Scattering from Lattice QCD

Journal club talk

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Department of Physics
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Outline

Why scattering (from LQCD)?

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We want to have a first principles way of studying nuclear reactions (lattice QCD!).

Scattering in Quantum Mechanics

SCATTERING IN NON-RELATIVISTIC QM

Schrödinger equation:

$$\Psi(\mathbf{r}, t) = \psi(\mathbf{r}) e^{iEt/\hbar} \qquad \left[-\frac{\hbar^2}{2m}\nabla^2 + V(\mathbf{r})\right]\psi(\mathbf{r}) = E\psi(\mathbf{r})$$

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$$\psi(\mathbf{r}) \propto e^{ikx} + f(\theta, \phi)\frac{e^{ikr}}{r}$$

$f(\theta, \phi)$ is called scattering amplitude.

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$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = |f(\theta,\phi)|^2 \qquad \sigma_{tot} = \frac{4\pi}{k}\,\mathrm{Im}\,f(0)$$

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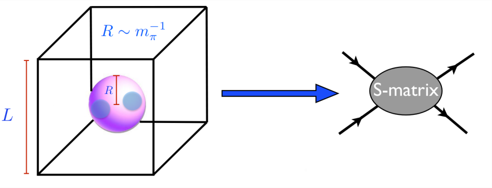
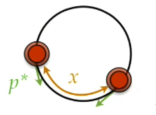


Can finite-volume quantities be related to (infinite-volume) scattering amplitudes?

Lüscher’s formalism

Lüscher's quantization condition

2 particles in 1D QM with a scattering potential Lüscher 1986



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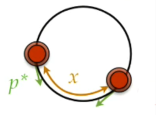

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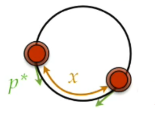

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$$\boxed{\det\left[F^{-1}(E_n; L) + \mathcal{M}\right] = 0}$$

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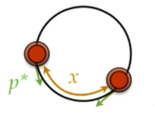

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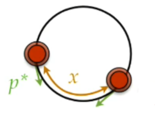

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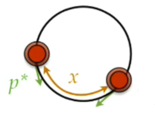

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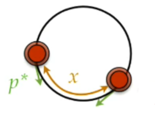

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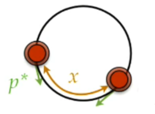

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Spectrum (lattice QCD) $\longrightarrow$ scattering amplitudes

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The finite-volume object F (for a single channel)

$$F_{lm;l'm'} = \left[\frac{1}{L^3}\sum_{\mathbf{k}} - \int \frac{d\mathbf{k}}{(2\pi)^3}\right] \frac{4\pi Y_{lm}(\hat{\mathbf{k}}^*) Y^*_{l'm'}(\hat{\mathbf{k}}^*)}{2\omega_k 2\omega_{Pk}(E-\omega_k-\omega_{Pk}+i\epsilon)} \left(\frac{k^*}{q^*}\right)^{l+l'}$$

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For elastic scattering of spinless particles with $\mathbf{P} = 2\pi\mathbf{n}/L$

$$F_{lm;l'm'} = \frac{2iq^*}{16\pi E^*}\left[\delta_{ll'}\delta_{mm'} + \frac{i}{\gamma\pi^{3/2}}\sum_{\bar{l},\bar{m}} v_{lm\bar{l}\bar{m}l'm'}\left(\frac{q^*L}{2\pi}\right)^{-(\bar{l}+1)} \mathcal{Z}^{\mathbf{n}}_{\bar{l}\bar{m}}\left(1;\left(\frac{q^*L}{2\pi}\right)^2\right)\right]$$

$\mathcal{Z}^{\mathbf{n}}_{\bar{l}\bar{m}}$ is the so-called Luscher zeta function.

