Hadronic Correlation and Spectral Functions at Finite Temperature

Olaf Kaczmarek

Universität Bielefeld

FAIR Lattice QCD Days
GSI Darmstadt, November 23, 2009
Dilepton rate directly related to vector spectral function:

\[
\frac{dW}{d\omega d^3p} = \frac{5\alpha^2}{27\pi^2} \frac{1}{\omega^2(e^{\omega/T} - 1)} \sigma_V(\omega, \vec{p}, T)
\]
Hard Probes in Heavy Ion Collisions – RHIC results

pp-data well understood by hadronic cocktail
low invariant mass region <150 MeV similar in Au-Au
large enhancement between 150-750 MeV
indications for thermal effects!? Also at higher $m_{ee}$?

Need to understand the contribution from QGP!
→ spectral functions from lattice QCD
Hard Probes in Heavy Ion Collisions - Photons

Direct and fragmentation photon relative contribution

- Hadron Gas Thermal $T_f$
- QGP Thermal $T_i$
- "Pre-equilibrium" ("secondary" or "cascading")

Jet Re-interaction $\sqrt{T_f T_i} s$

$pQCD$ Prompt $x/s$

Emission time

[Fleuret 2009]

**Photonrate** directly related to vector spectral function (at finite momentum):

$$\omega \frac{dN_\gamma}{d^4x d^3q} = \frac{5\alpha}{6\pi^2} \frac{1}{e^{\omega/T} - 1} \sigma_V (\omega = |\vec{p}|, T)$$
Transport Coefficients

The small $\omega$ limit of $\sigma_V$ is related to transport coefficients (Kubo-Formulas)

Light quark sector $\to$ electrical conductivity:

$$\sigma_{el} = \lim_{\omega \to 0} \frac{\sigma_{ii}^V(\omega, \vec{p} = 0, T)}{6\omega}$$

Heavy quark sector $\to$ heavy quark diffusion constant:

$$D = \frac{1}{3\chi^{00}} \lim_{\omega \to 0} \frac{\sigma_{ii}^V(\omega, \vec{p} = 0, T)}{\omega}$$

Free (non-interacting) spectral function [Karsch et al. 03, Aarts et al. 05]

$$\sigma_H = \frac{N_c}{8\pi^2} \Theta(\omega^2 - 4m^2) \frac{\omega^2}{\omega} \tanh\left(\frac{\omega}{4T}\right) \times \sqrt{1 - \left(\frac{2m}{\omega}\right)^2 \left(a_H + \left(\frac{2m}{\omega}\right)^2 b_H\right)}$$

$$+ \frac{N_c}{3} \frac{T^2}{2} f_H \omega \delta(\omega)$$

with interactions:

$$\delta(\omega) \to \frac{1}{\pi} \frac{\eta}{\omega^2 + \eta^2}$$

[Petreczky+Teaney 06, Aarts et al. 05]
Introduction

Hadronic correlators for light quarks ($m_q=0$)

  Screening masses in the thermodynamic and continuum limit

  Temporal correlators vs. free correlators

  Spectral functions and Dilepton rates

Charmonium hadronic correlators ($m_q=m_c$)

  Screening Masses below and above $T_c$

  Temporal correlators vs. free correlators

  Spectral functions below and above $T_c$

  Temporal correlators vs. reconstructed correlators

  Zero mode contributions
Hadronic correlators – Lattice setup

Thermal hadronic correlation functions

\[ J_H = \bar{q}(\tau, \vec{x}) \Gamma_H q(\tau, \vec{x}) \]

\[ G_H(\tau, T, \vec{p}) = \sum_{\vec{x}} e^{-i\vec{p}\vec{x}} \langle J_H(0, 0) J^\dagger_H(\tau, \vec{x}) \rangle \]

O(a)-improved Clover improved fermionic action

on large quenched lattice configurations up to \(128^3 \times 16/32/48\)

includes all the relevant physics (in the quenched limit)

how to extract it?

directly from the correlators?

