Hadronic Correlation and Spectral Functions in Lattice QCD

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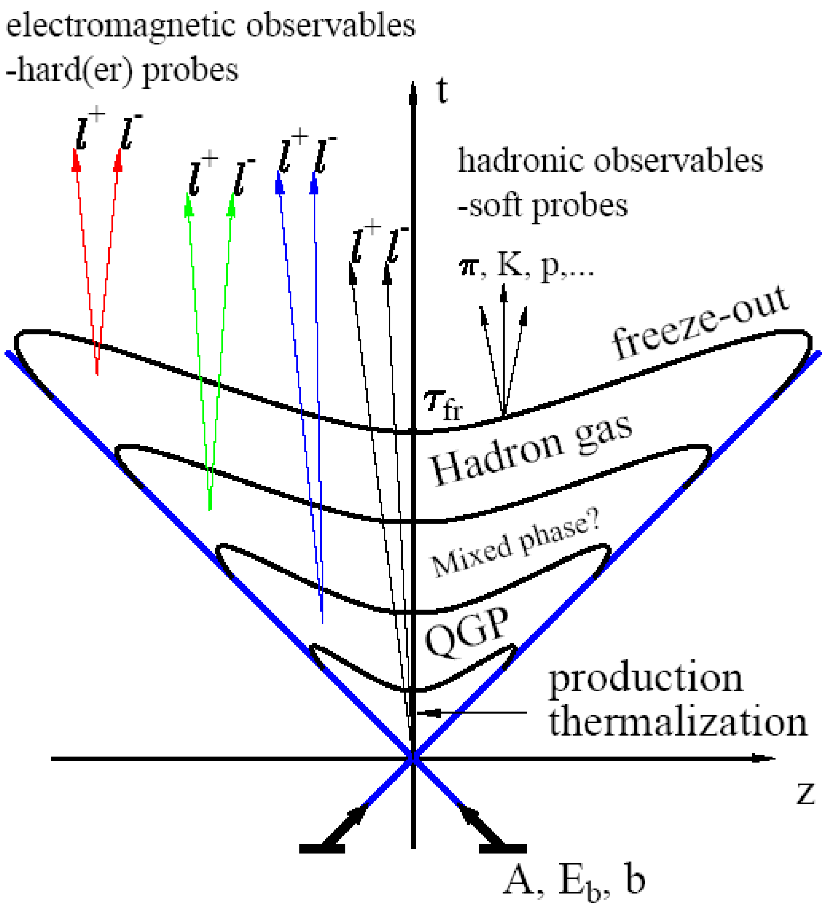
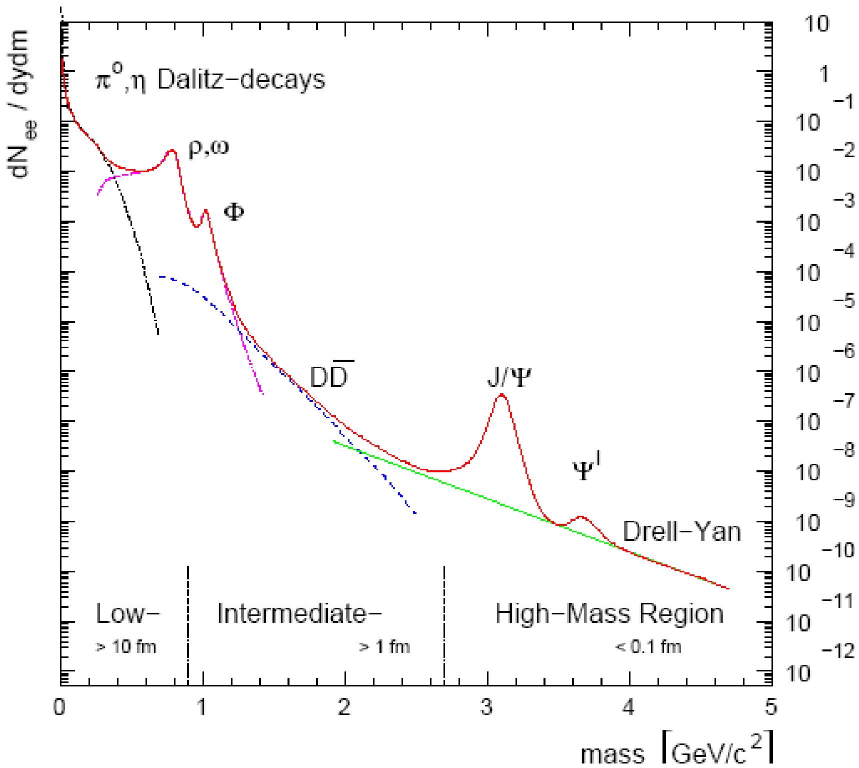
HIC for FAIR Workshop
Dense QCD Phases in Heavy Ion Collisions and Supernovae
Prerow, October 12, 2009

Hard Probes in Heavy Ion Collisions

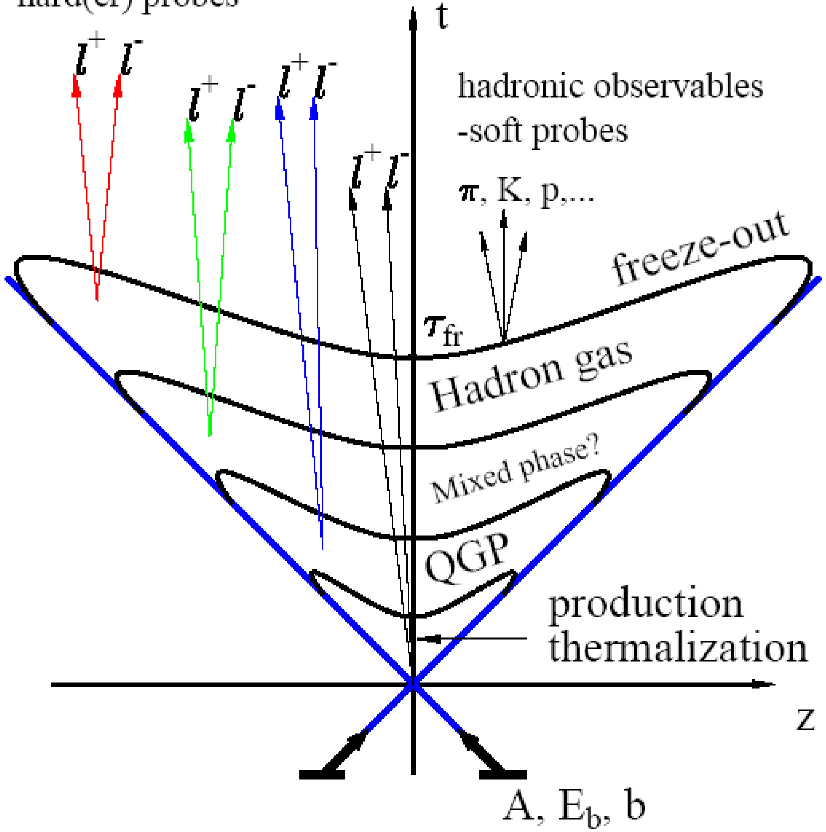


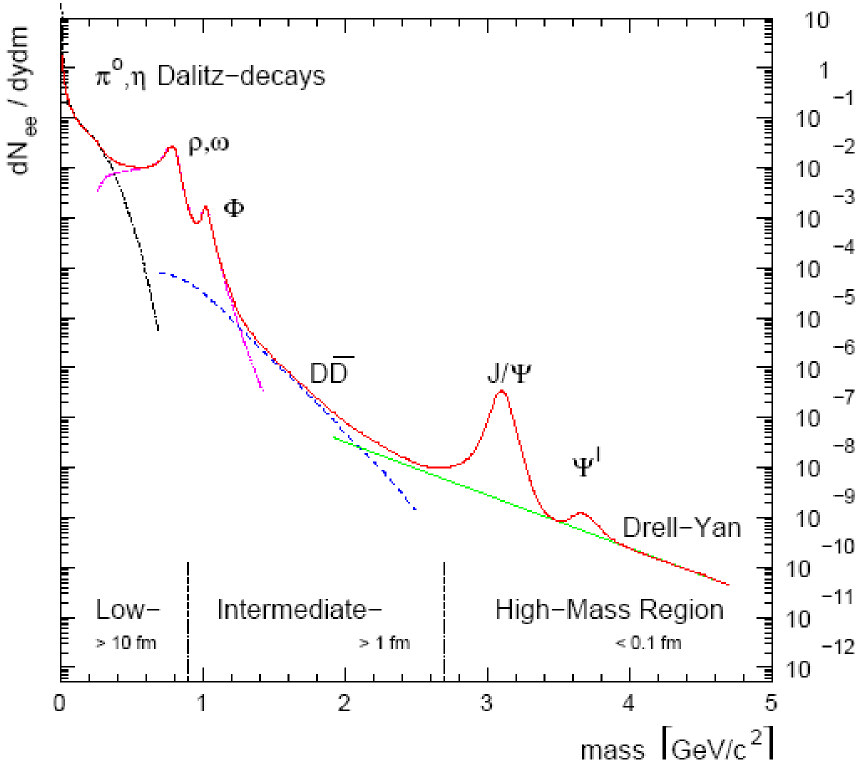


Dileptonrate directly related to vector spectral function:

$$\frac{\mathrm{d}W}{\mathrm{d}\omega \mathrm{d}^3p} = \frac{5\alpha^2}{27\pi^2} \frac{1}{\omega^2 (e^{\omega/T} - 1)} \sigma_V(\omega, \vec{p}, T)$$

Hard Probes in Heavy Ion Collisions





Photonrate directly related to vector spectral function (at finite momentum):

$$\omega \frac{dN_\gamma}{d^4x d^3q} = \frac{5\alpha}{6\pi^2} \frac{1}{e^{\omega/T} - 1} \sigma_V(\omega = |\vec{p}|, T)$$

Transport Coefficients

The small ω limit of σ_V is related to transport (Kubo-Formulas)

Light quark sector $\rightarrow$ **electrical conductivity**:

$$\sigma_{el} = \lim_{\omega \to 0} \frac{\sigma_V^{ii}(\omega, \vec{p} = 0, T)}{6\omega}$$

Heavy quark sector $\rightarrow$ **heavy quark diffusion constant**:

$$D = \frac{1}{3\chi^{00}} \lim_{\omega \to 0} \frac{\sigma_V^{ii}(\omega, \vec{p} = 0, T)}{\omega}$$

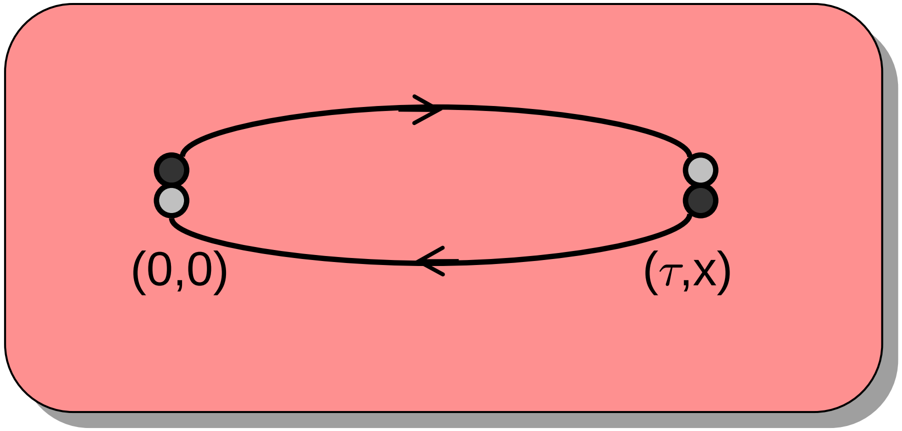
Free (non-interacting) spectral function [Karsch et al. 03, Aarts et al. 05]

$$\sigma_H = \frac{N_c}{8\pi^2} \Theta(\omega^2 - 4m^2)\ \omega^2\ \tanh\left(\frac{\omega}{4T}\right) \times \sqrt{1 - \left(\frac{2m}{\omega}\right)^2} \left[a_H + \left(\frac{2m}{\omega}\right)^2\ b_H\right] + \frac{N_c}{3} \frac{T^2}{2} f_H\ \omega\delta(\omega)$$

$$\delta(\omega) \rightarrow \frac{1}{\pi} \frac{\eta}{\omega^2 + \eta^2}$$

[Petreczky+Teany 06
Aarts et al. 05]

