# Quarkonium in Lattice QCD

## What can we learn from lattice correlators?

Olaf Kaczmarek

Universität Bielefeld

ECT* Workshop on
Heavy Quarkonium Production in Heavy-Ion Collisions
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# Introduction

**Heavy Quark Free Energies ($m_q=\infty$)**

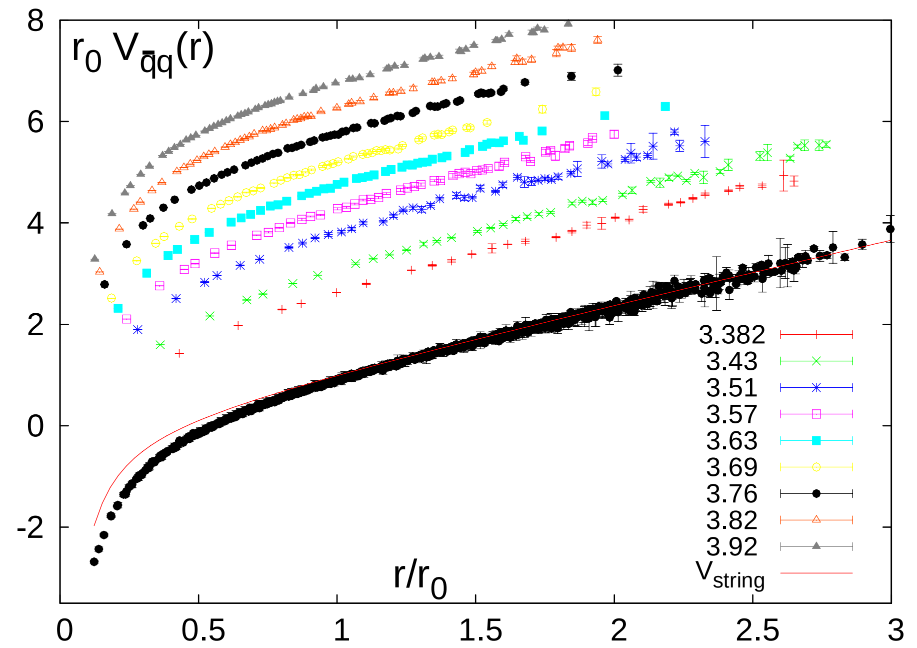
- Zero Temperature Potential (as reference)
- T-dependent Free energies and Screening
- Entropy vs. Internal Energy contributions

**Hadronic correlators for light quarks ($m_q=0$)**

- temporal correlators vs. free correlators
- screening masses in the thermodynamic and continuum limit

**Charmonium hadronic correlators ($m_q=m_c$)**

- temporal correlators vs. free correlators
- temporal correlators vs. reconstructed correlators (from 0.75 $T_c$ )
- charmonium screening masses
- zero mode contributions




## Large distance behaviour:

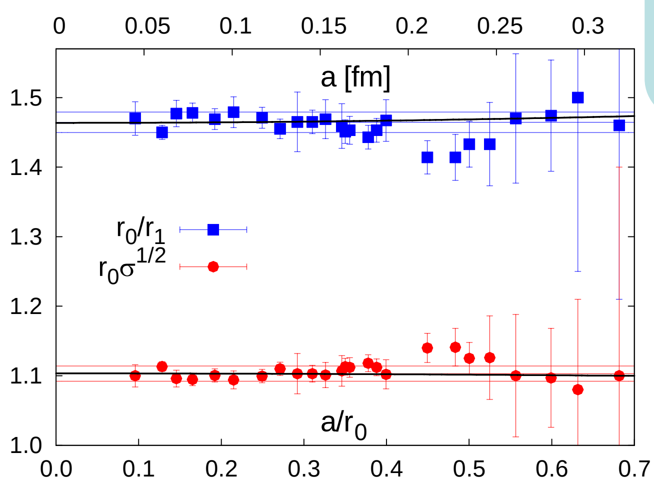
consistent with string model prediction

$$V(r) = -\frac{\pi}{12}\frac{1}{r} + \sigma r$$

used for renormalization

$$V(r) = -\log\left((Z_{\text{ren}}(\beta))^2 \frac{W(r,\tau)}{W(r,\tau+1)}\right)$$

cut-off effects are small

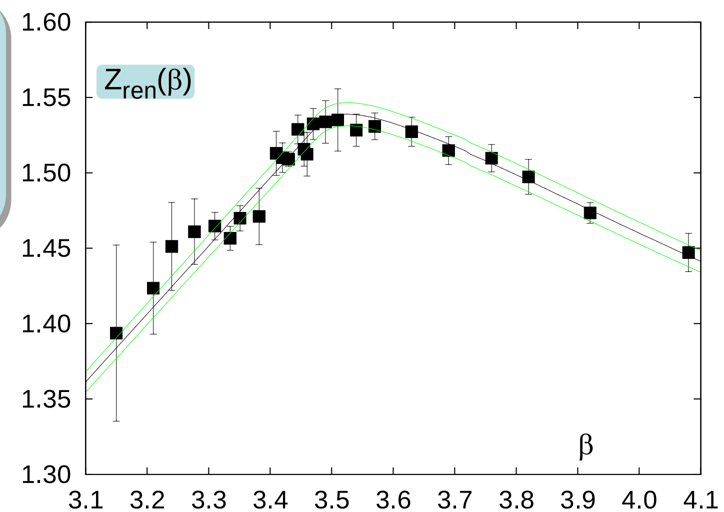


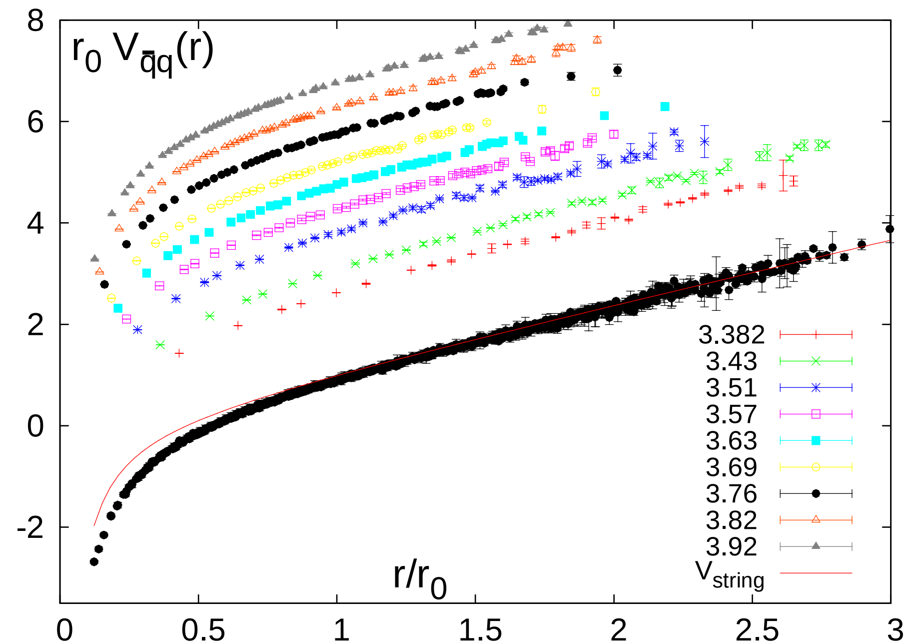

2+1-flavour QCD
highly improved
p4-staggered action
almost realistic quark masses
physical $m_s$, $m_\pi \approx$ 220 MeV

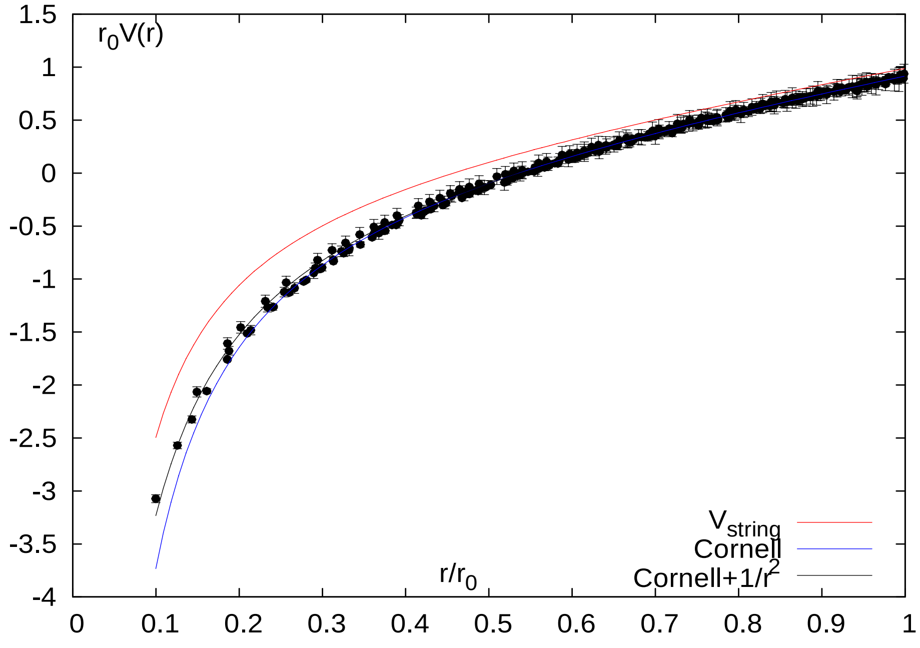
$$r^2 \frac{\mathrm{d}V_{\bar{q}q}(r)}{\mathrm{d}r}\bigg|_{r=r_0} = 1.65$$

$$r^2 \frac{\mathrm{d}V_{\bar{q}q}(r)}{\mathrm{d}r}\bigg|_{r=r_1} = 1.0$$

$$(r_0 = 0.469(7)\ \text{fm})$$






## Large distance behaviour:

consistent with string model prediction

$$V(r) = -\frac{\pi}{12}\frac{1}{r} + \sigma r$$

## Short distance behaviour:

deviations from string model

enhancement of the running coupling

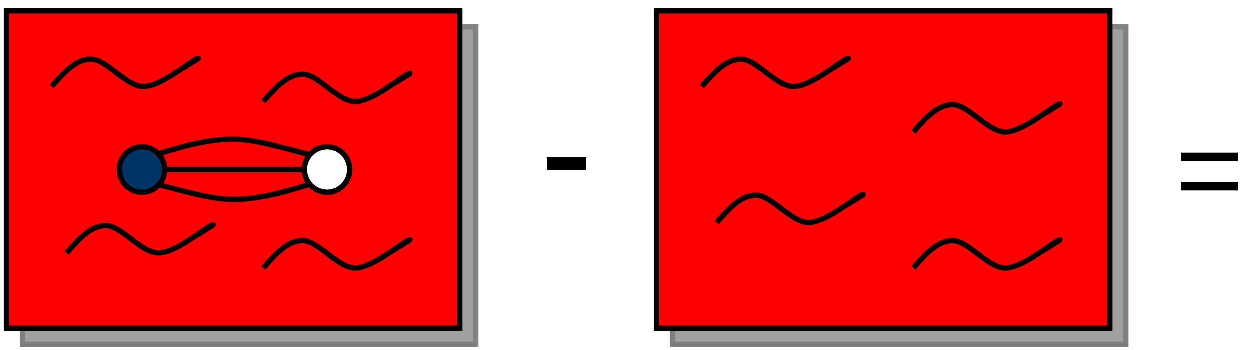
$$V(r) = -\frac{0.392(6)}{r} + \sigma r$$

r-dependent running coupling $\alpha$(r)

Heavy Quark Free Energies defined by Polyakov Loop Correlators

*L. McLerran, B. Svetitsky (1981)*

$$\frac{\mathcal{Z}_{\mathbf{Q\bar{Q}}}}{\mathcal{Z}(\mathbf{T})} \simeq \frac{1}{\mathcal{Z}(\mathbf{T})} \int \mathcal{D}\mathbf{A}\ldots\ \mathbf{L}(\mathbf{x})\ \mathbf{L}^{\dagger}(\mathbf{y})\ \exp\left( - \int_{\mathbf{0}}^{\mathbf{1/T}} \mathbf{dt} \int \mathbf{d^3x}\mathcal{L}[\mathbf{A},\ldots] \right)$$



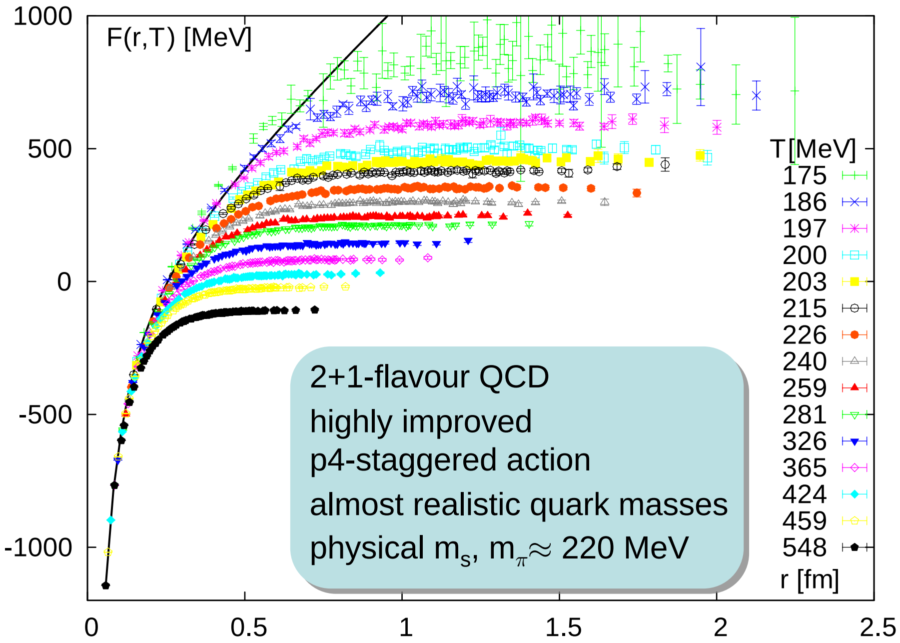
$$-\frac{\mathbf{F_{Q\bar{Q}}(r,T)}}{\mathbf{T}}$$

$\mathbf{Q\bar{Q} = 1, 8, av}$

measures the change in free energy induced by the $Q\bar{Q}$ pair

What can we learn about medium modifications of Quarkonium?

