Universität Bielefeld

General lecture: Additional strange hadrons from QCD thermodynamics and freeze-out in heavy ion collisions

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Strangeness in Quark Matter 2015
&
Helmholtz International Summer School “Dense Matter“

JINR Dubna
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Outline

1) Higher order cumulants of conserved charges in the strange sector

Bielefeld-BNL-Wuhan collaboration

- evidence for additional strange hadrons
- implications for strangeness freeze-out
- deconfinement of open strange hadrons

2) Higher order cumulants of conserved charges in the charm sector

Bielefeld-BNL-Wuhan collaboration

- evidence for additional charmed hadrons
- deconfinement of open charm hadrons

3) Hadronic correlation functions and screening masses

Bielefeld-BNL-Wuhan collaboration

- dissociation of quarkonia in the QGP

H.-T. Ding, OK, M. Laine and H.Ohno

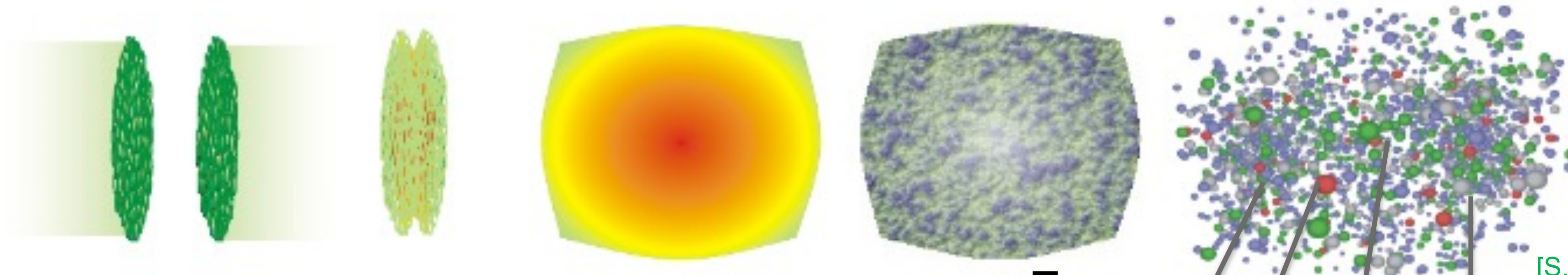
4) Color electric field correlation function

A.Francis, OK, M. Laine, T.Neuhaus, H.Ohno

- Heavy quark momentum diffusion coefficient κ

Motivation – Hadron yields at the freeze-out

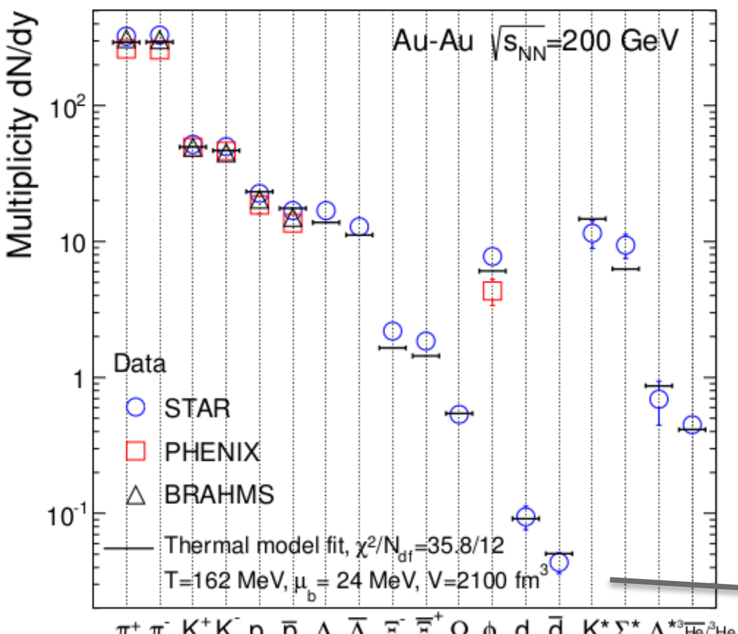
Heavy Ion Collision **QGP** **Expansion+Cooling** **Hadronization**



[S.Bass]

T_c, μ_c

Hadron Resonance Gas (HRG)
to describe
hadron yields at the freeze-out



[Andronic et.al., Nucl.Phys.A904(2013)535c]

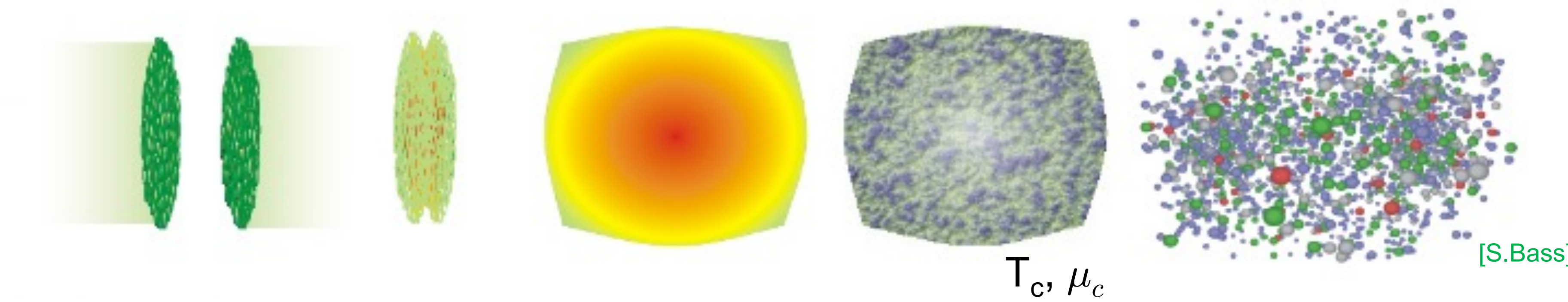
hadrons measured
in the detectors

"observed" mainly at the
freeze-out stage of the HIC

T^f, μ^f

Motivation – Hadron yields at the freeze-out

Heavy Ion Collision **QGP** **Expansion+Cooling** **Hadronization**



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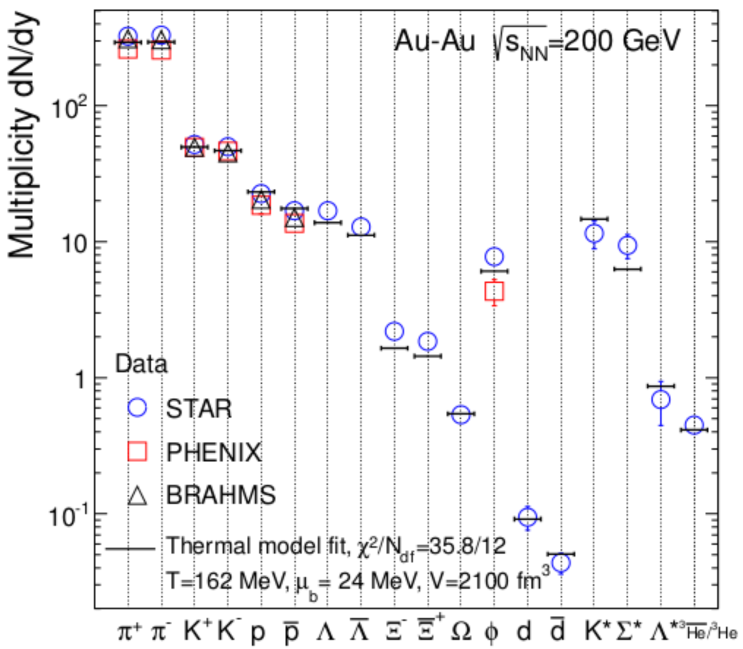


[Andronic et.al., Nucl.Phys.A904(2013)535c]

HRG: thermal gas of uncorrelated hadrons

partial pressure of each hadron:

$$\hat{P}_h \sim f(\hat{m}_h)\cosh\left[B_h\hat{\mu}_h + Q_h\hat{\mu}_Q + S_h\hat{\mu}_S + C_h\hat{\mu}_C\right]$$

total pressure given by the sum over all (known) hadrons

$$\hat{P}_{total} = \sum_{all\ \ hadrons} \hat{P}_h$$

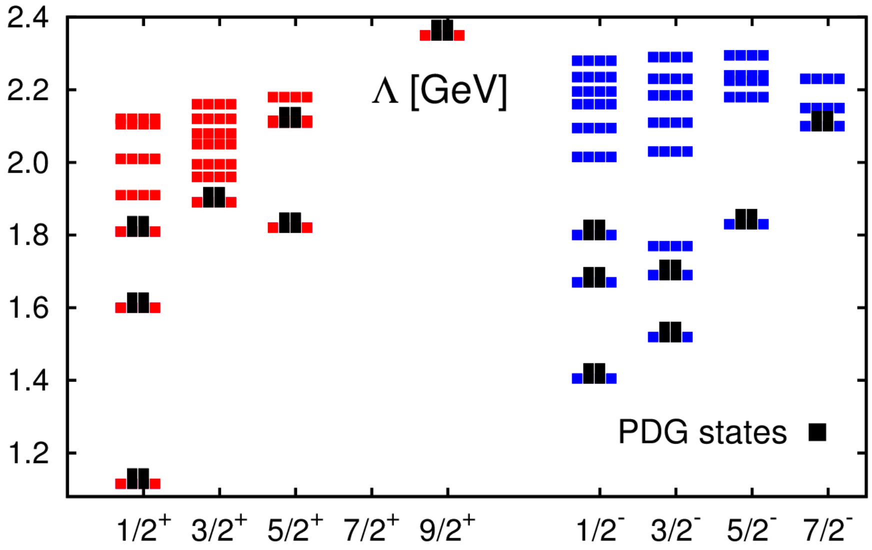
are we sensitive to this?

→ use thermodynamics instead of HRG
to describe freeze-out

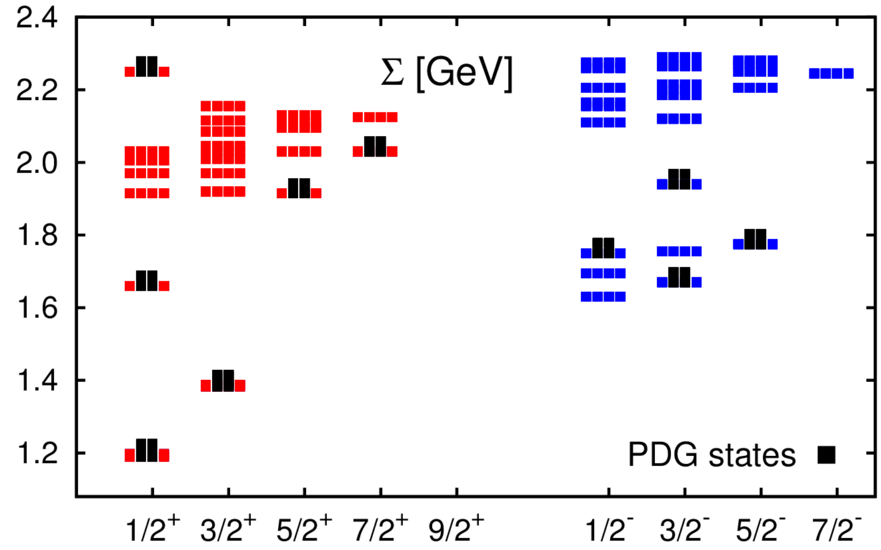
What do we know of the hadron spectrum?

strange baryons

Quark Model



Quark Model



[Capstick-Isgur, Phys.Rev.D34 (1986) 2809]

in the following

PDG will denote results using states listed in the particle data tables

QM will denote results using states calculated in the quark model

What do we know of the hadron spectrum?

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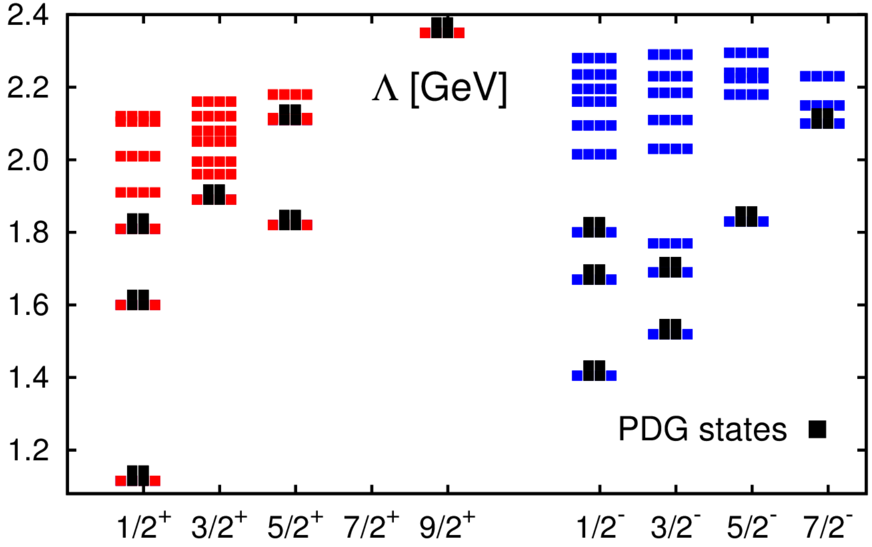
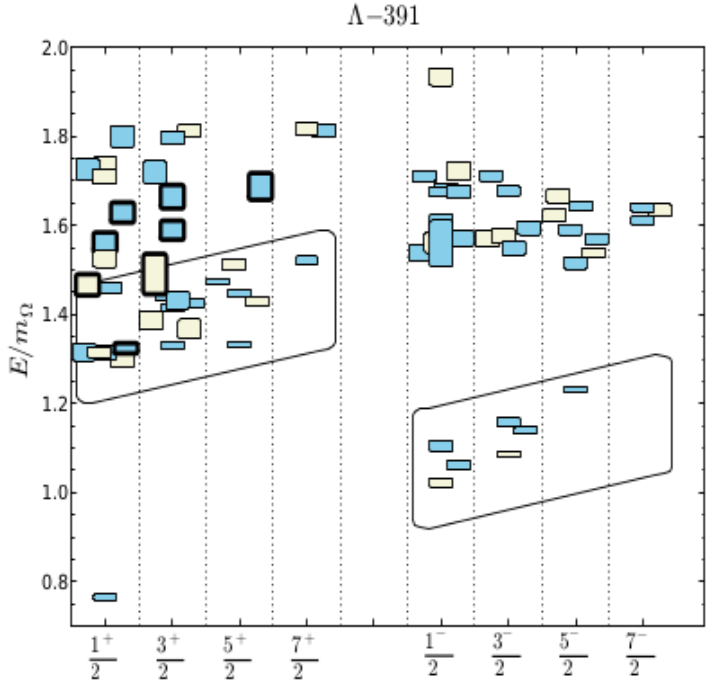


