General lecture: Additional strange hadrons from QCD thermodynamics and freeze-out in heavy ion collisions

Olaf Kaczmarek

University of Bielefeld

Strangeness in Quark Matter 2015
&
Helmholtz International Summer School “Dense Matter“

JINR Dubna
09.07.2015
Outline

1) Higher order cumulants of conserved charges in the strange sector
   - evidence for additional strange hadrons
   - implications for strangeness freeze-out
   - deconfinement of open strange hadrons

2) Higher order cumulants of conserved charges in the charm sector
   - evidence for additional charmed hadrons
   - deconfinement of open charm hadrons

3) Hadronic correlation functions and screening masses
   - dissociation of quarkonia in the QGP

4) Color electric field correlation function
   - Heavy quark momentum diffusion coefficient $\kappa$
Motivation – Hadron yields at the freeze-out

Heavy Ion Collision                      QGP             Expansion+Cooling             Hadronization

Hadron Resonance Gas (HRG) to describe hadron yields at the freeze-out

Hadrons measured in the detectors

“observed” mainly at the freeze-out stage of the HIC

Data

Hadron Resonance Gas (HRG) to describe hadron yields at the freeze-out

[S.Bass]

HRG: thermal gas of uncorrelated hadrons

Partial pressure of each hadron:

\[ \hat{P}_h \sim f(\hat{m}_h) \cosh \left[ B_h \hat{\mu}_h + Q_h \hat{\mu}_Q + S_h \hat{\mu}_S + C_h \hat{\mu}_C \right] \]

total pressure given by the sum over all (known) hadrons

\[ \hat{P}_{total} = \sum_{all \ hadrons} \hat{P}_h \]

→ use thermodynamics instead of HRG to describe freeze-out
What do we know of the hadron spectrum?

Strange baryons


In the following:

PDG will denote results using states listed in the particle data tables.

QM will denote results using states calculated in the quark model.
What do we know of the hadron spectrum?

**Quark Model**

![Graph showing states in the quark model for strange baryons](image1)

- **Legend**:
  - PDG states
  - Lattice QCD states

- **Reference**:

**Lattice QCD**

![Graph showing results from lattice QCD](image2)

- **Legend**:
  - PDG states
  - Lattice QCD states

- **Reference**:

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In the following:

- **PDG** will denote results using states listed in the particle data tables.
- **QM** will denote results using states calculated in the quark model.
Hadron Resonance Gas - contributions of additional states - strange

partial pressure $P$ of all open strange hadrons in a hadron resonance gas (HRG) can be separated into mesonic $P_M$ and baryonic $P_B$ components

$$P_{\text{tot}}^{S,X} = P_M^{S,X} + P_B^{S,X}$$

$$P_{M/B}^{S,X}(T, \vec{\mu}) = \frac{T^4}{2\pi^2} \sum_{i \in X} g_i \left( \frac{m_i}{T} \right)^2 K_2 \left( \frac{m_i}{T} \right) \times \cosh \left( B_i \hat{\mu}_B + Q_i \hat{\mu}_Q + S_i \hat{\mu}_S \right)$$

large enhancement of the partial baryonic pressure from additional strange baryons

large part of open strange mesons experimentally observed
thermodynamic quantities obtained from derivatives of the partition function

\[ Z(\beta, N_\sigma, N_\tau) = \int \prod_{x, \mu} dU_{x, \mu} e^{-S(U)} \]

\[ S(U) = \beta S_G(U) - S_F(U) \]

using trace of the energy momentum tensor:

\[ \Theta^{\mu\mu} = \epsilon - 3p = -\frac{T}{V} \frac{d \ln Z}{d \ln a} \]

\[ (\epsilon - 3p)/T^4 \equiv \frac{\Theta_G^{\mu\mu}(T)}{T^4} + \frac{\Theta_F^{\mu\mu}(T)}{T^4} , \]

\[ \frac{\Theta_G^{\mu\mu}(T)}{T^4} = R_\beta [\langle s_G \rangle_0 - \langle s_G \rangle_\tau] N_\tau^4 , \]

\[ \frac{\Theta_F^{\mu\mu}(T)}{T^4} = -R_\beta R_m [2m_l (\langle \bar{\psi}\psi \rangle_{l,0} - \langle \bar{\psi}\psi \rangle_{l,\tau}) \]

\[ + m_s (\langle \bar{\psi}\psi \rangle_{s,0} - \langle \bar{\psi}\psi \rangle_{s,\tau})] N_\tau^4 . \]