Thermal hadronic correlation functions



$$J_H = \bar{q}(\tau,\vec{x})\Gamma_H q(\tau,\vec{x})$$

$$G_H(\tau,T,\vec{p}) = \sum_{\vec{x}} e^{-i\vec{p}\vec{x}} \langle J_H(0,0) J_H^\dagger(\tau,\vec{x})\rangle$$

O(a)-improved Clover improved fermionic action

on large quenched lattice configurations up to $128^3 \times 16$ and 32

includes all the physics

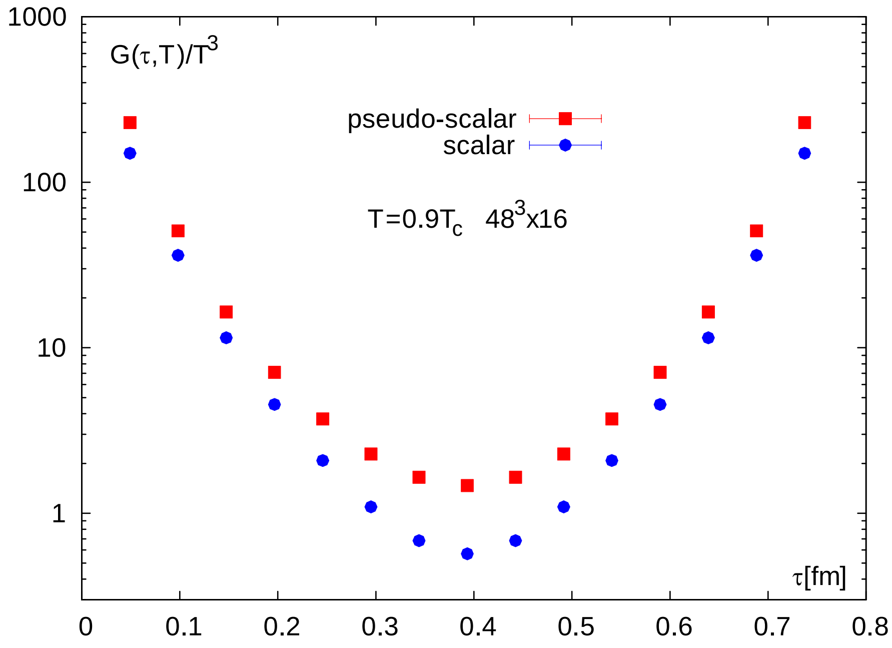
how to extract it?

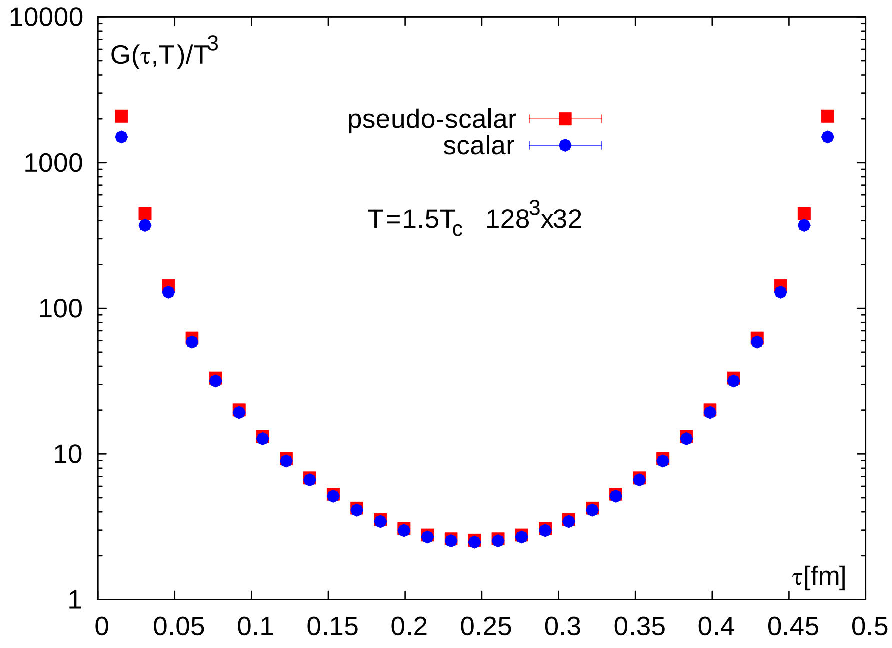
directly from the correlators?

spectral functions using MEM?

Light Quark Correlators – Chiral Symmetry

Temporal Correlators:





splitting below T_c due to

chiral and axial U(1) symmetry breaking

symmetry restauration above T_c

Light Quark Correlators – Chiral Symmetry

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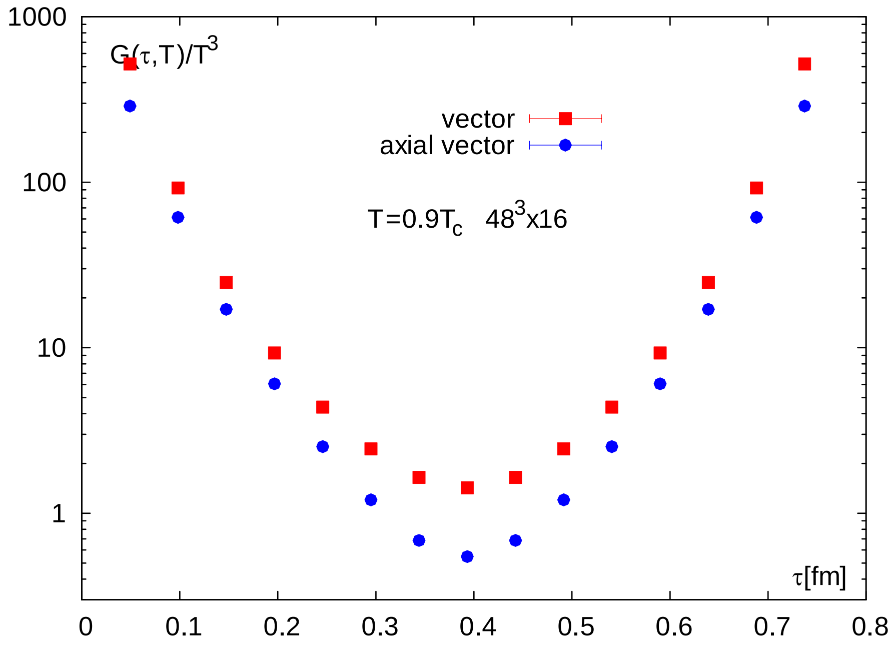

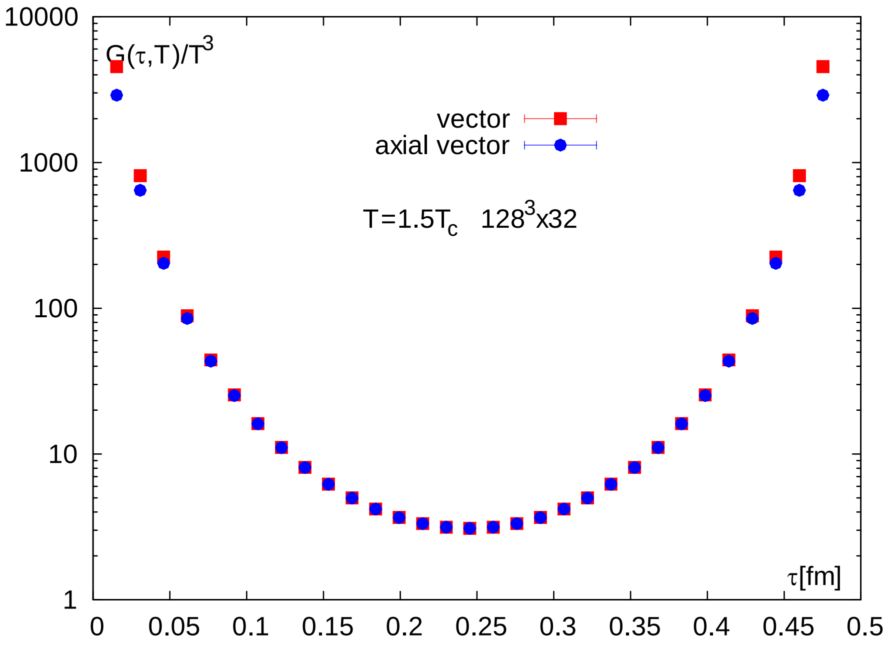

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Light Quark Screening Masses – Definition

use spatial correlators

$$G_H(z,T,\vec{p}_\perp) = \sum_{\tau,\vec{x}_\perp} e^{-i\vec{p}_\perp \vec{x}_\perp} \langle J_H(0,0) J_H^\dagger(\tau,\vec{x})\rangle$$

correlation function depends on the same spectral density,

but the relation is more involved

$$G_H(z) = \int_{-\infty}^{+\infty} \frac{\mathrm{d}p_z}{2\pi} e^{ip_z z} \int_{-\infty}^{+\infty} \mathrm{d}p_0 \frac{\sigma_H(p_0,\vec{0}_\perp,p_z)}{p_0} \xrightarrow[z\to\infty]{} \text{Ampl.} \times \exp(-m_{\text{screen}}\, z)$$

however, $m_{\text{screen}}(T) \neq m_{pole}(T)$ in general :

look for zeros of $G^{-1}(p) = p_0^2 + \vec{p}^{\,2} + m_0^2 + \Pi(p_0,\vec{p},T)$

$$\vec{p} = 0 : \qquad -p_0^2 = m_0^2 + \Pi(p_0,\vec{0},T) = (m_{pole}(T))^2$$

$$p_0 = 0 : \qquad -\vec{p}^{\,2} = m_0^2 + \Pi(0,\vec{p},T) = (m_{\text{screen}}(T))^2$$

$$\Longrightarrow m_{screen}(T) = \frac{m_{pole}(T)}{A(T)}$$

Light Quark Screening Masses – Thermodynamic Limit, V$\to\infty$

large collection of lattices ranging from $16^3\times 8$ to $128^3\times 16$

allowing for thermodynamic limit $V\to\infty$ at N_t=8,12 and 16

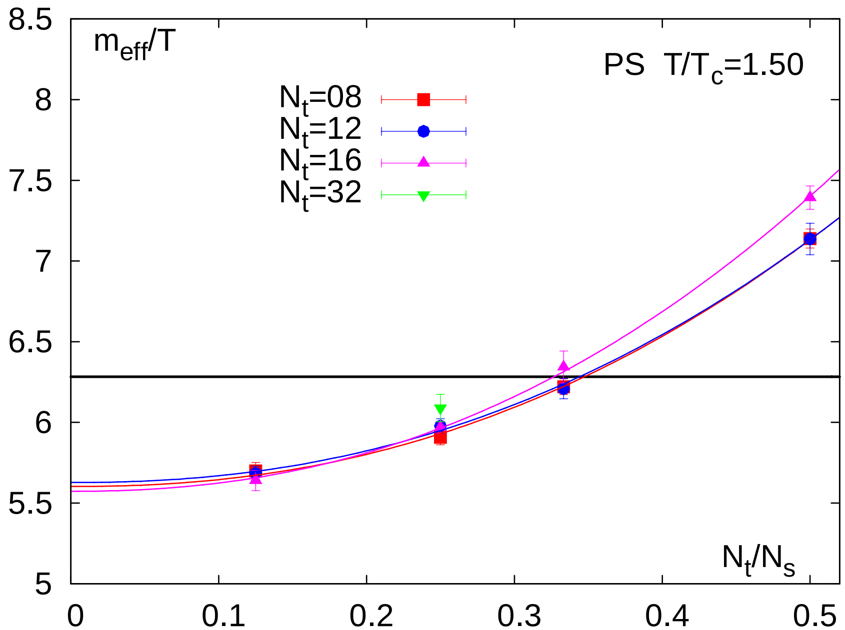
$$T \le T_c: \quad m_{\rm screen}(L,a) = m_{\rm screen}(a)\left[1+\gamma_V\left(\frac{N_\tau}{N_\sigma}\right)^3\right]$$

