Critical point for strong background magnetic fields

[1504.08280]

Gergely Endrődi

University of Regensburg

XQCD @ Wuhan, 22. September 2015

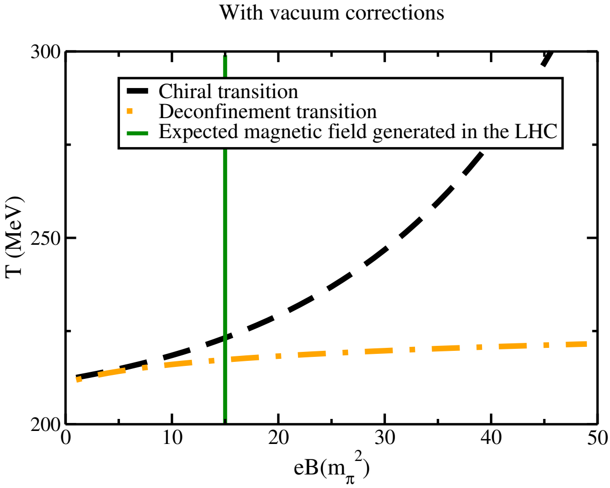
Outline

- introduction
  - a brief history of $B - T$ phase diagrams
  - open questions
- new lattice results
  - full QCD for strong magnetic fields
  - effective theory for $B \to \infty$ limit
- conclusions

Introduction

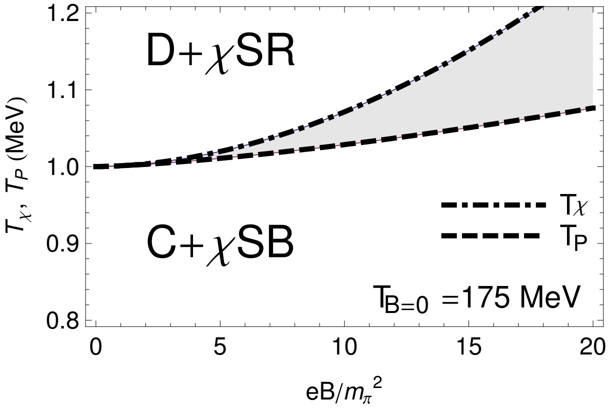
A brief history of $B - T$ phase diagrams

- 2010: linear σ model [Mizher, Chernodub, Fraga]



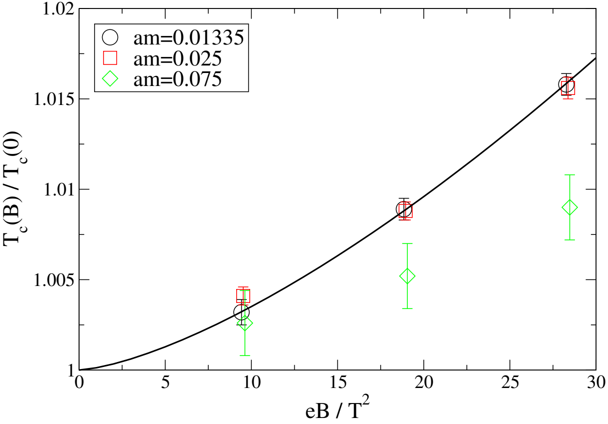
A brief history of $B - T$ phase diagrams

- 2010: PNJL model [Gatto, Ruggieri]



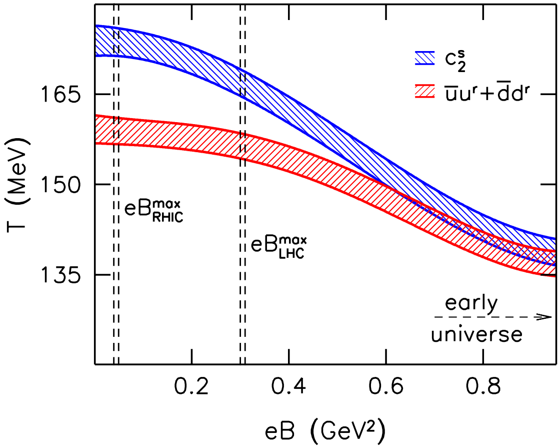
A brief history of $B - T$ phase diagrams

- 2010: lattice, coarse, heavy [D'Elia, Mukherjee, Sanfilippo]