# Heavy Quark Free Energies – Medium Modifications




Renormalization of F(r,T)

$$e^{-F_1(r,T)/T} = \left(Z_r(g^2)\right)^{2N_\tau} \langle \mathrm{Tr}(L_x L_y^\dagger)\rangle$$

alternative renormalization procedures all equivalent!

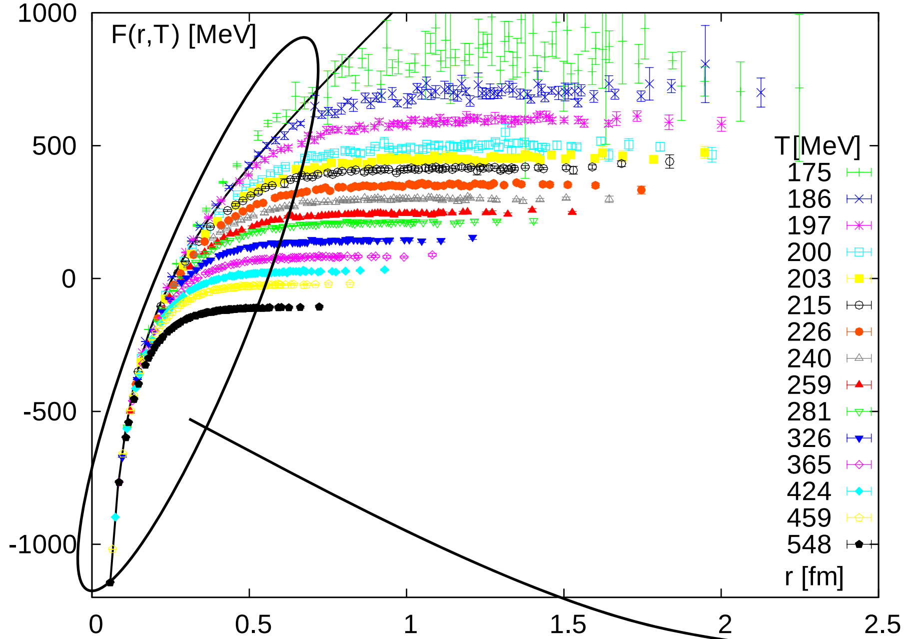
*(O. Kaczmarek et al., PLB543(2002)41,*
*S. Gupta et al., Phys.Rev.D77(2008)034503)*

no temperature effect at small distances

zero temperature (perturbative) at sufficiently small r : $F_1(r) \simeq -\alpha(r)/r$

medium modifications set in at smaller separarations with increasing T

# Heavy Quark Free Energies – Temperature dependent running coupling




## small distance behaviour: $\alpha_{eff}$(r,T)

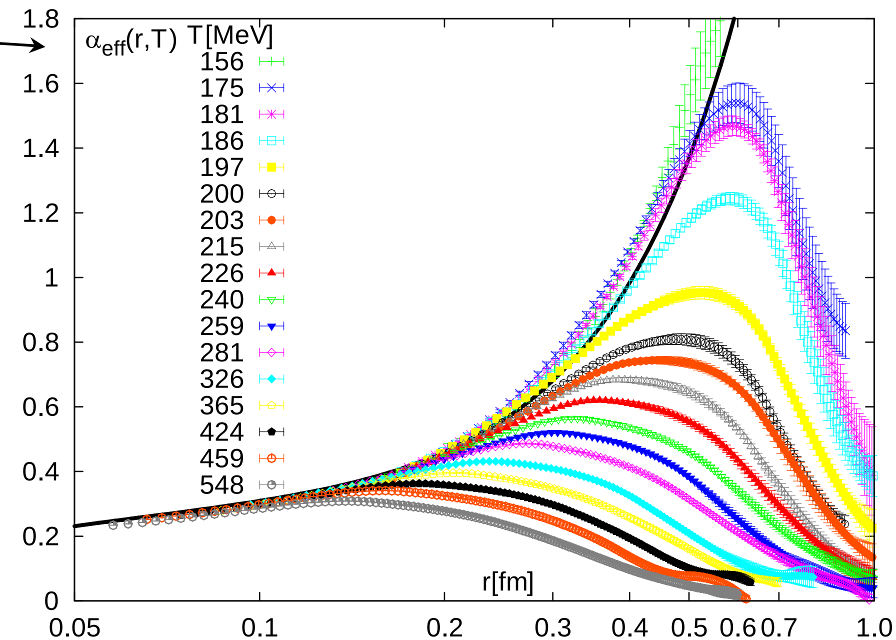
zero-T (perturbative) behaviour at small r

non-perturbative large values for r>0.4 fm

$$\alpha_{eff}(r,T) \simeq \frac{3}{4} r^2 \sigma$$

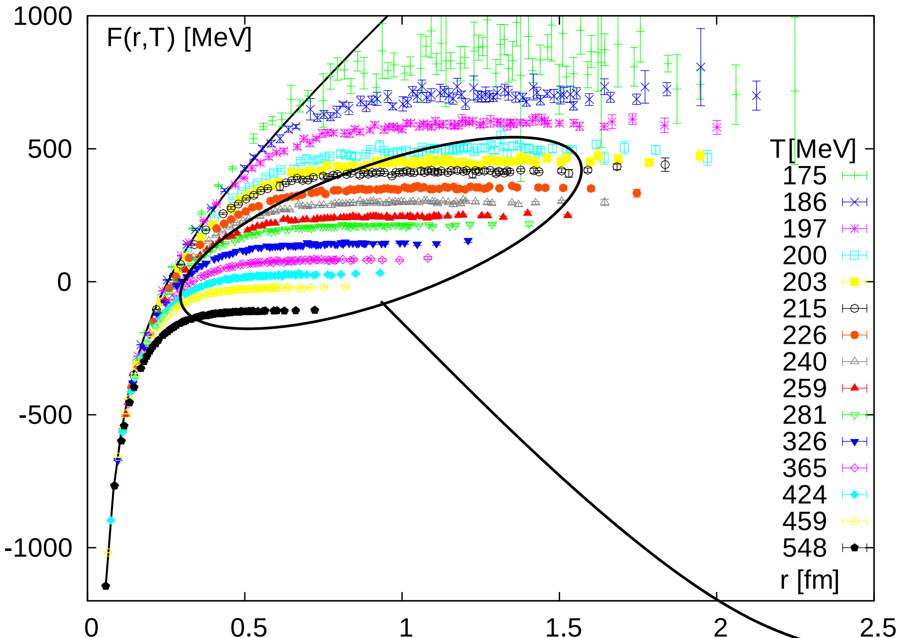
remnants of confinement forces above $T_c$

screening sets in at smaller r with incr. T




## T and r dependent running coupling:

$$\alpha_{eff}(r,T) = \frac{3}{4} r^2 \frac{\mathrm{d}F_1(r,T)}{\mathrm{d}r}$$




## Intermediate distances: screening

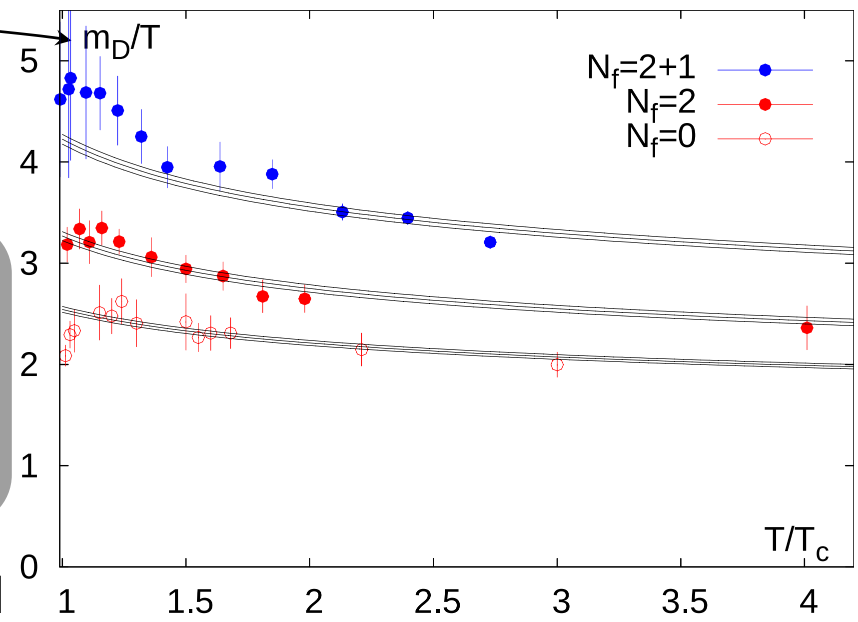
screening masses obtained from fits

$$F_1(r,T) - F_1(r=\infty,T) = -\frac{4}{3}\frac{\alpha}{r}e^{-m_D(T)r}$$

at large distances $rT \geq 1$

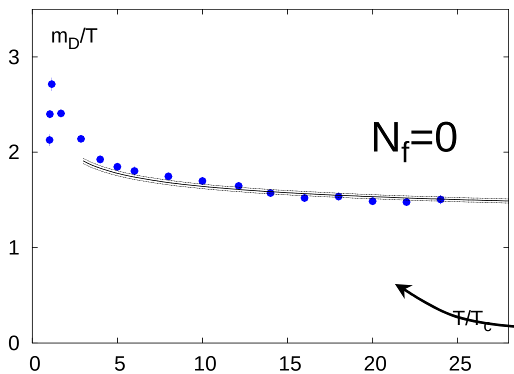
leading order perturbative result:

$$\frac{m_D(T)}{T} = A\left(1+\frac{N_f}{6}\right)^{1/2} g(T)$$




**perturbative limit reached very slowly**

**(logarithms at work)**

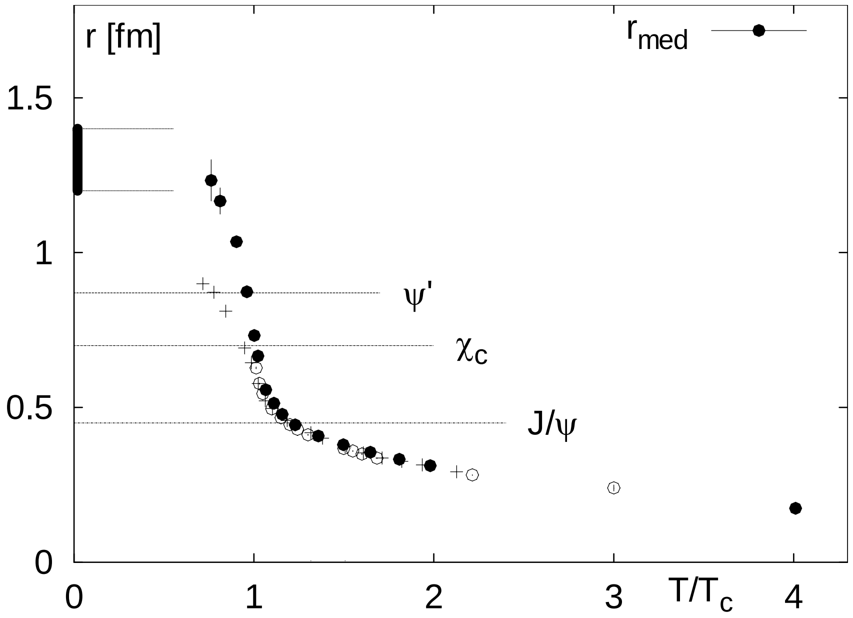


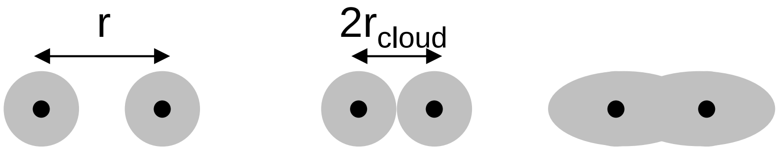

$$A_{N_f=2+1} = 1.66(2)$$
$$A_{N_f=2} = 1.42(2)$$
$$A_{N_f=0} = 1.52(2)$$

$$A_{N_f=0} = 1.39(2)$$

which g(T), i.e. which scale? $\mu=\mu(m_D)$? [A.Peshier 2006]

# Heavy Quark Free Energies – Debye Screening




define screening radius by

$$V_{T=0}(r_{med}) \equiv F_1(r=\infty, T)$$

equivalently $r_{med}=2/m_D$

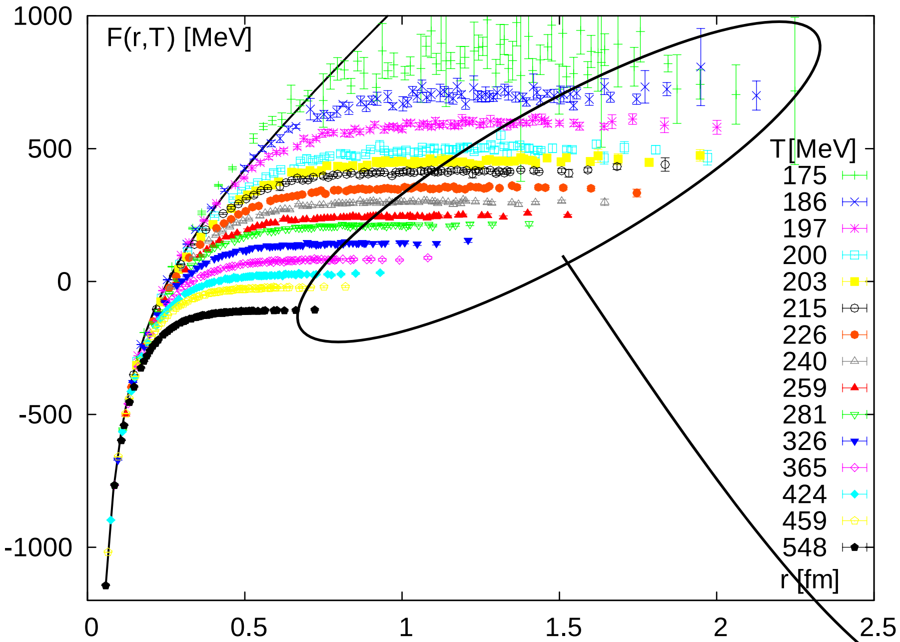
compare to mean charge radii of charmonium states

**thermal modifications on $\psi$' and $\chi_c$ already close to $T_c$**

**J/$\psi$ may survive above deconfinement**

The large distance behaviour of the finite temperature energies is rather related to screening than to the temperature dependence of masses of corresponding heavy light mesons!