spectral functions using MEM?
Temporal Correlators:

splitting below $T_c$ due to chiral and axial U(1) symmetry breaking
degenerate states at $1.5T_c$
symmetry restoration above $T_c$
deviations at small $\tau$ due to different cut-off effects
Temporal Correlators:

splitting below $T_c$ due to chiral and axial $U(1)$ symmetry breaking
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symmetry restauration above $T_c$
deviations at small $\tau$ due to different cut-off effects
use **Spatial Correlators**

\[ G_H(z, T, \vec{p}_\perp) = \sum_{\tau, \vec{x}_\perp} e^{-i\vec{p}_\perp \cdot \vec{x}_\perp} \langle J_H(0, 0) J_H^\dagger(\tau, \vec{x}) \rangle \]

correlation function depends on the same spectral density, but the relation is more involved

\[ G_H(z) = \int_{-\infty}^{+\infty} \frac{dp_z}{2\pi} e^{ip_z z} \int_{-\infty}^{+\infty} dp_0 \frac{\sigma_H(p_0, \vec{0}_\perp, p_z)}{p_0} \xrightarrow{z \to \infty} \text{Ampl.} \times \exp(-m_{\text{screen}} z) \]

however, \( m_{\text{screen}}(T) \neq m_{\text{pole}}(T) \) in general:

look for zeros of \( G^{-1}(p) = p_0^2 + \vec{p}^2 + m_0^2 + \Pi(p_0, \vec{p}, T) \)

\( \vec{p} = 0 : \quad -p_0^2 = m_0^2 + \Pi(p_0, \vec{0}, T) = (m_{\text{pole}}(T))^2 \)

\( p_0 = 0 : \quad -\vec{p}^2 = m_0^2 + \Pi(0, \vec{p}, T) = (m_{\text{screen}}(T))^2 \)

\[ \Rightarrow m_{\text{screen}}(T) = \frac{m_{\text{pole}}(T)}{A(T)} \]
Light Quark Screening Masses – Thermodynamic Limit, $V \to \infty$

large collection of lattices ranging from $16^3 \times 8$ to $128^3 \times 16$

allowing for thermodynamic limit $V \to \infty$ at $N_t=8, 12$ and $16$

\[
T \leq T_c : \quad m_{\text{screen}}(L, a) = m_{\text{screen}}(a) \left[ 1 + \gamma_V \left( \frac{N_T}{N_S} \right)^3 \right]
\]

\[
T > T_c : \quad m_{\text{screen}}(L, a) = m_{\text{screen}}(a) \left[ 1 + \gamma_V \left( \frac{N_T}{N_S} \right)^p \right]
\]

\[
T = \infty : \quad m_{\text{screen}}(L, a) = m_{\text{screen}}(a) \left[ 1 + \gamma_V \left( \frac{N_T}{N_S} \right)^1 \right]
\]

combined fit:

<table>
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<tr>
<th>$p$</th>
<th>PS</th>
<th>V</th>
</tr>
</thead>
<tbody>
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<td>1.5</td>
<td>$T_c$</td>
<td>2.22(10)</td>
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<tr>
<td>3.0</td>
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$$T \leq T_c : \quad m_{\text{screen}}(L, a) = m_{\text{screen}}(a) \left[ 1 + \gamma_V \left( \frac{N_T}{N_\sigma} \right)^3 \right]$$

$$T > T_c : \quad m_{\text{screen}}(L, a) = m_{\text{screen}}(a) \left[ 1 + \gamma_V \left( \frac{N_T}{N_\sigma} \right)^p \right]$$

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Continuum Limit

lattice spacing $a \to 0$

Non-perturbatively improved action

$\Rightarrow$ discretization errors $O(a^2)$
Thermodynamic and Continuum Limit:

\[ V \to \infty \quad \text{and} \quad a \to 0 \]

weak temperature dependence below \( T_c \)
data still below free limit \((2\pi T)\) at \( 3T_c \), vector closer to free case
perturbative limit reached from above [Laine, Vepsäläinen]
\( \to \) need higher temperatures to verify this
comparison with free (non-interacting) high temperature lattice correlator

vector and axial-vector close to free case

cut-off effects are well described by free lattice correlator

this explains the difference at small $\tau$

in the following only vector and pseudo-scalar are discussed
comparison with free (non-interacting) high temperature lattice correlator

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still strong correlations in scalar and pseudo-scalar channel!
comparison with free (non-interacting) high temperature lattice correlator

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Light Quark Correlators vs Free Correlators

Comparison with free (non-interacting) high temperature lattice correlator

Vector and axial-vector close to free case

Still strong correlations in scalar and pseudo-scalar channel

First 4-5 distances still dominated by cut-off effects
Light Quark Correlators vs Free Correlators

Comparison with free (non-interacting) high temperature lattice correlator

Vector and axial-vector close to free case

Still strong correlations in scalar and pseudo-scalar channel

First 5 points still dominated by cut-off effects!