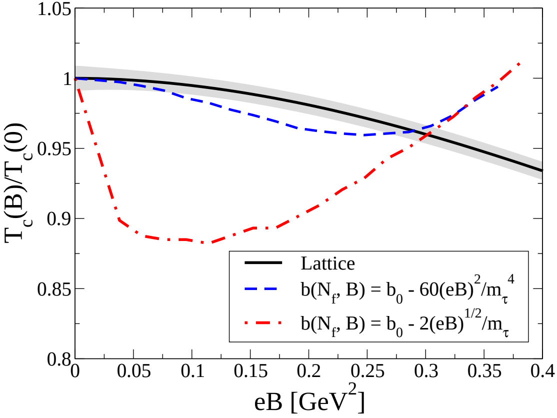
A brief history of $B-T$ phase diagrams

- 2011: lattice, cont.limit, physical
  [Bali, Bruckmann, Endrődi, Fodor, Katz, Krieg, Schäfer, Szabó]



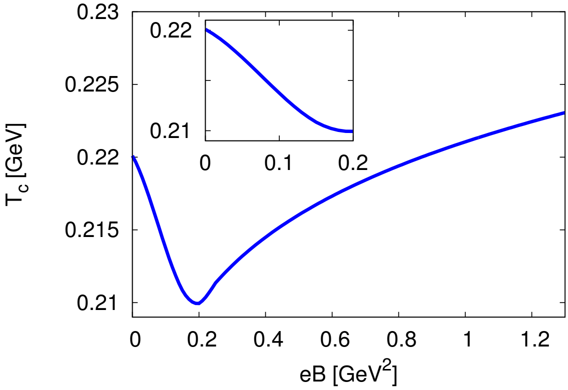
A brief history of $B - T$ phase diagrams

- 2014: parameterized models [Fraga, Mintz, Schaffner-Bielich]



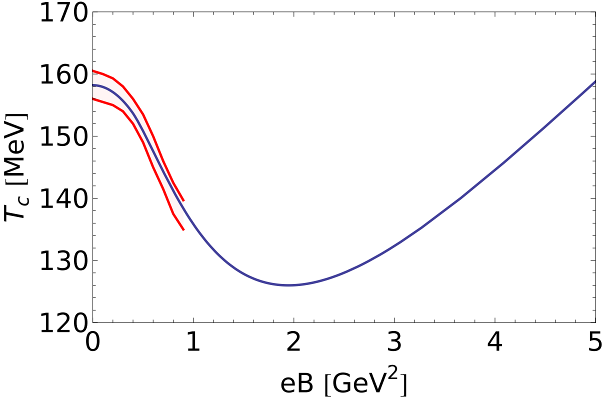
A brief history of $B - T$ phase diagrams

- 2014: FRG [Braun, Mian, Rechenberger]



A brief history of $B - T$ phase diagrams

- 2014: FRG+NJL [Mueller, Pawlowski]



Open questions

- for $eB < 1$ GeV^2 the phase diagram is known from lattice
  - $T_c(B)$ monotonously decreases
  - the transition is an analytic crossover
- what happens for $eB > 1$ GeV^2?
  - is there a turning point, where $T_c(B)$ starts increasing?
  - is there a splitting between the chiral/deconfinement transitions?
  - is there a splitting between the up/down chiral transitions?
  - does the transition become a real phase transition?

Open questions

- for $eB < 1$ GeV^2 the phase diagram is known from lattice
  - $T_c(B)$ monotonously decreases
  - the transition is an analytic crossover
- what happens for $eB > 1$ GeV^2?
  - is there a turning point, where $T_c(B)$ starts increasing?
  - is there a splitting between the chiral/deconfinement transitions?
  - is there a splitting between the up/down chiral transitions?
  - does the transition become a real phase transition?
- significance: guiding effective theories and low-energy models

Open questions

- for $eB < 1$ GeV^2 the phase diagram is known from lattice
  - $T_c(B)$ monotonously decreases
  - the transition is an analytic crossover
- what happens for $eB > 1$ GeV^2?
  - is there a turning point, where $T_c(B)$ starts increasing?
  - is there a splitting between the chiral/deconfinement transitions?
  - is there a splitting between the up/down chiral transitions?
  - does the transition become a real phase transition?
- significance: guiding effective theories and low-energy models
- aim: answer these questions using lattice simulations