# Heavy Quark Free Energies – Asymptotic behaviour




**Asymptotic behaviour:**

almost zero-T value close to $T_c$

strong descrease at critical temperature

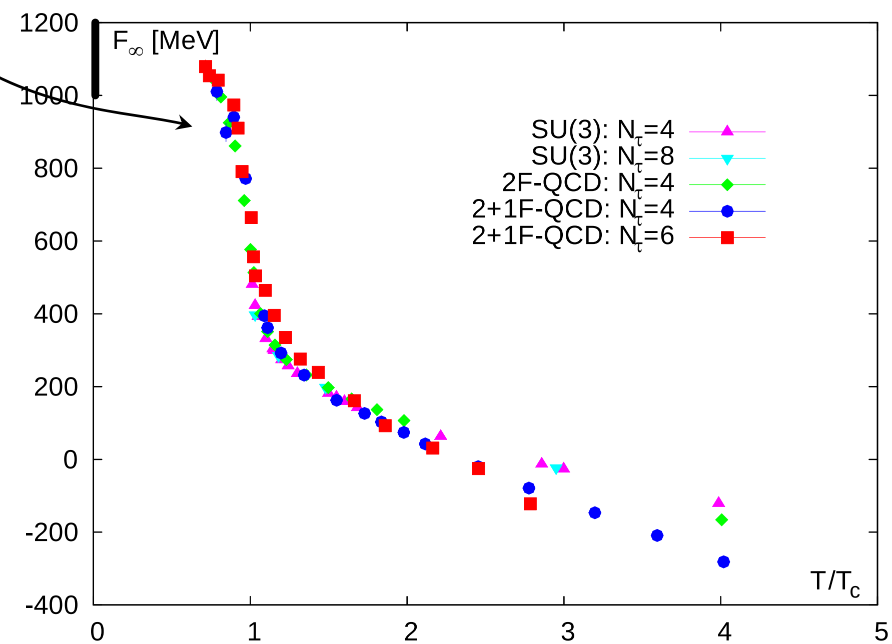
almost linear at high T: $F_1(\infty,T) \sim -T$

$F_1(r,T)$ not only determined by potential energy

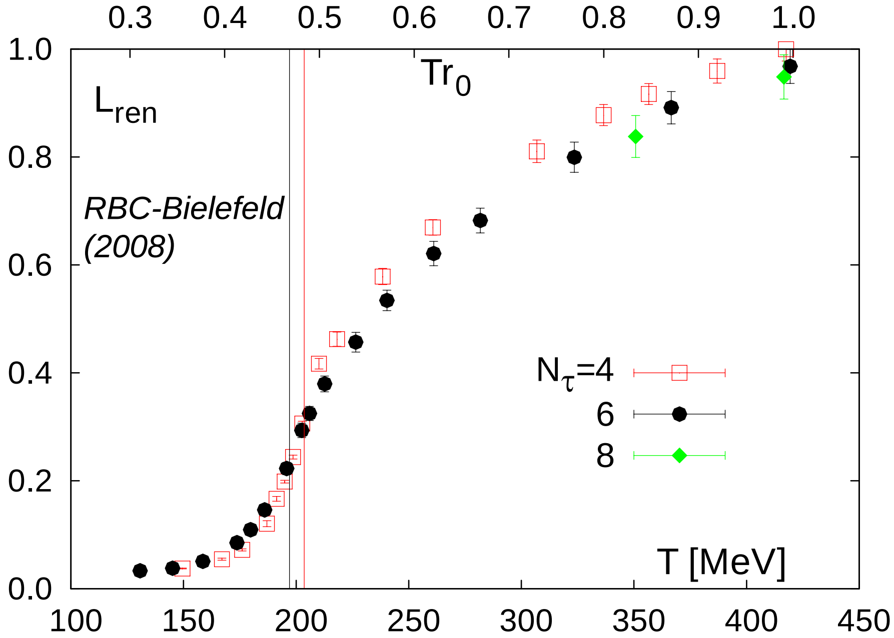
$$F_1(r,T) = U_1(r,T) - TS_1(r,T)$$

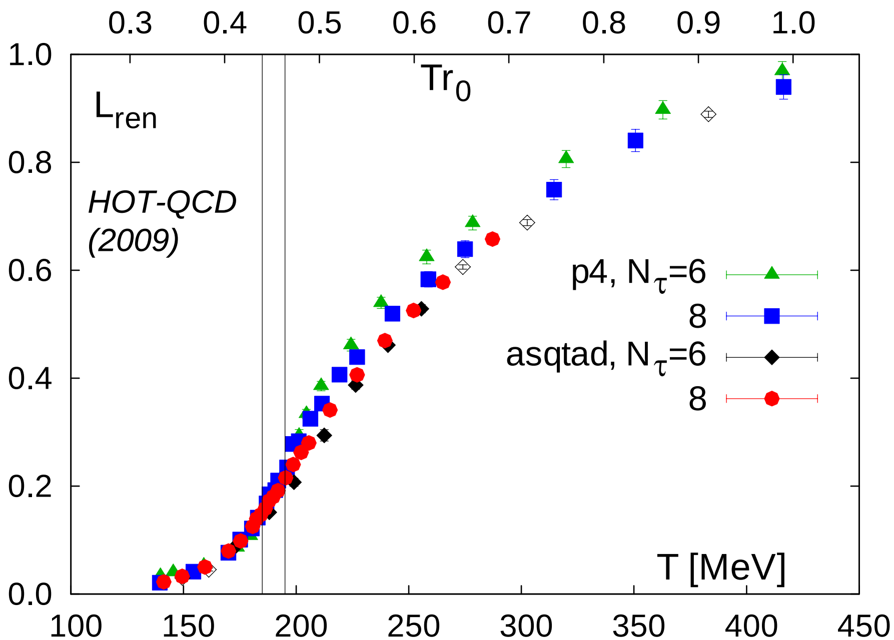
Entropy contributions become important

at finite temperature!



# Renormalized Polyakov Loop




Renormalization of free energies

$$e^{-\frac{F_1(r,T)}{T}} = \left(Z_R(g^2)\right)^{2N_\tau} \langle \mathrm{Tr}(L_x L_y^\dagger)\rangle$$

equivalent to renormalization

of Polyakov Loop

$$L_{\mathrm{ren}} = \left(Z_R(g^2)\right)^{N_\tau} L_{\mathrm{lattice}}$$

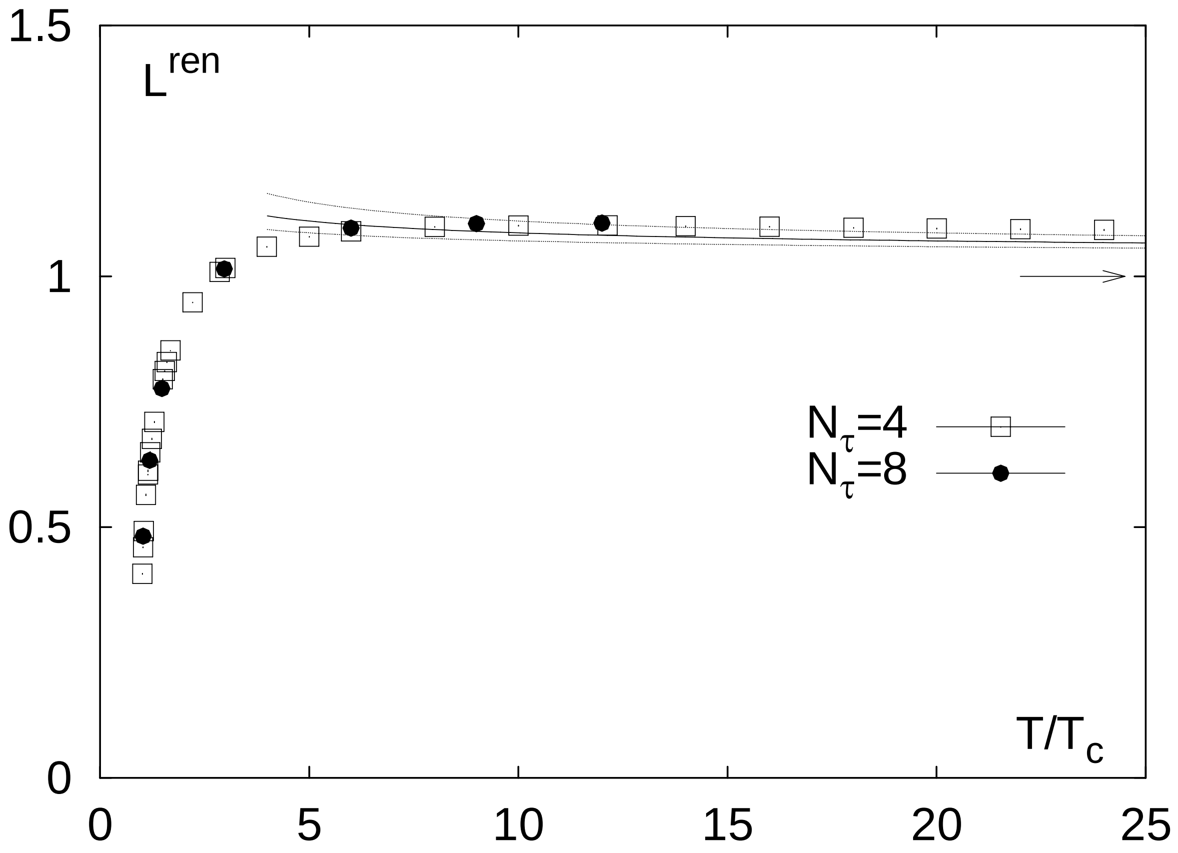
equivalent to definition by

large distance behaviour of $F_1(r,T)$

$$L_{\mathrm{ren}} = \exp\left(-\frac{F_1(r=\infty,T)}{2T}\right)$$

# Renormalized Polyakov Loop

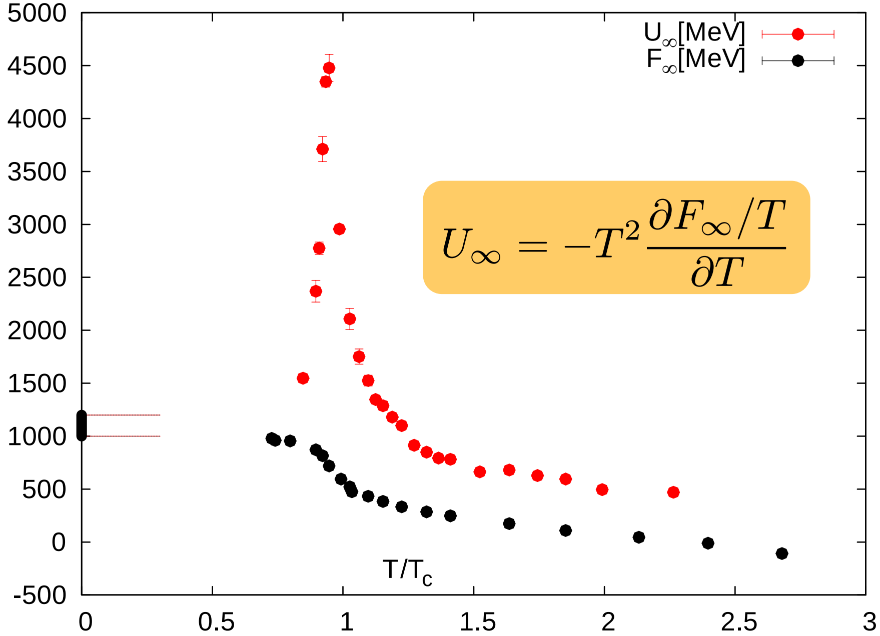
## SU(3) pure gauge result




high temperature limit, $L_{ren}$=1, reached from above as expected in PT

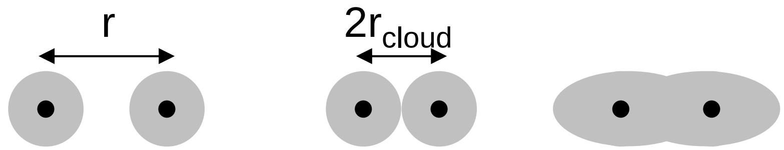
Non-perturbative effects dominate below 5 $T_c$