Nt=32 and 48 needed to extract continuum physics!
How to obtain continuous spectral function $\sigma(\omega, T)$ from discrete (and small) number of correlators?

$$G(\tau, T) = \int_0^\infty d\omega K(\tau, \omega, T)\sigma(\omega, T)$$

$$K(\tau, \omega, T) = \frac{\cosh \left( \frac{\omega(\tau - \frac{1}{2T})}{2T} \right)}{\sinh \left( \frac{\omega}{2T} \right)}$$

Best method on the market: Maximum Entropy Method (MEM)
based on Bayesian theorem [Asakawa et al. 01] → most probable spectral function
properly renormalized correlators as input

- non-perturbative renormalization constants for vector [Lüscher et al. 1997]

- TI perturbative renormalization constants for pseudo-scalar

Prior knowledge needed as input → default model $m(\omega)$
result should be independent of default model ← usually not the case
MEM – Free spectral function

Free (non-interacting) spectral function [Karsch et al. 03, Aarts et al. 05]

\[
\sigma_H = \frac{N_c}{8\pi^2} \Theta(\omega^2 - 4m^2) \frac{\omega^2}{4T} \tanh(\frac{\omega}{4T}) \\
\times \sqrt{1 - \left(\frac{2m}{\omega}\right)^2 \left[ a_H + \left(\frac{2m}{\omega}\right)^2 b_H \right]} + \frac{N_c}{3} \frac{T^2}{2} f_H \omega \delta(\omega)
\]
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\sigma_H = \frac{N_c}{8\pi^2} \Theta(\omega^2 - 4m^2) \omega^2 \tanh\left(\frac{\omega}{4T}\right)
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\times \sqrt{1 - \left(\frac{2m}{\omega}\right)^2} \left[ a_H + \left(\frac{2m}{\omega}\right)^2 b_H \right] + \frac{N_c}{3} \frac{T^2}{2} f_H \omega \delta(\omega)
\]

Lattice cut-off effects

\[
\omega_{max} = 2 \log(7 + ma)
\]
MEM – Free spectral function

Free (non-interacting) spectral function [Karsch et al. 03, Aarts et al. 05]

\[
\sigma_H = \frac{N_c}{8\pi^2} \Theta(\omega^2 - 4m^2) \omega^2 \tanh\left(\frac{\omega}{4T}\right)
\times \sqrt{1 - \left(\frac{2m}{\omega}\right)^2} \left[ a_H + \frac{2m}{\omega} b_H \right]
\]

\[
+ \frac{N_c}{3} \frac{T^2}{2} f_H \omega \delta(\omega)
\]

zero mode contribution at \(\omega \approx 0\) [Umeda 07]

with interactions:

\[
\delta(\omega) \rightarrow \frac{1}{\pi} \frac{\eta}{\omega^2 + \eta^2}
\]

[Petreczky+Teaney 06
Aarts et al. 05]
large $\omega$ behaviour well described by free lattice SPF

cut-Off effects are under control and well separated form physical interesting region

Vector SPF close to free case except at small $\omega$

still large correlations in the Pseudoscalar sector

small $\omega$ region accessible $\rightarrow$ hope to extract transport properties $\rightarrow$ higher statistics needed
large $\omega$ behaviour well described by free lattice SPF

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Vector SPF close to free case except at small $\omega$

still large correlations in the Pseudoscalar sector

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→ higher statistics needed
Dilepton rates directly related to vector spectral function:

\[
\frac{dW}{d\omega d^3p} = \frac{5\alpha^2}{27\pi^2} \frac{1}{\omega^2 (e^{\omega/T} - 1)} \sigma_V(\omega, \vec{p}, T)
\]

Born rate approached at large \(\omega/T\)
consistent with HTL calculations at intermediate \(\omega/T\)
better behaved \(\sim 1/\omega^2\) at small \(\omega/T\) [Moore et al., Teaney, ...]
higher statistics needed to resolve details at small \(\omega/T\)
consistent behavior at both lattice spacings \(\rightarrow\) continuum physics
screening masses at 1.50 T_c already close to the free case

$$m_{\text{free}}(T) = 2\sqrt{(\pi T)^2 + m_c^2}.$$ 

does this tell us anything about dissociation?