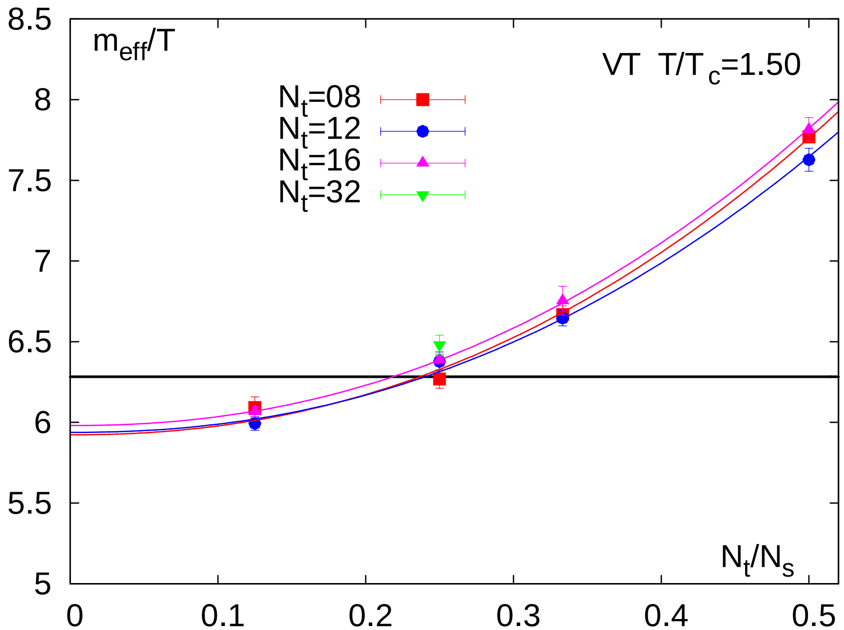
$$T > T_c: \quad m_{\rm screen}(L,a) = m_{\rm screen}(a)\left[1+\gamma_V\left(\frac{N_\tau}{N_\sigma}\right)^p\right]$$

$$T = \infty: \quad m_{\rm screen}(L,a) = m_{\rm screen}(a)\left[1+\gamma_V\left(\frac{N_\tau}{N_\sigma}\right)^1\right]$$

combined fit:

p	PS	V
$1.5\ Tc$	$2.22(10)$	$2.18(13)$
$3.0\ T_c$	$2.06(7)$	$2.05(11)$





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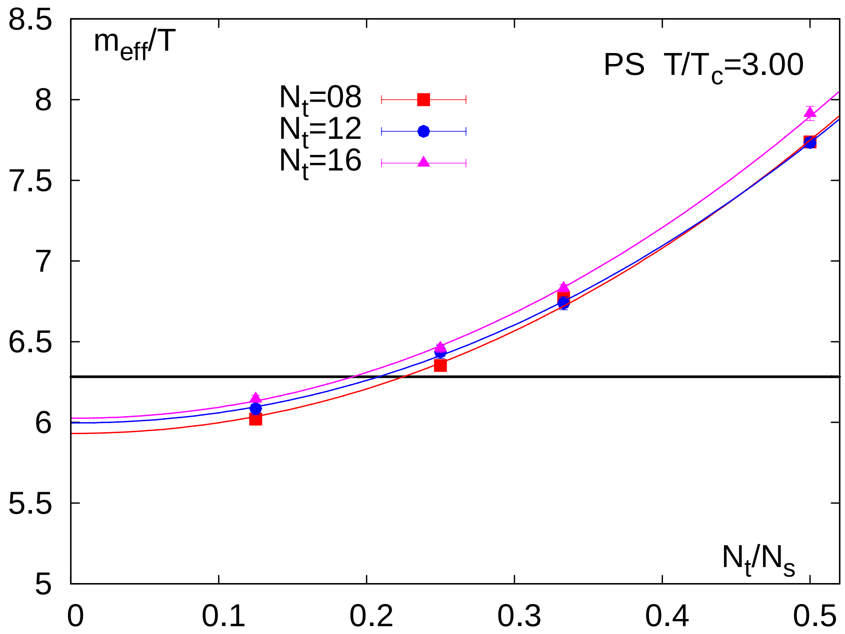
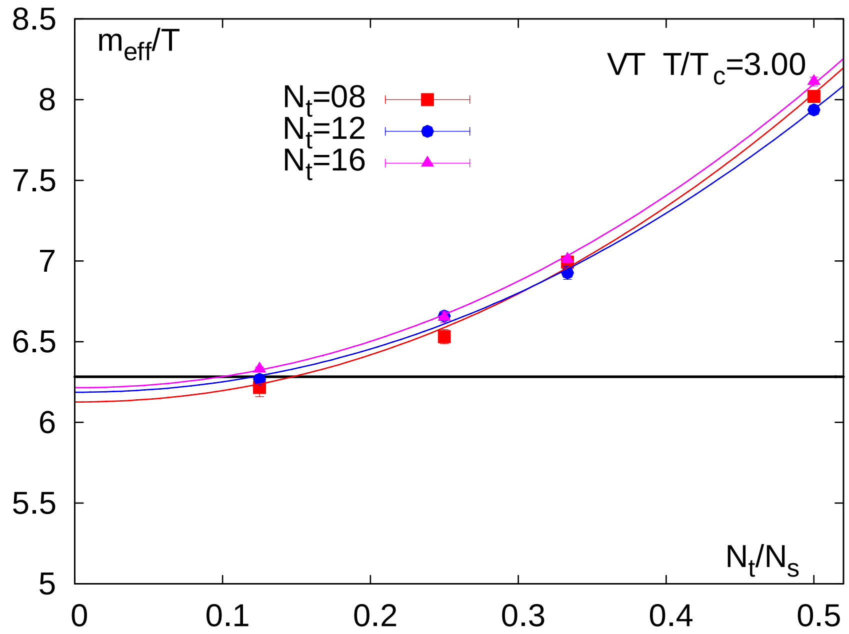