Strategy

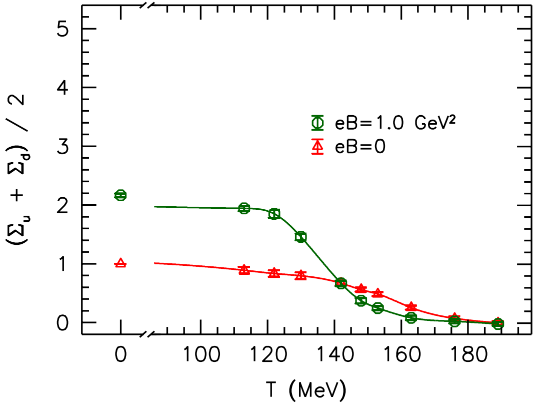
- largest possible field on a finite lattice is

$$eB_{\max} \approx a^{-2} \quad \Rightarrow \quad eB_{\max}/T^2 \approx N_t^2$$

- how to go even beyond?
  - exploit that eB is the largest scale and calculate the relevant effective theory
- strategy
  - simulate full QCD at $eB = 3.25\ \mathrm{GeV}^2$
  - simulate the effective theory at $B \to \infty$

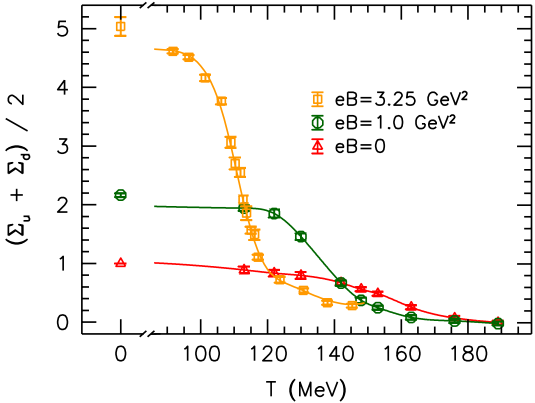
Lattice results – full QCD

Quark condensates



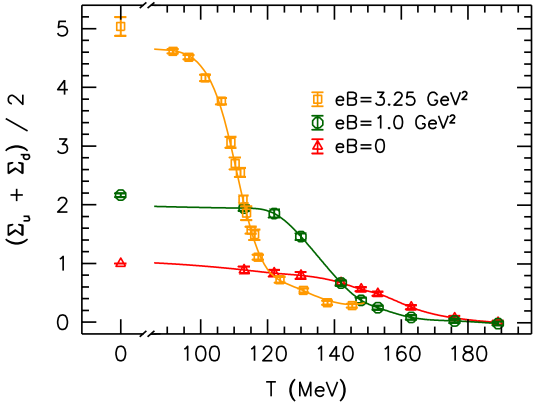
- average of up and down quark condensates:
  T_c=inflection point

Quark condensates



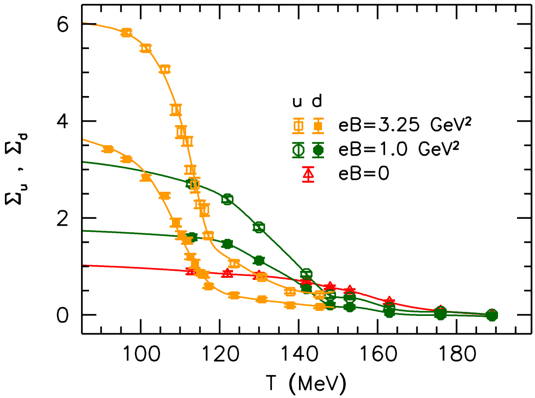
- average of up and down quark condensates:
  T_c=inflection point

Quark condensates



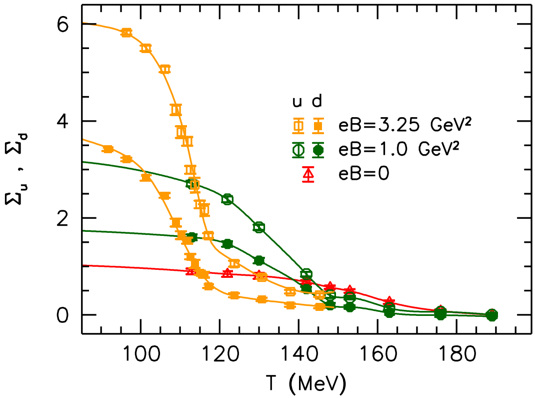
- average of up and down quark condensates:
  T_c=inflection point
  - is there a turning point, where $T_c(B)$ starts increasing? No.