[Capstick-Isgur, Phys.Rev.D34 (1986) 2809]

Lattice QCD



[Edwards et al., Phys.Rev.D87 (2013) 054506]

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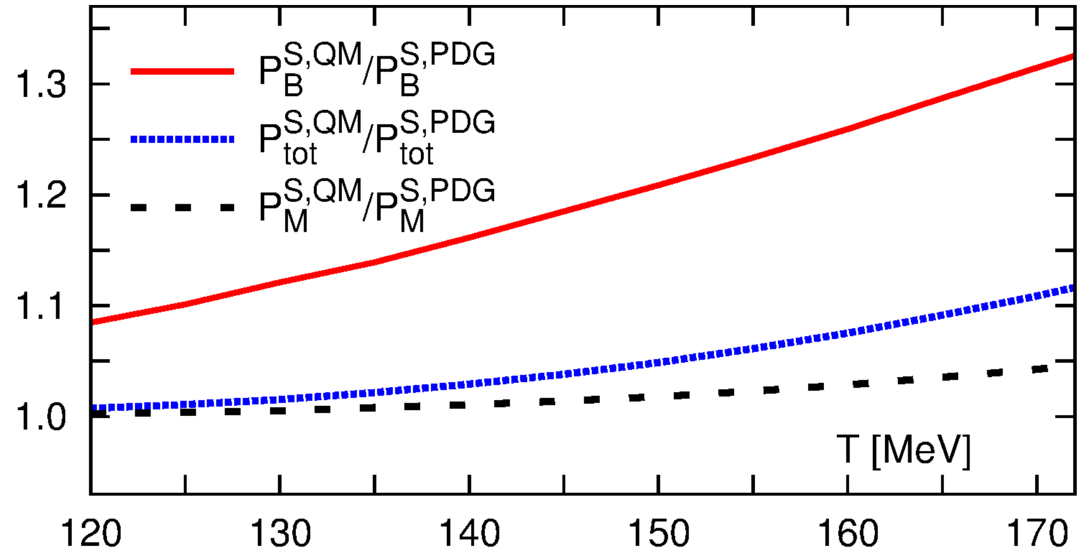
Hadron Resonance Gas - contributions of additional states - strange

partial pressure *P* of all open strange hadrons in a hadron resonance gas (HRG) can be separated into mesonic P_M and baryonic P_B components

$$P^{S,X}_{\mathrm{tot}} = P^{S,X}_{M} + P^{S,X}_{B}$$

$$X = \begin{cases} \text{QM resonances} \\ \text{PDG resonances} \end{cases}$$

$$P^{S,X}_{M/B}(T,\vec{\mu}) = \frac{T^4}{2\pi^2} \sum_{i\in X} g_i \left(\frac{m_i}{T}\right)^2 K_2(m_i/T) \times \cosh\left(B_i\hat{\mu}_B + Q_i\hat{\mu}_Q + S_i\hat{\mu}_S\right)$$



large enhancement of the partial baryonic pressure from additional strange baryons

large part of open strange mesons experimentally observed

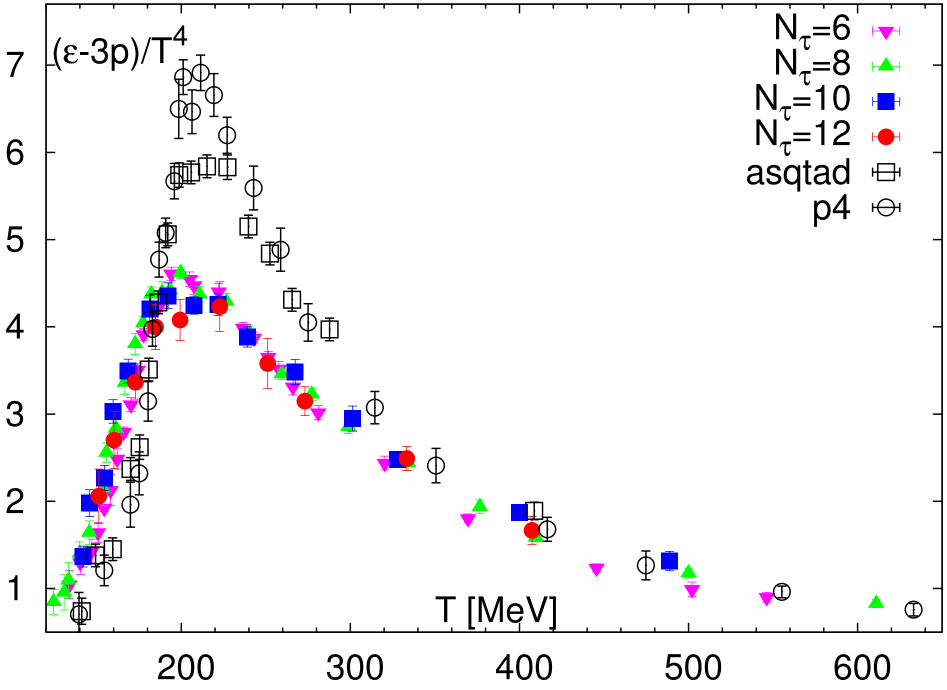
Equation of state of (2+1)-flavor QCD

thermodynamic quantities obtained from derivatives of the partition function

$$Z(\beta, N_\sigma, N_\tau) = \int \prod_{x,\mu} dU_{x,\mu} e^{-S(U)} \qquad S(U) = \beta S_G(U) - S_F(U)$$

using trace of the energy momentum tensor: $$\Theta^{\mu\mu} = \epsilon - 3p = -\frac{T}{V}\frac{d\ln Z}{d\ln a}$$

HISQ: [A. Bazavov et al. (hotQCD), PRD90 (2014) 094503]



$$\frac{\epsilon - 3p}{T^4} \equiv \frac{\Theta^{\mu\mu}_G(T)}{T^4} + \frac{\Theta^{\mu\mu}_F(T)}{T^4} \,,$$

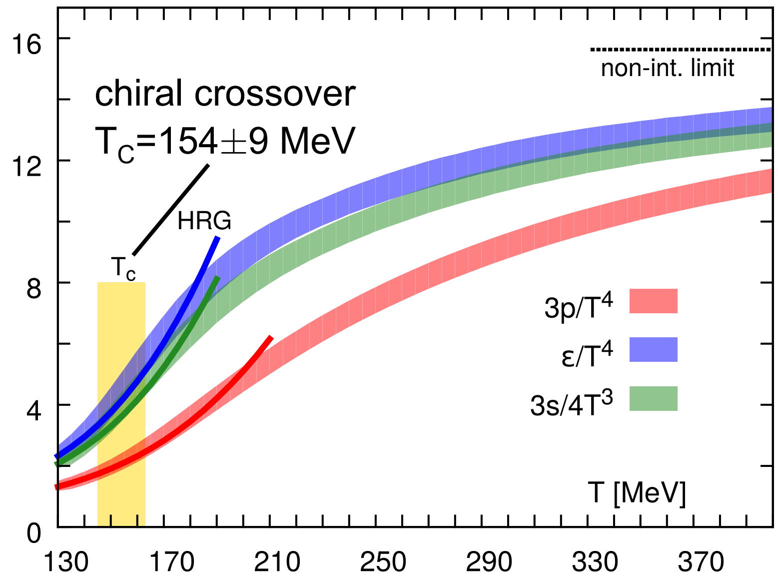
$$\frac{\Theta^{\mu\mu}_G(T)}{T^4} = R_\beta \left[\langle s_G\rangle_0 - \langle s_G\rangle_\tau\right] N_\tau^4 \,,$$

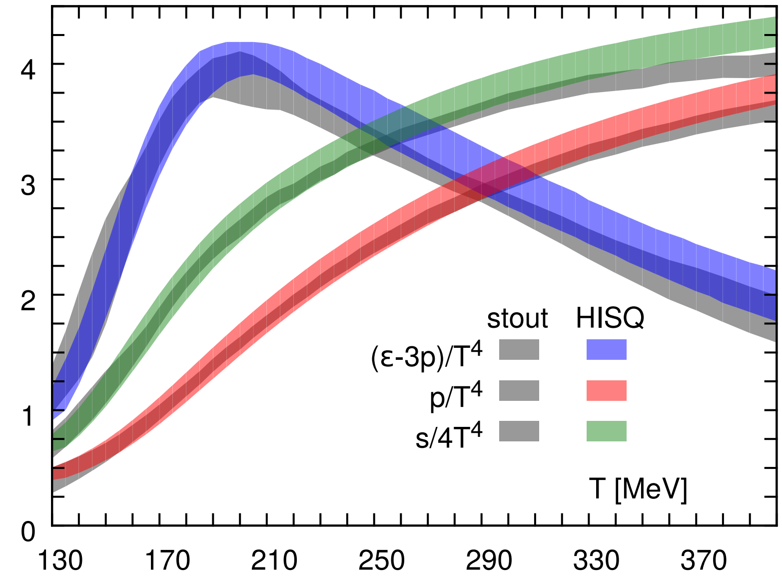
$$\frac{\Theta^{\mu\mu}_F(T)}{T^4} = -R_\beta R_m [2m_l \left(\langle\bar\psi\psi\rangle_{l,0} - \langle\bar\psi\psi\rangle_{l,\tau}\right) + m_s \left(\langle\bar\psi\psi\rangle_{s,0} - \langle\bar\psi\psi\rangle_{s,\tau}\right)] N_\tau^4 \,.$$

pressure calculated using integral method: $$\frac{p(T)}{T^4} = \frac{p_0}{T_0^4} + \int_{T_0}^{T} dT' \frac{\Theta^{\mu\mu}}{T'^5}$$

Equation of state of (2+1)-flavor QCD - $\mu_B/T = 0$

continuum extrapolated results of **pressure** & **energy density** & **entropy density**





HISQ: [A. Bazavov et al. (hotQCD), PRD90 (2014) 094503]

stout: [S. Borsanyi et al., PLB730, 99 (2014)]

consistent results from hotQCD (HISQ) and Budapest-Wuppertal (stout)

hadron resonance gas (HRG) model using all known hadronic resonances from PDG

describes the EoS quite well up to cross-over region

QCD results systematically above HRG

room for additional resonances not listed in the PDG

Cumulants of net-charge fluctuations

Taylor expansion of pressure in terms of chemical potentials related to conserved charges

$$\frac{P}{T^4} = \sum_{i,j,k=0}^{\infty} \frac{1}{i!j!k!} \chi_{ijk}^{BQS}(T) \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k$$

defines generalized susceptibilities: $\chi_{ijk}^{BQS} = \left. \frac{\partial^{(i+j+k)}[P(T,\hat{\mu}_B,\hat{\mu}_Q,\hat{\mu}_S)/T^4]}{\partial \hat{\mu}_B^i \partial \hat{\mu}_Q^j \partial \hat{\mu}_S^k} \right|_{\vec{\mu}=0}$

with $\hat{\mu}_X = \mu_X/T$

generalized susceptibilities calculated at zero μ	cumulants of net-charge fluctuations measured at the freeze out

Lattice QCD

$$\begin{aligned}
VT^3\chi_2^X &= \langle(\delta N_X)^2\rangle \\
VT^3\chi_4^X &= \langle(\delta N_X)^4\rangle - 3\langle(\delta N_X)^2\rangle^2 \\
VT^3\chi_6^X &= \langle(\delta N_X)^6\rangle \\
&\quad -15\langle(\delta N_X)^4\rangle\langle(\delta N_X)^2\rangle \\
&\quad +30\langle(\delta N_X)^2\rangle^3
\end{aligned}$$

Experiment

$$\delta N_X \equiv N_X - \langle N_x \rangle$$

Cumulants of net-charge fluctuations

higher order cumulants characterize the shape of conserved charge distributions

$$S_q \sigma_q = \frac{\chi_3^q}{\chi_2^q}$$

$$\kappa_q \sigma_q^2 = \frac{\chi_4^q}{\chi_2^q}$$

$$q = B, Q, S$$

mean: $\langle \delta N_q \rangle \equiv \langle N_q - N_{\bar{q}} \rangle$

variance: $\sigma_q^2 \equiv \langle (\delta N_q)^2 \rangle - \langle \delta N_q \rangle^2$

skewness: $S_q \equiv \langle (\delta N_q)^3 \rangle / \sigma_q^3$

kurtosis: $\kappa_q \equiv \langle (\delta N_q)^4 \rangle / \sigma_q^4 - 3$

generalized susceptibilities calculated at zero μ

cumulants of net-charge fluctuations measured at the freeze out

Lattice QCD

Experiment

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\end{aligned}$$

$$\delta N_X \equiv N_X - \langle N_x \rangle$$

Equation of state of (2+1)-flavor QCD - $\mu_B/T > 0$

Taylor expansion of pressure in terms of chemical potentials related to conserved charges

$$\frac{P}{T^4} = \sum_{i,j,k=0}^{\infty} \frac{1}{i!j!k!} \chi_{ijk}^{BQS}(T) \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k$$