HISQ: [A. Bazavov et al. (hotQCD), PRD90 (2014) 094503]

pressure calculated using integral method:

\[ \frac{p(T)}{T^4} = \frac{p_0}{T_0^4} + \int_{T_0}^{T} dT' \frac{\Theta^{\mu\mu}}{T'^5} \]
Equation of state of (2+1)-flavor QCD - $\mu_B/T = 0$

continuum extrapolated results of pressure & energy density & entropy density

HISQ: [A. Bazavov et al. (hotQCD), PRD90 (2014) 094503]  
stout: [S. Borsanyi et al., PLB730, 99 (2014)]

consistent results from hotQCD (HISQ) and Budapest-Wuppertal (stout)

hadron resonance gas (HRG) model using all known hadronic resonances from PDG

describes the EoS quite well up to cross-over region

QCD results systematically above HRG

room for additional resonances not listed in the PDG
Cumulants of net-charge fluctuations

Taylor expansion of pressure in terms of chemical potentials related to conserved charges:

\[
\frac{P}{T^4} = \sum_{i,j,k=0}^{\infty} \frac{1}{i!j!k!} \chi_{ijk}^{BQS}(T) \left( \frac{\mu_B}{T} \right)^i \left( \frac{\mu_Q}{T} \right)^j \left( \frac{\mu_S}{T} \right)^k
\]

defines generalized susceptibilities:

\[
\chi_{ijk}^{BQS} = \left. \frac{\partial^{(i+j+k)} [P(T, \hat{\mu}_B, \hat{\mu}_Q, \hat{\mu}_S)/T^4]}{\partial \hat{\mu}_B^i \partial \hat{\mu}_Q^j \partial \hat{\mu}_S^k} \right|_{\bar{\mu}=0}
\]

with \( \hat{\mu}_X = \mu_X/T \)

generalized susceptibilities calculated at zero \( \mu \)
cumulants of net-charge fluctuations measured at the freeze out

Lattice QCD

\[
VT^3 \chi_2^X = \langle (\delta N_X)^2 \rangle
\]

\[
VT^3 \chi_4^X = \langle (\delta N_X)^4 \rangle - 3 \langle (\delta N_X)^2 \rangle^2
\]

\[
VT^3 \chi_6^X = \langle (\delta N_X)^6 \rangle - 15 \langle (\delta N_X)^4 \rangle \langle (\delta N_X)^2 \rangle + 30 \langle (\delta N_X)^2 \rangle^3
\]

\( \delta N_X \equiv N_X - \langle N_x \rangle \)

Experiment
Cumulants of net-charge fluctuations

higher order cumulants characterize the shape of conserved charge distributions

\[ S_q \sigma_q = \frac{\chi_3^q}{\chi_2^q} \quad q = B, Q, S \]
\[ \kappa_q \sigma_q^2 = \frac{\chi_4^q}{\chi_2^q} \]

mean:
\[ \langle \delta N_q \rangle \equiv \langle N_q - \bar{N}_q \rangle \]

variance:
\[ \sigma_q^2 \equiv \langle (\delta N_q)^2 \rangle - \langle \delta N_q \rangle^2 \]

skewness:
\[ S_q \equiv \langle (\delta N_q)^3 \rangle / \sigma_q^3 \]

kurtosis:
\[ \kappa_q \equiv \langle (\delta N_q)^4 \rangle / \sigma_q^4 - 3 \]

generalized susceptibilities calculated at zero \( \mu \)

cumulants of net-charge fluctuations measured at the freeze out

Lattice QCD

\[ VT^3 \chi_2^X \]
\[ VT^3 \chi_4^X \]
\[ VT^3 \chi_6^X \]

\[ \langle (\delta N_X)^2 \rangle \]
\[ \langle (\delta N_X)^4 \rangle - 3 \langle (\delta N_X)^2 \rangle^2 \]
\[ \langle (\delta N_X)^6 \rangle - 15 \langle (\delta N_X)^4 \rangle \langle (\delta N_X)^2 \rangle + 30 \langle (\delta N_X)^2 \rangle^3 \]

\[ \delta N_X \equiv N_X - \langle N_x \rangle \]
Taylor expansion of pressure in terms of chemical potentials related to conserved charges

\[ \frac{P}{T^4} = \sum_{i,j,k=0}^{\infty} \frac{1}{i!j!k!} \chi_{ijk}^{BQS}(T) \left( \frac{\mu_B}{T} \right)^i \left( \frac{\mu_Q}{T} \right)^j \left( \frac{\mu_S}{T} \right)^k \]

defines generalized susceptibilities:

\[ \chi_{ijk}^{BQS} = \left. \frac{\partial^{i+j+k}[P(T, \hat{\mu}_B, \hat{\mu}_Q, \hat{\mu}_S)/T^4]}{\partial \hat{\mu}_B^i \partial \hat{\mu}_Q^j \partial \hat{\mu}_S^k} \right|_{\hat{\mu}=0} \]

for \( \mu_Q = \mu_S = 0 \) this simplifies to

\[ \Delta P(T) = \frac{P(T, \mu_B) - P(T, 0)}{T^4} = \frac{\chi_2^B}{2} \left( \frac{\mu_B}{T} \right)^2 \left( 1 + \frac{1}{12} \frac{\chi_4^B}{\chi_2^B} \left( \frac{\mu_B}{T} \right)^2 \right) + \mathcal{O}(\mu_B^6) \]