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$$\det\left[\begin{pmatrix} F_{00} & F_{01} & \dots \\ F_{10} & F_{11} & \dots \\ \vdots & \vdots & \ddots \end{pmatrix}^{-1} + \begin{pmatrix} \mathcal{M}_S & \mathcal{M}_{SD} & \\ \mathcal{M}_{DS} & \mathcal{M}_D & \dots \\ \vdots & \vdots & \ddots \end{pmatrix}\right] = 0$$

Hadrons on the lattice

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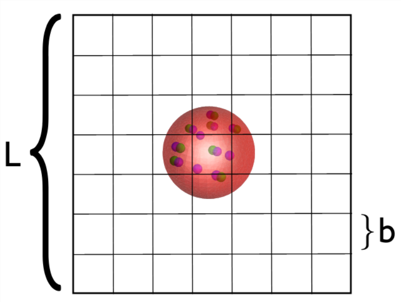

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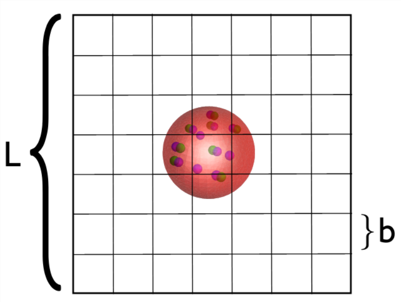

b: lattice spacing.

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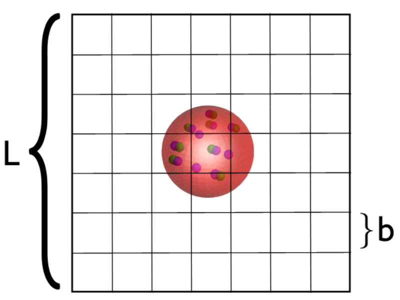

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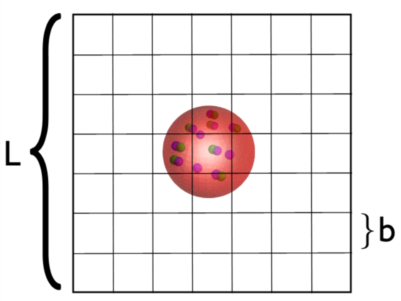

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- Typical volume corrections $\sim e^{-m_\pi L}$.

Spectra from LQCD

From lattice QCD

$$\langle \mathcal{O} \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}U \, \mathcal{O} \det M(U) e^{-S_g[\bar{U}]}$$

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For instance $\langle \mathcal{O}_{\pi^-} \mathcal{O}_{\pi^+} \rangle =$ Wick contractions...

Lattice simulations (for few-body physics)

LQCD SIMULATIONS: BINDING ENERGIES

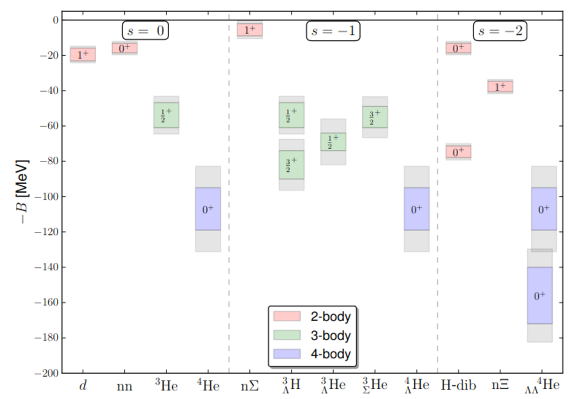


Figure 1: Binding energy of light nuclei and hyper nuclei. $N_f = 3$, using clover discretization and three lattice volumes $L \sim 3.4$ fm, 4.5 fm and 6.7 fm. Plot from Beane, Chang, S. D. Cohen, et al. 2013.