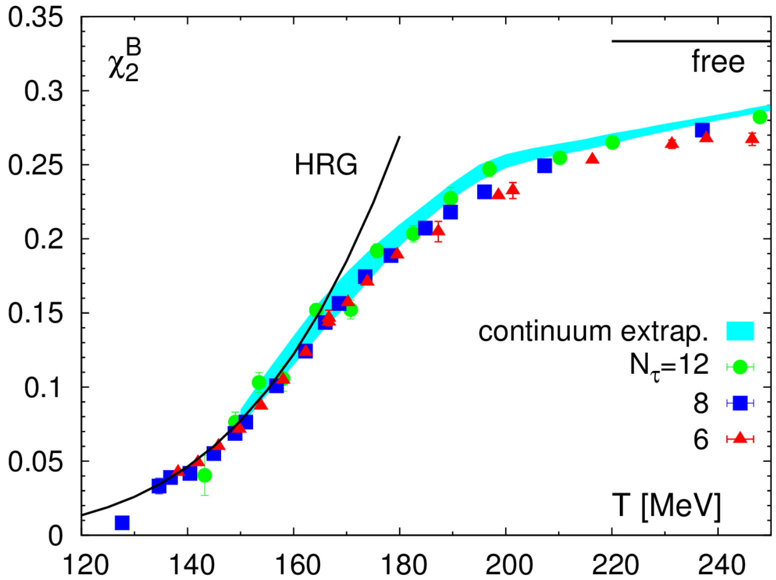
defines generalized susceptibilities: $\chi_{ijk}^{BQS} = \left.\frac{\partial^{(i+j+k)}[P(T,\hat{\mu}_B,\hat{\mu}_Q,\hat{\mu}_S)/T^4]}{\partial\hat{\mu}_B^i \partial\hat{\mu}_Q^j \partial\hat{\mu}_S^k}\right|_{\vec{\mu}=0}$

for $\mu_Q = \mu_S = 0$ this simplifies to

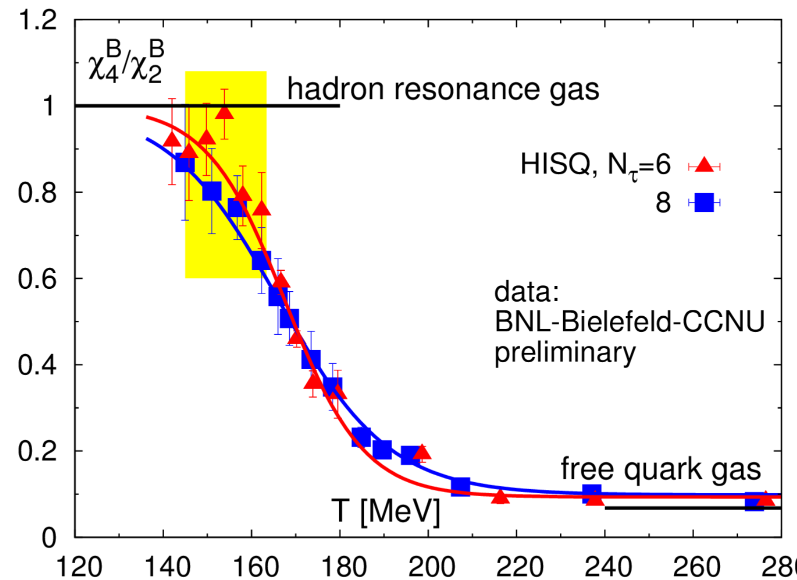
variance of net-baryon number distribution

kurtosis*variance $\kappa_B \sigma_B^2$

$$\frac{\Delta P(T)}{T^4} = \frac{P(T,\mu_B) - P(T,0)}{T^4} = \frac{\chi_2^B}{2}\left(\frac{\mu_B}{T}\right)^2 \left(1 + \frac{1}{12}\frac{\chi_4^B}{\chi_2^B}\left(\frac{\mu_B}{T}\right)^2\right) + \mathcal{O}\left(\mu_B^6\right)$$



good agreement with HRG in crossover region



deviations from HRG in crossover region

Equation of state of (2+1)-flavor QCD – strange sector

Taylor expansion of pressure in terms of chemical potentials related to conserved charges

$$\frac{P}{T^4} = \sum_{i,j,k=0}^{\infty} \frac{1}{i!j!k!} \chi_{ijk}^{BQS}(T) \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k$$

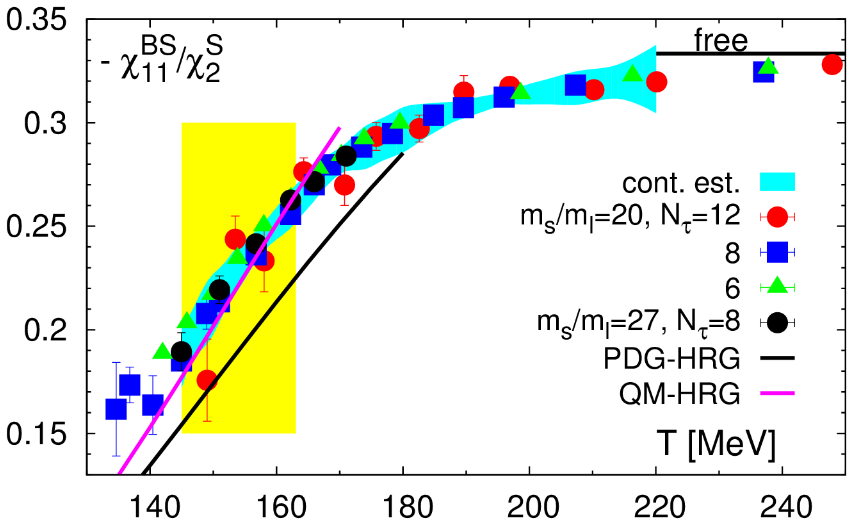
defines generalized susceptibilities: $\chi_{ijk}^{BQS} = \left. \frac{\partial^{(i+j+k)}[P(T,\hat\mu_B,\hat\mu_Q,\hat\mu_S)/T^4]}{\partial\hat\mu_B^i \partial\hat\mu_Q^j \partial\hat\mu_S^k} \right|_{\vec\mu=0}$

correlations of strangeness with baryon number fluctuations:

$$\chi_{11}^{BS} = \left. \frac{\partial^2[P(T,\hat\mu_B,\hat\mu_Q,\hat\mu_S)/T^4]}{\partial\hat\mu_B \partial\hat\mu_S} \right|_{\vec\mu=0}$$

second cumulant of net strangeness fluctuations:

$$\chi_2^S = \left. \frac{\partial^2[P(T,\hat\mu_B,\hat\mu_Q,\hat\mu_S)/T^4]}{\partial\hat\mu_S^2} \right|_{\vec\mu=0}$$



suitable ratios like

$$\frac{\chi_{11}^{BS}}{\chi_2^S}$$

χ_{11}^{BS} — partial pressure of strange baryons (in a hadron gas)

χ_2^S — dominated by strange mesons

are sensitive probes of the strangeness carrying degrees of freedom

Equation of state of (2+1)-flavor QCD – strange sector

Taylor expansion of pressure in terms of chemical potentials related to conserved charges

$$\frac{P}{T^4} = \sum_{i,j,k=0}^{\infty} \frac{1}{i!j!k!} \chi_{ijk}^{BQS}(T) \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k$$

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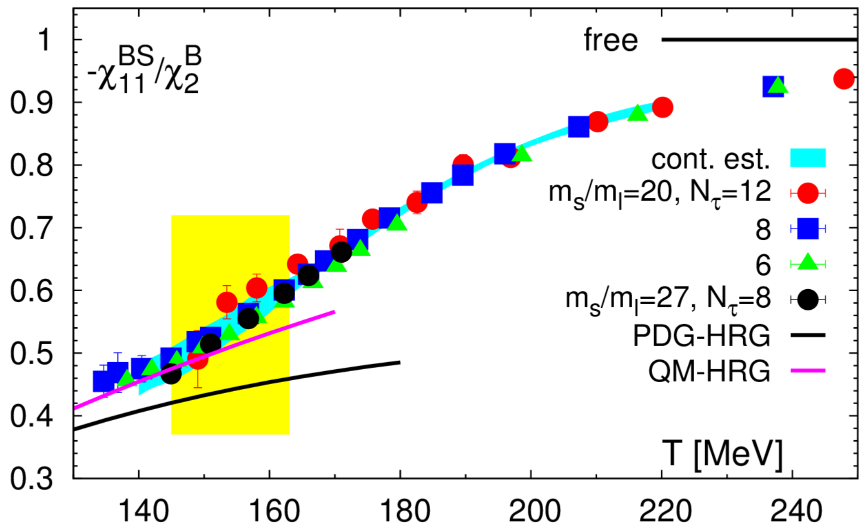
second cumulant of net baryon number fluctuations:

$$\chi_{2}^{B} = \left. \frac{\partial^2[P(T,\hat{\mu}_B,\hat{\mu}_Q,\hat{\mu}_S)/T^4]}{\partial\hat{\mu}_B^2} \right|_{\vec{\mu}=0}$$



suitable ratios like

$$\frac{\chi_{11}^{BS}}{\chi_2^B}$$

are sensitive probes of the strangeness carrying degrees of freedom

Thermodynamic contributions of strange baryons

$$\chi_{klm}^{BQS} = \left. \frac{\partial^{(k+l+m)}[P(T,\hat{\mu}_B,\hat{\mu}_Q,\hat{\mu}_S)/T^4]}{\partial\hat{\mu}_B^k \partial\hat{\mu}_Q^l \partial\hat{\mu}_S^m} \right|_{\vec{\mu}=0}$$

individual pressure-observables

for open strange mesons (P_M^S in HRG):

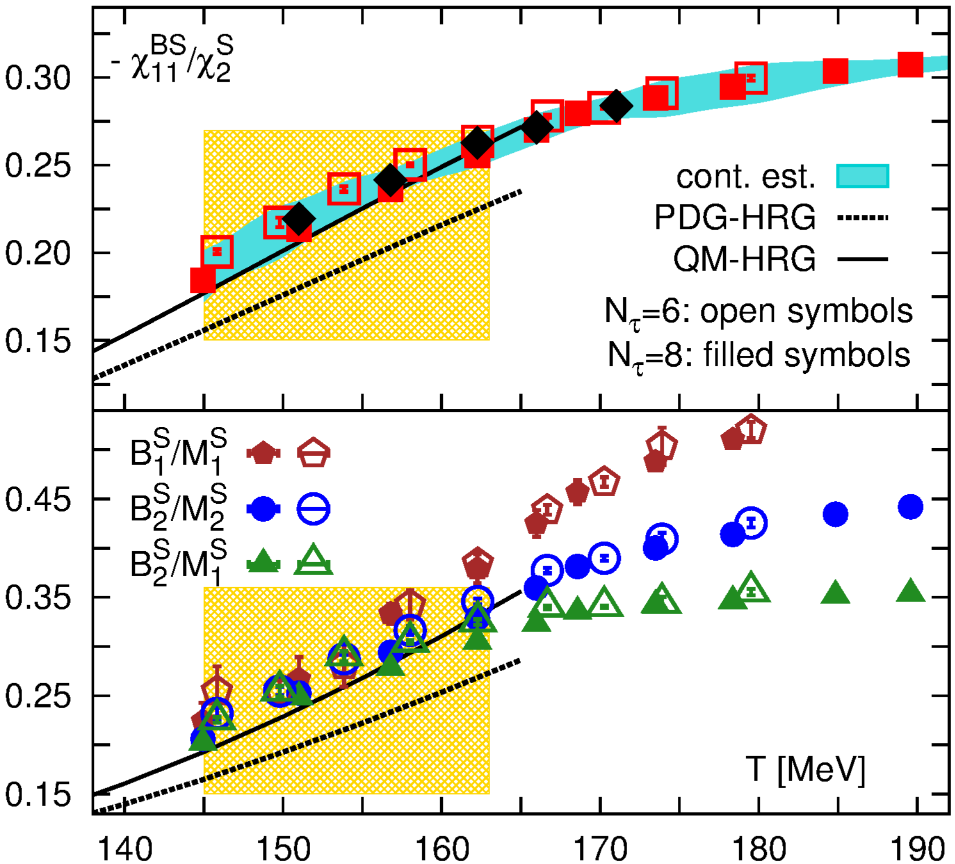
$$M_1^S = \chi_2^S - \chi_{22}^{BS}$$

$$M_2^S = \frac{1}{12}\left(\chi_4^S + 11\chi_2^S\right) + \frac{1}{2}\left(\chi_{11}^{BS} + \chi_{13}^{BS}\right)$$

for strange baryons (P_B^S in HRG):

$$B_1^S = -\frac{1}{6}\left(11\chi_{11}^{BS} + 6\chi_{22}^{BS} + \chi_{13}^{BS}\right)$$

$$B_2^S = \frac{1}{12}\left(\chi_4^S - \chi_2^S\right) - \frac{1}{3}\left(4\chi_{11}^{BS} - \chi_{13}^{BS}\right)$$



all give identical results in a gas of uncorrelated hadrons

yield widely different results when the degrees of freedom are quarks

→ QM-HRG model calculations are in good agreement with LQCD up to the chiral crossover region