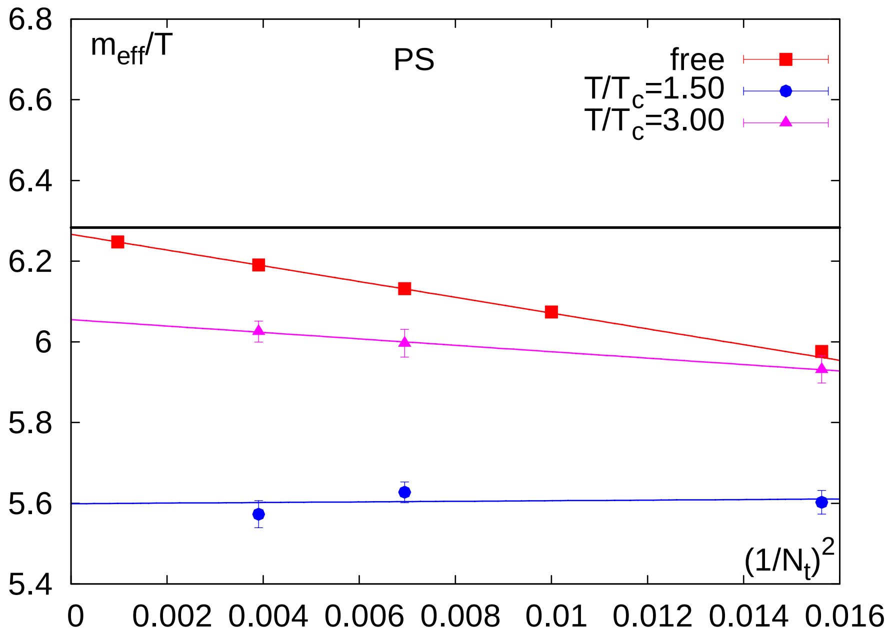
Continuum Limit

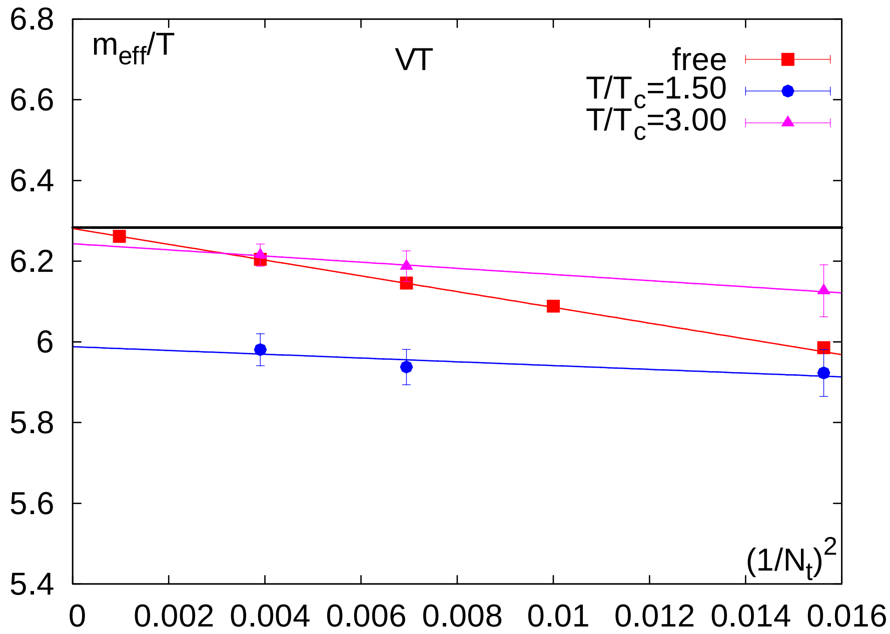
lattice spacing $a \to 0$

Non-perturbatively improved action

$\Rightarrow$ discretization errors $O(a^2)$

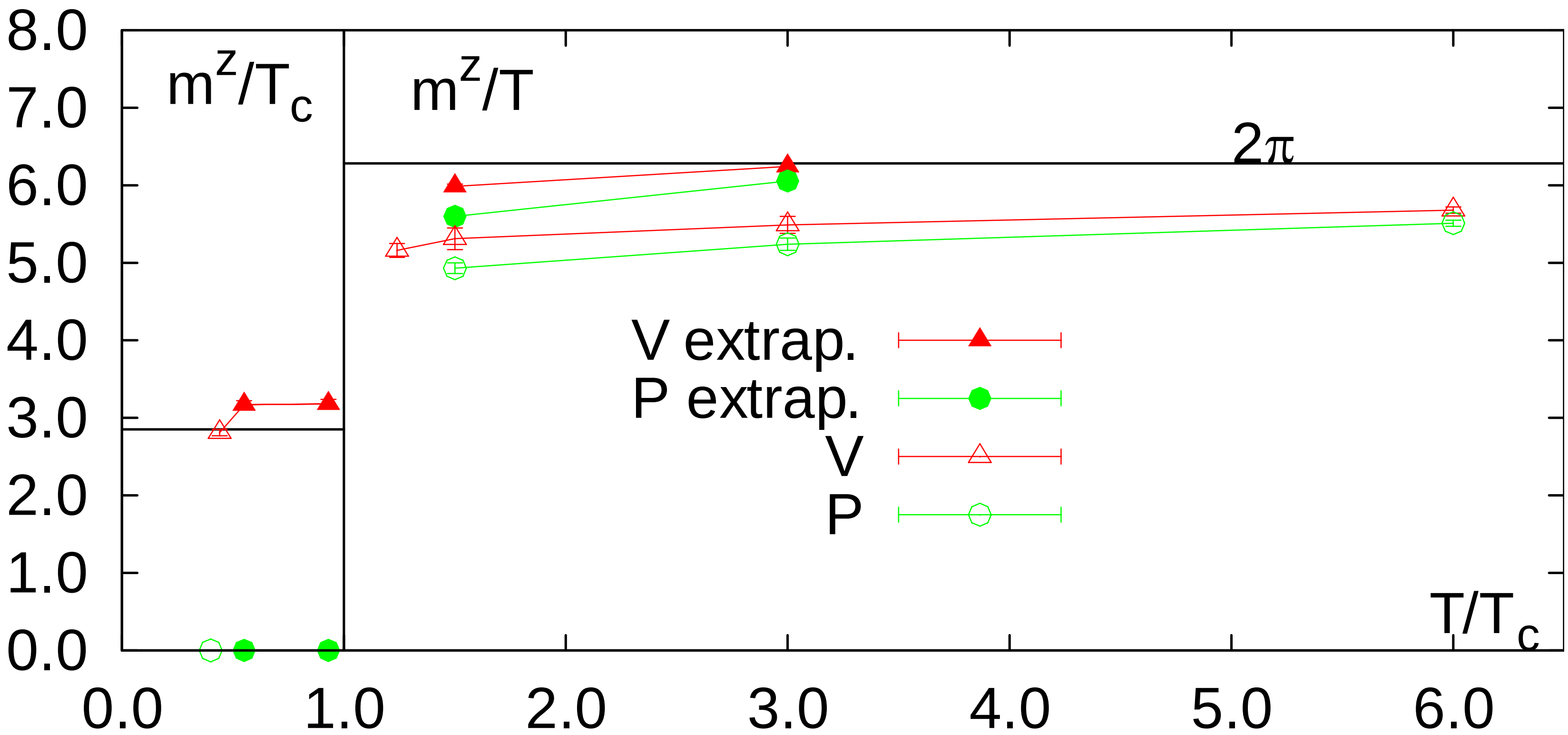
$$T = \infty: \quad \frac{m^z(a)}{T} = \frac{m^z}{T} - \lambda \left(\frac{1}{N_\tau}\right)^2$$





Thermodynamic and Continuum Limit:

$V \to \infty$ and $a \to 0$



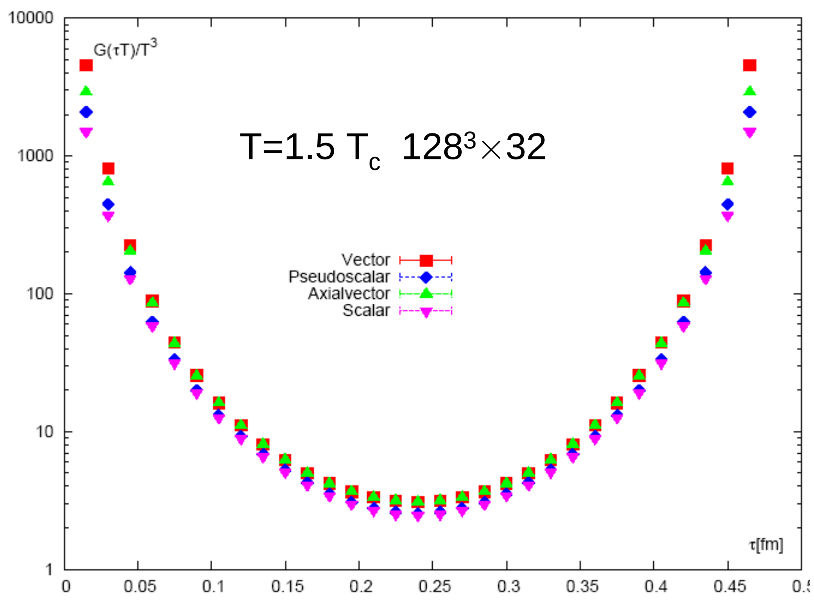
weak temperature dependence below T_c

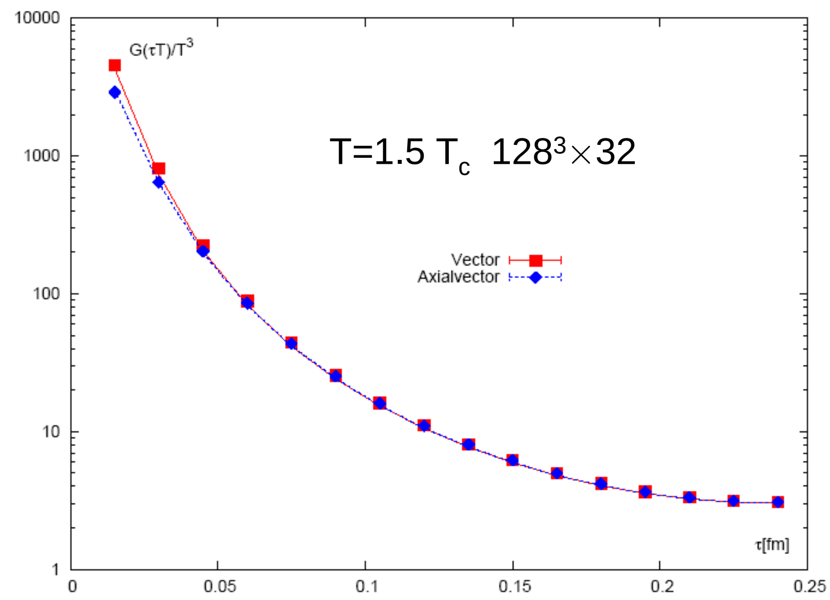
data still below free limit ($2\pi T$) at 3 T_c, vector closer to free case

perturbative limit reached from above [Laine, Vepsäläinen]

$\to$ need higher temperatures

Light Quark Correlators vs Free Correlators





comparison with free high temperature lattice correlator

vector and axial-vector close to free case

Cut-off effects are well described by free lattice correlator

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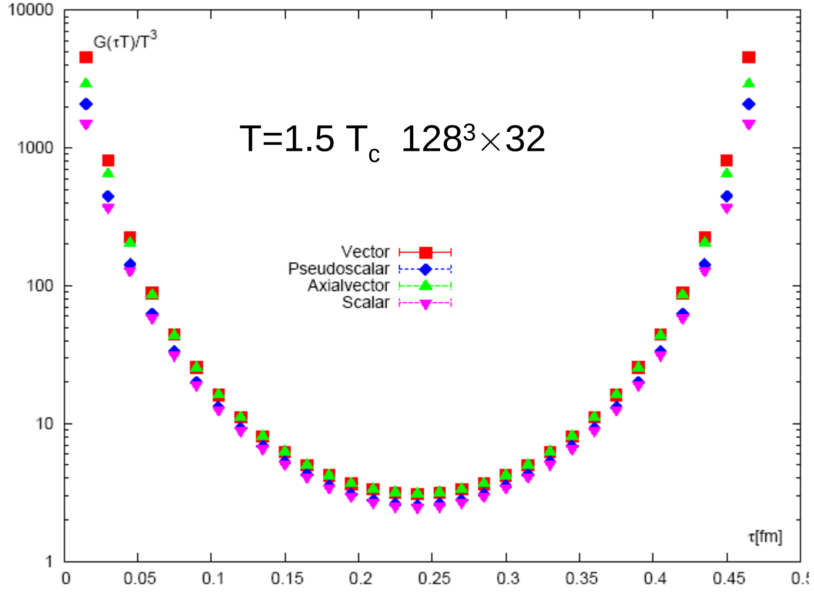

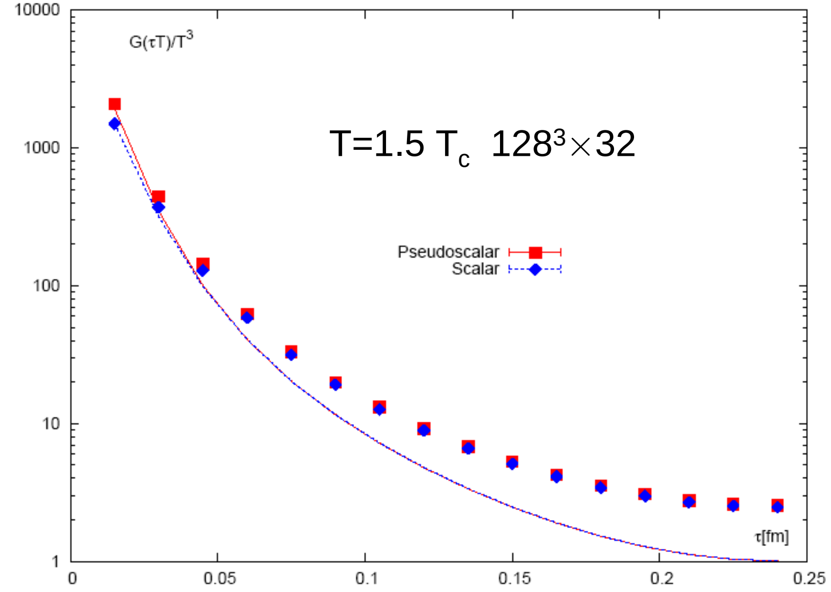

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still strong correlations in scalar and pseudo-scalar channel

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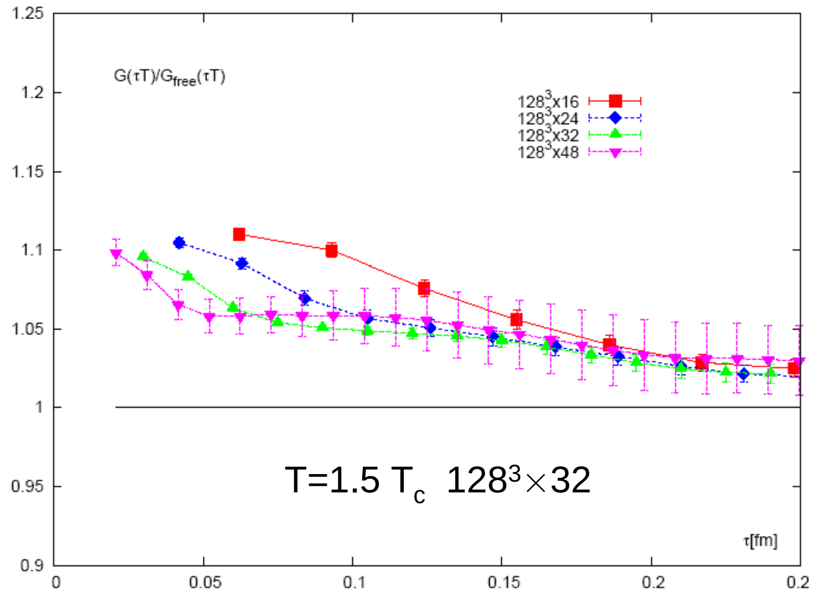