Equation of state of (2+1)-flavor QCD - \( \mu_B/T > 0 \)

variance of net-baryon number distribution

kurtosis*variance \( \kappa_B \sigma_B^2 \)

good agreement with HRG in crossover region

deviations from HRG in crossover region
Taylor expansion of pressure in terms of chemical potentials related to conserved charges:

$$\frac{P}{T^4} = \sum_{i,j,k=0}^{\infty} \frac{1}{i!j!k!} \chi_{ijk}^{BQS}(T) \left( \frac{\mu_B}{T} \right)^i \left( \frac{\mu_Q}{T} \right)^j \left( \frac{\mu_S}{T} \right)^k$$

defines generalized susceptibilities:

$$\chi_{ijk}^{BQS} = \frac{\partial^{(i+j+k)}[P(T, \hat{\mu}_B, \hat{\mu}_Q, \hat{\mu}_S)/T^4]}{\partial \hat{\mu}_B^i \partial \hat{\mu}_Q^j \partial \hat{\mu}_S^k} \bigg|_{\bar{\mu}=0}$$

**correlations of strangeness with baryon number fluctuations:**

$$\chi_{11}^{BS} = \frac{\partial^2[P(T, \hat{\mu}_B, \hat{\mu}_Q, \hat{\mu}_S)/T^4]}{\partial \hat{\mu}_B \partial \hat{\mu}_S} \bigg|_{\bar{\mu}=0}$$

**second cumulant of net strangeness fluctuations:**

$$\chi_2^S = \frac{\partial^2[P(T, \hat{\mu}_B, \hat{\mu}_Q, \hat{\mu}_S)/T^4]}{\partial \hat{\mu}_S^2} \bigg|_{\bar{\mu}=0}$$

suitable ratios like

$$\frac{\chi_{11}^{BS}}{\chi_2^S}$$

partial pressure of strange baryons

in a hadron gas

dominated by strange mesons

are sensitive probes of the strangeness carrying degrees of freedom
Taylor expansion of pressure in terms of chemical potentials related to conserved charges defines generalized susceptibilities:

\[
\frac{P}{T^4} = \sum_{i,j,k=0}^{\infty} \frac{1}{i!j!k!} \chi_{ijk}^{BQS}(T) \left( \frac{\mu_B}{T} \right)^i \left( \frac{\mu_Q}{T} \right)^j \left( \frac{\mu_S}{T} \right)^k
\]

defines generalized susceptibilities: \( \chi_{ijk}^{BQS} = \left. \frac{\partial^{(i+j+k)}[P(T, \hat{\mu}_B, \hat{\mu}_Q, \hat{\mu}_S)/T^4]}{\partial \hat{\mu}_B^i \partial \hat{\mu}_Q^j \partial \hat{\mu}_S^k} \right|_{\bar{\mu}=0} \)

correlations of strangeness with baryon number fluctuations:

\[
\chi_{11}^{BS} = \left. \frac{\partial^2[P(T, \hat{\mu}_B, \hat{\mu}_Q, \hat{\mu}_S)/T^4]}{\partial \hat{\mu}_B \partial \hat{\mu}_S} \right|_{\bar{\mu}=0}
\]

second cumulant of net baryon number fluctuations:

\[
\chi_2^B = \left. \frac{\partial^2[P(T, \hat{\mu}_B, \hat{\mu}_Q, \hat{\mu}_S)/T^4]}{\partial \hat{\mu}_B^2} \right|_{\bar{\mu}=0}
\]

suitable ratios like

\[
\frac{\chi_{11}^{BS}}{\chi_2^B}
\]

are sensitive probes of the strangeness carrying degrees of freedom
Thermodynamic contributions of strange baryons

\[
\chi_{klm}^{BQS} = \left. \frac{\partial^{(k+l+m)}[P(T, \tilde{\mu}_B, \tilde{\mu}_Q, \tilde{\mu}_S)/T^4]}{\partial \tilde{\mu}_B^k \partial \tilde{\mu}_Q^l \partial \tilde{\mu}_S^m} \right|_{\tilde{\mu}=0}
\]

individual pressure-observables

for open strange mesons \( P_M^S \) in HRG:

\[
M_1^S = \chi_2^S - \chi_{22}^B
\]

\[
M_2^S = \frac{1}{12} (\chi_4^S + 11\chi_2^S) + \frac{1}{2} (\chi_{11}^S + \chi_{13}^S)
\]

for strange baryons \( P_B^S \) in HRG:

\[
B_1^S = -\frac{1}{6} (11\chi_{11}^S + 6\chi_{22}^S + \chi_{13}^S)
\]

\[
B_2^S = \frac{1}{12} (\chi_4^S - \chi_2^S) - \frac{1}{3} (4\chi_{11}^S - \chi_{13}^S)
\]

all give identical results in a gas of uncorrelated hadrons

yield widely different results when the degrees of freedom are quarks

→ QM-HRG model calculations are in good agreement with LQCD up to the chiral crossover region

→ evidence for the existence of additional strange baryons

and their thermodynamic importance below the QCD crossover
Implications for strangeness freeze-out

Initial nuclei in a heavy ion collision are net strangeness free + iso-spin asymmetry

\[ \langle n_Q \rangle = r \langle n_B \rangle \]

→ the HRG at the chemical freeze-out must also be strangeness neutral

→ thermal parameters \( T, \mu_B \) and \( \mu_S \) are related

\[ \frac{\mu_S}{\mu_B} = s_1(T) + s_3(T) \left( \frac{\mu_B}{T} \right)^2 + \mathcal{O}(\mu_B^4) \]

small for \( \mu_B \lesssim 200 \text{MeV} \)

\[ \left( \frac{\mu_S}{\mu_B} \right)_{LO} \equiv s_1(T) = -\frac{\chi_{11}^{BS}}{\chi_2^S} - \frac{\chi_{11}^{QS}}{\chi_2^S} \frac{\mu_Q}{\mu_B} \]

small correction from nonzero electric charge chemical potential

Lattice QCD results well reproduced by QM-HRG in the crossover region for a given \( \mu_S/\mu_B \)

QM-HRG would give a smaller temperature compared to PDG-HRG
relative yields of strange anti-baryons ($\bar{H}_S$) to baryons ($H_S$) can be used to determine freeze-out parameters $\mu_B^f/T^f$ and $\mu_S^f/\mu_B^f$ from experiment

$$R_H \equiv \frac{\bar{H}_S}{H_S} = e^{-2(\mu_B^f/T^f)(1-(\mu_S^f/\mu_B^f)|S|)}$$

Implications for strangeness freeze-out

only assumes that hadron yields are thermal

compare results for $\mu_B/T$ and $\mu_S/\mu_B$ to Lattice QCD to obtain freeze-out $T$
relative yields of strange anti-baryons ($\bar{H}_S$) to baryons ($H_S$) can be used to determine freeze-out parameters $\mu_B^f/T^f$ and $\mu_S^f/\mu_B^f$ from experiment

$$R_H \equiv \frac{\bar{H}_S}{H_S} = e^{-2(\mu_B^f/T^f)(1-(\mu_S^f/\mu_B^f)|S|)}$$

and compared to Lattice QCD or HRG to determine freeze-out temperature $T^f$:

Implications for strangeness freeze-out

Experiment STAR/NA57
relative yields of strange anti-baryons ($\bar{H}_S$) to baryons ($H_S$) can be used to determine freeze-out parameters $\mu_B^f/T^f$ and $\mu_S^f/\mu_B^f$ from experiment

$$R_H \equiv \frac{\bar{H}_S}{H_S} = e^{-2(\mu_B^f/T^f)(1-(\mu_S^f/\mu_B^f)|S|)}$$

and compared to Lattice QCD or HRG to determine freeze-out temperature $T^f$:

**Implications for strangeness freeze-out**

**Lattice QCD**

- $T=155(5)$ MeV
- $T=145(2)$ MeV

freeze-out temperature obtained from a comparison of experimental data and Lattice QCD results

**Experiment STAR/NA57**

- $T=170$ MeV
- $T=166$ MeV
- $T=162$ MeV
- $T=149$ MeV
- $T=147$ MeV
- $T=145$ MeV
- $T=155$ MeV
- $T=152.5$ MeV
- $T=150$ MeV

- $T=155(5)$ MeV
- $T=145(2)$ MeV
- $39$ GeV (STAR prelim.)
- $17.3$ GeV (NA57)
relative yields of strange anti-baryons ($\bar{H}_S$) to baryons ($H_S$) can be used to determine freeze-out parameters $\mu_B^f/T^f$ and $\mu_S^f/\mu_B^f$ from experiment

$$R_H \equiv \frac{\bar{H}_S}{H_S} = e^{-2(\mu_B^f/T^f)(1-(\mu_S^f/\mu_B^f)|S|)}$$

and compared to Lattice QCD or HRG to determine freeze-out temperature $T^f$:

**Implications for strangeness freeze-out**

- **Lattice QCD**
  - $T=155(5)$ MeV
  - $T=145(2)$ MeV

- **QM-HRG**
  - $T=158(3)$ MeV
  - $T=147(2)$ MeV

freeze-out temperature obtained from QM-HRG agrees well with Lattice QCD results

important to include so-far undiscovered strange baryons in HRG
What do we know of the hadron spectrum?