→ evidence for the existence of additional strange baryons

and their thermodynamic importance below the QCD crossover

Implications for strangeness freeze-out

initial nuclei in a heavy ion collision are net strangeness free + iso-spin asymmetry

$$\langle n_Q \rangle = r \langle n_B \rangle$$

→ the HRG at the chemical freeze-out must also be strangeness neutral

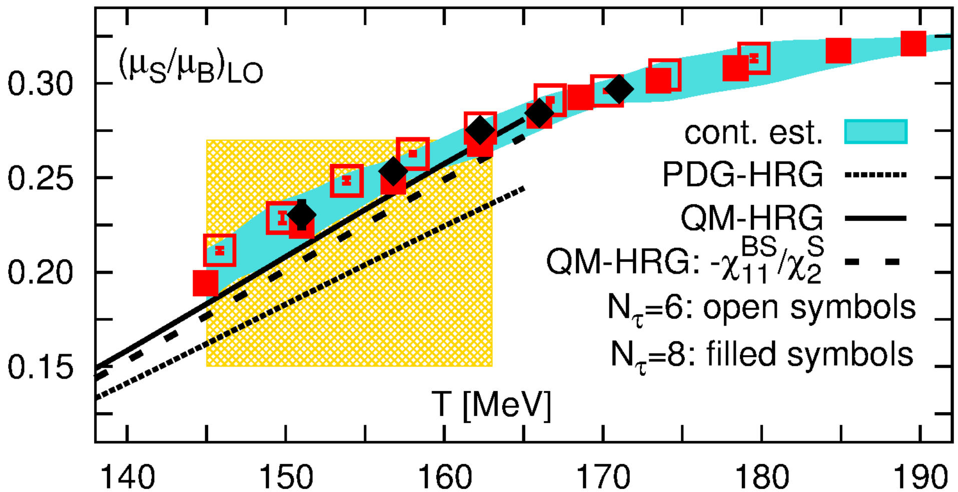
→ thermal parameters T, μ_B and μ_S are related

$$\frac{\mu_S}{\mu_B} = s_1(T) + s_3(T) \left(\frac{\mu_B}{T}\right)^2 + \mathcal{O}(\mu_B^4)$$

small for $\mu_B \lesssim 200 MeV$

$$\left(\frac{\mu_S}{\mu_B}\right)_{\rm LO} \equiv s_1(T) = -\frac{\chi_{11}^{BS}}{\chi_2^S} - \frac{\chi_{11}^{QS}}{\chi_2^S}\frac{\mu_Q}{\mu_B}$$

small correction from nonzero electric charge chemical potential



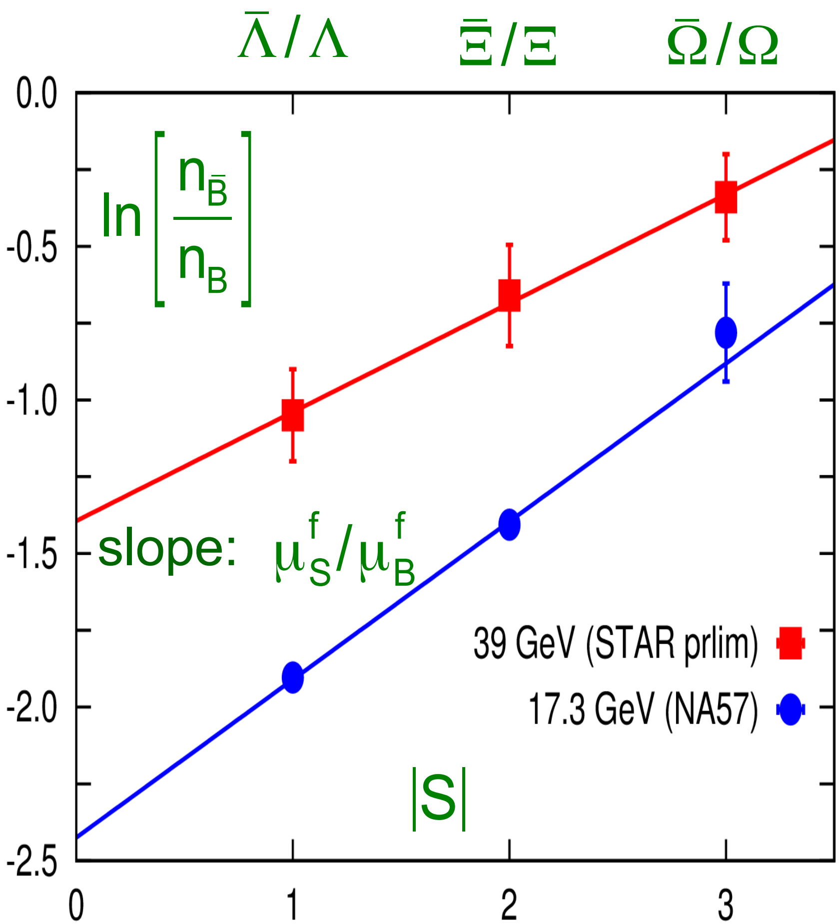
Lattice QCD results well reproduced by QM-HRG in the crossover region

for a given μ_S/μ_B QM-HRG would give a smaller temperature compared to PDG-HRG

Implications for strangeness freeze-out

relative yields of strange anti-baryons ($\overline{H}_S$) to baryons (H_S) can be used to determine freeze-out parameters μ_B^f/T^f and μ_S^f/μ_B^f from experiment

$$R_H \equiv \frac{\overline{H}_S}{H_S} = \mathrm{e}^{-2(\mu_B^f/T^f)\left(1-(\mu_S^f/\mu_B^f)|S|\right)}$$



only assumes that hadron yields are thermal

compare results for μ_B/T and μ_S/μ_B to Lattice QCD

to obtain **freeze-out *T***

Implications for strangeness freeze-out

relative yields of strange anti-baryons ($\overline{H}_S$) to baryons (H_S) can be used to determine freeze-out parameters μ_B^f/T^f and μ_S^f/μ_B^f from experiment

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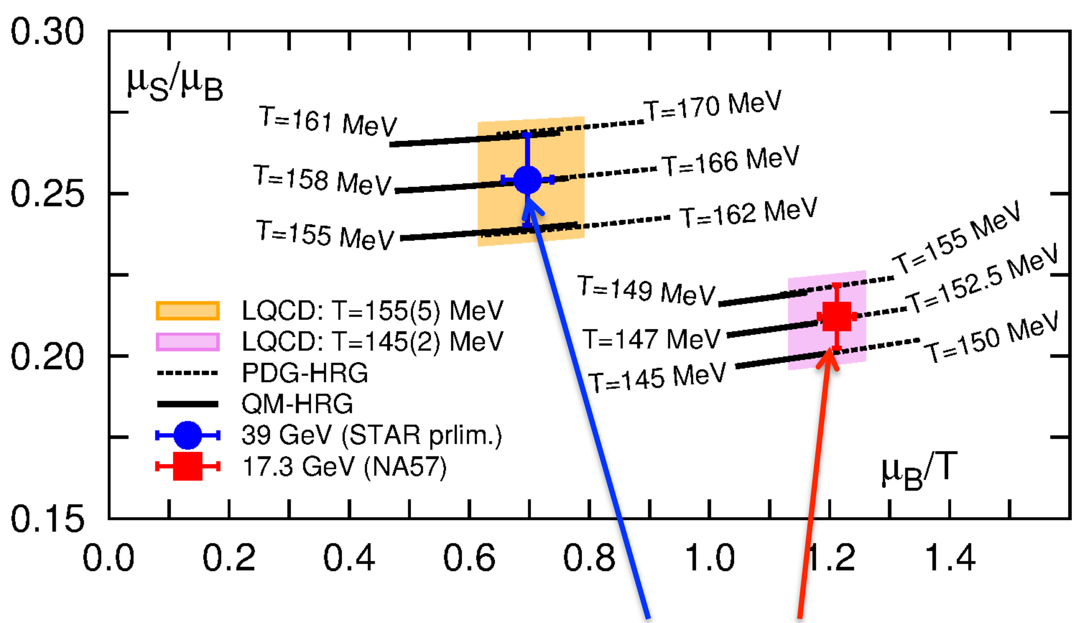
and compared to Lattice QCD or HRG to determine freeze-out temperature T^f:



Experiment STAR/NA57

Implications for strangeness freeze-out

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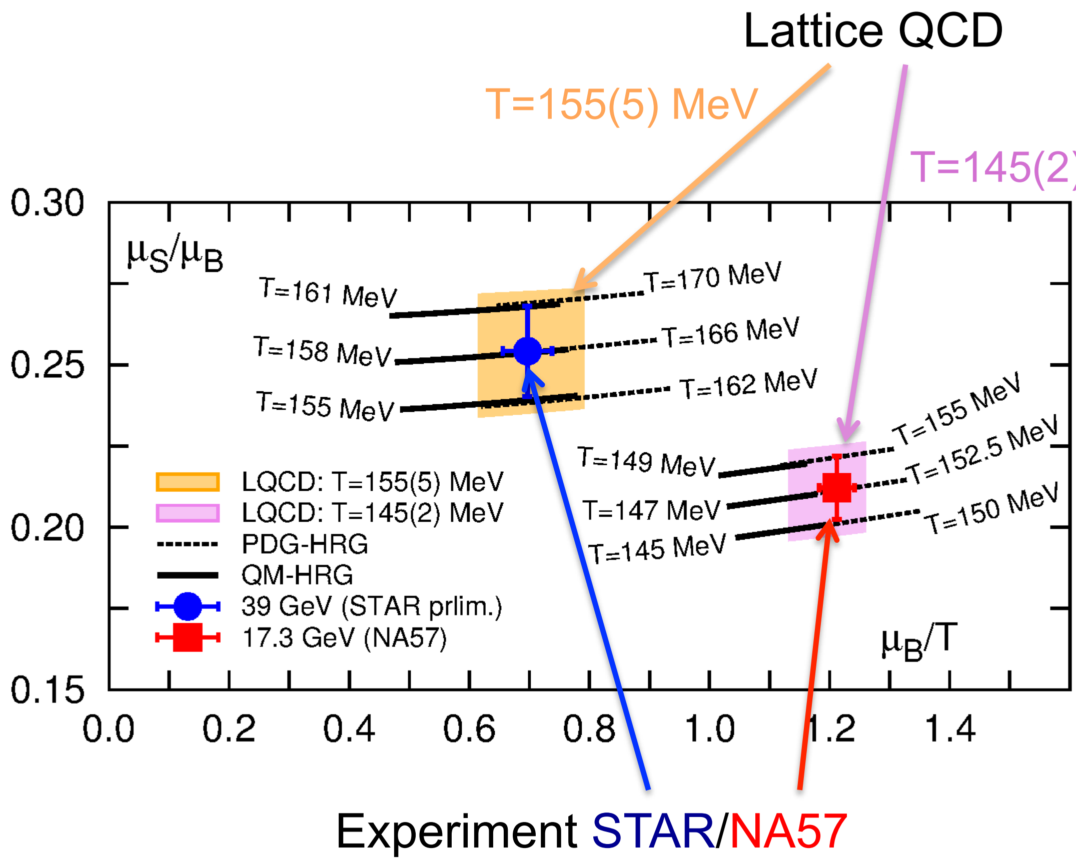


freeze-out temperature obtained from a comparison of

experimental data

and

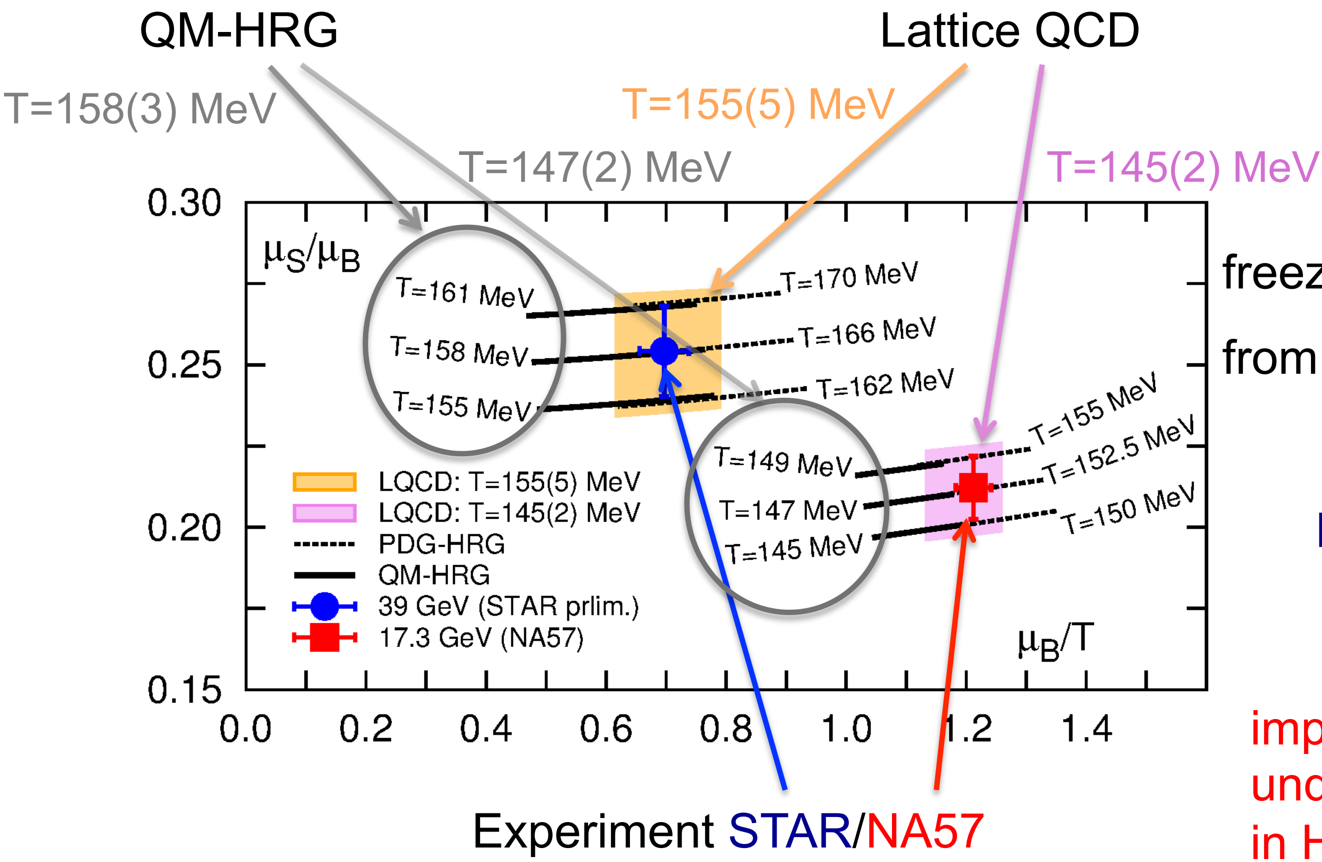
Lattice QCD results

Implications for strangeness freeze-out

relative yields of strange anti-baryons ($\overline{H}_S$) to baryons (H_S) can be used to determine freeze-out parameters μ_B^f/T^f and μ_S^f/μ_B^f from experiment

$$R_H \equiv \frac{\overline{H}_S}{H_S} = \mathrm{e}^{-2(\mu_B^f/T^f)\left(1-(\mu_S^f/\mu_B^f)|S|\right)}$$

and compared to Lattice QCD or HRG to determine freeze-out temperature T^f:



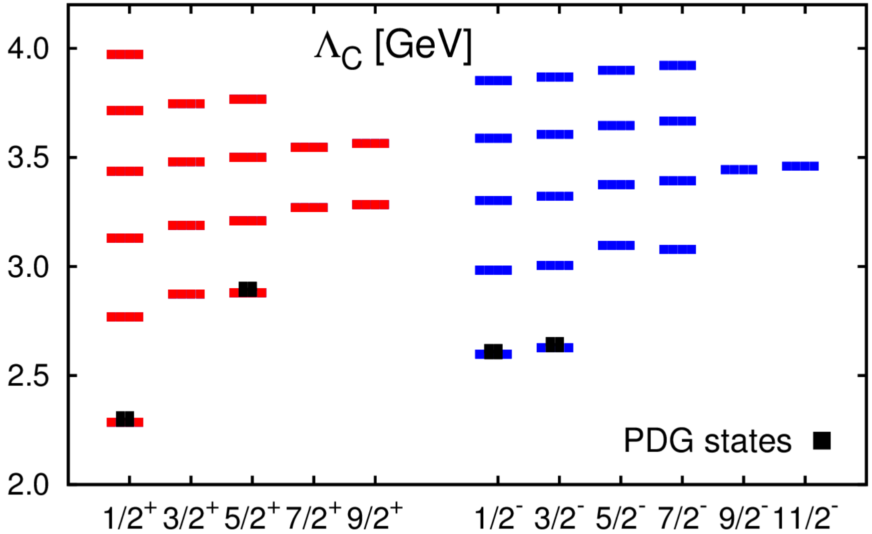
freeze-out temperature obtained from QM-HRG agrees well with Lattice QCD results

important to include so-far undiscovered strange baryons in HRG

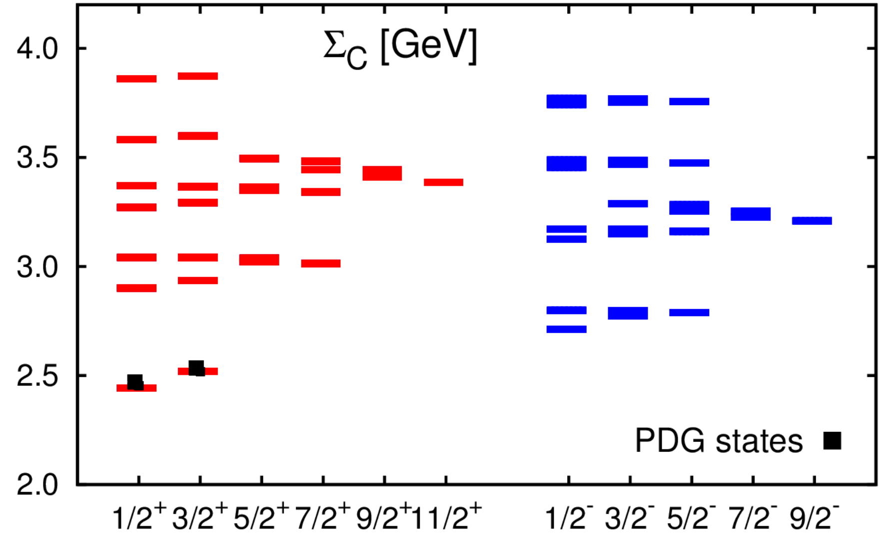
What do we know of the hadron spectrum?

charm baryons

Quark Model



Quark Model



[Capstick-Isgur, Phys.Rev.D34 (1986) 2809]

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QM-3 all resonances up to 3.0 GeV

QM-3.5 all resonances up to 3.5 GeV

What do we know of the hadron spectrum?

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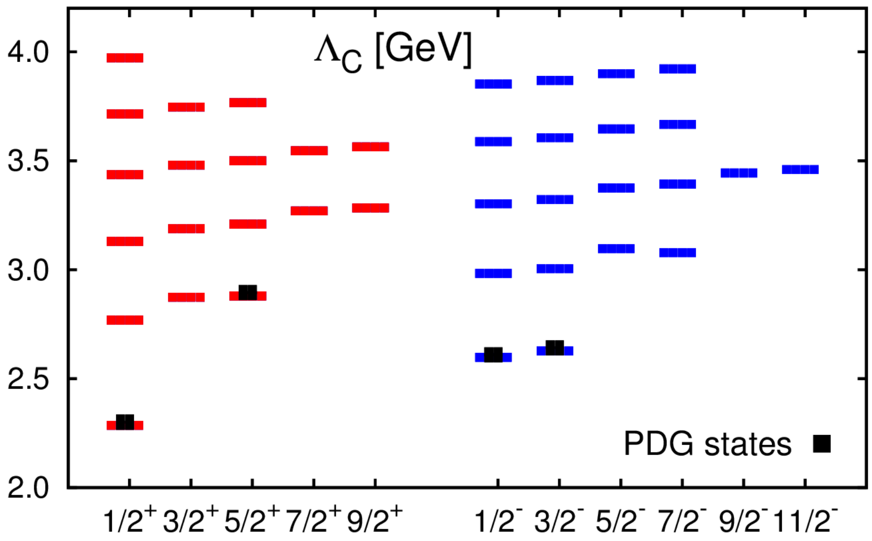
Quark Model



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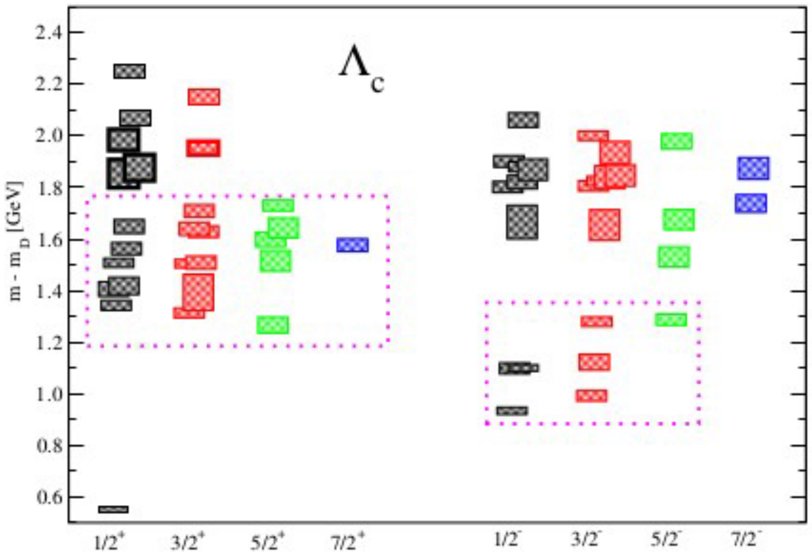
Lattice QCD



[Padmanath et al., arXiv 1311.4806]

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QM-3 all resonances up to 3.0 GeV

QM-3.5 all resonances up to 3.5 GeV

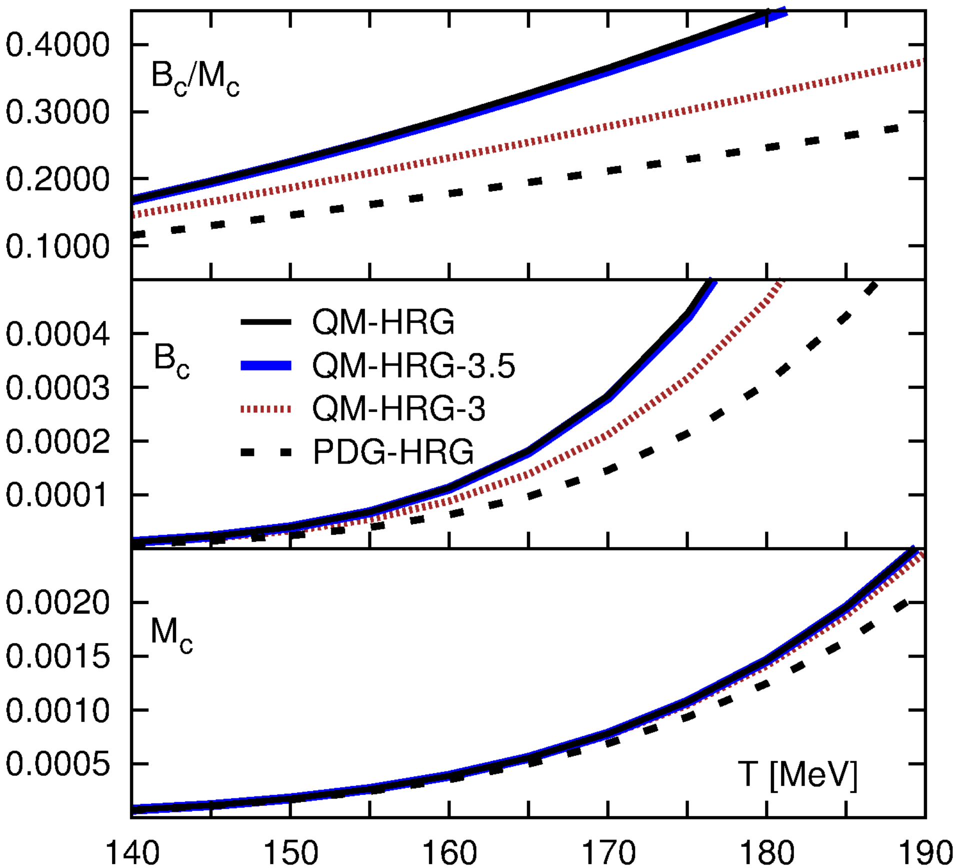
Hadron Resonance Gas - contributions of additional states - charm

partial pressure P of all open charm hadrons

can be separated into mesonic P_M and baryonic P_B components

$$P_{\mathrm{tot}}^{C,X} = P_M^{C,X} + P_B^{C,X}$$

$$P_{M/B}^{C,X}(T,\vec{\mu}) = \frac{T^4}{2\pi^2} \sum_{i\in X} g_i \left(\frac{m_i}{T}\right)^2 K_2(m_i/T) \times \cosh\left(B_i\hat{\mu}_B + Q_i\hat{\mu}_Q + S_i\hat{\mu}_S + C_i\hat{\mu}_C\right)$$



$$X = \begin{cases} \text{QM resonances} \\ \text{QM-3.5 resonances up to 3.5 GeV} \\ \text{QM-3 resonances up to 3.0 GeV} \\ \text{PDG resonances} \end{cases}$$

Correlations of conserved charges – open charm sector

Taylor expansion of pressure in terms of chemical potentials related to conserved charges

$$\frac{P}{T^4} = \sum_{k,l,m,n=0}^{\infty} \frac{1}{k!l!m!n!} \chi_{klmn}^{BQSC}(T) \left(\frac{\mu_B}{T}\right)^k \left(\frac{\mu_Q}{T}\right)^l \left(\frac{\mu_S}{T}\right)^m \left(\frac{\mu_C}{T}\right)^n$$

generalized susceptibilities of conserved charges

$$\chi_{klmn}^{BQSC} = \left.\frac{\partial^{(k+l+m+n)}[P(\hat{\mu}_B, \hat{\mu}_Q, \hat{\mu}_S, \hat{\mu}_C)/T^4]}{\partial \hat{\mu}_B^k \partial \hat{\mu}_Q^l \hat{\mu}_S^m \partial \hat{\mu}_C^n}\right|_{\vec{\mu}=0}$$

are sensitive to the underlying degrees of freedom

charm contributions to pressure in a hadron gas:

$$P^C = P_M^C \cosh(\hat{\mu}_C) + \sum_{k=1,2,3} P_B^{C=k} \cosh(B\hat{\mu}_B + k\hat{\mu}_C)$$

partial pressure of open-charm mesons and charmed baryons depends on hadron spectra

$$\chi_{mn}^{BC} = B^m P_B^{C=1} + B^m 2^n P_B^{C=2} + B^m 3^n P_B^{C=3} \simeq B^m P_B^{C=1}$$

relative contribution of C=2 and C=3 baryons negligible

ratios independent of the detailed spectrum and sensitive to special sectors:

charmed baryon sector

$$\frac{\chi_{mn}^{BC}}{\chi_{m+1,n-1}^{BC}} = B^{-1}$$

=1 when DoF are hadronic

=3 when DoF are quarks

$$\frac{\chi_{mn}^{BC}}{\chi_{m,n+2}^{BC}} = 1$$ always

Correlations of conserved charges – open charm sector

[A.Bazavov, H.T.Ding, P.Hegde, OK et al., PLB737 (2014) 210]

2+1 flavor HISQ with almost physical quark masses

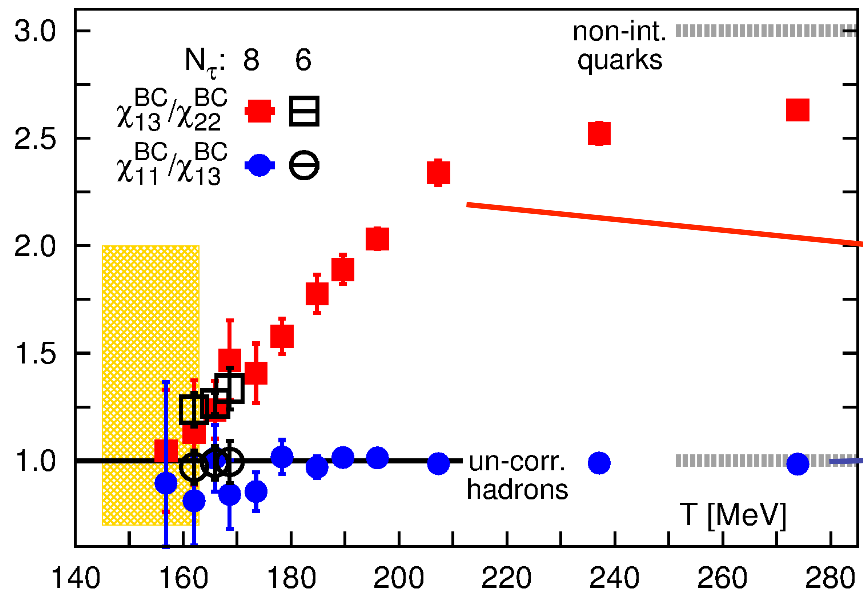
$32^3 \times 8$ and $24^3 \times 6$ lattices with $m_l = m_s/20$ and physical m_s and quenched charm quarks

generalized susceptibilities of conserved charges

$$\chi_{klmn}^{BQSC} = \left.\frac{\partial^{(k+l+m+n)}[P(\hat{\mu}_B, \hat{\mu}_Q, \hat{\mu}_S, \hat{\mu}_C)/T^4]}{\partial\hat{\mu}_B^k \partial\hat{\mu}_Q^l \hat{\mu}_S^m \partial\hat{\mu}_C^n}\right|_{\vec{\mu}=0}$$

are sensitive to the underlying degrees of freedom

charmed baryon sector



$$\frac{\chi_{mn}^{BC}}{\chi_{m+1,n-1}^{BC}} = B^{-1}$$

$$\frac{\chi_{mn}^{BC}}{\chi_{m,n+2}^{BC}} = 1$$

→ indications that charmed baryons start to dissolve already close to the chiral crossover