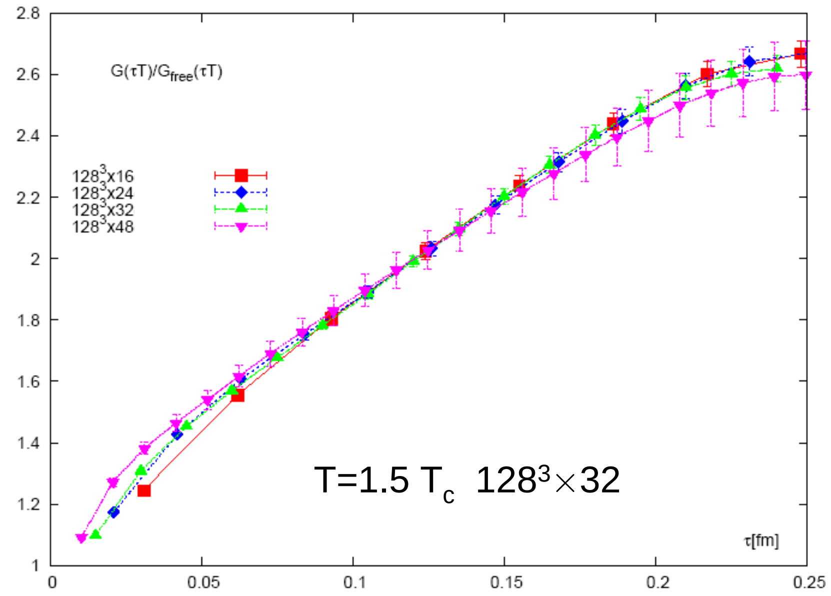

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Spectral Functions – Maximum Entropy Method

How to obtain continuous spectral function $\sigma(\omega$,T)

from discrete (and small) number of correlators?

$$G(\tau, T) = \int_0^\infty \mathrm{d}\omega K(\tau, \omega, T)\sigma(\omega, T)$$

$$K(\tau, \omega, T) = \frac{\cosh\left(\omega(\tau - \frac{1}{2T})\right)}{\sinh\left(\frac{\omega}{2T}\right)}$$

Best method on the market: Maximum Entropy Method (MEM)

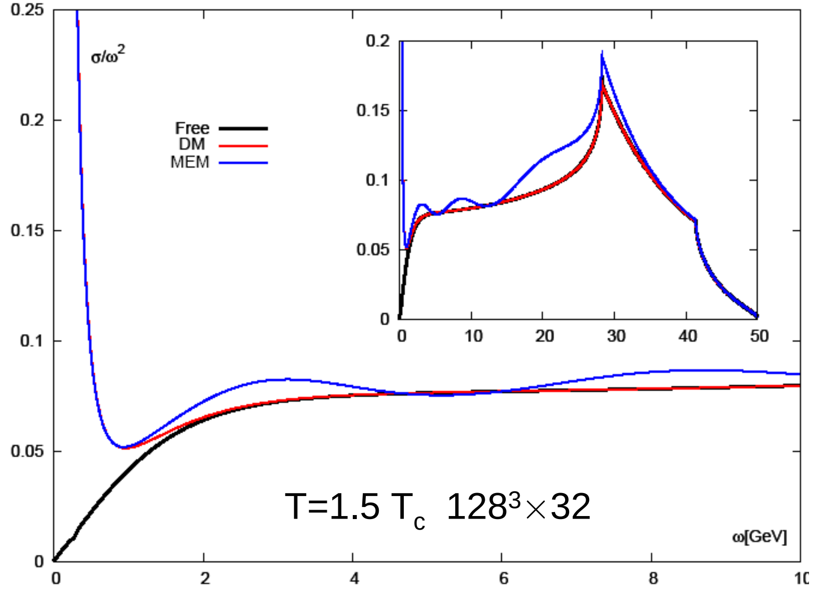
based on Bayesian theorem [Asakawa et al. 01] $\rightarrow$ most probable spectral function

prior knowledge needed as input $\rightarrow$ default model m(ω)

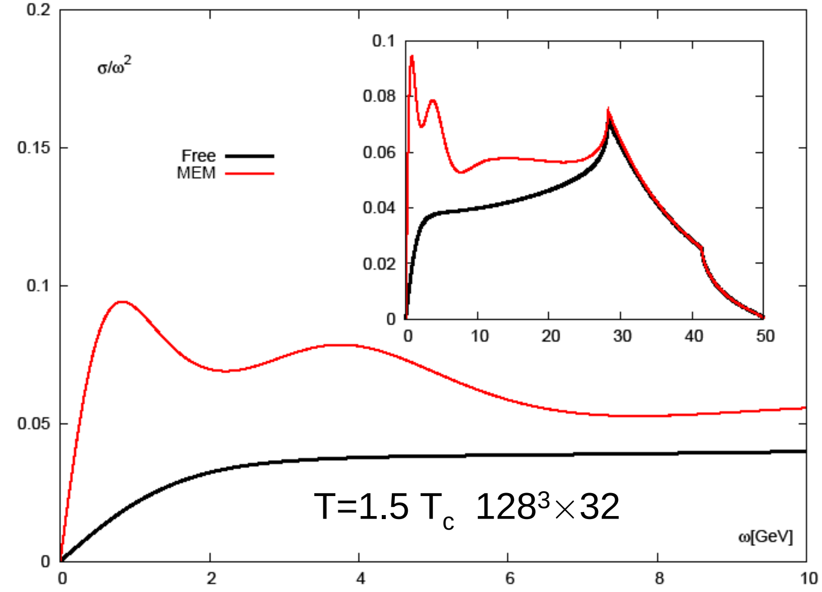
result should be independent of default model $\leftarrow$ usually not the case

Spectral Function – Light Quark Sector

Vector



Pseudoscalar



Large ω behaviour well described by free lattice SPF

Cut-Off effects are under control and well separated form physical interesting region

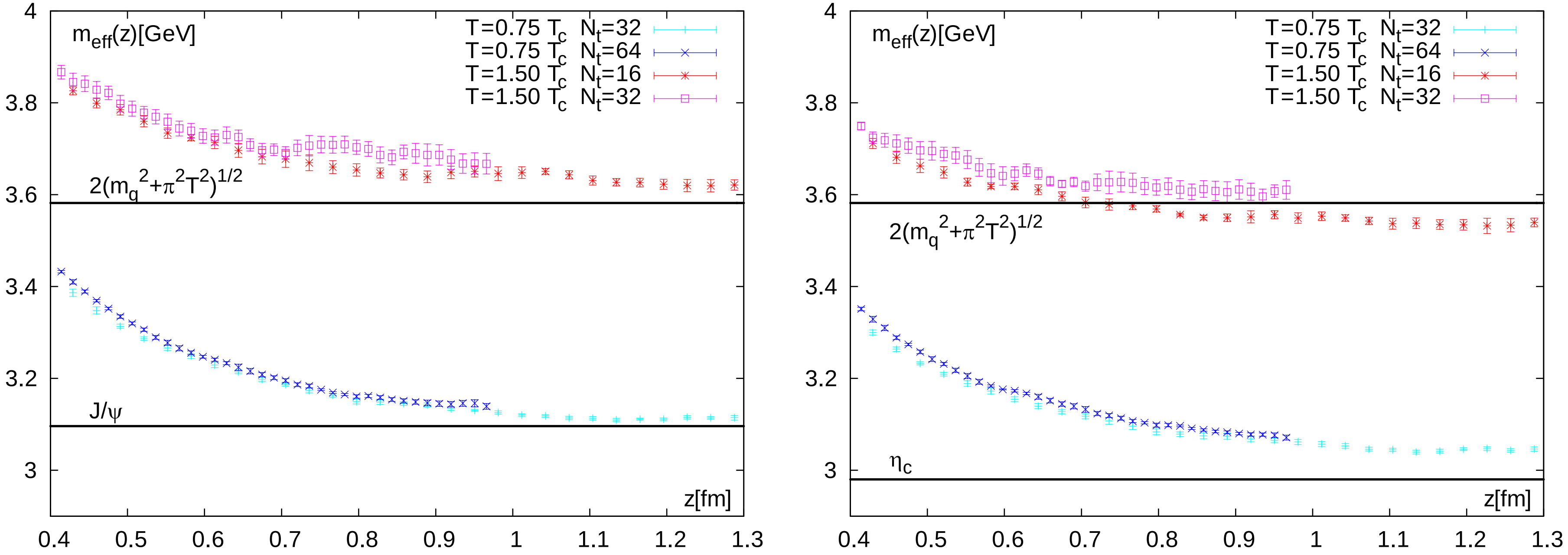
Vector SPF close to free case except at small ω

Still large correlations in the Pseudoscalar sector

Small ω region accessible $\rightarrow$ hope to extract transport properties

Next step: Dileptonrate (in the contiuum limit)

Charmonium Correlators – Screening Masses



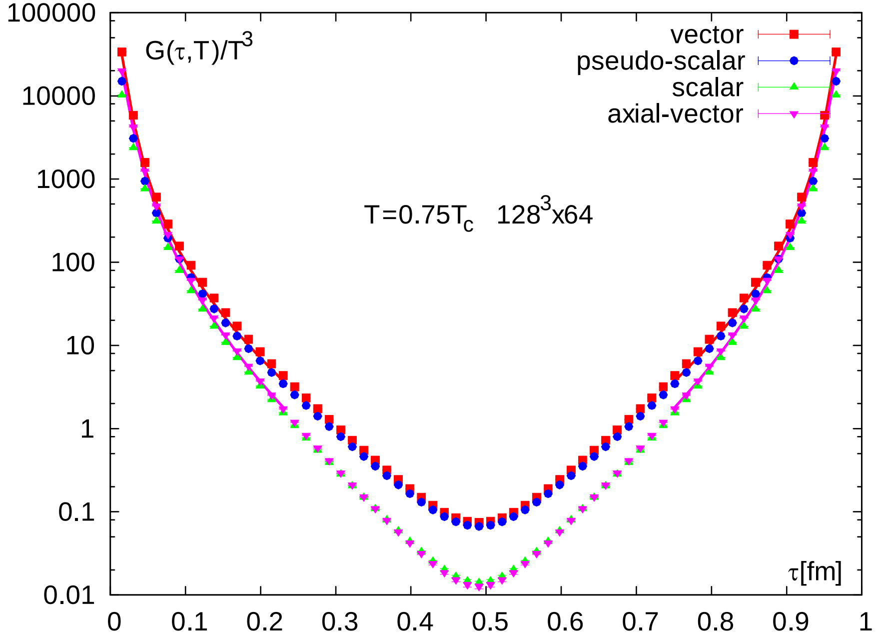
screening masses at 1.50 T_c already close to the free case $m^{scr}_{free}(T) = 2\sqrt{(\pi T)^2 + m_c^2}.$

