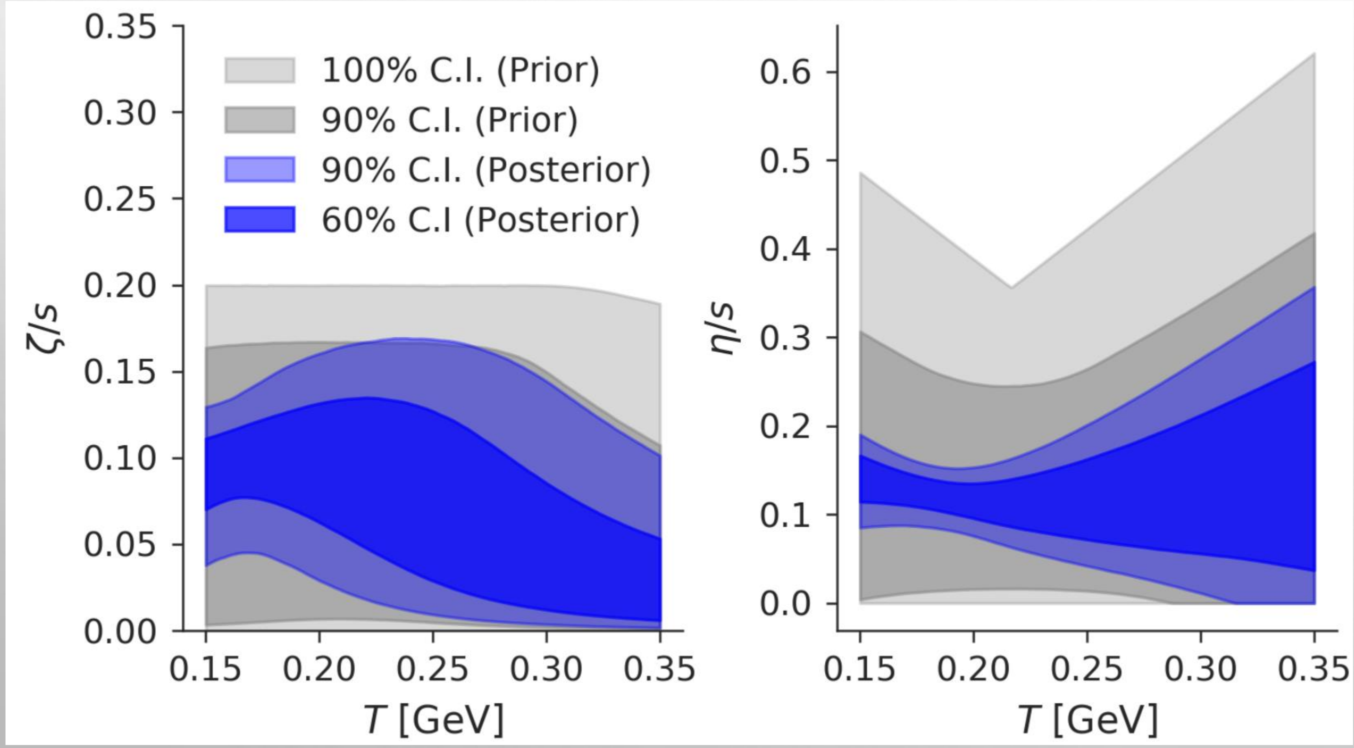


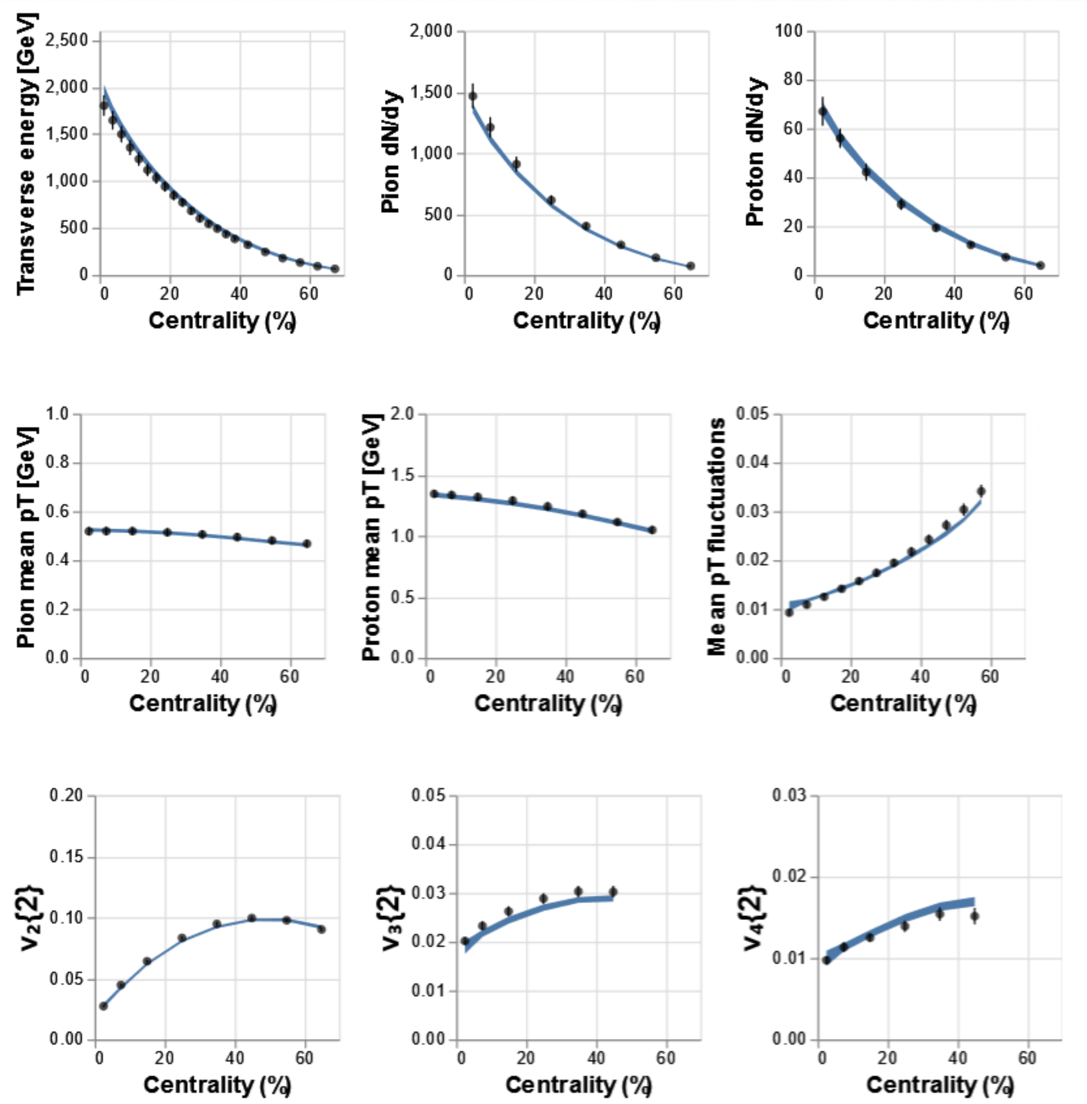
Local Fluctuations!

P. Carzon, et al., *Phys.Rev.*C 105 (2022) 3, 034908
M. Martinez, et al., arXiv:1911.10272

CAN WE REALLY EXTRACT NON-EQUILIBRIUM PROPERTIES?