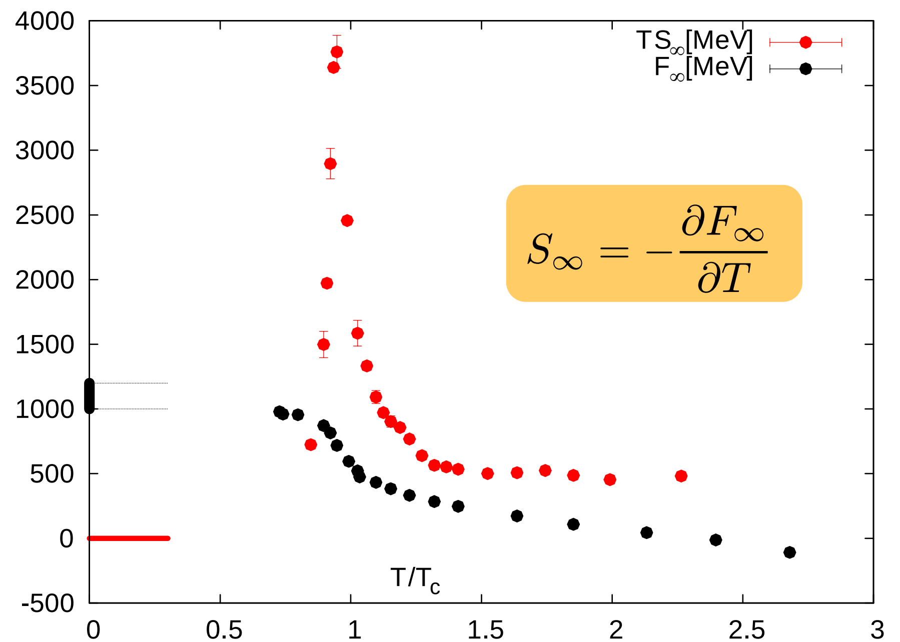
# Internal Energy vs. Entropy contributions – asymptotic behaviour




## High Temperature behaviour:

$$
\begin{aligned}
\mathbf{F_\infty(T)} &\simeq -\frac{\mathbf{4}}{\mathbf{3}}\mathbf{m_D(T)}\alpha\mathbf{(T)} \simeq -\mathcal{O}(g^3 T) \\
\mathbf{TS_\infty(T)} &\simeq +\frac{\mathbf{4}}{\mathbf{3}}\mathbf{m_D(T)}\alpha\mathbf{(T)} \\
\mathbf{U_\infty(T)} &\simeq -\mathbf{4m_D(T)}\alpha\mathbf{(T)}\frac{\beta\mathbf{(g)}}{\mathbf{g}} \\
&\simeq -\mathcal{O}(g^5 T)
\end{aligned}
$$

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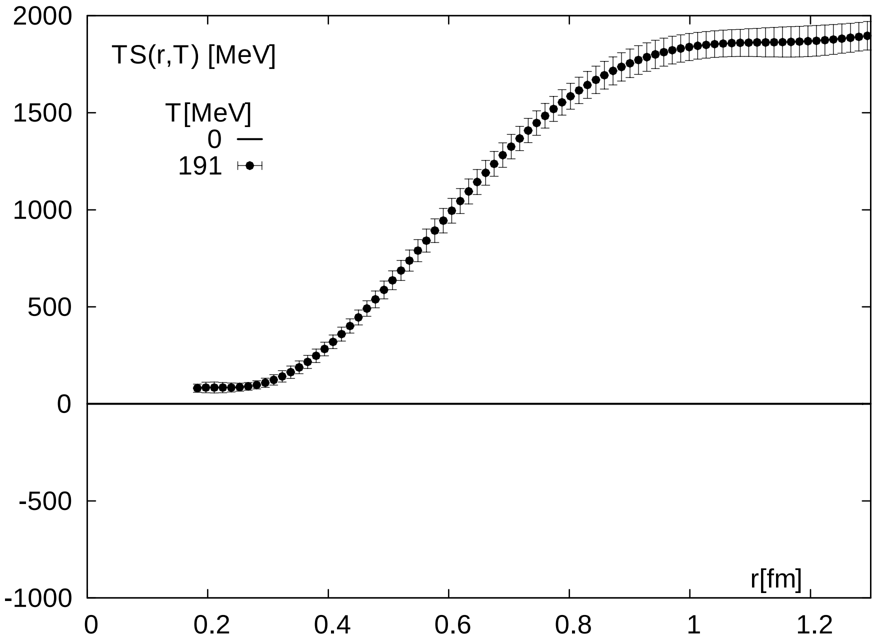
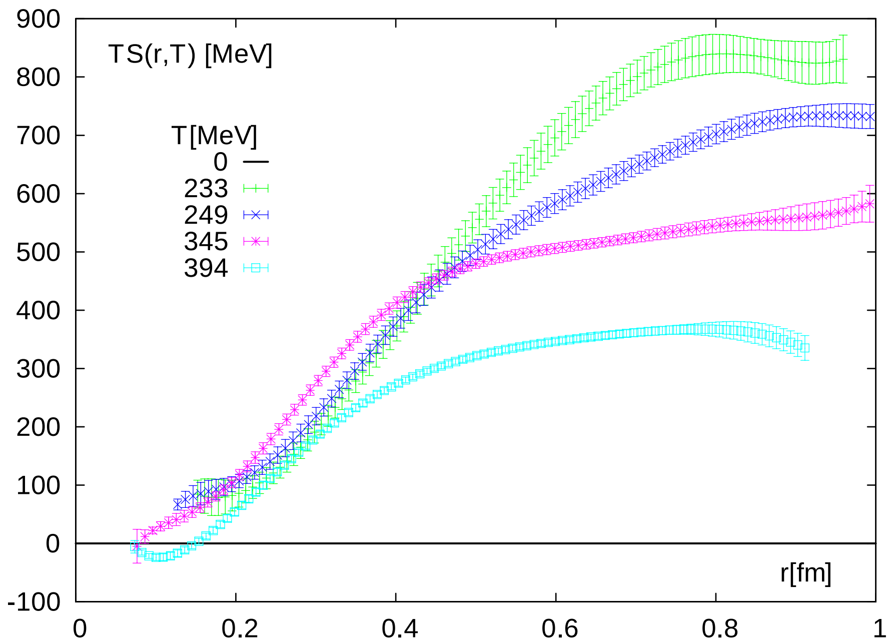
# r-dependent internal energy contributions




$$F_1(r,T) = U_1(r,T) - TS_1(r,T)$$

$$S_1(r,T) = \frac{\partial F_1(r,T)}{\partial T}$$

$$U_1(r,T) = -T^2 \frac{\partial F_1(r,T)/T}{\partial T}$$

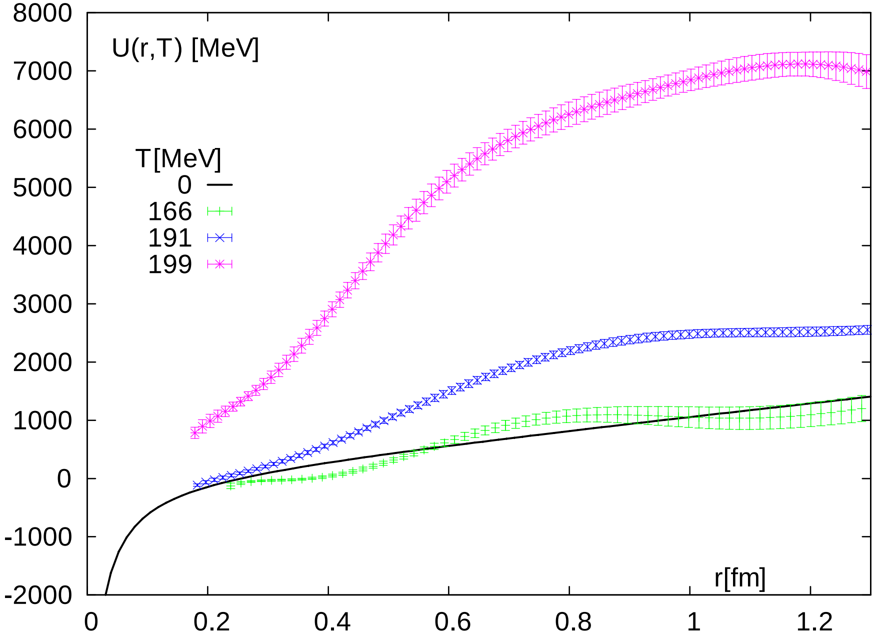
Entropy contributions vanish in the limit r$\rightarrow$ 0

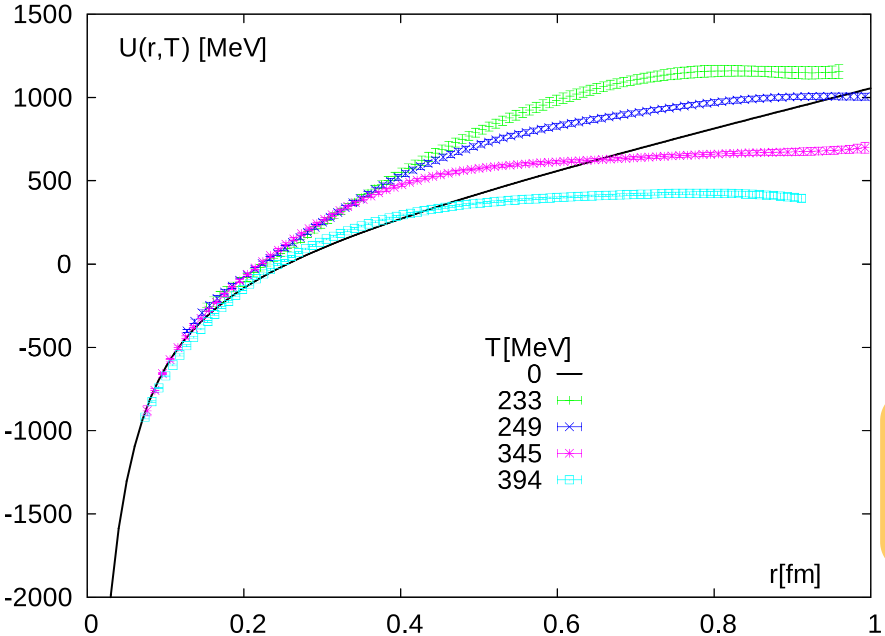
$$F_1(r \ll 1, T) = U_1(r \ll 1, T) \equiv V_1(r)$$

But they play an important role at intermediate r

especially close to $T_c$

# r-dependent Entropy contributions

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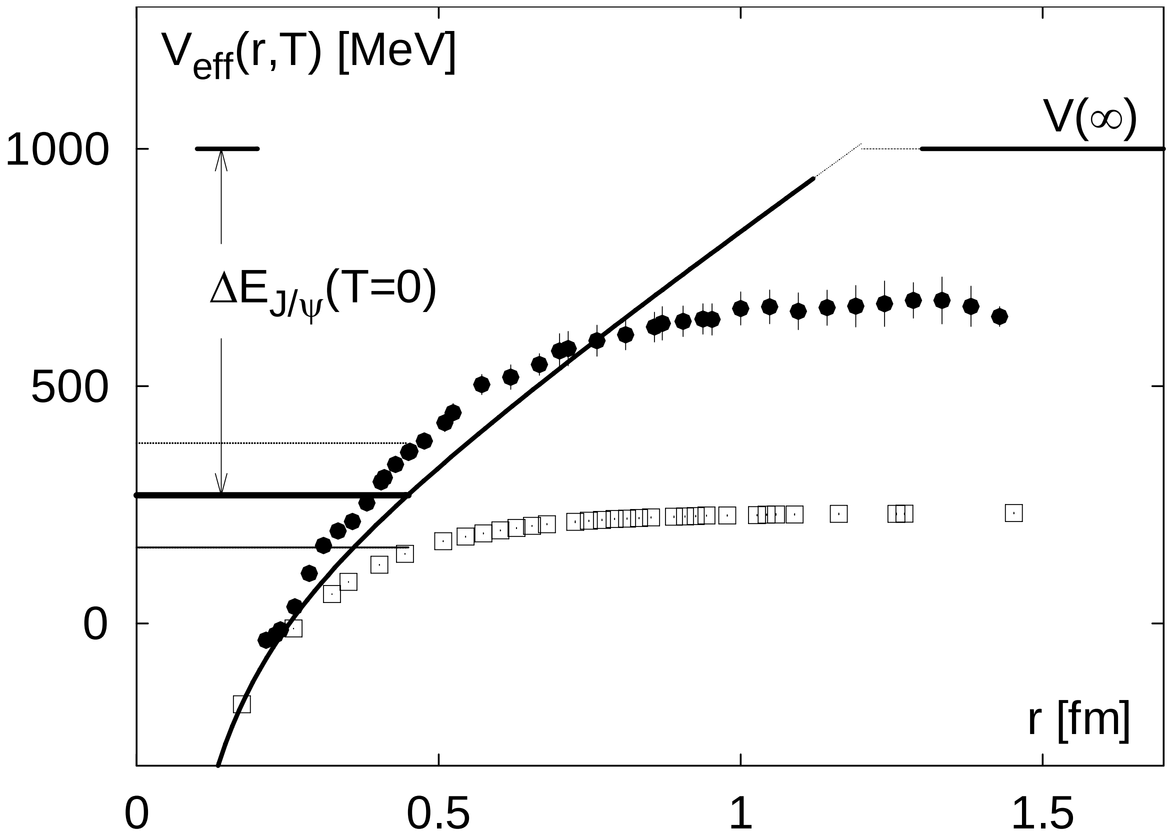
But they play an important role at intermediate r

especially close to $T_c$

**Implications on Heavy Quark bound states ?**

**What is the correct $V_{eff}(r,T)$ ?**

# Heavy Quark Bound States – Potential Models




steeper slope of $V_{eff}(r,T) = U_1(r,T)$

→ $J/\psi$ stronger bound using $V_{eff}(r,T)=U_1(r,T)$

→ dissociation at higher temperatures compared to $V_{eff}(r,T)=F_1(r,T)$

# Heavy Quark Free Energies – Many Open Questions

**What is the correct effective potential at finite temperature?**

$V_{eff}(r,T) = F_1(r,T)$ or $U_1(r,T)$ or linear combination?