LQCD SIMULATIONS: PHASE SHIFTS

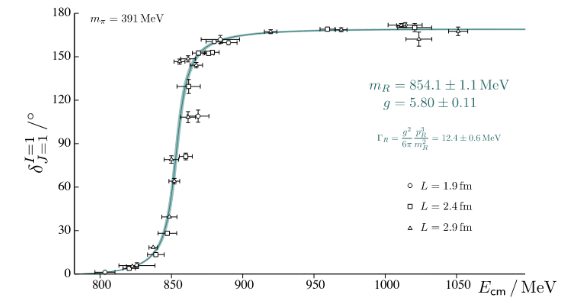
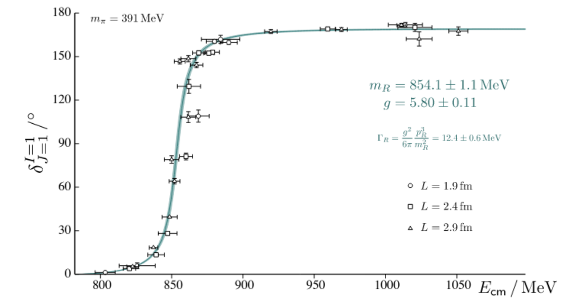


Figure 2: Isospin-1, P-wave $\pi\pi$ elastic scattering phase shift and Breit-Wigner parametrization for $m_\pi = 391$ MeV. Plot from Dudek et al. 2013.

LQCD simulations: phase shifts



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Although at unphysical quark masses!

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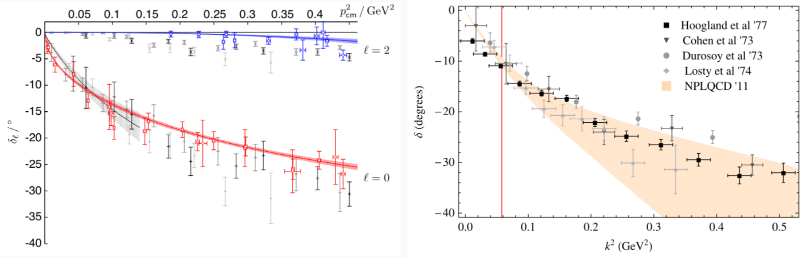

Figure 3: Left plot Dudek et al. 2012: $I = 2$ $\pi\pi$ elastic S-wave (red) and D-wave (blue) scattering phase shift (for $m_\pi = 396$ MeV). Shown in grey the experimental data. Right plot Beane, Chang, Detmold, et al. 2012: same process, but only $l = 0$.

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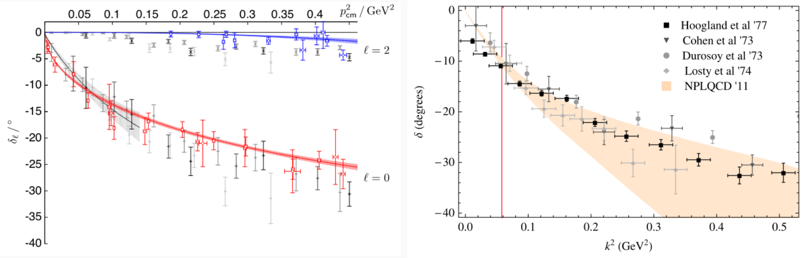

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Experimental data from Hoogland et al. 1977, D. Cohen et al. 1973, Zieminski et al. 1974 and Durusoy et al. 1973.

LQCD simulations: form factors

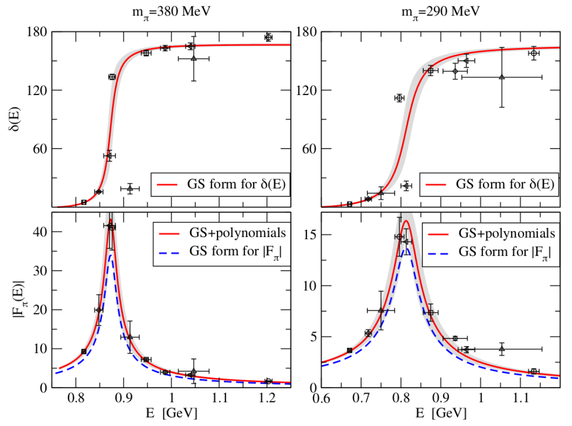

Figure 4: Upper panels: Scattering phases together with the fits to the GS form. Lower panels: Modulus of the pion form factor together with the GS-model curves (blue dashed) and the fits to (red solid). Plots from Feng et al. 2015.