need to understand the momentum dependence of $G_H(\tau, T, p)$ and $\sigma(\omega, T, p)$ in detail

thermodynamic and continuum limit not performed yet
non-degenerate states still at $1.50 \, T_c$

(almost) close to free correlators at (very) small separations

largest distance 0.25 fm due to compact temporal direction

only small distance regime (0.1-0.25 fm) relevant

for thermal effects

for bound state effects
non-degenerate states still at 1.50 $T_c$

(almost) close to free correlators at (very) small separations

largest distance 0.25 fm due to compact temporal direction

only small distance regime relevant

for thermal effects

for bound state effects
N_σ=128 and N_τ=64 (a^{-1}≈13 GeV) → cut-off effects well separated

Pronounced ground state peak close to J/ψ mass

no zero mode contributions observed below T_c in all channels
\[ V \beta = 7.457 \]

\[ \sigma(\omega, T)/\omega^2 \]

\[ 128^3 \times 64 \quad T = 0.75 \, T_c \]

\[ \omega \, [\text{GeV}] \]

\[ \rho_{\sigma} = 128 \quad \text{and} \quad n_{\tau} = 64 \quad (a^{-1} \approx 13 \, \text{GeV}) \rightarrow \text{cut-off effects well separated} \]

pronounced ground state peak close to J/\(\psi\) mass

no zero mode contributions observed below \(T_c\) in all channels

first peak independent of default model
N$_\sigma$ = 128 and N$_\tau$ = 48 ($a^{-1} \approx 19 \text{ GeV}$) → cut-off effects well separated

small $\omega$ region accessible → hope to extract transport properties

default model dependence only in small $\omega$ region → higher statistics needed

no default model dependence in the intermediate $\omega$ region

no zero-mode contribution in pseudo-scalar channel observed

no pronounced peak → bound states melted? / threshold enhancement?
Charmonium Correlators – Zero Mode Contributions

\[ G_{\text{rec}}(\tau, T) = \int \sigma_0(\omega, 0.75T_c) K(\omega, \tau, T) \]
\[ G_{\text{low}}^{\text{rec}}(\tau, T) = \int_{2m_c}^{0} \sigma_T(\omega, T) K(\omega, \tau, T) \]
\[ G_{\mu\mu}(\tau, T) = G_{ii}(\tau, T) + G_{44}(\tau, T) \]

- main T-effect due to zero-mode contribution
- well described by small \( \omega \)-part of \( \sigma_T(\omega, T) \)
- smaller than \( G_{00}(\tau, T) = \chi(T)T \)
- no zero-mode contribution in PS-channel

(similar to discussions by Umeda, Petreczky)
Charmonium Correlators – Zero Mode Contributions

\[ G_{rec}(\tau, T) = \int \sigma_0(\omega, 0.75T_c)K(\omega, \tau, T) \]
\[ G_{rec}^{low}(\tau, T) = \int_0^{2m_c} \sigma_T(\omega, T)K(\omega, \tau, T) \]

- larger zero-mode contribution in S-wave
- larger T-effect in the S-wave states

systematic uncertainties in reconstruction and low-\(\omega\) part of spectral function
high quality data and small lattice spacing + large momenta (volume) needed
Conclusions and Outlook

Hadronic correlators for light quarks \( (m_q=0) \)

- Thermodynamic and Continuum Limit of screening masses!
- Spectral functions \( \rightarrow \) Dilepton rates, Transport coefficients?
- Momentum dependence needs to be analyzed \( \rightarrow \) Photon rates
- Comparison with HTL calculations and experiment

Charmonium hadronic correlators \( (m_q=m_c) \)

- What can we learn from Hadronic correlators on Dissociation?
- Momentum dependence vs. Spatial correlators/screening masses?
- Spectral functions \( \rightarrow \) Dilepton rates, Transport coefficients?
- Momentum dependence needs to be analyzed \( \rightarrow \) Photon rates
- Comparison with HTL/NRQCD calculations and experiment
Many Thanks to

HengTong Ding
Anthony Francis
Helmut Satz
+
RBC-Bielefeld