**Quark Model**

**charm baryons**


**Quark Model**

- PDG will denote results using states listed in the particle data tables
- QM will denote results using states calculated in the quark model
- QM-3 all resonances up to 3.0 GeV
- QM-3.5 all resonances up to 3.5 GeV
What do we know of the hadron spectrum?

**Quark Model**

charm baryons

**Lattice QCD**


[Padmanath et al., arXiv 1311.4806]

PDG will denote results using states listed in the particle data tables

QM will denote results using states calculated in the quark model

QM-3 all resonances up to 3.0 GeV

QM-3.5 all resonances up to 3.5 GeV
partial pressure $P$ of all open charm hadrons

can be separated into mesonic $P_M^C$ and baryonic $P_B^C$ components

$$P_{tot}^C, X = P_M^C, X + P_B^C, X$$

$$P_M^C, X(T, \mu) = \frac{T^4}{2\pi^2} \sum_{i \in X} g_i \left( \frac{m_i}{T} \right)^2 K_2\left(\frac{m_i}{T}\right) \times \cosh\left(B_i \hat{\mu}_B + Q_i \hat{\mu}_Q + S_i \hat{\mu}_S + C_i \hat{\mu}_C\right)$$

$$X = \begin{cases} 
\text{QM resonances} \\
\text{QM-3.5 resonances up to 3.5 GeV} \\
\text{QM-3 resonances up to 3.0 GeV} \\
\text{PDG resonances}
\end{cases}$$
Correlations of conserved charges – open charm sector

Taylor expansion of pressure in terms of chemical potentials related to conserved charges:

\[
\frac{P}{T^4} = \sum_{k,l,m,n=0}^{\infty} \frac{1}{k!l!m!n!} \chi_{k l m n}^{B Q S C} (T) \left( \frac{\mu_B}{T} \right)^k \left( \frac{\mu_Q}{T} \right)^l \left( \frac{\mu_S}{T} \right)^m \left( \frac{\mu_C}{T} \right)^n
\]

generalized susceptibilities of conserved charges

\[
\chi_{k l m n}^{B Q S C} = \frac{\partial^{(k+l+m+n)} [P(\hat{\mu}_B, \hat{\mu}_Q, \hat{\mu}_S, \hat{\mu}_C)/T^4]}{\partial \hat{\mu}_B^k \partial \hat{\mu}_Q^l \partial \hat{\mu}_S^m \partial \hat{\mu}_C^n} \bigg|_{\mu = 0}
\]

are sensitive to the underlying degrees of freedom

charm contributions to pressure in a hadron gas:

\[
P^C = P^C_M \cosh(\hat{\mu}_C) + \sum_{k=1,2,3} P^C_B \cosh(B \hat{\mu}_B + k \hat{\mu}_C)
\]

partial pressure of open-charm mesons and charmed baryons depends on hadron spectra

\[
\chi_{m n}^{B C} = B^m P_B^{C=1} + B^m 2^n P_B^{C=2} + B^m 3^n P_B^{C=3} \quad \sim \quad B^m P_B^{C=1}
\]

relative contribution of C=2 and C=3 baryons negligible

ratios independent of the detailed spectrum and sensitive to special sectors:

charmed baryon sector

\[
\frac{\chi_{m n}^{B C}}{\chi_{m+1, n-1}^{B C}} = B^{-1}
\]

=1 when DoF are hadronic

=3 when DoF are quarks

\[
\frac{\chi_{m n}^{B C}}{\chi_{m, n+2}^{B C}} = 1 \quad \text{always}
\]
2+1 flavor HISQ with almost physical quark masses

$32^3 \times 8$ and $24^3 \times 6$ lattices with $m_l = m_s/20$ and physical $m_s$ and quenched charm quarks

generalized susceptibilities of conserved charges

$$\chi^{BQSC}_{klnm} = \frac{\partial^{(k+l+m+n)}[P(\hat{\mu}_B, \hat{\mu}_Q, \hat{\mu}_S, \hat{\mu}_C)/T^4]}{\partial \hat{\mu}_B \partial \hat{\mu}_Q \partial \hat{\mu}_S \partial \hat{\mu}_C} \bigg|_{\hat{\mu}=0}$$

are sensitive to the underlying degrees of freedom

→ indications that charmed baryons start to dissolve already close to the chiral crossover
Correlations of conserved charges – open charm sector