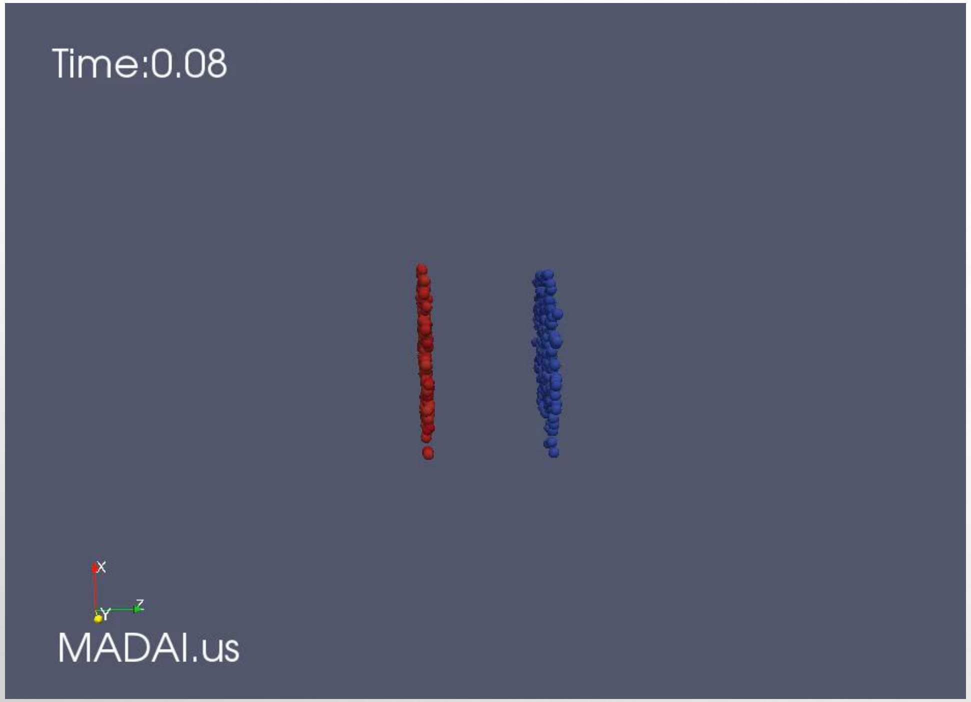
Yes! Quite systematically using tools of Bayesian Analysis





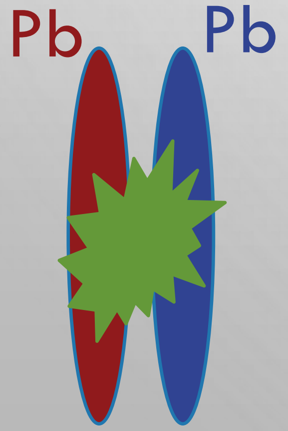
http://eg1.jetscape.wayne.edu:443/

WHAT IS A HEAVY-ION COLLISION? WHY STUDY THEM?



Born out of particle physics, heavy-ion collisions are *the way* to study high-energy, many-body, Quantum Chromodynamics

Prof. Donald W. Kerst with the world's first betatron, built at the University of Illinois in 1940



I will argue that research in HIC has connections to:

Condensed Matter Physics

Cosmology

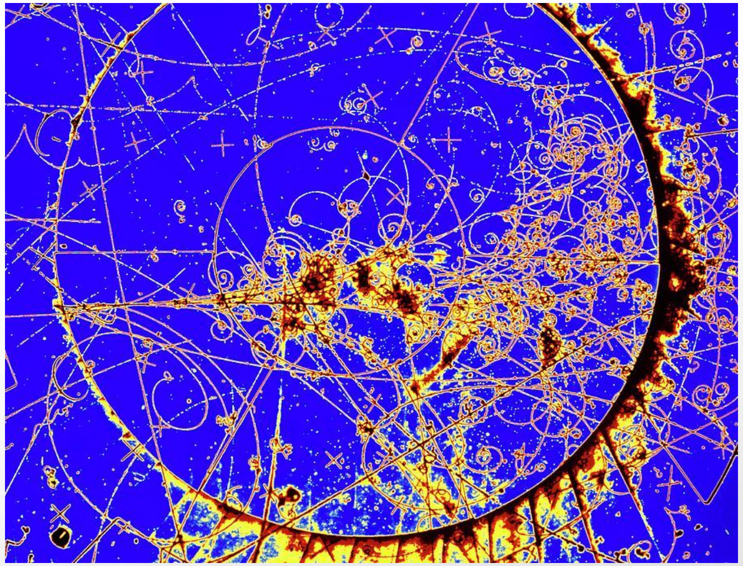
Particle Physics and Field Theory

Nuclear Astrophysics

And more!