Quark condensates



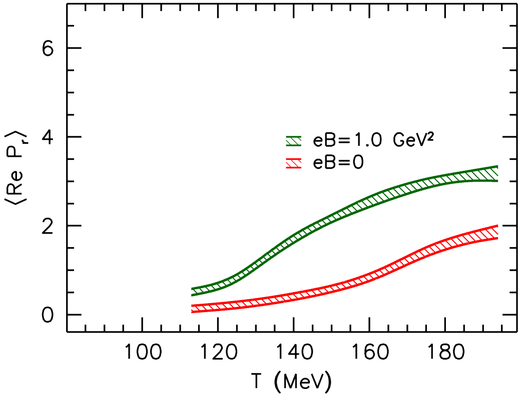
- up and down quark condensates separately

Quark condensates



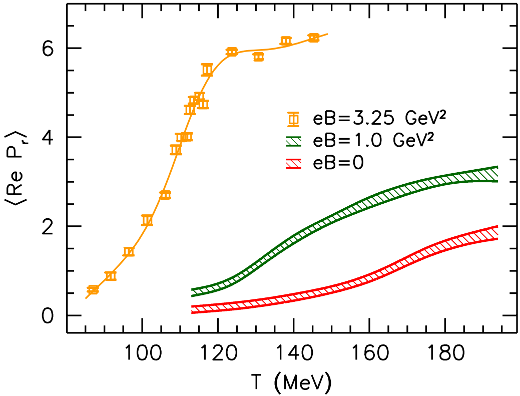
- up and down quark condensates separately
  - is there a splitting between the up/down chiral transitions? No.

Polyakov loop



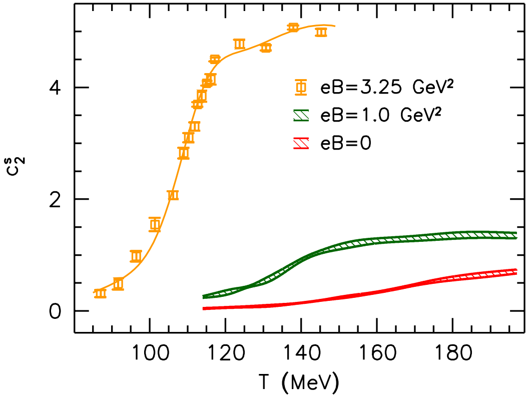
- Polyakov loop: T_c=inflection point

Polyakov loop



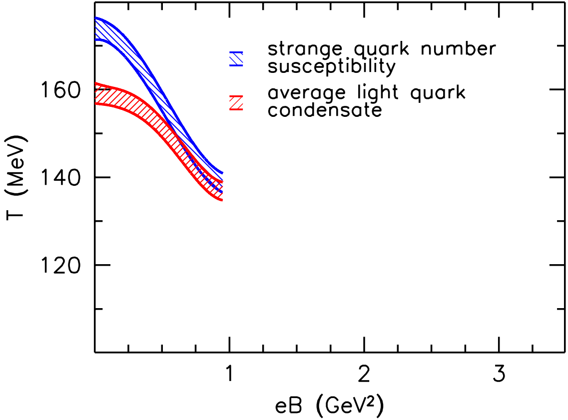
- Polyakov loop: T_c=inflection point
  - is there a splitting between the chiral/deconfinement transitions? No.

Strange quark number susceptibility

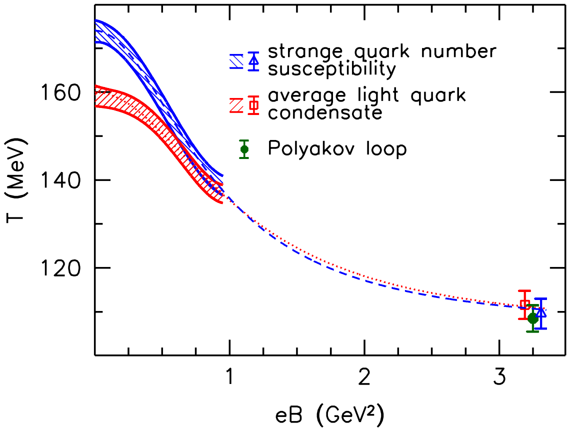


- is there a splitting between the chiral/deconfinement transitions? No.