Taylor expansion of pressure in terms of chemical potentials related to conserved charges

\[
\frac{P}{T^4} = \sum_{k,l,m,n=0}^{\infty} \frac{1}{k!l!m!n!} \chi_{klnn}^{BQSC}(T) \left( \frac{\mu_B}{T} \right)^k \left( \frac{\mu_Q}{T} \right)^l \left( \frac{\mu_S}{T} \right)^m \left( \frac{\mu_C}{T} \right)^n
\]

generalized susceptibilities of conserved charges

\[
\chi_{klnn}^{BQSC} = \left. \frac{\partial^{(k+l+m+n)}[P(\hat{\mu}_B, \hat{\mu}_Q, \hat{\mu}_S, \hat{\mu}_C)/T^4]}{\partial \hat{\mu}_B^k \partial \hat{\mu}_Q^l \partial \hat{\mu}_S^m \partial \hat{\mu}_C^n} \right|_{\hat{\mu}=0}
\]

are sensitive to the underlying degrees of freedom

charm contributions to pressure in a hadron gas:

\[
P^C = P^C_M \cosh(\hat{\mu}_C) + \sum_{k=1,2,3} P^C_B = k \cosh(B\hat{\mu}_B + k\hat{\mu}_C)
\]

partial pressure of open-charm mesons and charmed baryons depends on hadron spectra

\[
\chi_{klnn}^{B=1,C} = P^C_B = 1 + 2^n P^C_B = 2 + 3^n P^C_B = 3 \approx P^C_B
\]

\[
\chi^C_k = P^C_M + 2^n P^C_B = 2 + 3^n P^C_B = 3 \approx P^C_M + P^C_B = 1
\]

ratios independent of the detailed spectrum and sensitive to special sectors:

partial pressure of open-charm mesons:

\[
P^C_M = \chi^C_2 - \chi^B_{2C} = \chi^C_4 - \chi^{BC}_{13}
\]

\[
\frac{\chi^C_4}{\chi^C_2} = 1
\]
Correlations of conserved charges – open charm sector

[A.Bazavov, H.T.Ding, P.Hegde, OK et al., PLB737 (2014) 210]

2+1 flavor HISQ with almost physical quark masses

32³ × 8 and 24³ × 6 lattices with \( m_l = m_s/20 \) and physical \( m_s \) and quenched charm quarks

**Generalized susceptibilities of conserved charges**

\[
\chi^{BQ \, SC}_{klmn} = \frac{\partial^{(k+l+m+n)}[P(\hat{\mu}_B, \hat{\mu}_Q, \hat{\mu}_S, \hat{\mu}_C)/T^4]}{\partial \hat{\mu}_B \partial \hat{\mu}_Q \partial \hat{\mu}_S \partial \hat{\mu}_C} \bigg|_{\bar{\mu}=0}
\]

are sensitive to the underlying degrees of freedom

\[ \rightarrow \] indications that open charm mesons start to dissolve already close to the chiral crossover

\[ P^C_M = \chi^C_2 - \chi^{BC}_{22} = \chi^C_4 - \chi^{BC}_{13} \]

\[ \frac{\chi^C_4}{\chi^C_2} = 1 \]
Signatures for deconfinement of light/strange/charm baryons


\[ \rightarrow \text{charmed hadrons start to deconfine around the chiral crossover region} \]

\[ \rightarrow \text{strange hadrons start to deconfine around the chiral crossover region} \]
Signatures for additional charm baryons

Charmed pressure ratios are sensitive to the charm hadron spectrum

\[ R_{13}^{BC} = \frac{\chi_{13}^{BC}}{M_C} = \frac{B_C}{M_C} \]

\[ M_C \approx \chi_4^C - \chi_{13}^{BC} \]

**Charmed baryon to meson ratio**

\[ R_{13}^{BQC} = \frac{\chi_{112}^{BQC}}{M_{QC}} \]

\[ M_{QC} \approx \chi_{13}^Q - \chi_{112}^{BQC} \]

**Charged charmed baryon to meson ratio**

\[ R_{13}^{SC} = \frac{-\chi_{112}^{BSC}}{M_{SC}} \]

\[ M_{SC} \approx \chi_{13}^S - \chi_{112}^{BSC} \]