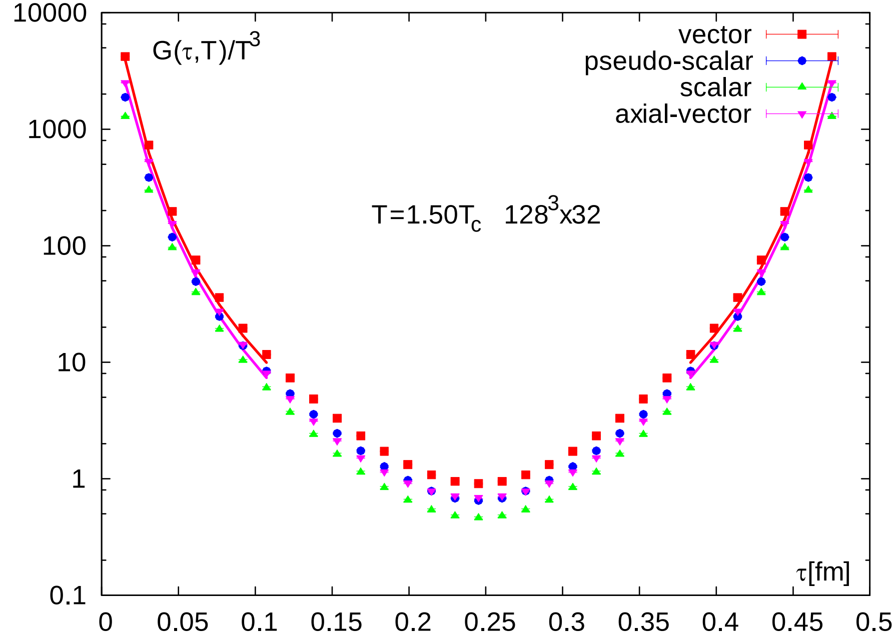
does this tell us anything about dissociation?

need to understand the momentum dependence of $G_H(\tau,T,p)$ and $\sigma(\omega,T,p)$ in detail

thermodynamic and continuum limit not performed yet

Charmonium Correlators – Temporal Correlators





non-degenerate states still at 1.50 T_c

(almost) close to free correlators at (very) small separations

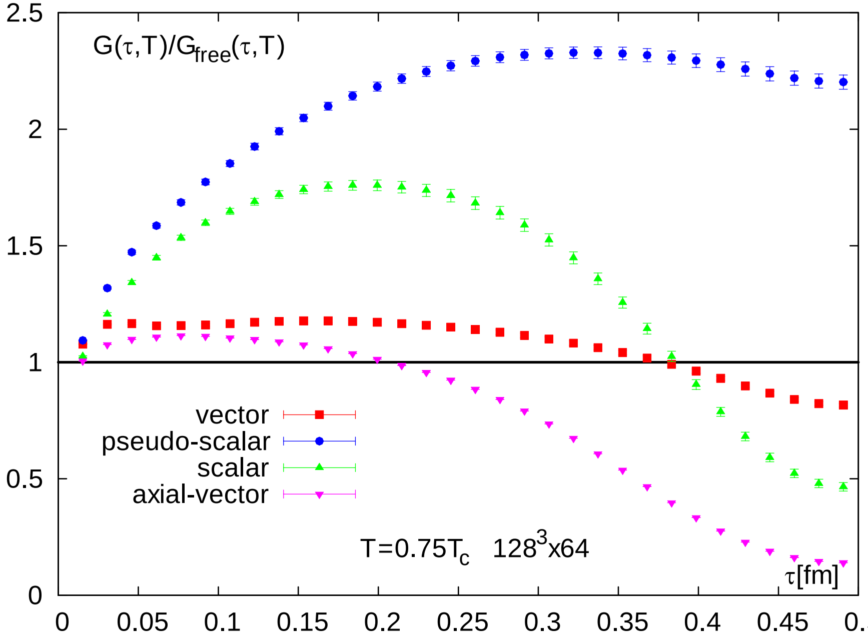
largest distance 0.25 fm due to compact temporal direction

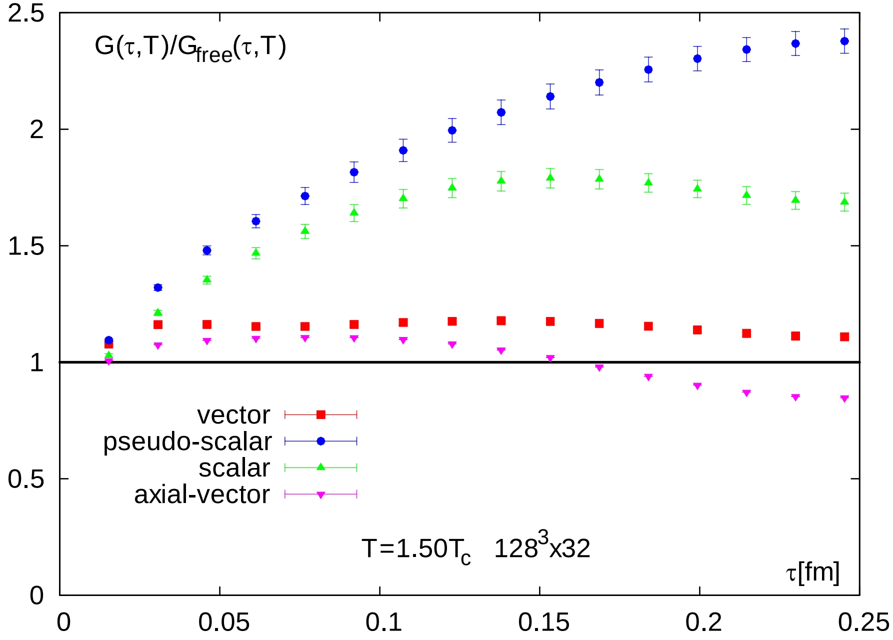
only small distance regime (0.1-0.25 fm) relevant

for thermal effects

for bound state effects

Charmonium Correlators vs Free Correlators





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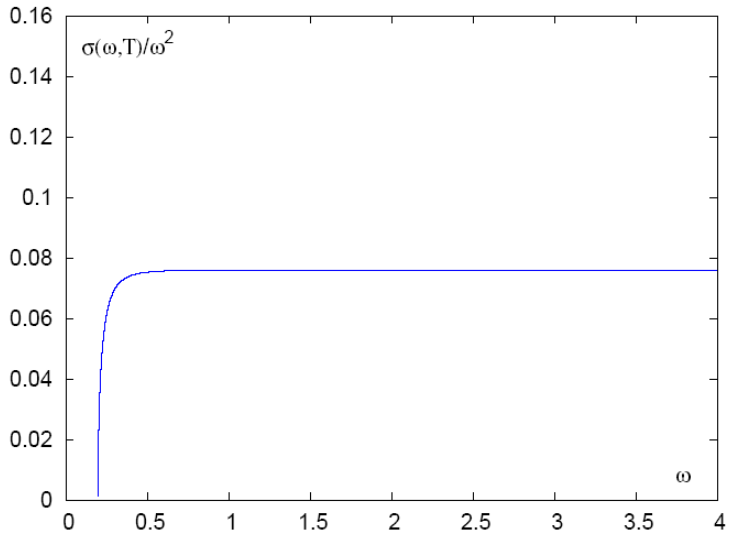
MEM – Free spectral function

Free (non-interacting) spectral function [Karsch et al. 03, Aarts et al. 05]

$$\sigma_H = \frac{N_c}{8\pi^2}\Theta(\omega^2 - 4m^2)\ \omega^2 \tanh(\frac{\omega}{4T}) \times\sqrt{1-(\frac{2m}{\omega})^2}\left[a_H + (\frac{2m}{\omega})^2\ b_H\right] + \frac{N_c}{3}\ \frac{T^2}{2}\ f_H\ \omega\delta(\omega)$$



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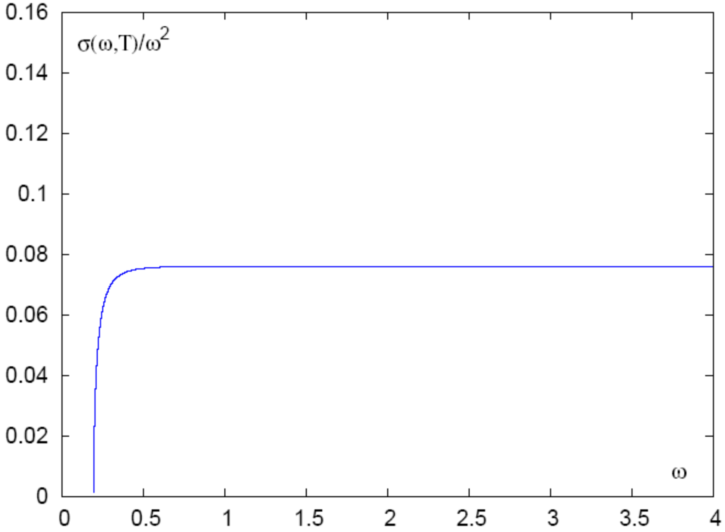
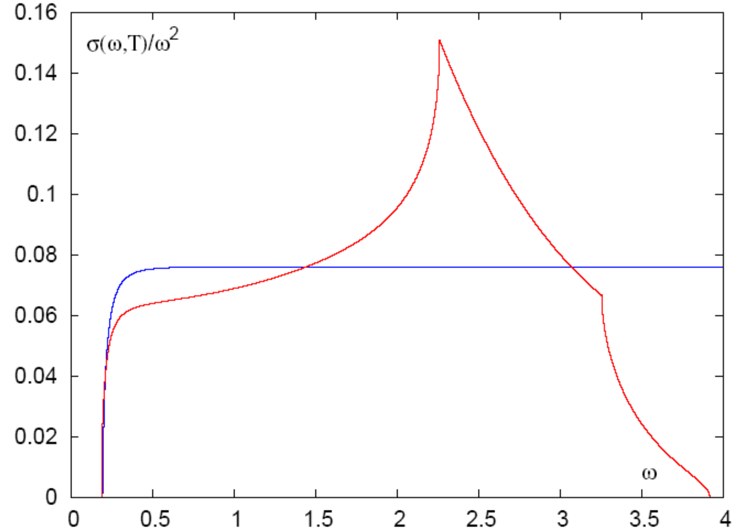
$$+\ \frac{N_c}{3}\ \frac{T^2}{2}\ f_H\ \omega\delta(\omega)$$

Lattice cut-off effects

$$\omega_{max} = 2\log(7+ma)$$





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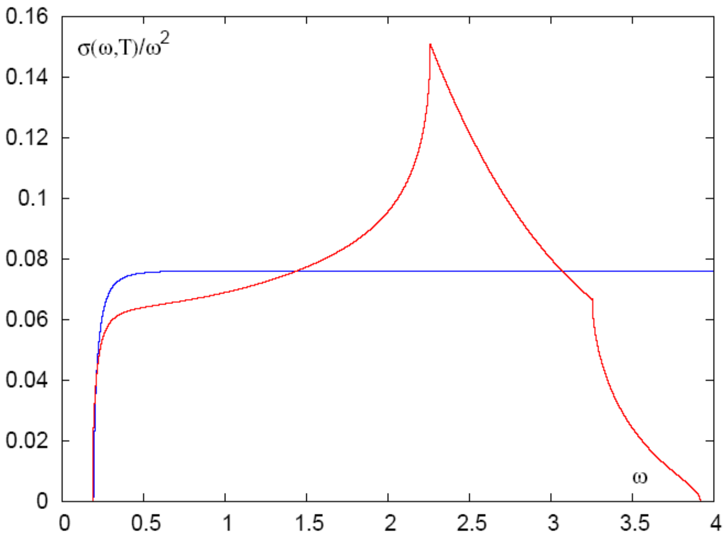
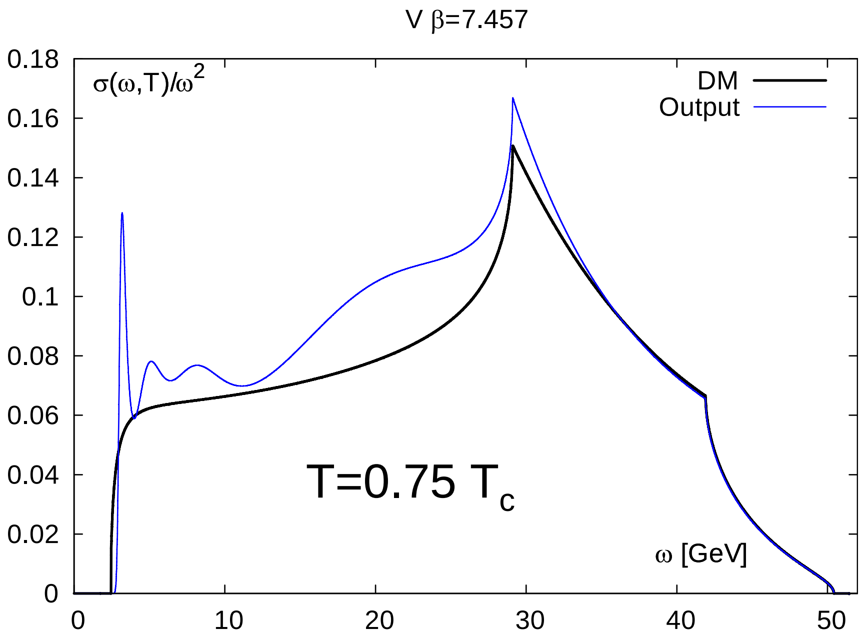
zero mode contribution at $\omega \simeq 0$ [Umeda 07]