Phase diagram

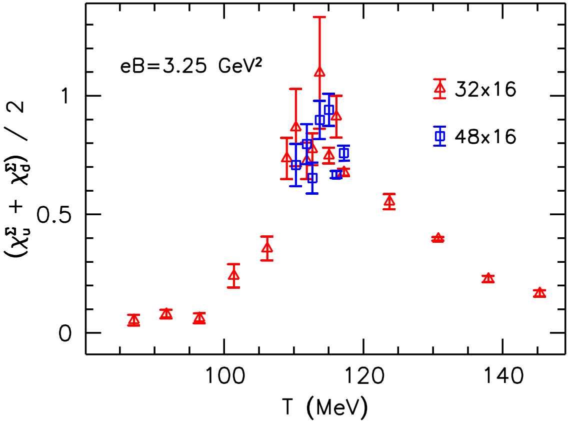


Phase diagram



- summarizing T_c from all observables at $eB = 3.25$ GeV^2

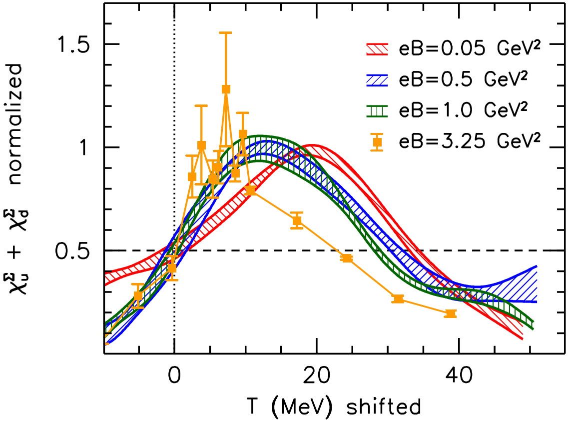
Nature of transition: chiral susceptibility



- peak height independent of volume → analytic crossover (real phase transition would show singularity as $V \to \infty$)

Strength of the transition

- is there a tendency for strengthening/weakening?

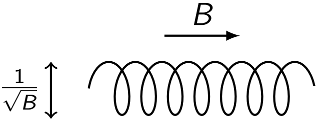


  - the peak gets slowly but significantly narrower
  - maybe there is a critical point at even stronger B?

Lattice results – effective theory

The effective theory

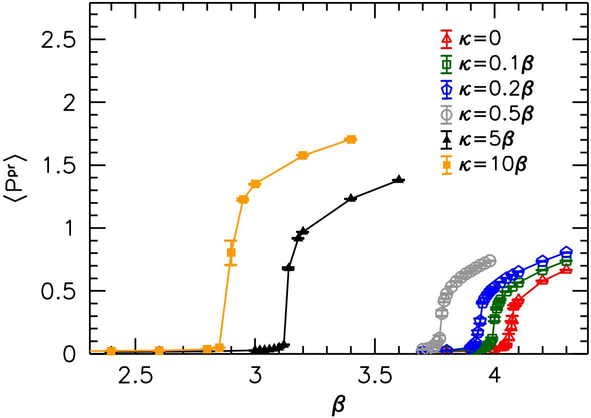
- what happens to $\mathcal{L}_{\rm QCD}$ at $eB \gg \Lambda_{\rm QCD}^2$?
  - first guess: asymptotic freedom says $\alpha_s \to 0$ i.e. complete decoupling of quarks and gluons
  - but: B breaks rotational symmetry and effectively reduces the dimension of the theory for quarks



- gluons also inherit this spatial anisotropy, $\kappa(B) \propto B$
  [Miransky, Shovkovy 2002; Endrődi 1504.08280]