Correlations of conserved charges – open charm sector

Taylor expansion of pressure in terms of chemical potentials related to conserved charges

$$\frac{P}{T^4} = \sum_{k,l,m,n=0}^{\infty} \frac{1}{k!l!m!n!} \chi_{klmn}^{BQSC}(T) \left(\frac{\mu_B}{T}\right)^k \left(\frac{\mu_Q}{T}\right)^l \left(\frac{\mu_S}{T}\right)^m \left(\frac{\mu_C}{T}\right)^n$$

generalized susceptibilities of conserved charges

$$\chi_{klmn}^{BQSC} = \left. \frac{\partial^{(k+l+m+n)}[P(\hat{\mu}_B, \hat{\mu}_Q, \hat{\mu}_S, \hat{\mu}_C)/T^4]}{\partial \hat{\mu}_B^k \partial \hat{\mu}_Q^l \hat{\mu}_S^m \partial \hat{\mu}_C^n} \right|_{\vec{\mu}=0}$$

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charm contributions to pressure in a hadron gas:

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partial pressure of open-charm mesons and charmed baryons depends on hadron spectra

$$\chi_{mn}^{B=1,C} = P_B^{C=1} + 2^n P_B^{C=2} + 3^n P_B^{C=3} \simeq P_B^{C=1}$$

$$\chi_k^C = P_M^C + 2^n P_B^{C=2} + 3^n P_B^{C=3} \simeq P_M^C + P_B^{C=1}$$

ratios independent of the detailed spectrum and sensitive to special sectors:

partial pressure of open-charm mesons:

open charm meson sector

$$P_M^C = \chi_2^C - \chi_{22}^{BC} = \chi_4^C - \chi_{13}^{BC}$$

$$\frac{\chi_4^C}{\chi_2^C} = 1$$

Correlations of conserved charges – open charm sector

[A.Bazavov, H.T.Ding, P.Hegde, OK et al., PLB737 (2014) 210]

2+1 flavor HISQ with almost physical quark masses

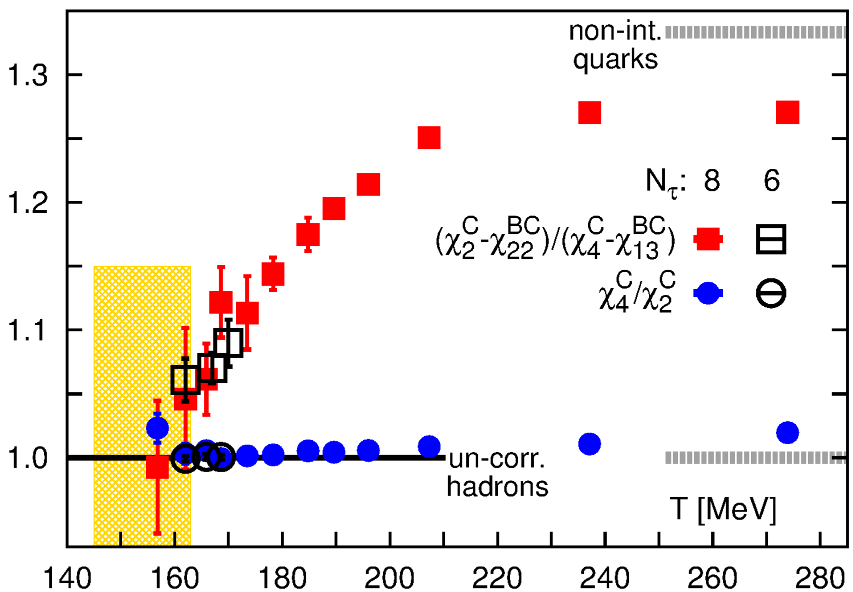
$32^3\times 8$ and $24^3\times$ 6 lattices with $m_l = m_s/20$ and physical m_s and quenched charm quarks

generalized susceptibilities of conserved charges

$$\chi_{klmn}^{BQSC} = \left.\frac{\partial^{(k+l+m+n)}[P(\hat{\mu}_B, \hat{\mu}_Q, \hat{\mu}_S, \hat{\mu}_C)/T^4]}{\partial\hat{\mu}_B^k \partial\hat{\mu}_Q^l \hat{\mu}_S^m \partial\hat{\mu}_C^n}\right|_{\vec{\mu}=0}$$

are sensitive to the underlying degrees of freedom

open charm meson sector



partial pressure of open-charm mesons:

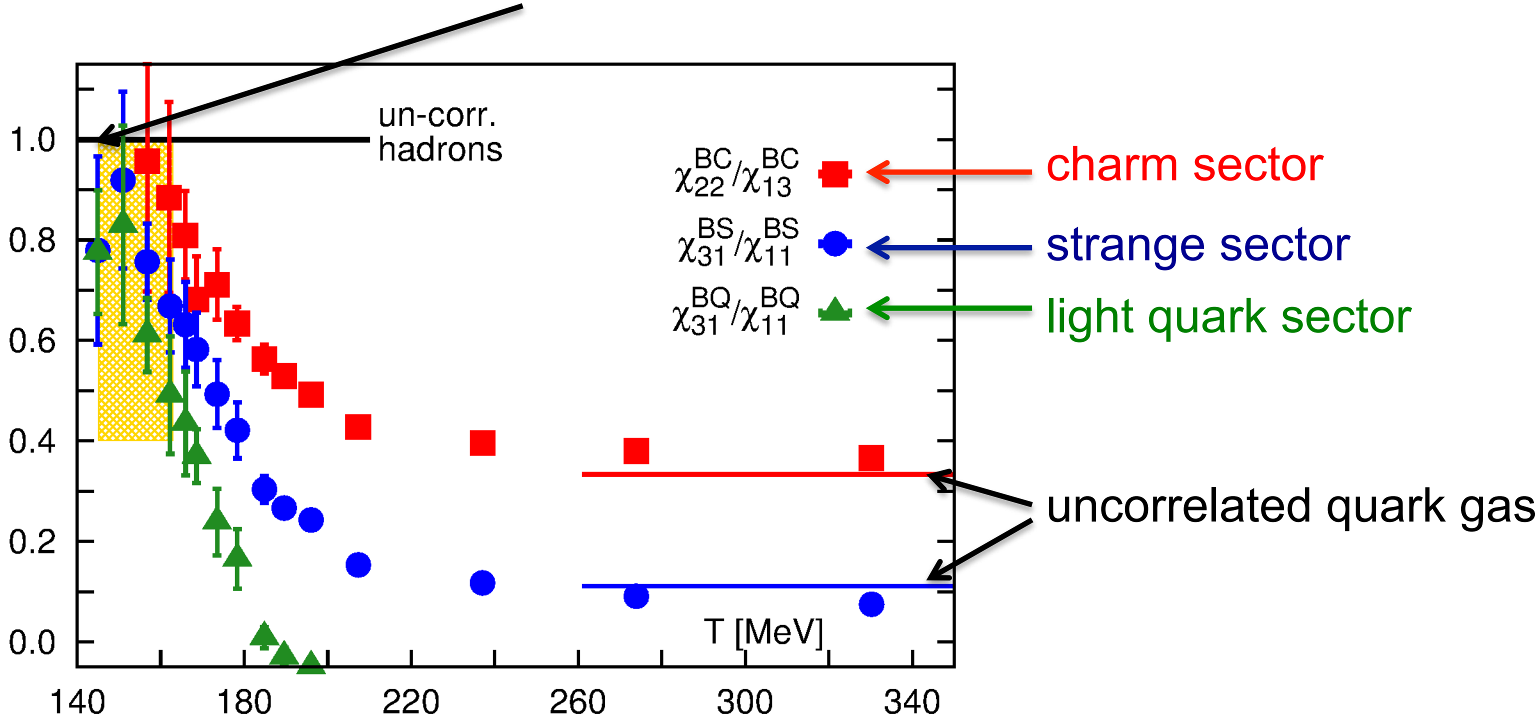
$$P_M^C = \chi_2^C - \chi_{22}^{BC} = \chi_4^C - \chi_{13}^{BC}$$

$$\frac{\chi_4^C}{\chi_2^C} = 1$$

→ **indications that open charm mesons start to dissolve already close to the chiral crossover**

Signatures for deconfinement of light/strange/charm baryons

ratios of BC, BS, and BQ correlations
unity in a gas of uncorrelated hadrons



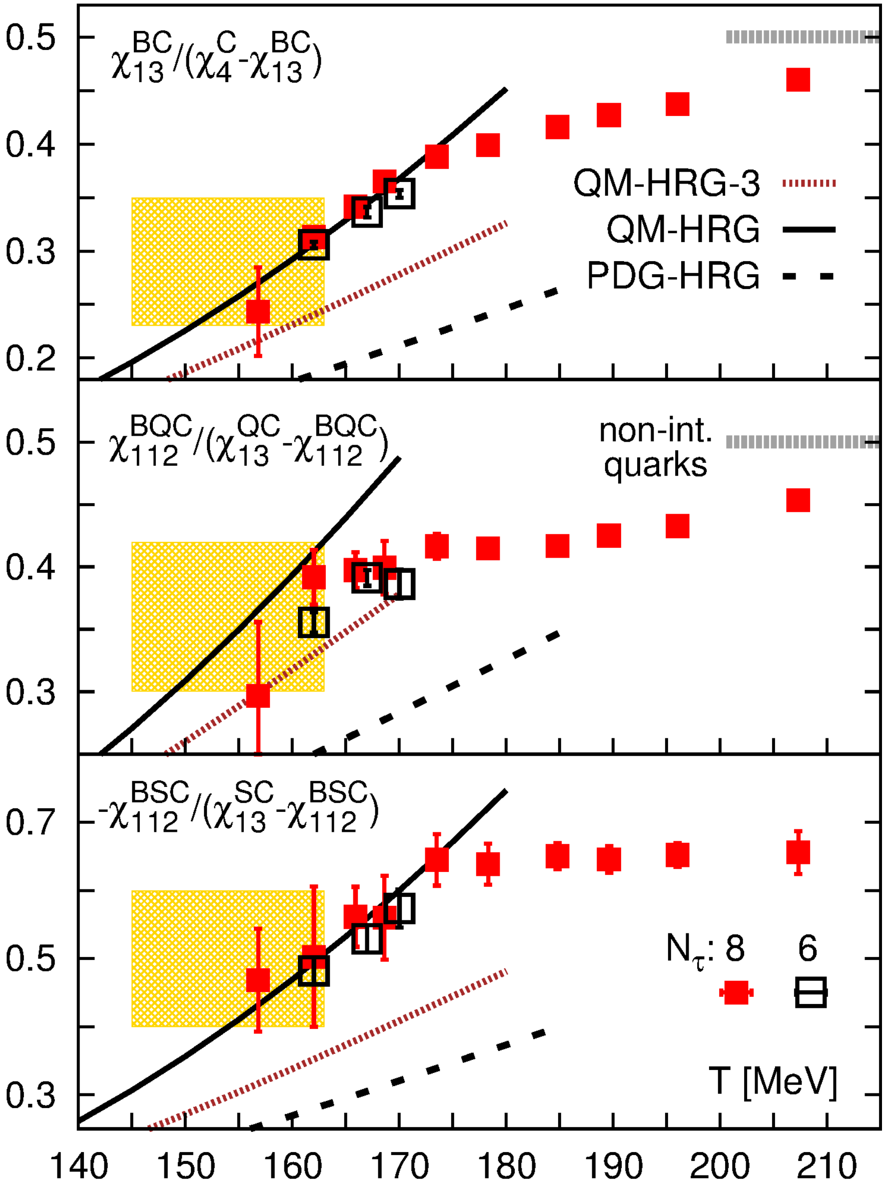
A. Bazavov et al., Phys.Lett.B 737 (2014) 210]

→ **charmed hadrons start to deconfine around the chiral crossover region**

→ **strange hadrons start to deconfine around the chiral crossover region**

Signatures for additional charm baryons

charmed pressure ratios are sensitive to the charm hadron spectrum



A. Bazavov et al., Phys.Lett.B 737 (2014) 210]

charmed baryon to meson ratio

$$R_{13}^{BC} = \frac{\chi_{13}^{BC}}{M_C} = \frac{B_C}{M_C}$$

$$M_C \simeq \chi_4^C - \chi_{13}^{BC}$$

charged charmed baryon to meson ratio

$$R_{13}^{QC} = \frac{\chi_{112}^{BQC}}{M_{QC}}$$

$$M_{QC} \simeq \chi_{13}^{QC} - \chi_{112}^{BQC}$$

strange charmed baryon to meson ratio

$$R_{13}^{SC} = -\frac{\chi_{112}^{BSC}}{M_{SC}}$$

$$M_{SC} \simeq \chi_{13}^{SC} - \chi_{112}^{BSC}$$

→ important to include so-far undiscovered open charm hadrons in HRG

Spatial correlation function and screening masses

[“Signatures of charmonium modification in spatial correlation functions“, F.Karsch, E.Laermann, S.Mukherjee, P.Petreczky, (2012) arXiv:1203.3770]

Correlation functions along the **spatial direction**

$$G(z,T) = \int dxdy \int_0^{1/T} d\tau \langle J(x,y,z,\tau) J(0,0,0,0) \rangle$$

are related to the meson spectral function at **non-zero spatial momentum**

$$G(z,T) = \int_{-\infty}^{\infty} dp_z e^{ip_z z} \int_0^{\infty} d\omega \frac{\sigma(\omega, p_z, T)}{\omega}$$

exponential decay defines **screening mass M_{scr} :** $\mathbf{G(z,T)} \underset{z \gg 1/T}{\longrightarrow} \mathbf{e^{-M_{scr} z}}$

bound state contribution

$$\sigma(\omega, p_z, T) \sim \delta(\omega^2 - p_z^2 - M^2)$$

high-T limit (non-interacting free limit)

$$\sigma(\omega, p_z, T) \sim \sigma_{free}(\omega, p_z, T)$$

$\boxed{M_{scr} = M}$ $\longrightarrow$ indications for medium modifications/dissociation $\longleftarrow$ $\boxed{M_{scr} = 2\sqrt{(\pi T)^2 + {m_c}^2}}$