**Gauge or operator dependence of color singlet free energy?** [Jahn+Philipsen]

**Do potential models at finite T make sense at all?**

is a two-particle (Schrödinger) Ansatz justified?

**What quark mass should be used?**

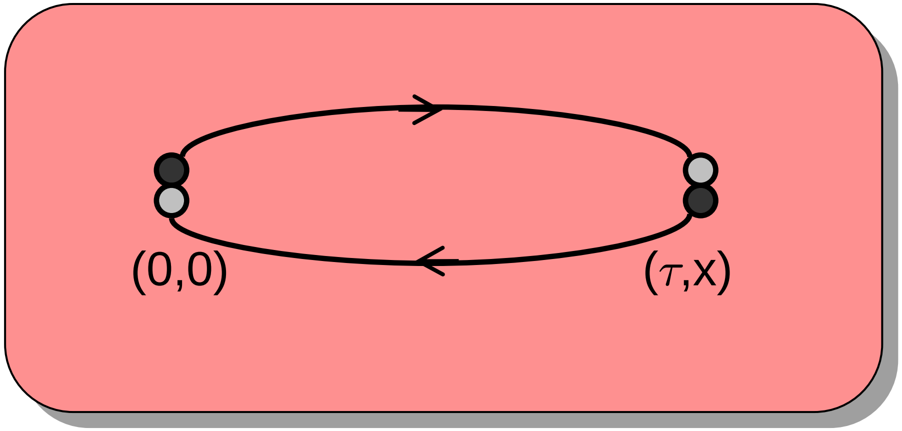
$m_c(\mu)$ depends on the scale (PDG: $m_c(m_c) = 1.27$ GeV)

what is the charm quark mass in the medium?

**What are the relevant processes and are they included in a potential model?**

scattering and Landau damping $\rightarrow$ real-time potential

**Thermal hadronic correlation functions**




$$J_H = \bar{q}(\tau,\vec{x})\Gamma_H q(\tau,\vec{x})$$

$$G_H(\tau,T,\vec{p}) = \sum_{\vec{x}} e^{-i\vec{p}\vec{x}} \langle J_H(0,0) J_H^{\dagger}(\tau,\vec{x})\rangle$$

O(a)-improved Clover improved fermionic action

on large quenched lattice configurations up to $128^3 \times 16$ and 32

includes all the physics

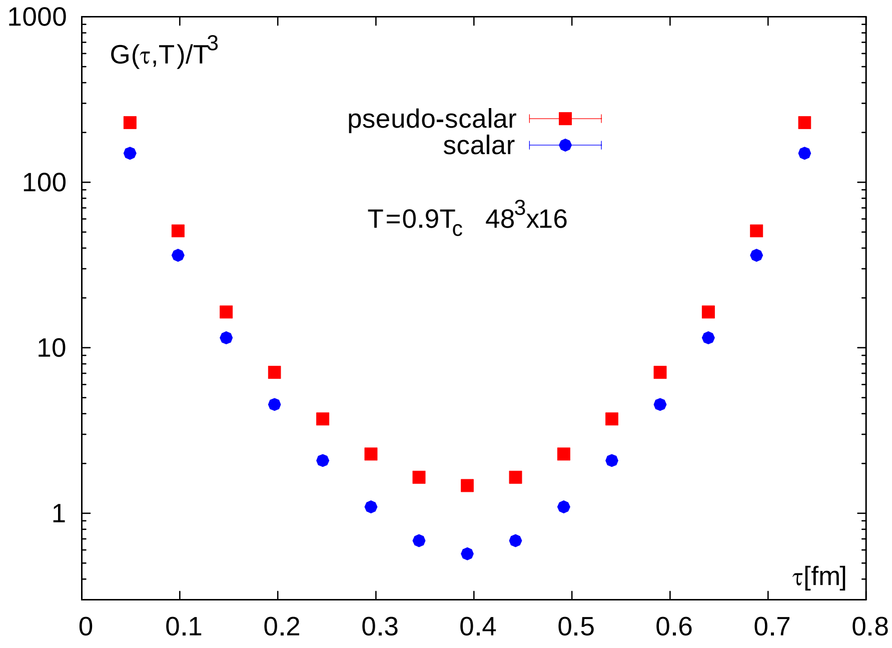
how to extract it?

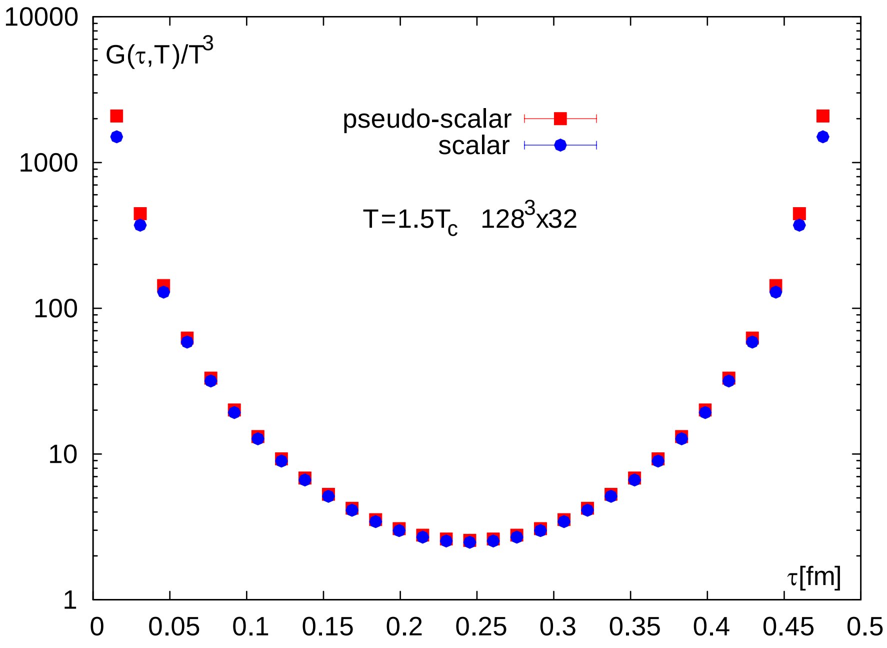
directly from the correlators?

spectral functions using MEM?

# Light Quark Correlators – Chiral Symmetry

## Temporal Correlators:




splitting below $T_c$ due to

chiral and axial U(1) symmetry breaking

symmetry restauration above $T_c$

# Light Quark Correlators – Chiral Symmetry

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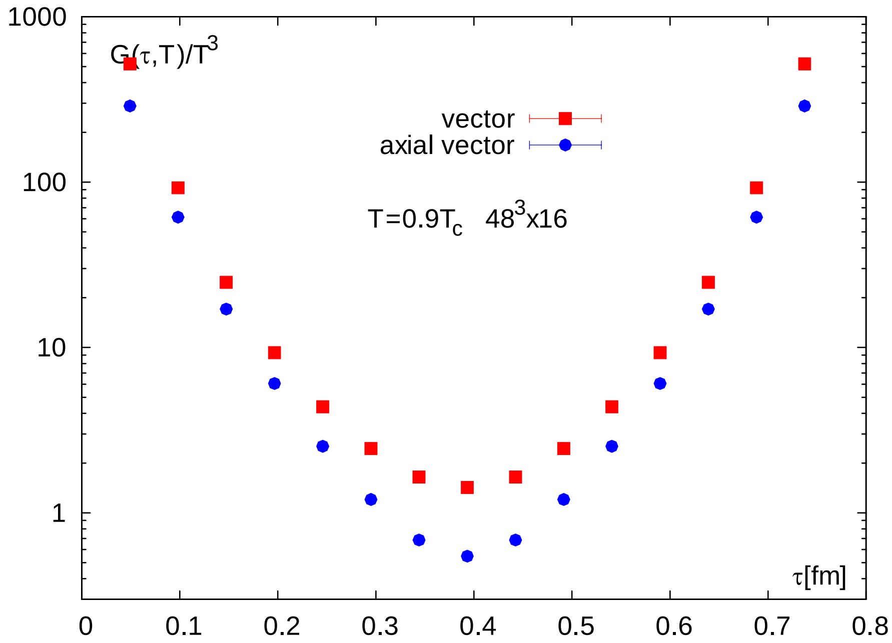


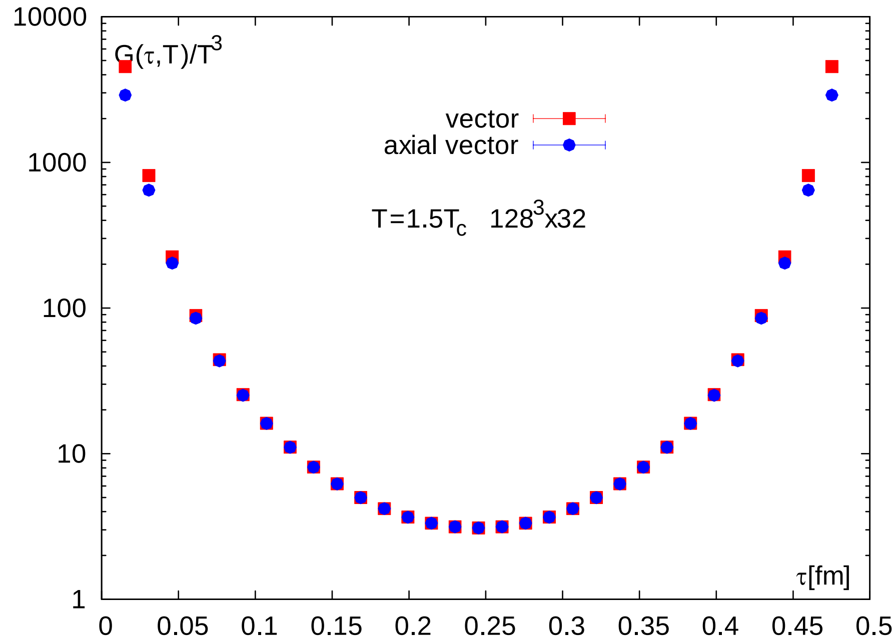


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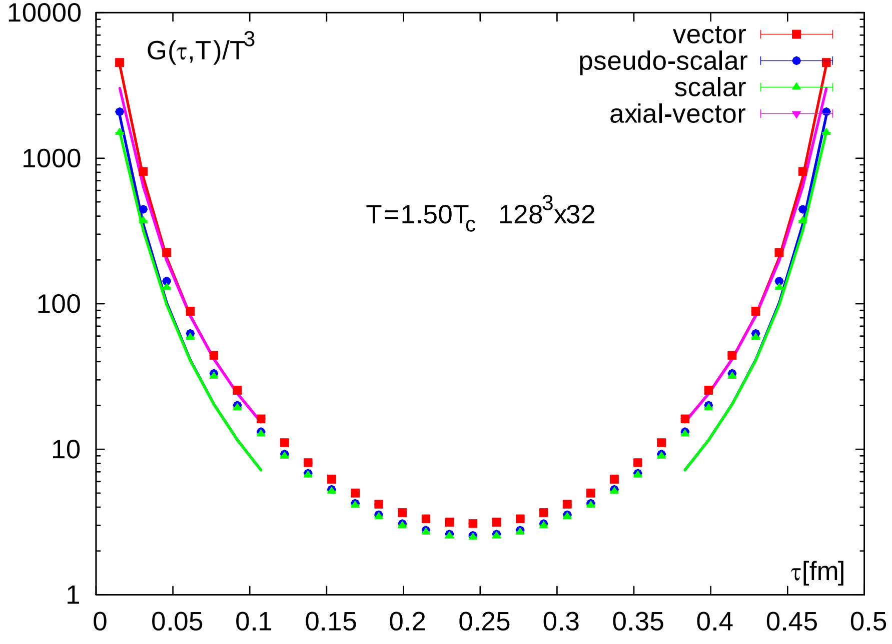
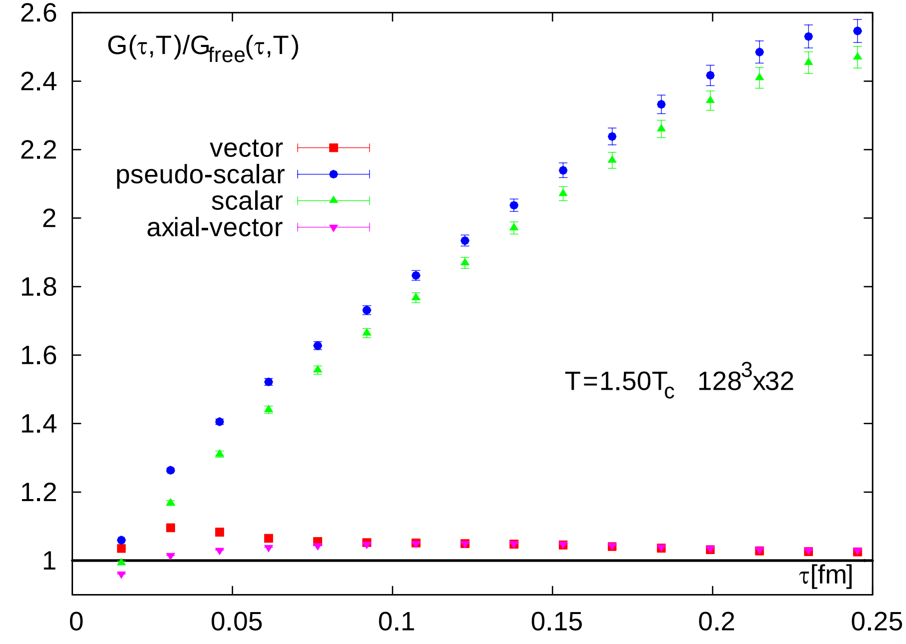
# Light Quark Correlators vs Free Correlators