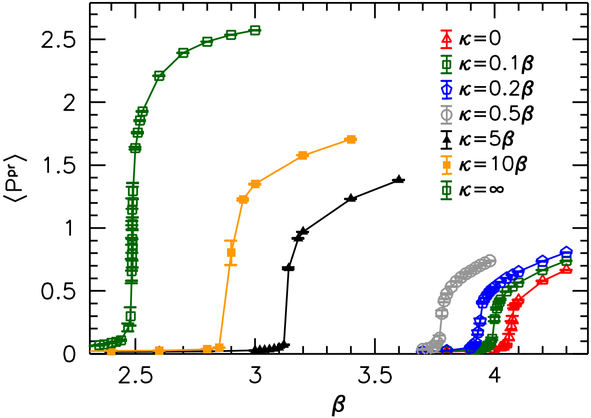
$$\mathcal{L}_{\rm QCD} \xrightarrow{B\to\infty} \operatorname{tr}\mathcal{B}_{\parallel}^2 + \operatorname{tr}\mathcal{B}_{\perp}^2 + [1+\kappa(B)]\operatorname{tr}\mathcal{E}_{\parallel}^2 + \operatorname{tr}\mathcal{E}_{\perp}^2$$

Polyakov loop



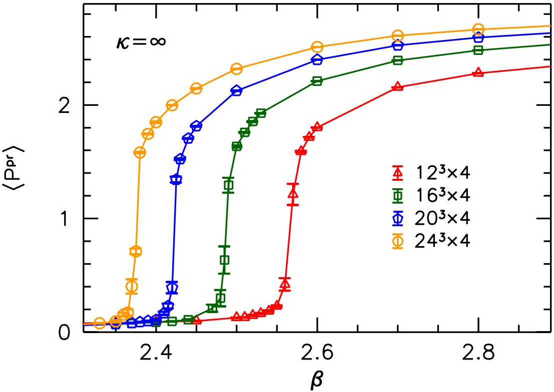
- finite κ: usual action, just multiply $z - t$ plaquettes by $(1 + \kappa)$
  - large κ leads to large autocorrelation times

Polyakov loop



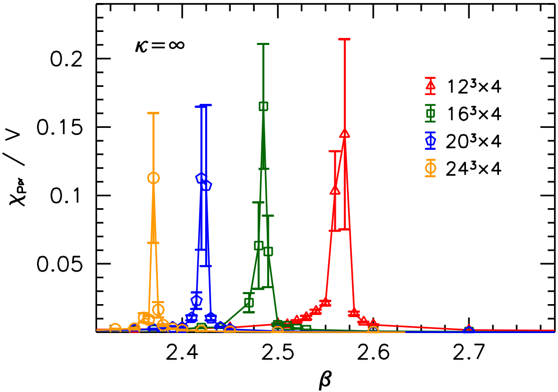
- finite κ: usual action, just multiply $z - t$ plaquettes by $(1 + \kappa)$
  - large κ leads to large autocorrelation times
- $\kappa = \infty$ reduces independent dofs to local Polyakov loops $L_t(x, y)$ and local spatial Polyakov loops $L_z(x, y)$

Finite size scaling at $\kappa = \infty$



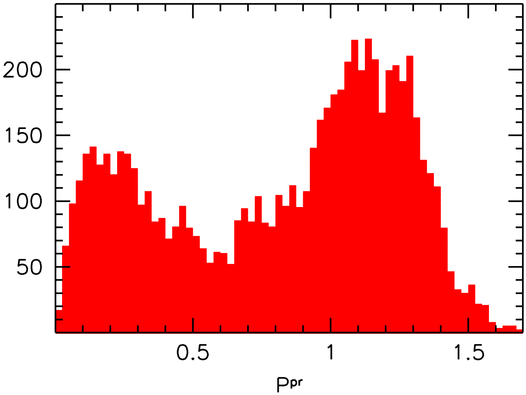
- Polyakov loop on different volumes: jump gets sharper

Finite size scaling at $\kappa = \infty$



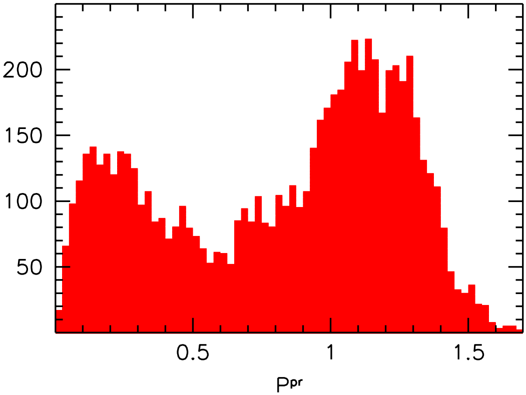
- Polyakov loop on different volumes: jump gets sharper
- Polyakov loop susceptibility peak height scales with V

Finite size scaling at $\kappa = \infty$



- Polyakov loop on different volumes: jump gets sharper
- Polyakov loop susceptibility peak height scales with V
- histogram shows double peak-structure at T_c

Finite size scaling at $\kappa = \infty$



- Polyakov loop on different volumes: jump gets sharper
- Polyakov loop susceptibility peak height scales with V
- histogram shows double peak-structure at T_c
  - does the transition become a real phase transition? Yes.