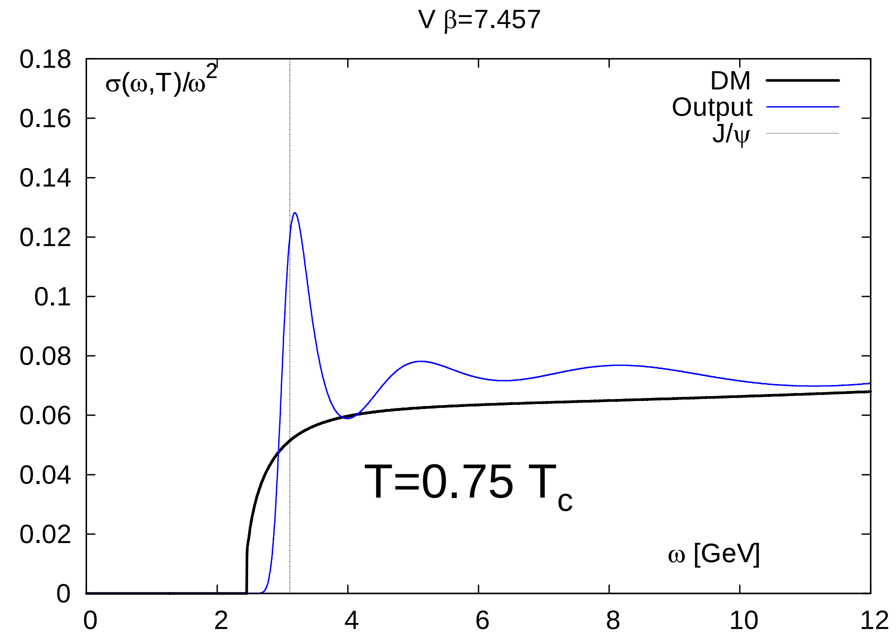
$$\delta(\omega) \to \frac{1}{\pi}\frac{\eta}{\omega^2+\eta^2}$$

[Petreczky+Teany 06 Aarts et al. 05]



MEM – Spectral function below T_c



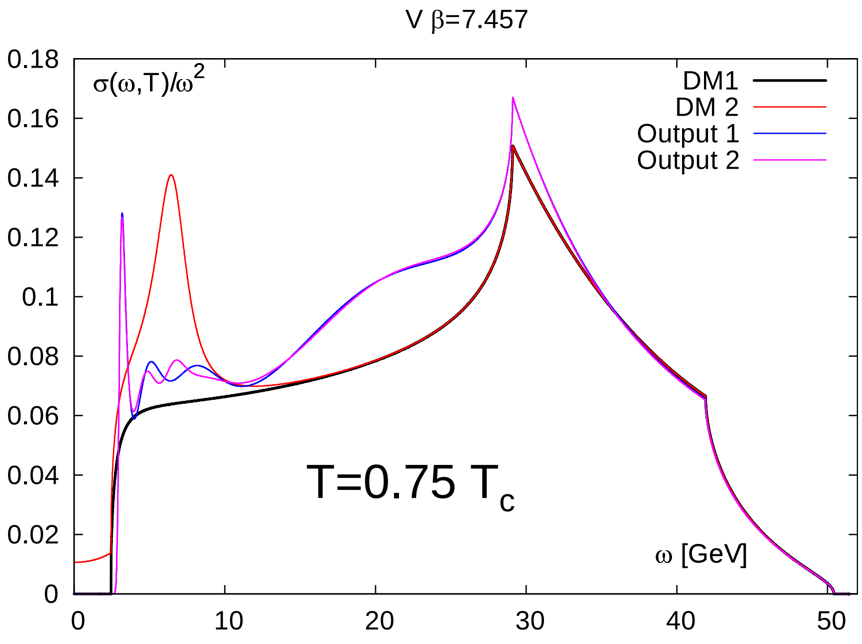


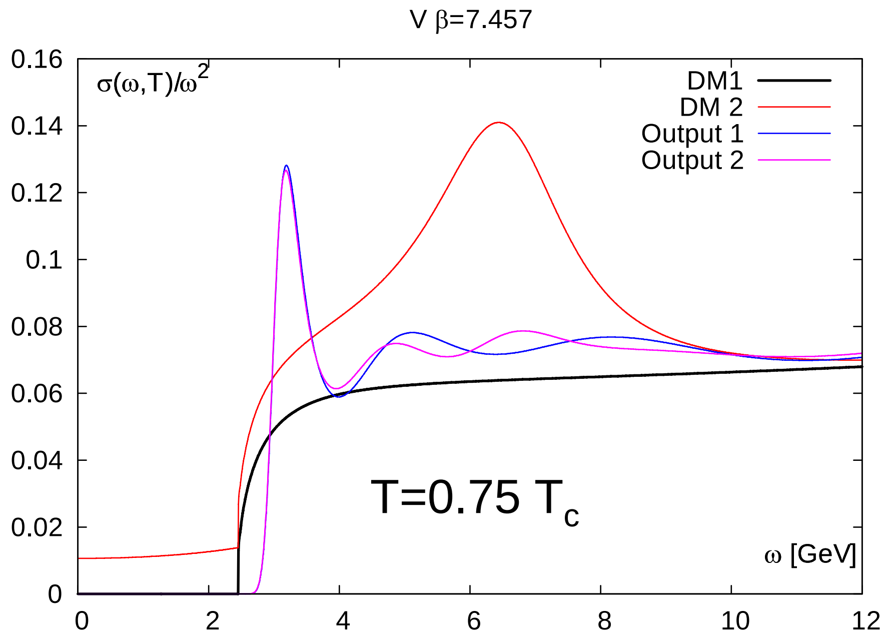
N_σ=128 and N_τ=64 $\rightarrow$ cut-off effects well separated

Pronounced ground state peak close to J/ψ mass

no zero mode contributions observed below T_c

MEM – Spectral function below T_c





N_σ=128 and N_τ=64 $\rightarrow$ cut-off effects well separated

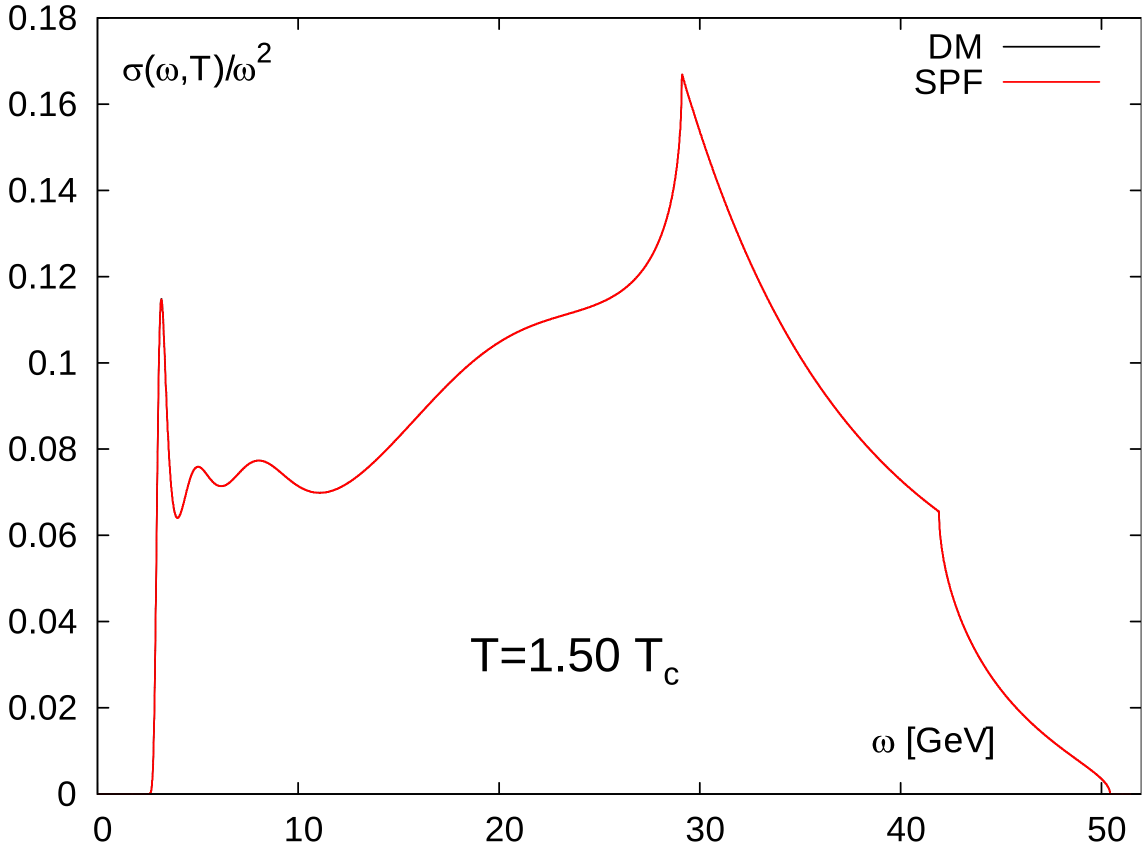
pronounced ground state peak close to J/ψ mass