comparison with free high temperature lattice correlator

vector and axial-vector close to free case

still strong correlations in scalar and pseudo-scalar channel

# Light Quark Screening Masses – Definition

use spatial correlators

$$G_H(z,T,\vec{p}_\perp) = \sum_{\tau,\vec{x}_\perp} e^{-i\vec{p}_\perp \vec{x}_\perp} \langle J_H(0,0) J_H^\dagger(\tau,\vec{x}) \rangle$$

correlation function depends on the same spectral density,

but the relation is more involved

$$G_H(z) = \int_{-\infty}^{+\infty} \frac{\mathrm{d}p_z}{2\pi} e^{ip_z z} \int_{-\infty}^{+\infty} \mathrm{d}p_0 \frac{\sigma_H(p_0,\vec{0}_\perp,p_z)}{p_0} \quad \xrightarrow[z\to\infty]{} \text{Ampl.} \times \exp(-m_{\text{screen}}\, z)$$

however, $m_{\text{screen}}(T) \neq m_{pole}(T)$ in general :

look for zeros of $G^{-1}(p) = p_0^2 + \vec{p}^2 + m_0^2 + \Pi(p_0,\vec{p},T)$

$$\vec{p} = 0: \qquad -p_0^2 = m_0^2 + \Pi(p_0,\vec{0},T) = (m_{pole}(T))^2$$

$$p_0 = 0: \qquad -\vec{p}^2 = m_0^2 + \Pi(0,\vec{p},T) = (m_{\text{screen}}(T))^2$$

$$\Longrightarrow m_{screen}(T) = \frac{m_{pole}(T)}{A(T)}$$

large collection of lattices ranging from $16^3\times 8$ to $128^3\times 16$

allowing for thermodynamic limit $V\to\infty$ at $N_t$=8,12 and 16

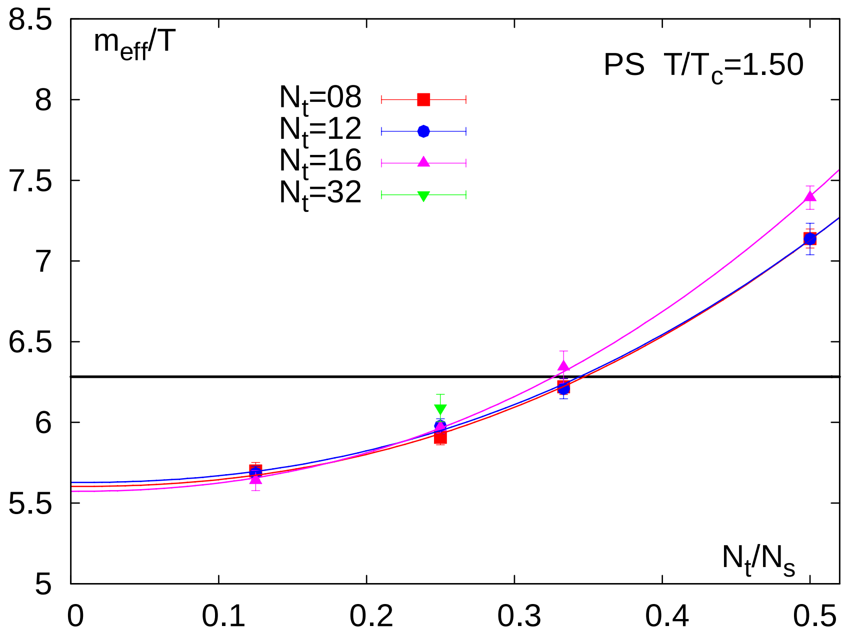
$$T \le T_c: \quad m_{\rm screen}(L,a) = m_{\rm screen}(a)\left[1+\gamma_V\left(\frac{N_\tau}{N_\sigma}\right)^{3}\right]$$

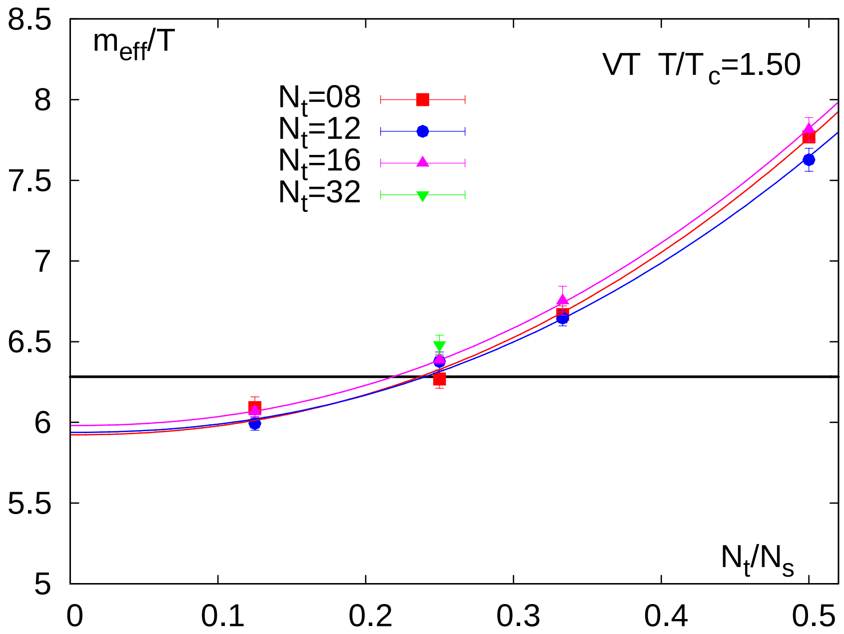
$$T > T_c: \quad m_{\rm screen}(L,a) = m_{\rm screen}(a)\left[1+\gamma_V\left(\frac{N_\tau}{N_\sigma}\right)^{p}\right]$$

$$T = \infty: \quad m_{\rm screen}(L,a) = m_{\rm screen}(a)\left[1+\gamma_V\left(\frac{N_\tau}{N_\sigma}\right)^{1}\right]$$

combined fit:

| $p$ | PS | V |
|---|---|---|
| 1.5 $Tc$ | 2.22(10) | 2.18(13) |
| 3.0 $T_c$ | 2.06(7) | 2.05(11) |



# Light Quark Screening Masses – Thermodynamic Limit, $V \to \infty$

large collection of lattices ranging from $16^3 \times 8$ to $128^3 \times 16$

allowing for thermodynamic limit $V \to \infty$ at $N_t$=8,12 and 16

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$$T = \infty : \quad m_{\mathrm{screen}}(L,a) = m_{\mathrm{screen}}(a) \left[ 1 + \gamma_V \left( \frac{N_\tau}{N_\sigma} \right)^1 \right]$$

combined fit:

| $p$ | PS | V |
|---|---|---|
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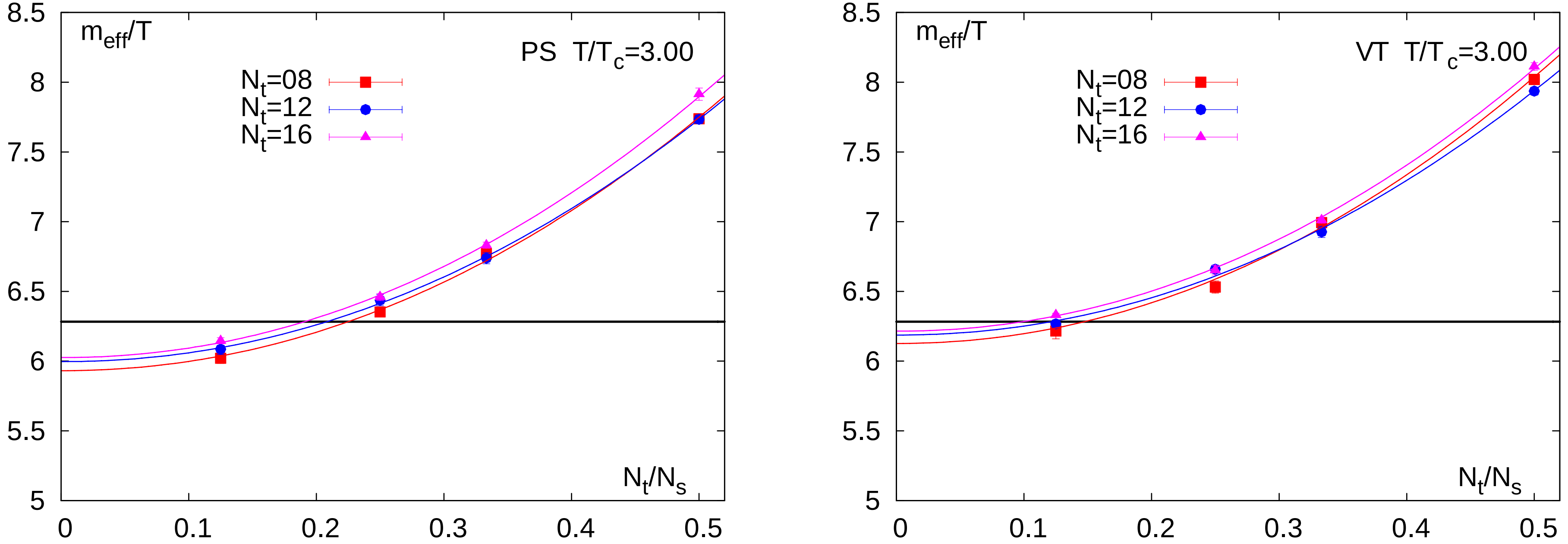