SOME HISTORICAL CONTEXT…

Bubble Chamber Particle Shower



Imagine: You are Enrico Fermi, trying to come up with some way to explain particle production **without QCD or confinement**

Thought Experiment:

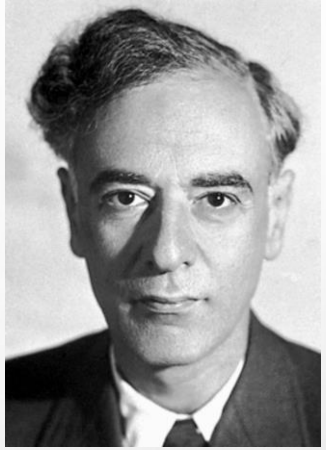
$T = 0$	$T = T'$	$T \gg m_\pi$
$\|0\rangle$	$n_\gamma \sim T^3$ (Black Body)	?

"THERMAL" COLLISIONS

- Produced particles thermally (black body) – Fermi
- Extremely energy dense, *hydrodynamic evolution – Landau*
- Freeze out T ~ 150 MeV – Pomeranchuk

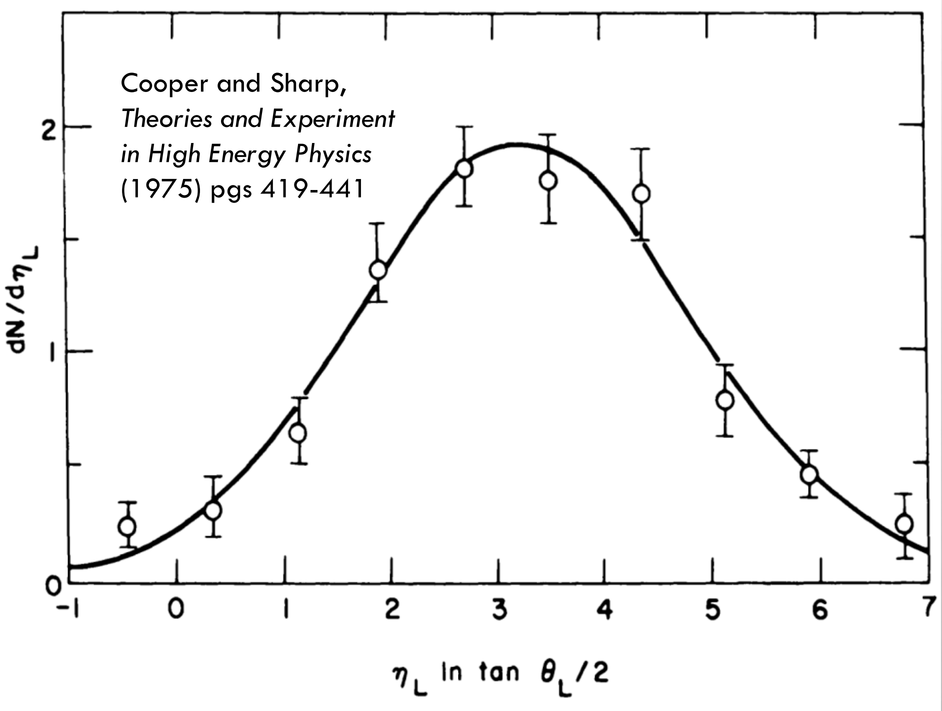
The Fermi*-Pomeranchuk-Landau Picture





*Fermi (1950) arguably preceded by Weisskopf, Phys. Rev. 52, 295 (1937) and Koppe, Phys. Rev. 76, 688 (1949)

Explained particle production from experiment well!



To this day, this qualitative picture remains