→ important to include so-far undiscovered open charm hadrons in HRG

Correlation functions along the **spatial direction**

\[
G(z, T) = \int dx dy \int_0^{1/T} d\tau \langle J(x, y, z, \tau) J(0, 0, 0, 0) \rangle
\]

are related to the meson spectral function at **non-zero spatial momentum**

\[
G(z, T) = \int_{-\infty}^{\infty} dp_z e^{ip_z z} \int_0^{\infty} d\omega \frac{\sigma(\omega, p_z, T)}{\omega}
\]

exponential decay defines **screening mass** \(M_{\text{scr}}\): \(G(z, T) \xrightarrow{z \gg 1/T} e^{-M_{\text{scr}} z}\)

- **bound state contribution**
  \[\sigma(\omega, p_z, T) \sim \delta(\omega^2 - p_z^2 - M^2)\]
  \[M_{\text{scr}} = M\]

- **high-T limit (non-interacting free limit)**
  \[\sigma(\omega, p_z, T) \sim \sigma_{\text{free}}(\omega, p_z, T)\]
  \[M_{\text{scr}} = 2\sqrt{(\pi T)^2 + m_c^2}\]

Spatial correlation functions and screening masses

[2+1 flavor HISQ with almost physical quark masses]

48³ × 12 lattices with m_ν = m_s/20 and physical m_s

“s\bar{s} and s\bar{c} possibly dissolve close to crossover temperature”

“c\bar{c} in line with the sequential melting of charmonium states”
exponential decay defines screening mass $M_{\text{scr}}$:

$$G(z, T) \rightarrow e^{-M_{\text{scr}}z} \quad \text{for } z \gg 1/T$$

bound state contribution

$$\sigma(\omega, p_z, T) \sim \delta(\omega^2 - p_z^2 - M^2)$$

high-T limit (non-interacting free limit)

$$\sigma(\omega, p_z, T) \sim \sigma_{\text{free}}(\omega, p_z, T)$$

indications for medium modifications/dissociation

$$M_{\text{scr}} = M$$

ongoing study in quenched QCD to understand the sequential melting of charmonium and bottomonium states in the QGP (see last week’s lecture notes)
Vector spectral function – hard to separate different scales

Different contributions and scales enter in the spectral function

- **continuum at large frequencies**
- **possible bound states at intermediate frequencies**
- **transport contributions at small frequencies**
- in addition cut-off effects on the lattice

notoriously difficult to extract from correlation functions

\[
G(\tau, \vec{p}, T) = \int_0^\infty \frac{d\omega}{2\pi} \rho(\omega, \vec{p}, T) K(\tau, \omega, T)
\]

(narrow) transport peak at small $\omega$: $\rho(\omega \ll T) \simeq 2\chi_{00} \frac{T}{M} \frac{\omega \eta}{\omega^2 + \eta^2}$, $\eta = \frac{T}{MD}$
Heavy Quark Effective Theory (HQET) in the large quark mass limit

for a single quark in medium

leads to a (pure gluonic) “color-electric correlator”

\[ G_E(\tau) \equiv -\frac{1}{3} \sum_{i=1}^{3} \frac{\langle \text{Re Tr} \left[ U \left( \frac{1}{T}; \tau \right) gE_i(\tau, 0) U(\tau; 0) gE_i(0, 0) \right] \rangle}{\langle \text{Re Tr}[U(\frac{1}{T}; 0)] \rangle} \]

Heavy quark (momentum) diffusion:

\[ \kappa = \lim_{\omega \to 0} \frac{2T \rho_E(\omega)}{\omega} \quad D = \frac{2T^2}{\kappa} \]
can be related to the thermalization rate:

$$\eta_D = \frac{\kappa}{2M_{kin}T} \left( 1 + O \left( \frac{\alpha_s^{3/2}T}{M_{kin}} \right) \right)$$

NLO in perturbation theory: [Caron-Huot, G. Moore, JHEP 0802 (2008) 081]

very poor convergence

→ Lattice QCD study required in the relevant temperature region

in contrast to a narrow transport peak, from this a smooth limit is expected qualitatively similar behavior also found in AdS/CFT [S.Gubser, Nucl.Phys.B790 (2008)175]
Heavy Quark Momentum Diffusion Constant – Lattice results

finest lattices still quite noisy at large $\tau T$

but only small cut-off effects at intermediate $\tau T$

cut-off effects become visible at small $\tau T$

need to extrapolate to the continuum

perturbative behavior in the limit $\tau T \to 0$

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Quenched Lattice QCD

$T \simeq 1.5 T_c$

$V \simeq (2\text{fm})^3$

allows to perform continuum extrapolation, $a \to 0 \leftrightarrow N_t \to \infty$, at fixed $T=1/a N_t$
Heavy Quark Momentum Diffusion Constant – Continuum extrapolation

finest lattices still quite noisy at large $\tau T$ but only
small cut-off effects at intermediate $\tau T$
cut-off effects become visible at small $\tau T$
need to extrapolate to the continuum
perturbative behavior in the limit $\tau T \to 0$

well behaved continuum extrapolation for $0.05 \leq \tau T \leq 0.5$
finest lattice already close to the continuum
coarser lattices at larger $\tau T$ close to the continuum

how to extract the spectral function from the correlator?
\( \omega \ll T \): linear behavior motivated at small frequencies

\[
\rho_{IR}(\omega) = \frac{\kappa \omega}{2T}
\]