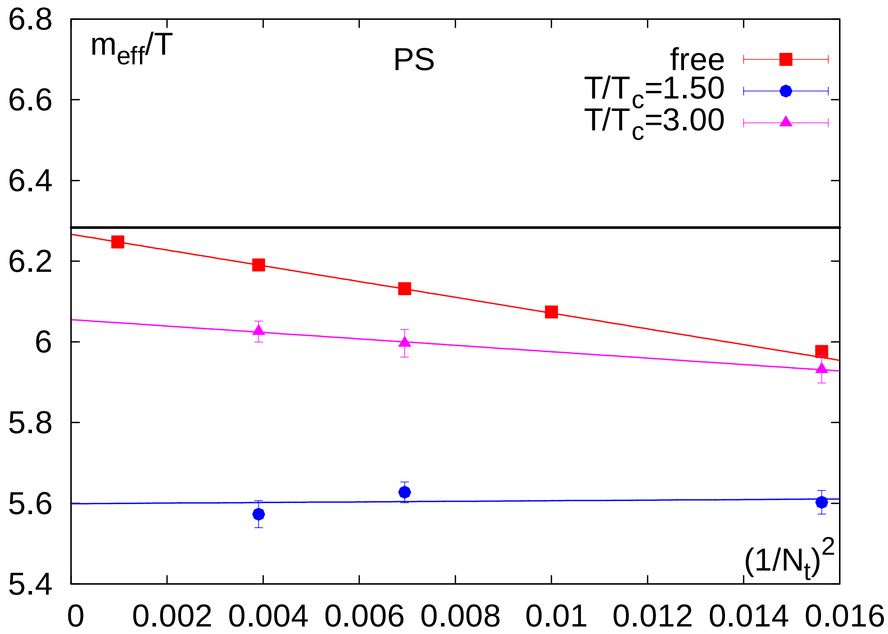
# Continuum Limit

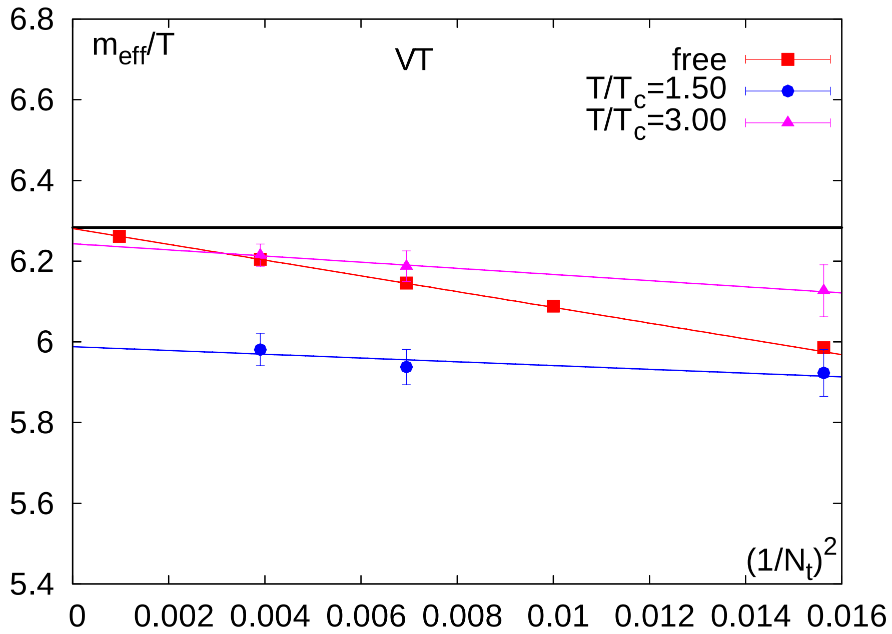
lattice spacing $a \to 0$

Non-perturbatively improved action

$\Rightarrow$ discretization errors $O(a^2)$

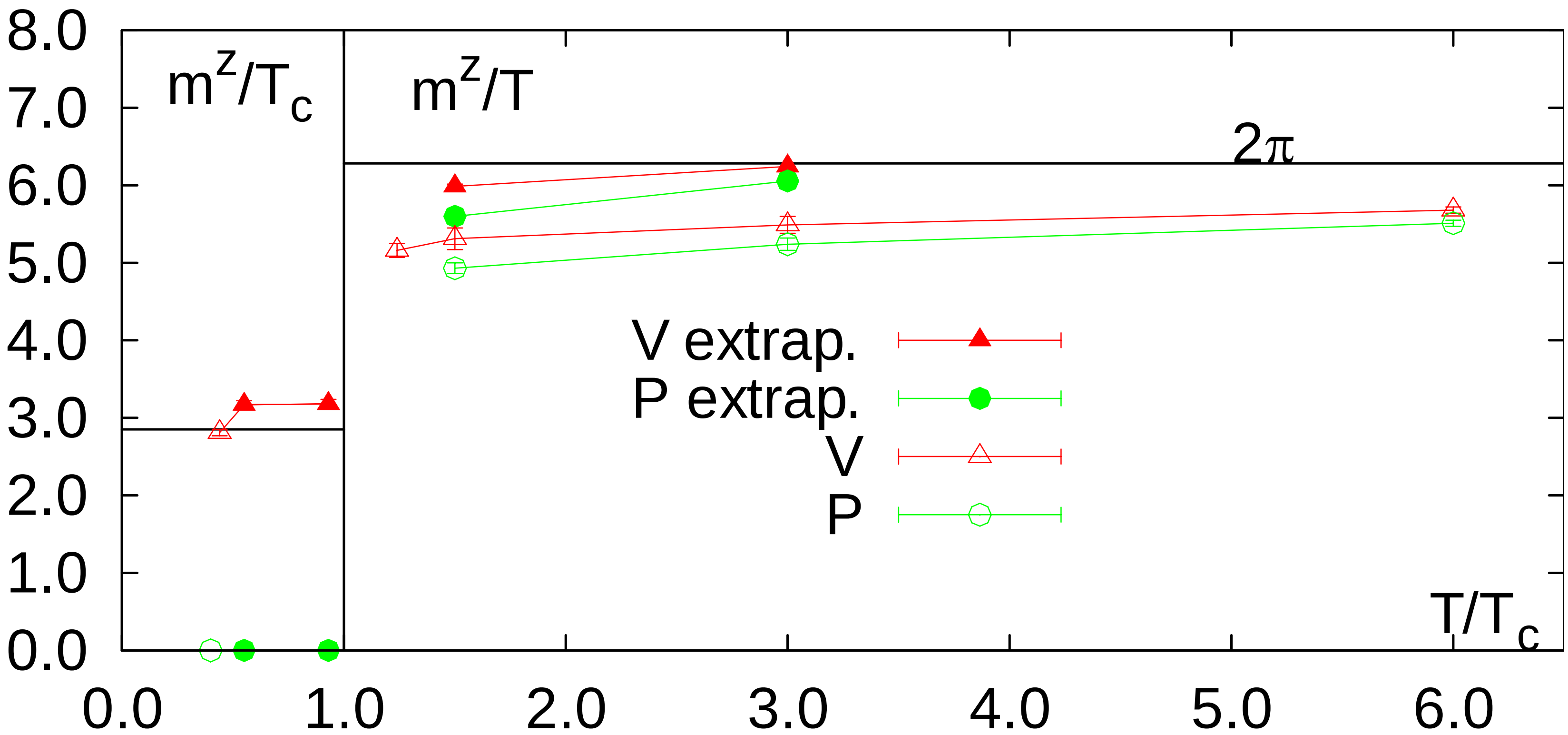
$$T = \infty : \quad \frac{m^z(a)}{T} = \frac{m^z}{T} - \lambda \left(\frac{1}{N_\tau}\right)^2$$



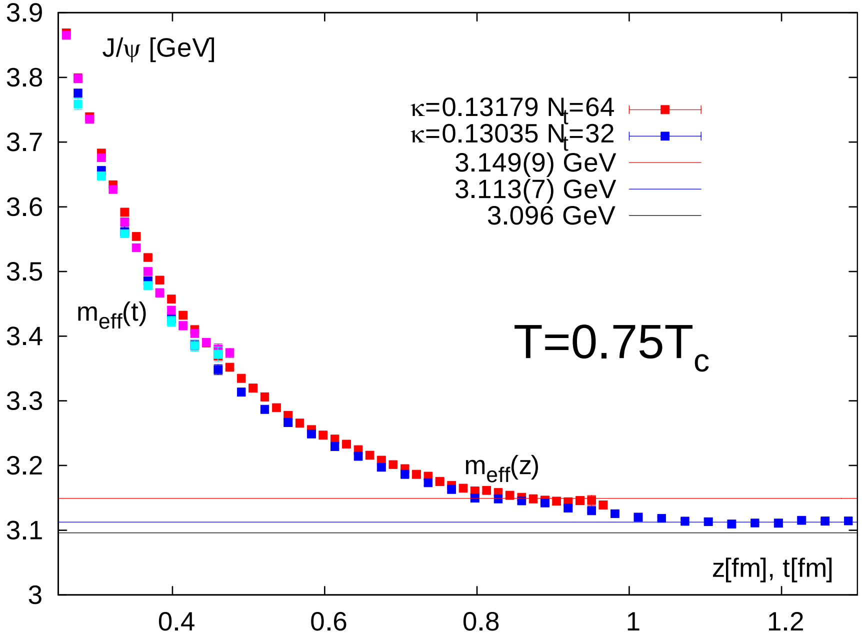
**Thermodynamic and Continuum Limit:**

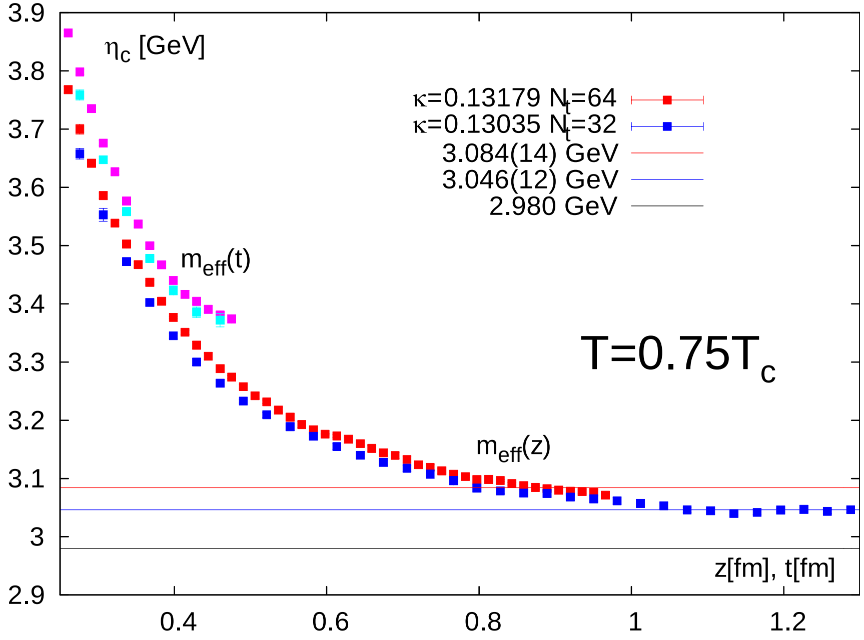
$V \to \infty$ and $a \to 0$




- weak temperature dependence below $T_c$
- data still below free limit ($2\pi T$) at 3 $T_c$, vector closer to free case
- at variance with [Laine, Vepsäläinen] $\to$ need higher temperatures

# Charmonium Correlators – Quark Mass Tuning



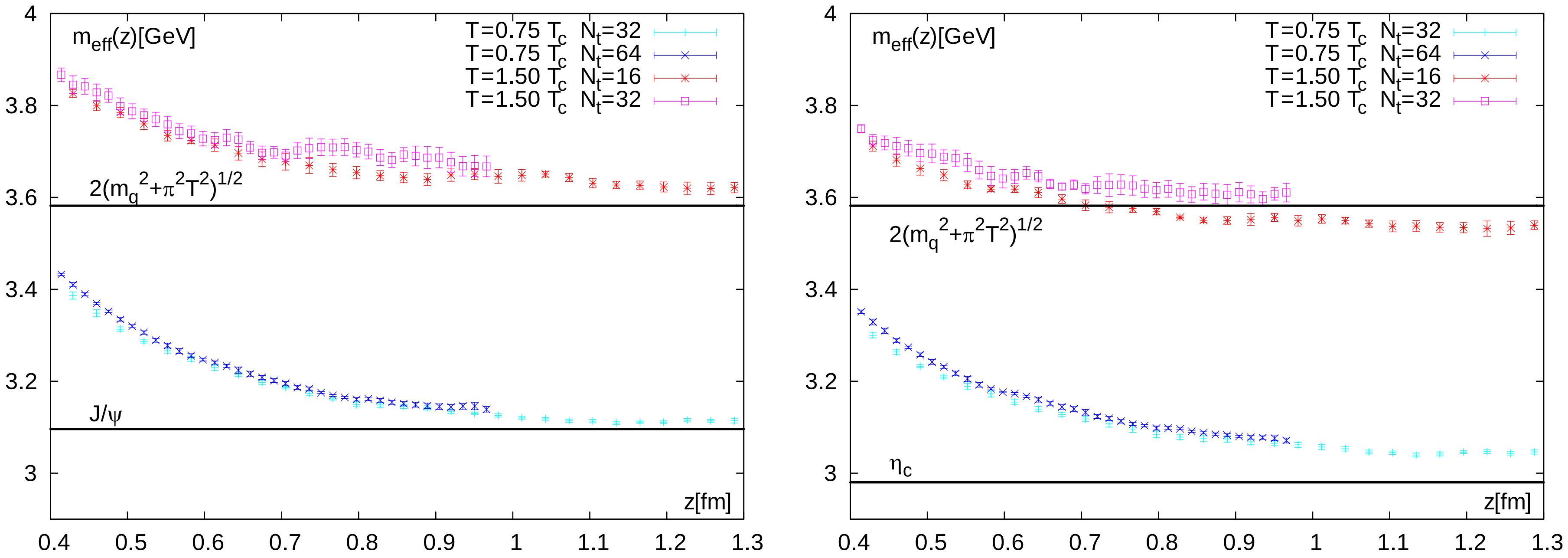

we use the spatial screening mass for quark mass tuning, as $m_{screen} = m_{pole}$ at small T

$$\frac{G(z)}{G(z+1)} = \frac{\cosh\left[m_{eff}(z)\left(\frac{N_\sigma}{2} - z\right)\right]}{\cosh\left[m_{eff}(z)\left(\frac{N_\sigma}{2} - z - 1\right)\right]}$$

Lattice parameters and quark masses from the Axial-Ward-Identity and in the $\overline{\text{MS}}$ scheme at the scale of the quark mass:

| $\beta$ | $a$[fm] | $1/a$[GeV] | $\kappa$ | $m_{AWI}(\mu = 1/a)$[GeV] | $m_{\overline{MS}}(\mu = m_c)$[GeV] |
|---|---|---|---|---|---|
| 6.872 | 0.031 | 6.43 | 0.13035 | 0.858(2) | 1.286(12) |
| 7.457 | 0.015 | 12.86 | 0.13179 | 0.841(1) | 1.282(9) |

# Charmonium Correlators – Screening Masses



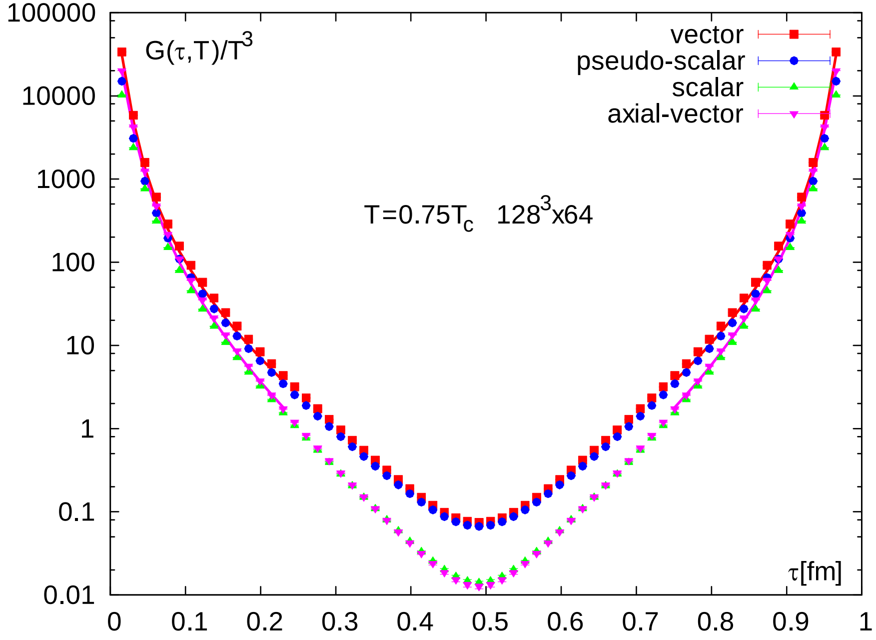

screening masses at 1.50 $T_c$ already close to the free case $m^{scr}_{free}(T) = 2\sqrt{(\pi T)^2 + m_c^2}.$