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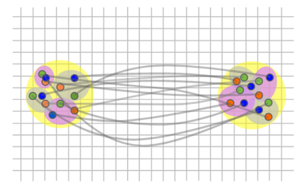
- Too many Wick contractions ($\sim n_u! n_d! n_s$)



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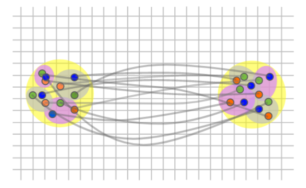

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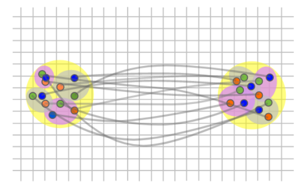

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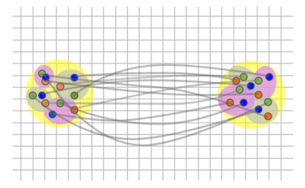

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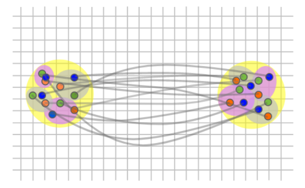

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State-of-art in the 3-body sector:

- Spinless particles
- Identical particles

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- Spin 0 particle $+$ spin $1/2$ particle (pion-nucleon) Göckeler et al. 2012;

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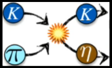
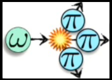
In the most general case of $\det_j[F^{-1} + \mathcal{M}] = 0$:

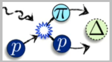
$$j = \{l, m_l, J, m_J, \text{channels}, \text{flavors}, ...\}$$

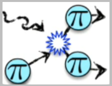
DEVELOPMENTS

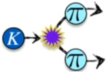
(LABELS: ✓ UNDER CONTROL ! PROGRESS MADE ? NOT WELL DEVELOPED)

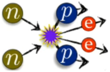












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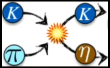

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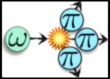

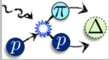

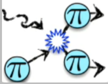

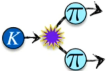

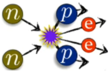

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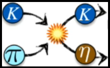

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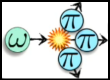

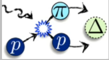

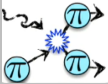

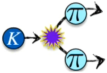

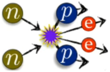

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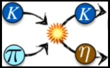

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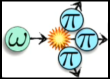

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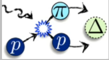

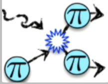

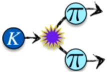

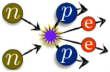

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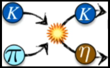

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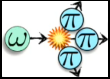

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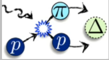

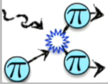

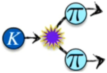

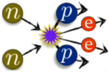

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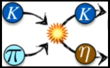

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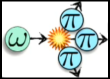

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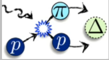

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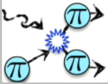

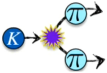

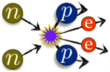

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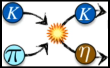

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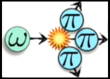

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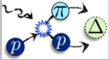

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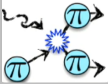

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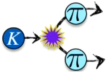

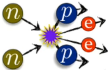

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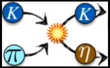

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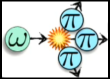

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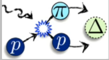

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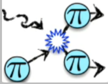

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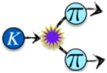

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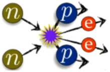

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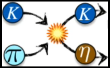

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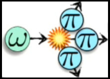

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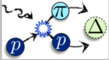

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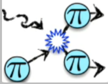

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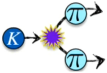

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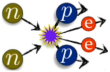

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From a talk by Raul Briceno at the Institute for Nuclear Theory, Seattle, 2014

Summary & Discussions

SUMMARY & DISCUSSIONS

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(Dean's) Questions

- Luscher's formalism at finite temperature?
- Best fermion actions for scattering? (Wilson? Domain wall? staggered?)
- Other methods for scattering from LQCD? (The potential method Aoki, Hatsuda, and Ishii 2010)

BIBLIOGRAPHY I

References

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