\( \omega \gg T \): vacuum perturbative results and leading order thermal correction:

\[
\rho_{UV}(\omega) = \left[ \rho_{UV}(\omega) \right]_{T=0} + \mathcal{O} \left( \frac{g^4 T^4}{\omega} \right)
\]

using a renormalization scale \( \bar{\mu}_\omega = \omega \) for \( \omega \gg \Lambda_{MS} \) leading order becomes

\[
\rho_{UV}(\omega) = \Phi_{UV}(\omega) \left[ 1 + \mathcal{O} \left( \frac{1}{\ln(\omega/\Lambda_{MS})} \right) \right]
\]

\[
\Phi_{UV}(\omega) = \frac{g^2(\bar{\mu}_\omega) C_F \omega^3}{6\pi}, \quad \bar{\mu}_\omega \equiv \max(\omega, \pi T)
\]

here we used 4-loop running of the coupling

model the spectral function using these asymptotics with two free parameters

\[
\rho_{model}(\omega) \equiv \max \left\{ A\Phi_{UV}(\omega), \frac{\omega \kappa}{2T} \right\}
\]
including thermal corrections

\[ \rho_{UV}(\omega) = \frac{g^2(\bar{\mu}_\omega)C_F\omega^3}{6\pi} \]

already closer to the data

but contributions from the transport visible at large separations

Model spectral function: transport contribution + UV-asymptotics

\[ \rho_{\text{model}}(\omega) \equiv \max \left\{ A\rho_{UV}(\omega), \frac{\omega \kappa}{2T} \right\} \]

\[ G_{\text{model}}(\tau) \equiv \int_0^\infty \frac{d\omega}{\pi} \rho_{\text{model}}(\omega) \frac{\cosh \left( \frac{1}{2} - \tau T \right) \omega}{\sinh \frac{\omega}{2T}} \]
Model spectral function: transport contribution + UV-asymptotics

\[ \rho_{\text{model}}(\omega) \equiv \max \left\{ A \rho_{\text{UV}}(\omega), \frac{\omega \kappa}{2T} \right\} \]

used to fit the continuum extrapolated data

\[ G_{\text{model}}(\tau) \equiv \int_0^\infty \frac{d\omega}{\pi} \rho_{\text{model}}(\omega) \frac{\cosh \left( \frac{\omega}{2} - \tau T \right)}{\sinh \frac{\omega}{2T}} \]

\[ \frac{\kappa}{T^3} = \lim_{\omega \to 0} \frac{2T \rho_E(\omega)}{\omega} \approx 2.31(7) \]

result of the fit to \( \rho_{\text{model}}(\omega) \)

\[ A \rho_{\text{UV}}(\omega) \]

small but relevant contribution at \( \tau T > 0.2 \)!
model corrections to $\rho_{IR}$ by a power series in $\omega$

analysis of the systematic uncertainties

$\rightarrow$ continuum estimate of $\kappa$:

$$\frac{\kappa}{T^3} = \lim_{\omega \to 0} \frac{2T \rho_E(\omega)}{\omega} = 1.8...3.4$$
Lattice QCD results on heavy quark diffusion coefficients

\[ D = \frac{2T^2}{\kappa} \]

charm quark (Ding et al.)
static quark (Banerjee et al.)
static quark (Kaczmarek et al.)
AdS/CFT

charm quark mass quenched approximation no continuum yet

heavy quark mass limit quenched approximation continuum extrapolated

next goals: continuum extrapolation for charm and bottom + including dynamical quarks

\[ \rho_{E} / (\omega T^2) \]

\[ \frac{\sigma(\omega)}{(\omega T)} \]

continuum extrapolated

\[ 0.35 \]

\[ 0.3 \]

\[ 0.25 \]

\[ 0.2 \]

\[ 0.15 \]

\[ 0.1 \]

\[ 0.05 \]

0.05

0.1

0.15

0.2

0.25

0.3

0.35

0

0.5

1

1.5

2

2.5

\[ V_{ii} \]

1.46 T_{c}

2.20 T_{c}

2.93 T_{c}

\[ \omega / T \]

[AdS/CFT]

[H.T.Ding, OK et al., PRD86(2012)014509]

[Banerjee et al., PRD85(2012) 014510]

[A.Francis, OK et al., 2015]