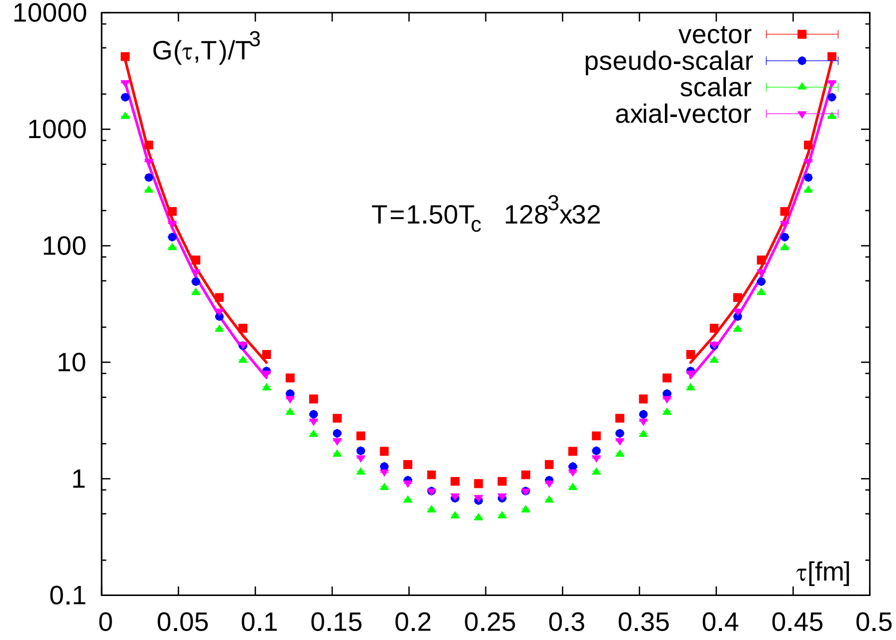
does this tell us anything about dissociation?

need to understand the momentum dependence of $G_H(\tau,T,p)$ and $\sigma(\omega,T,p)$ in detail

thermodynamic and continuum limit not performed yet

# Charmonium Correlators – Temporal Correlators




non-degenerate states still at 1.50 $T_c$

(almost) close to free correlators at (very) small separations

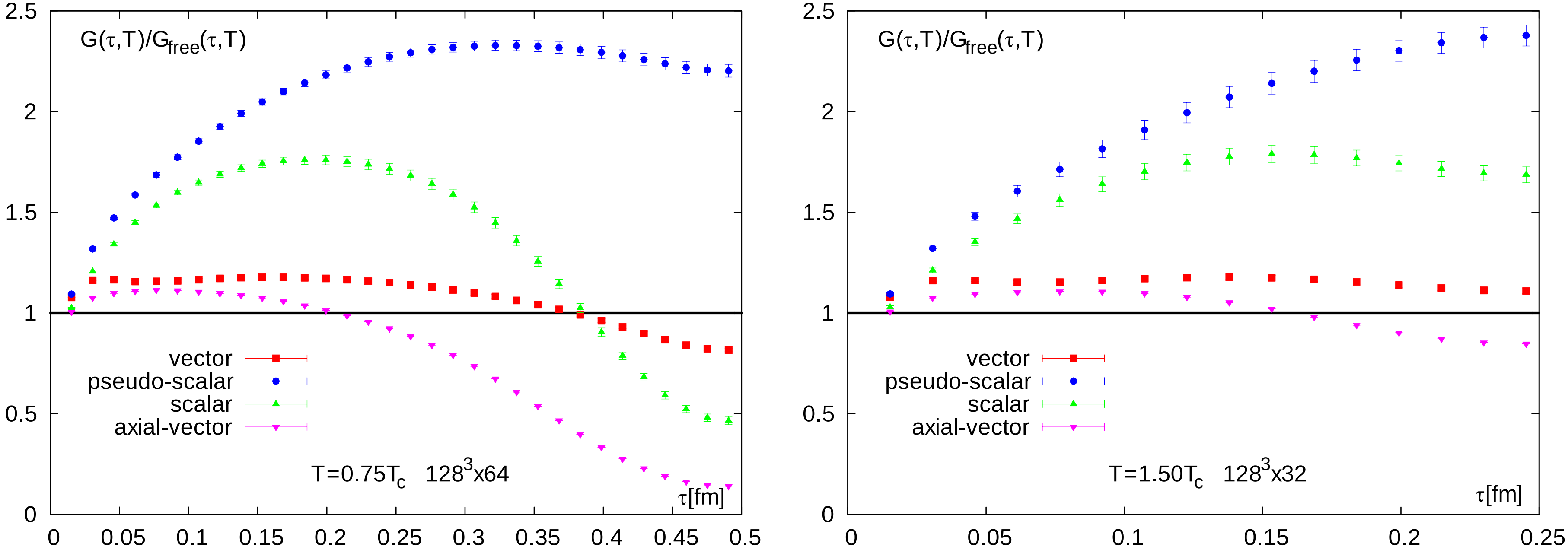
largest distance 0.25 fm due to compact temporal direction

only small distance regime (0.1-0.25 fm) relevant

for thermal effects

for bound state effects

# Charmonium Correlators vs Free Correlators

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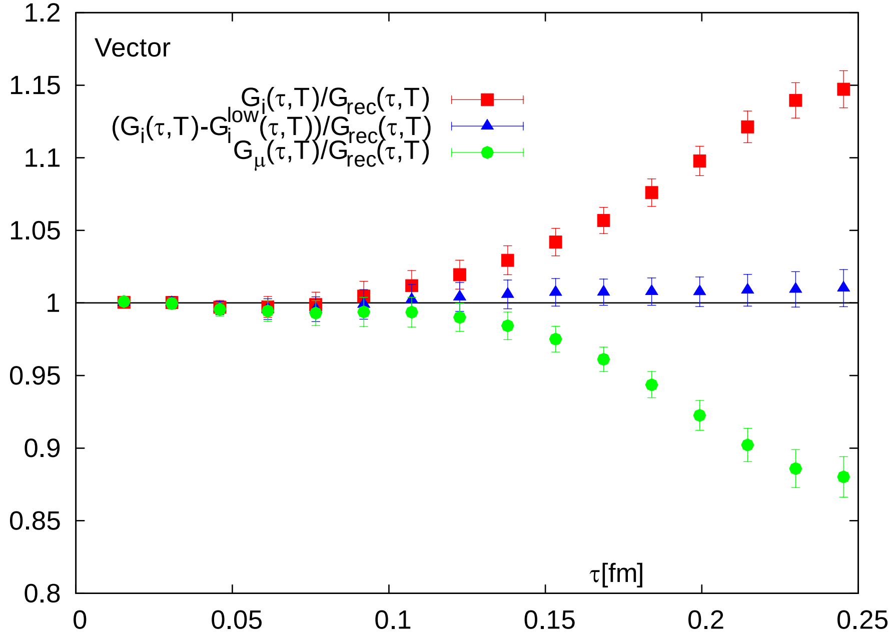
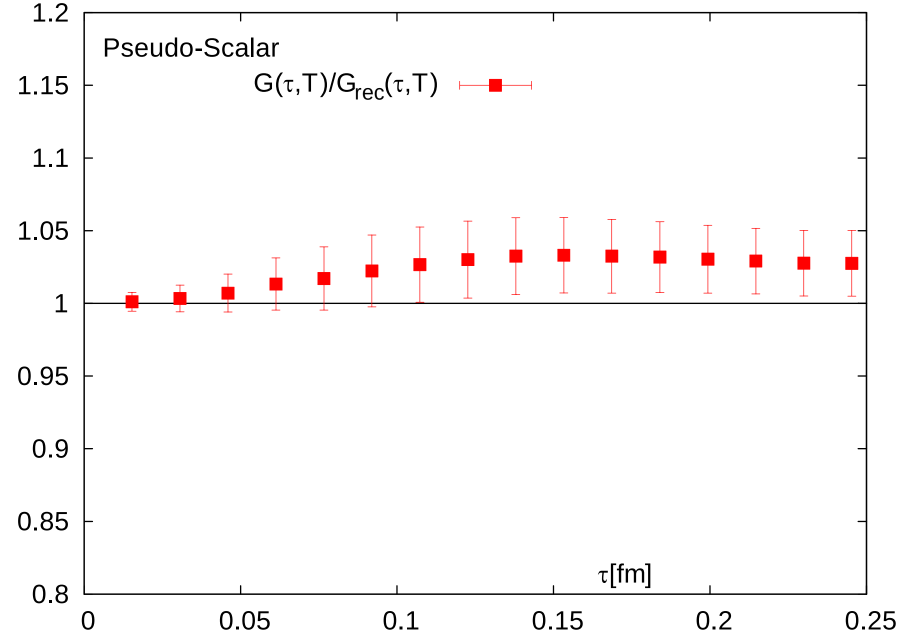
# Charmonium Correlators – Zero Mode Contributions




$$G_{rec}(\tau,T) = \int \sigma_0(\omega,0.75T_c)K(\omega,\tau,T)$$

$$G_{rec}^{low}(\tau,T) = \int_0^{2m_c} \sigma_T(\omega,T)K(\omega,\tau,T)$$

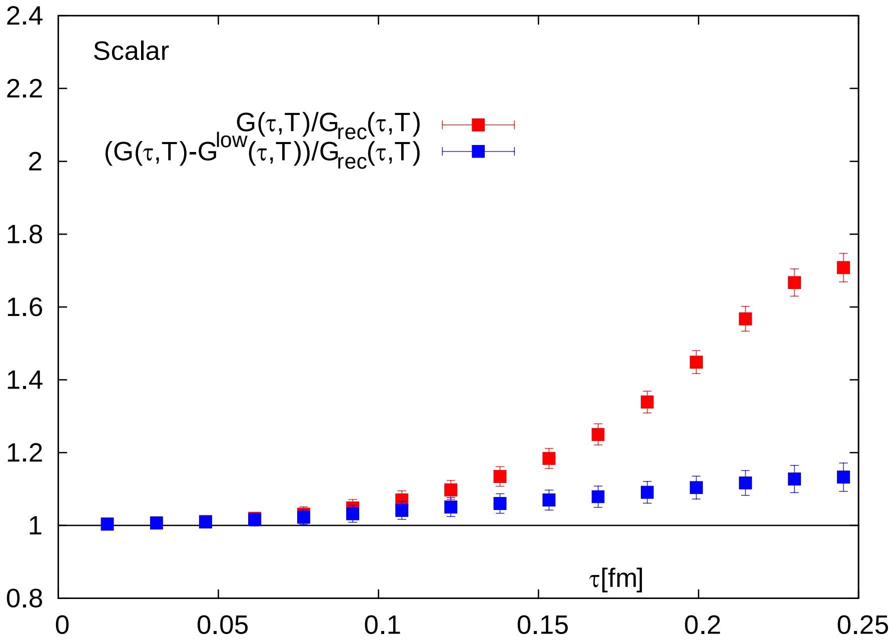
$$G_{\mu\mu}(\tau,T) = G_{ii}(\tau,T) + G_{44}(\tau,T)$$

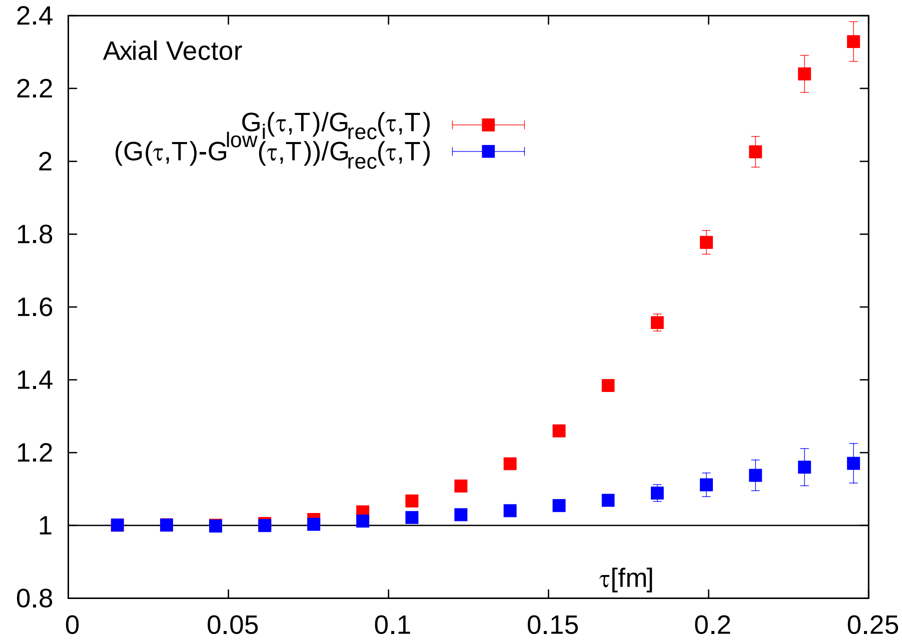
(see talk of HengTong Ding for SPF)

- main T-effect due to zero-mode contribution
- well described by small $\omega$-part of $\sigma_T(\omega$,T)
- smaller than $G_{44}(\tau$,T)=$\chi$(T)T
- no zero-mode contribution in PS-channel

(similar to discussions by Umeda, Petreczky)

# Charmonium Correlators – Zero Mode Contributions




$$G_{rec}(\tau,T) = \int \sigma_0(\omega, 0.75T_c) K(\omega,\tau,T)$$

$$G_{rec}^{low}(\tau,T) = \int_0^{2m_c} \sigma_T(\omega,T) K(\omega,\tau,T)$$

- larger zero-mode contribution in S-wave
- larger T-effect in the S-wave states

systematic uncertainties in reconstruction and low-$\omega$ part of spetral function

high quality data and small lattice spacing + large momenta (volume) needed

# Conclusions and Outlook

## Heavy Quark Free Energies ($m_q=\infty$)

Detailed knowledge of F, U, S and Screening effects!

But still many open questions! Application on Quarkonium?

## Hadronic correlators for light quarks ($m_q=0$)

Thermodynamic and Continuum Limit of screening masses!

Spectral functions $\rightarrow$ Dilepton rates, Transport coefficients?

## Charmonium hadronic correlators ($m_q=m_c$)

Can we really trust any of the spectral functions obtained with MEM?

What can we learn from Hadronic correlators on Dissociation?

Momentum dependence vs. Spatial correlators/screening masses?

Spectral functions $\rightarrow$ Dilepton rates, Transport coefficients?

# Many Thanks to

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