Spatial correlation functions and screening masses

[A.Bazavov, F.Karsch, Y.Maezawa et al., PRD91 (2015) 054503]

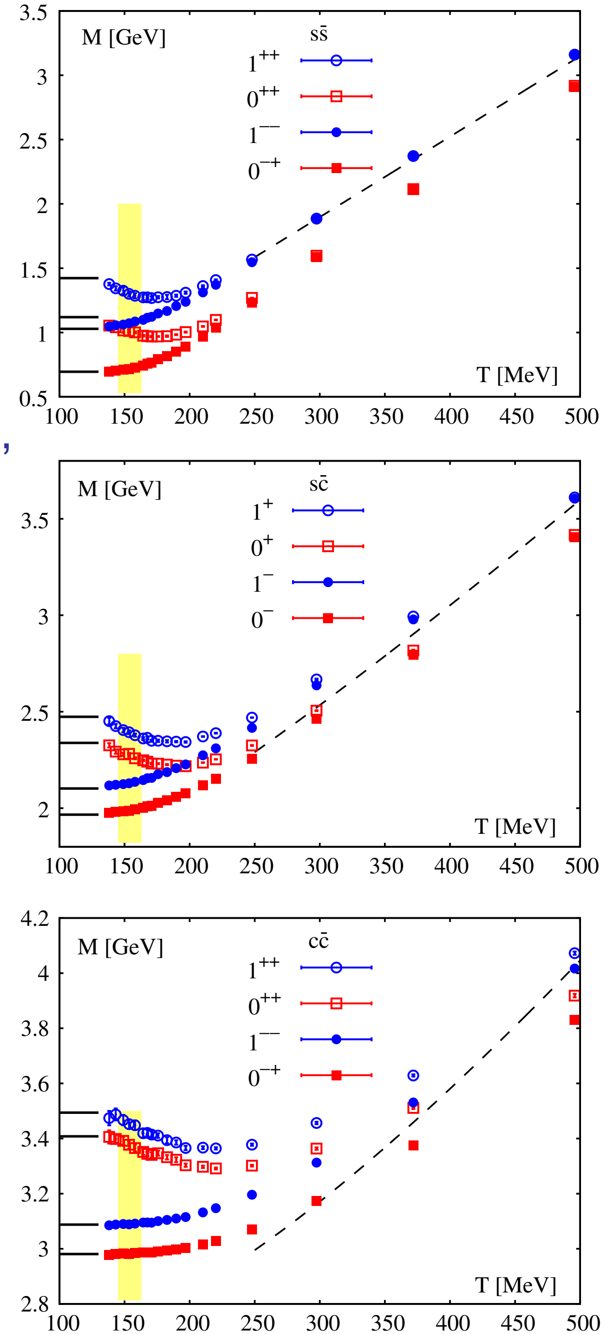
2+1 flavor HISQ with almost physical quark masses

$48^3 \times 12$ lattices with $m_l = m_s/20$ and physical m_s

"$s\bar{s}$ and $s\bar{c}$ possibly dissolve close to crossover temperature"

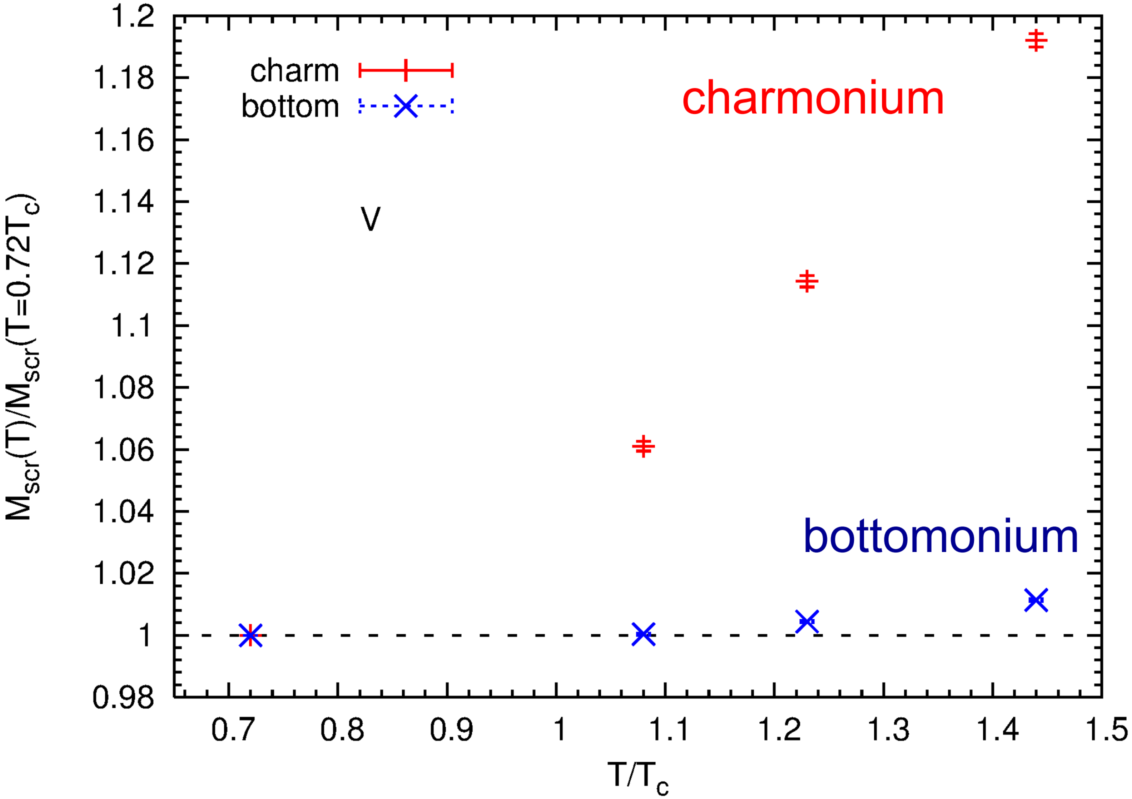
"$c\bar{c}$ in line with the sequential melting of charmonium states"

	$-\tilde{\phi}(x)$	Γ	J^{PC}	$s\bar{s}$	$s\bar{c}$	$c\bar{c}$
M_-^{S}	1	$\gamma_4\gamma_5$	0^{-+}	$\eta_{s\bar{s}}$	D_s	η_c
M_+^{S}		1	0^{++}		D_{s0}^*	χ_{c0}
M_-^{PS}	$(-1)^{x+y+z}$	γ_5	0^{-+}	$\eta_{s\bar{s}}$	D_s	η_c
M_+^{PS}		γ_4	0^{+-}	–		–
M_-^{AV}	$(-1)^x$, $(-1)^y$	$\gamma_i\gamma_4$	1^{--}	ϕ	D_s^*	J/ψ
M_+^{AV}		$\gamma_i\gamma_5$	1^{++}	$f_1(1420)$	D_{s1}	χ_{c1}
M_-^{V}	$(-1)^{x+z}$, $(-1)^{y+z}$	γ_i	1^{--}	ϕ	D_s^*	J/ψ
M_+^{V}		$\gamma_j\gamma_k$	1^{+-}			h_c



Spatial Correlation Functions and Screening Masses

[H.T.Ding, H.Ohno, OK, M.Laine, T.Neuhaus, work in progress]



ongoing study in quenched QCD to understand the sequential melting of charmonium and bottomonium states in the QGP

(see last week's lecture notes)

exponential decay defines **screening mass M_{scr}** : $\mathbf{G(z,T)} \underset{z \gg 1/T}{\longrightarrow} \mathbf{e^{-M_{scr} z}}$

bound state contribution

$$\sigma(\omega, p_z, T) \sim \delta(\omega^2 - p_z^2 - M^2)$$

high-T limit (non-interacting free limit)

$$\sigma(\omega, p_z, T) \sim \sigma_{free}(\omega, p_z, T)$$

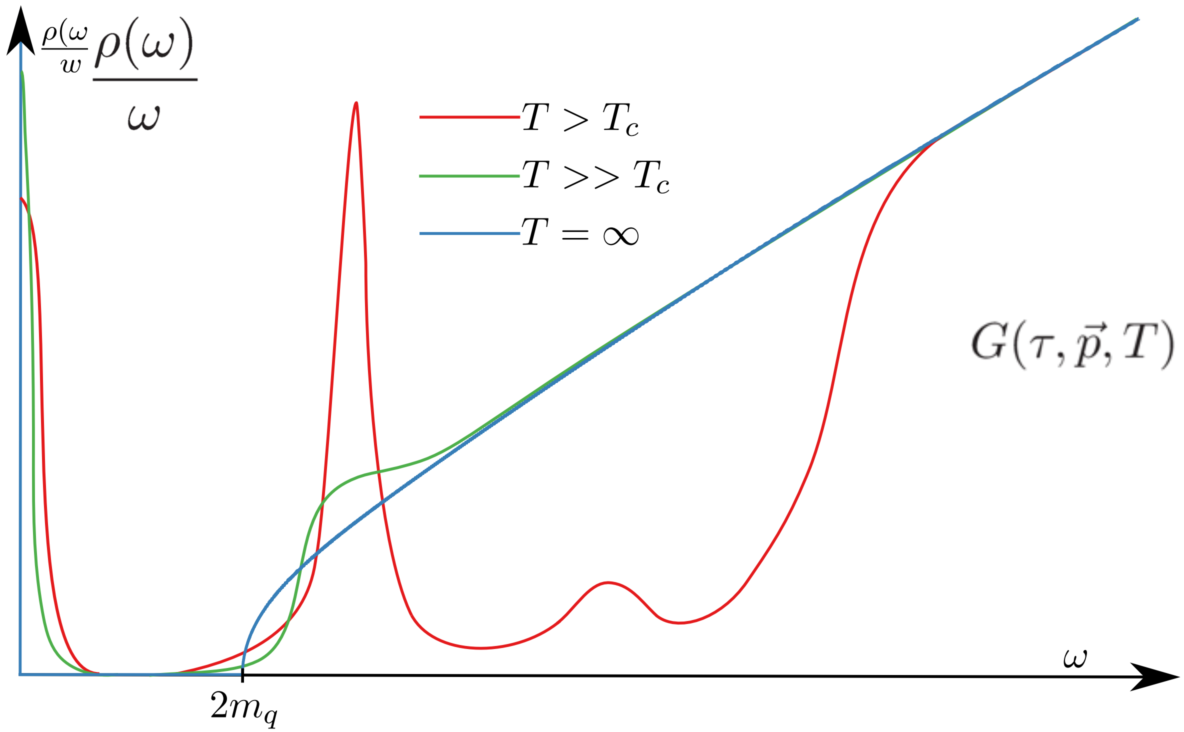
$\boxed{M_{scr} = M}$ $\longrightarrow$ indications for medium modifications/dissociation $\longleftarrow$ $\boxed{M_{scr} = 2\sqrt{(\pi T)^2 + m_c{}^2}}$

Vector spectral function – hard to separate different scales

Different contributions and scales enter in the spectral function

- continuum at large frequencies
- possible bound states at intermediate frequencies
- transport contributions at small frequencies
- in addition cut-off effects on the lattice

notoriously difficult to extract from correlation functions



$$G(\tau,\vec{p},T) = \int_0^\infty \frac{d\omega}{2\pi} \rho(\omega,\vec{p},T) K(\tau,\omega,T)$$

(narrow) transport peak at small ω: $\rho(\omega \ll T) \simeq 2\chi_{00} \frac{T}{M} \frac{\omega\eta}{\omega^2+\eta^2}\,, \qquad \eta = \frac{T}{MD}$

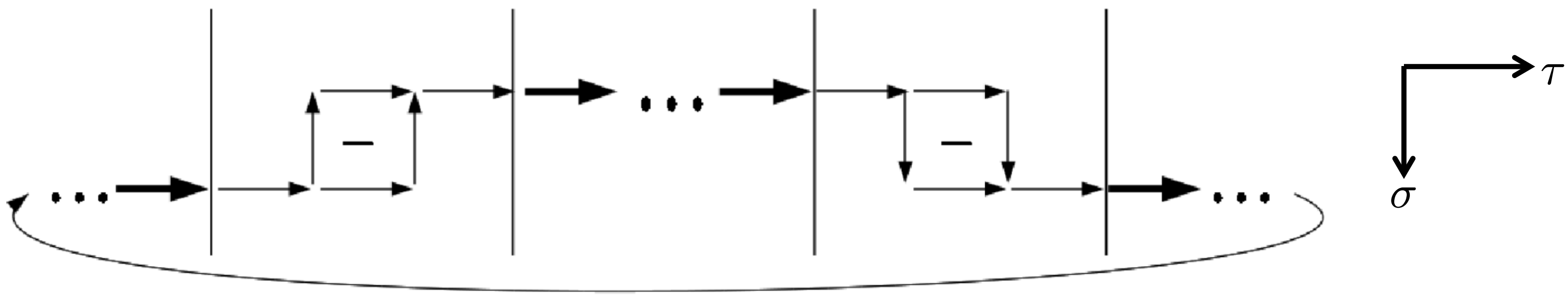
Heavy Quark Momentum Diffusion Constant – Single Quark in the Medium

Heavy Quark Effective Theory (HQET) in the large quark mass limit

for a single quark in medium

leads to a (pure gluonic) "color-electric correlator"

[J.Casalderrey-Solana, D.Teaney, PRD74(2006)085012, S.Caron-Huot,M.Laine,G.D. Moore,JHEP04(2009)053]



$$G_{\mathrm{E}}(\tau) \equiv -\frac{1}{3}\sum_{i=1}^{3}\frac{\left\langle \mathrm{Re\,Tr}\left[U(\tfrac{1}{T};\tau)\, gE_i(\tau,\mathbf{0})\, U(\tau;0)\, gE_i(0,\mathbf{0})\right]\right\rangle}{\left\langle \mathrm{Re\,Tr}[U(\tfrac{1}{T};0)]\right\rangle}$$