no zero mode contributions observed below T_c

first peak independent of default model

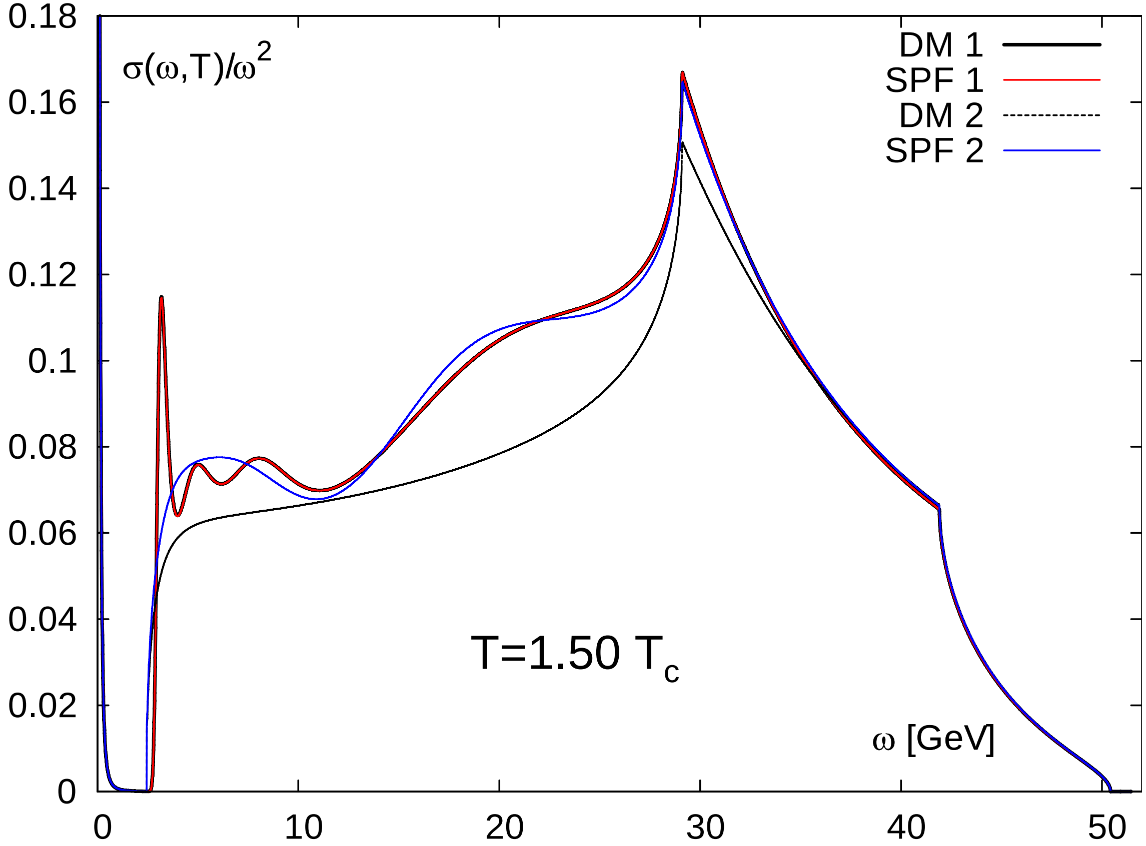
MEM – Spectral function above T_c

Use $0.75T_c$ SPF as default model at $1.50T_c$

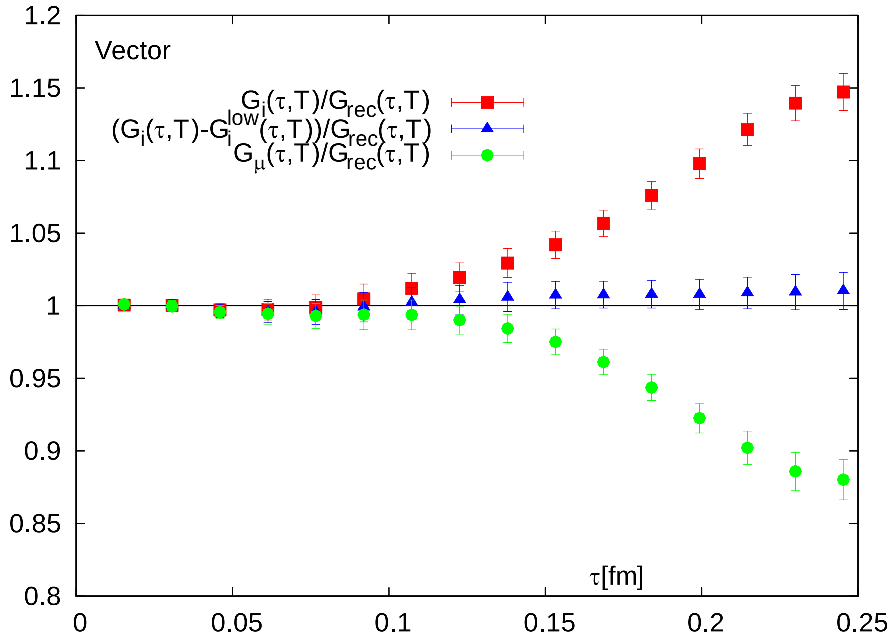


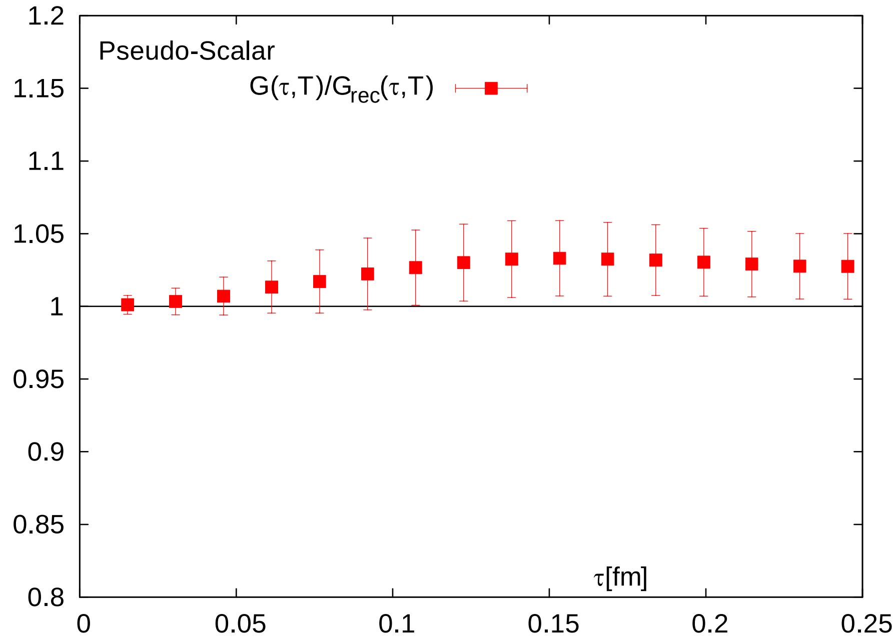
MEM – Spectral function above T_c

Use free SPF as default model at $1.50T_c$



Charmonium Correlators – Zero Mode Contributions





$$G_{rec}(\tau,T) = \int \sigma_0(\omega, 0.75T_c) K(\omega,\tau,T)$$

$$G_{rec}^{low}(\tau,T) = \int_0^{2m_c} \sigma_T(\omega,T) K(\omega,\tau,T)$$

$$G_{\mu\mu}(\tau,T) = G_{ii}(\tau,T) + G_{44}(\tau,T)$$

- main T-effect due to zero-mode contribution
- well described by small ω-part of $\sigma_T(\omega,T)$
- smaller than $G_{44}(\tau,T)=\chi(T)T$
- no zero-mode contribution in PS-channel

(similar to discussions by Umeda, Petreczky)

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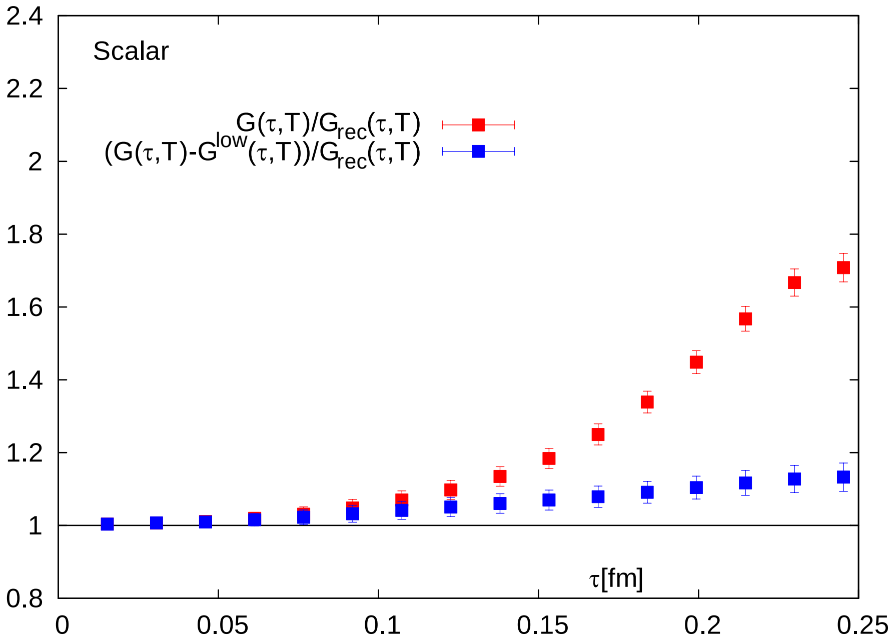

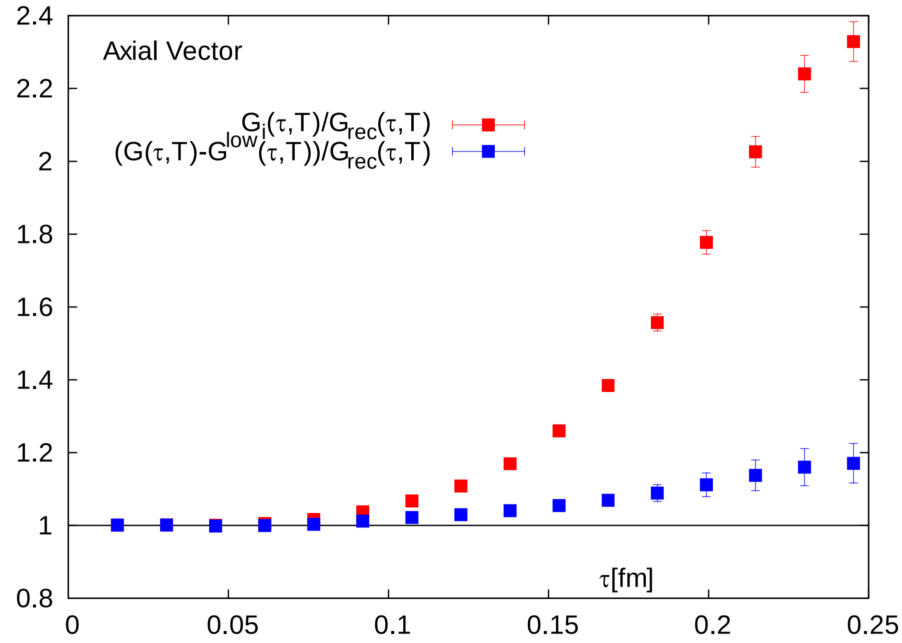

$$G_{rec}(\tau,T) = \int \sigma_0(\omega, 0.75T_c) K(\omega,\tau,T)$$

$$G^{low}_{rec}(\tau,T) = \int_0^{2m_c} \sigma_T(\omega,T) K(\omega,\tau,T)$$

- larger zero-mode contribution in S-wave
- larger T-effect in the S-wave states

systematic uncertainties in reconstruction and low-ω part of spetral function

high quality data and small lattice spacing + large momenta (volume) needed

Conclusions and Outlook

Hadronic correlators for light quarks (m_q=0)

Thermodynamic and Continuum Limit of screening masses!

Spectral functions $\rightarrow$ Dilepton rates, Transport coefficients?

Momentum dependence needs to be analyzed $\rightarrow$ Photonrates

Comparison with HTL calculations and experiment

Charmonium hadronic correlators (m_q=m_c)

Can we really trust any of the spectral functions obtained with MEM?

What can we learn from Hadronic correlators on Dissociation?

Momentum dependence vs. Spatial correlators/screening masses?

Spectral functions $\rightarrow$ Dilepton rates, Transport coefficients?

Momentum dependence needs to be analyzed $\rightarrow$ Photonrates

Comparison with HTL/NRQCD calculations and experiment

Many Thanks to

RBC-Bielefeld

+

HengTong Ding

Anthony Francis

Helmut Satz