Implications

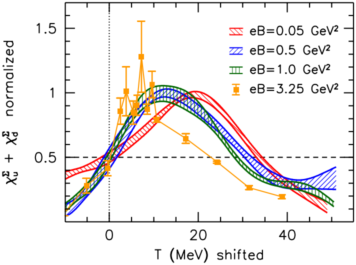
Critical point

- analytical crossover for $0 \leq eB \leq 3.25$ GeV^2
  first-order transition for $B \to \infty$

▸ there must be a critical point in between

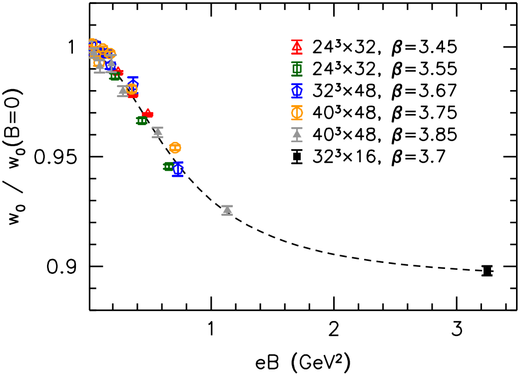
- estimate: extrapolate width of susceptibility peak to 0

$$\underline{eB_{\rm CP} = 10(2)\ {\rm GeV}^2}$$



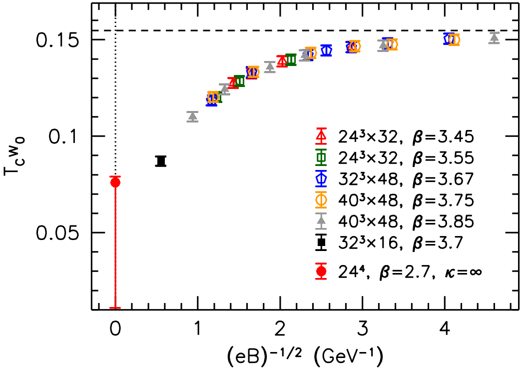
Critical temperature

- to get $T_c(B \to \infty)$ in physical units, we need lattice scale a but: no a priori known dimensionful quantity at $B \to \infty$
  - attempt to use a pure gluonic quantity: w_0
    [cf. Kitazawa, this morning]



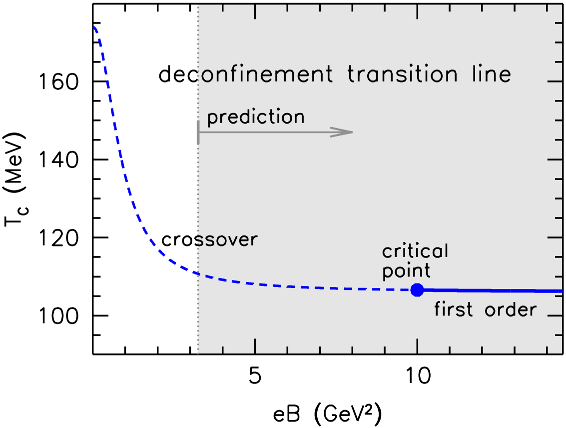
Critical temperature

- to get $T_c(B \to \infty)$ in physical units, we need lattice scale a
  but: no a priori known dimensionful quantity at $B \to \infty$
  - attempt to use a pure gluonic quantity: w_0
    [cf. Kitazawa, this morning]



- *assuming that* $w_0(B)$ flattens out as $B \to \infty$
  $\to$ T_c reduces monotonously

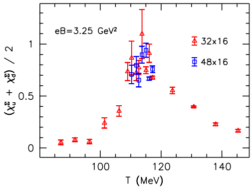
Final conclusion



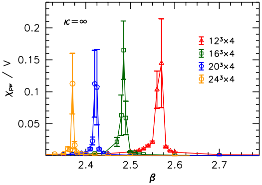
Summary

Summary

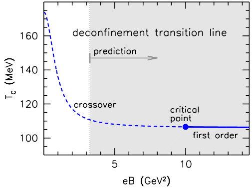
- analytic crossover even at $eB = 3.25\ \mathrm{GeV}^2$



- first-order phase transition at $B \to \infty$



- critical point, estimated location $eB_{\mathrm{CP}} = 10(2)\ \mathrm{GeV}^2$



Appendix

Deriving the effective theory

- take the fermionic action in Euclidean spacetime

$$\mathcal{L}^q = -\log\det[\not{D}(B,\mathcal{B},\mathcal{E}) + m]$$

  and expand it for $B^2 \gg \mathrm{tr}\mathcal{B}^2, \mathrm{tr}\mathcal{E}^2, m^4$

  - assumption: $\mathcal{B}$, $\mathcal{E}$ covariantly constant i.e. $D_\mu\mathcal{E} = D_\mu\mathcal{B} = 0$
  - to lowest order: enough to consider B and $\mathcal{B}_\parallel$ *or* B and $\mathcal{E}_\parallel$ etc.

- the case with B and $\mathcal{B}_\parallel$: Landau-problem in the $\perp$ plane

$$\mathcal{L}^q(B,\mathcal{B}_\parallel) = \frac{1}{8\pi^2}\sum_c m^2\,(B+\mathcal{B}_{\parallel c})\int\frac{\mathrm{d}s}{s^2}\,e^{-s}\,\coth\frac{(B+\mathcal{B}_{\parallel c})s}{m^2}$$

  - for $\mathcal{B}_{\parallel c} \gg B$ this becomes independent of $\mathcal{B}_\parallel$

    $\Rightarrow \mathcal{B}_\parallel$ decouples from the quarks

Deriving the effective theory

- similarly, $\mathcal{B}_\perp$ and $\mathcal{E}_\perp$ also decouple
- the case with B and $\mathcal{E}_\parallel$:
  Landau-problem in the $\perp$ plane **and** in the $\parallel$ t plane

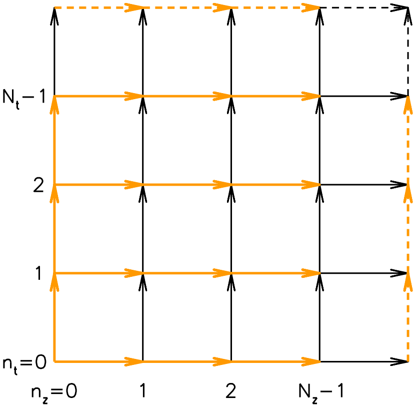
$$\mathcal{L}^q(B,\mathcal{E}_\parallel) = \frac{1}{8\pi^2}\sum_c B\,\mathcal{E}_{\parallel c}\int \frac{\mathrm{d}s}{s}\,e^{-s}\,\coth\frac{Bs}{m^2}\,\coth\frac{\mathcal{E}_{\parallel c}s}{m^2}$$

- for $\mathcal{E}_{\parallel c} \gg B$ a non-trivial dependence remains

$$\mathcal{L}^q(B,\mathcal{E}_\parallel) = \kappa(B)\,\mathrm{tr}\,\mathcal{E}_\parallel^2 + \mathcal{O}(\mathcal{E}^4), \qquad \kappa(B) \equiv \frac{1}{24\pi^2}\frac{|B|}{m^2}$$

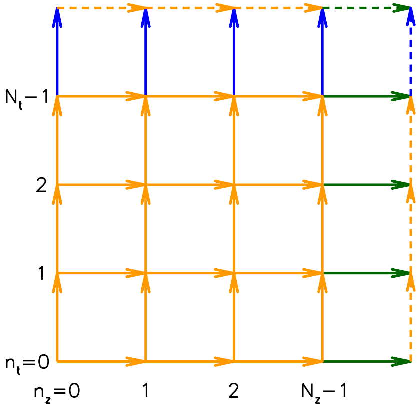
$\Rightarrow$ magnetized quarks contribute only to $\mathrm{tr}\,\mathcal{E}_\parallel^2$

Simulating at $\kappa = \infty$



- gauge fix to maximal tree ( $= \mathbb{1}$)

Simulating at $\kappa = \infty$



- gauge fix to maximal tree ($\longrightarrow = \mathbb{1}$)
- exploit that all plaquettes are $\mathbb{1}$ ($\longrightarrow = L_z$, $\longrightarrow = L_t$)