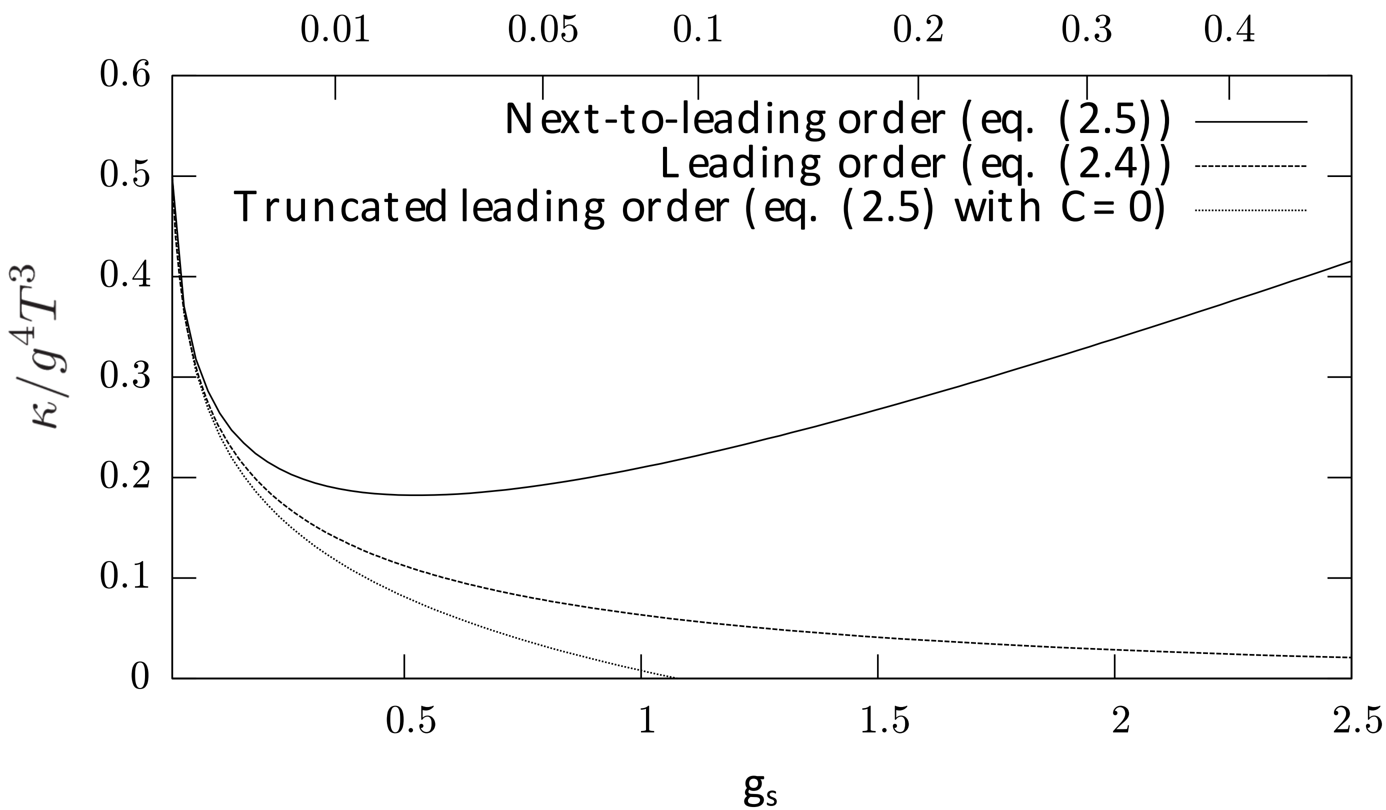
Heavy quark (momentum) diffusion:

$$\kappa = \lim_{\omega\to 0}\frac{2T\rho_{\mathrm{E}}(\omega)}{\omega} \qquad D = \frac{2T^2}{\kappa}$$

Heavy Quark Momentum Diffusion Constant – Perturbation Theory

can be related to the thermalization rate: $$\eta_D = \frac{\kappa}{2M_{kin}T}\left(1 + O\left(\frac{\alpha_s^{3/2}T}{M_{kin}}\right)\right)$$

NLO in perturbation theory: [Caron-Huot, G.Moore, JHEP 0802 (2008) 081]

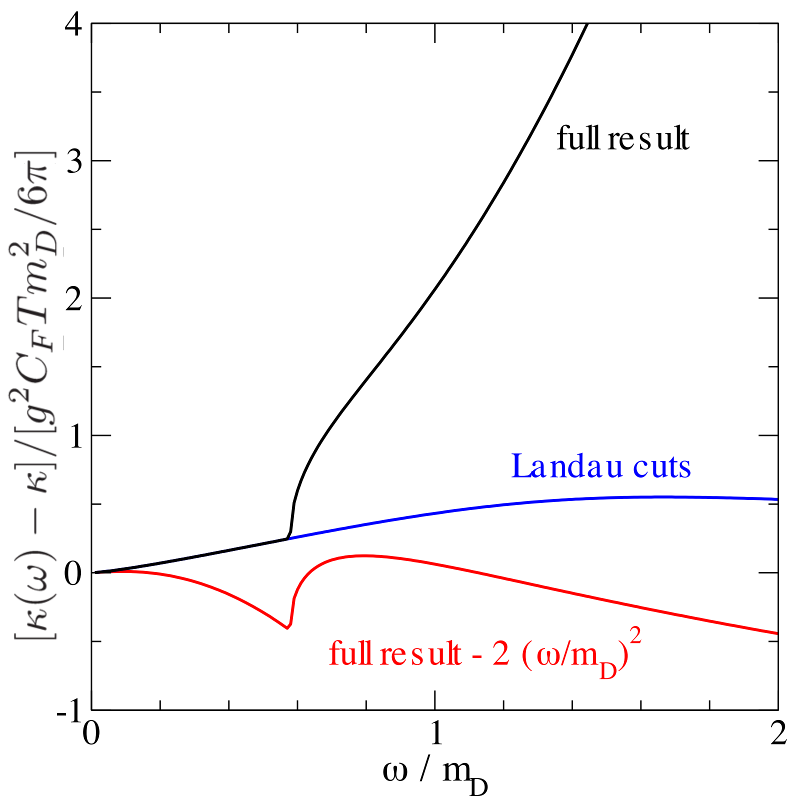


very poor convergence

→ **Lattice QCD study required in the relevant temperature region**

Heavy Quark Momentum Diffusion Constant – Perturbation Theory

NLO spectral function in perturbation theory: [Caron-Huot, M.Laine, G.Moore, JHEP 0904 (2009) 053]



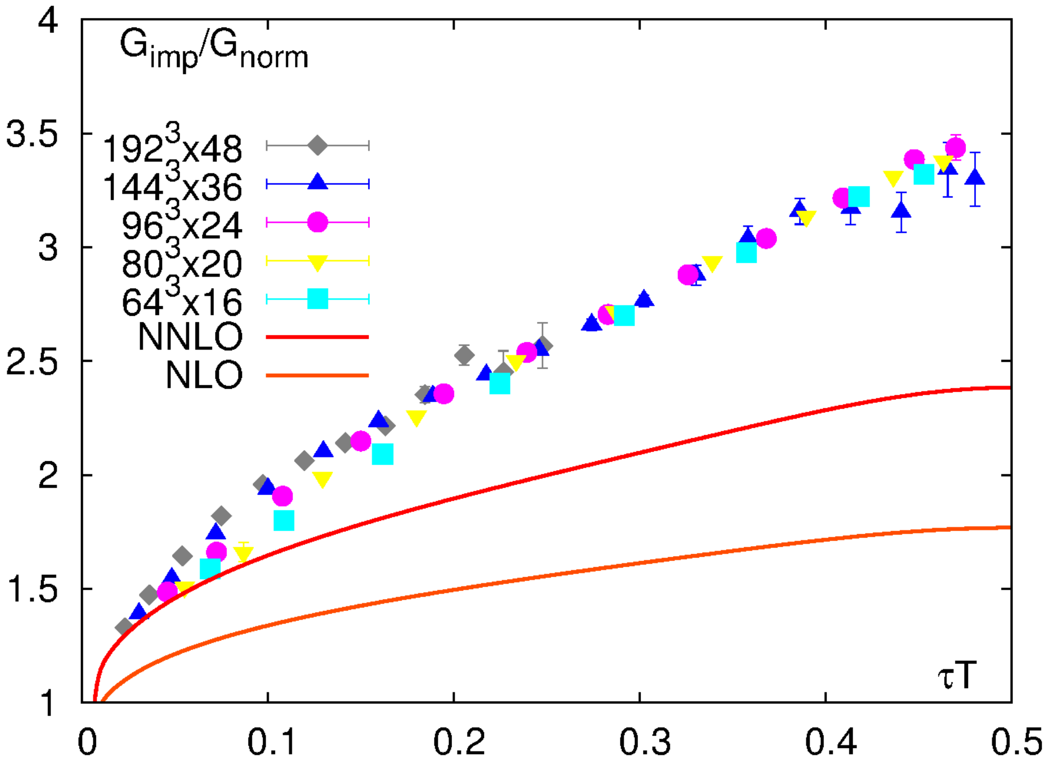
in contrast to a narrow transport peak, from this a smooth limit

$$\kappa/T^3 = \lim_{\omega\to 0} \frac{2T\rho_{\mathrm{E}}(\omega)}{\omega}$$

is expected

qualitatively similar behavior also found in AdS/CFT [S.Gubser, Nucl.Phys.B790 (2008)175]

Heavy Quark Momentum Diffusion Constant – Lattice results



finest lattices still quite noisy at large τT

but only

small cut-off effects at intermediate τT

cut-off effects become visible at small τT

need to extrapolate to the continuum

perturbative behavior in the limit τT $\to$ 0

Quenched Lattice QCD

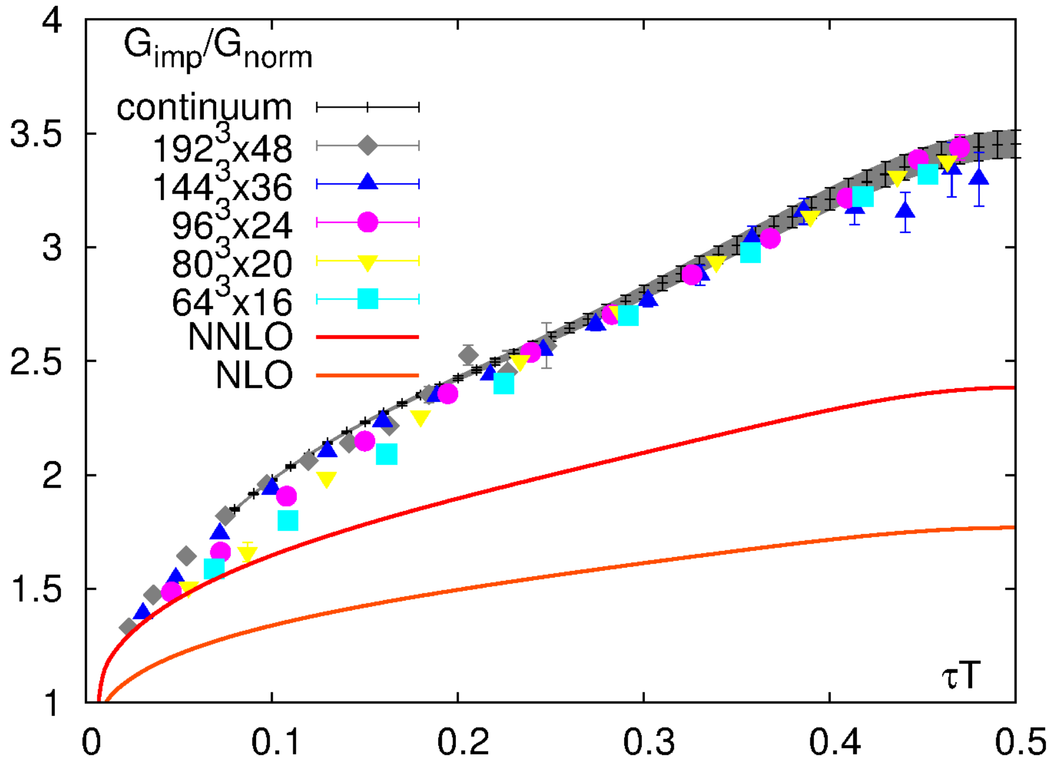
$T \simeq 1.5T_c$

$V \simeq (2\text{fm})^3$

N_σ	N_τ	β	$1/a[\text{GeV}]$	$a[\text{fm}]$	#Confs
64	16	6.872	7.16	0.03	172
80	20	7.035	8.74	0.023	180
96	24	7.192	10.4	0.019	160
144	36	7.544	15.5	0.013	693
192	48	7.793	20.4	0.010	223

allows to perform continuum extrapolation, a$\to$ 0 $\leftrightarrow$ $N_t \to \infty$ **, at fixed** T=1/a N_t

Heavy Quark Momentum Diffusion Constant – Continuum extrapolation



finest lattices still quite noisy at large τT

but only

small cut-off effects at intermediate τT

cut-off effects become visible at small τT

need to extrapolate to the continuum

perturbative behavior in the limit $\tau T \rightarrow 0$

well behaved continuum extrapolation for $0.05 \leq \tau T \leq 0.5$

finest lattice already close to the continuum

coarser lattices at larger τT close to the continuum

how to extract the spectral function from the correlator?

Heavy Quark Momentum Diffusion Constant – IR and UV asymptotics

$\omega \ll$T: linear behavior motivated at small frequencies

$$\rho_{\mathrm{IR}}(\omega) = \frac{\kappa\omega}{2T}$$

$\omega \gg$T: vacuum perturbative results and leading order thermal correction:

$$\rho_{\mathrm{UV}}(\omega) = \left[\rho_{\mathrm{UV}}(\omega)\right]_{T=0} + \mathcal{O}\left(\frac{g^4 T^4}{\omega}\right)$$

using a renormalization scale $\bar{\mu}_\omega = \omega$ for $\omega \gg \Lambda_{\overline{MS}}$ leading order becomes

$$\rho_{\mathrm{UV}}(\omega) = \Phi_{UV}(\omega)\left[1 + \mathcal{O}\left(\frac{1}{\ln(\omega/\Lambda_{\overline{MS}})}\right)\right]$$

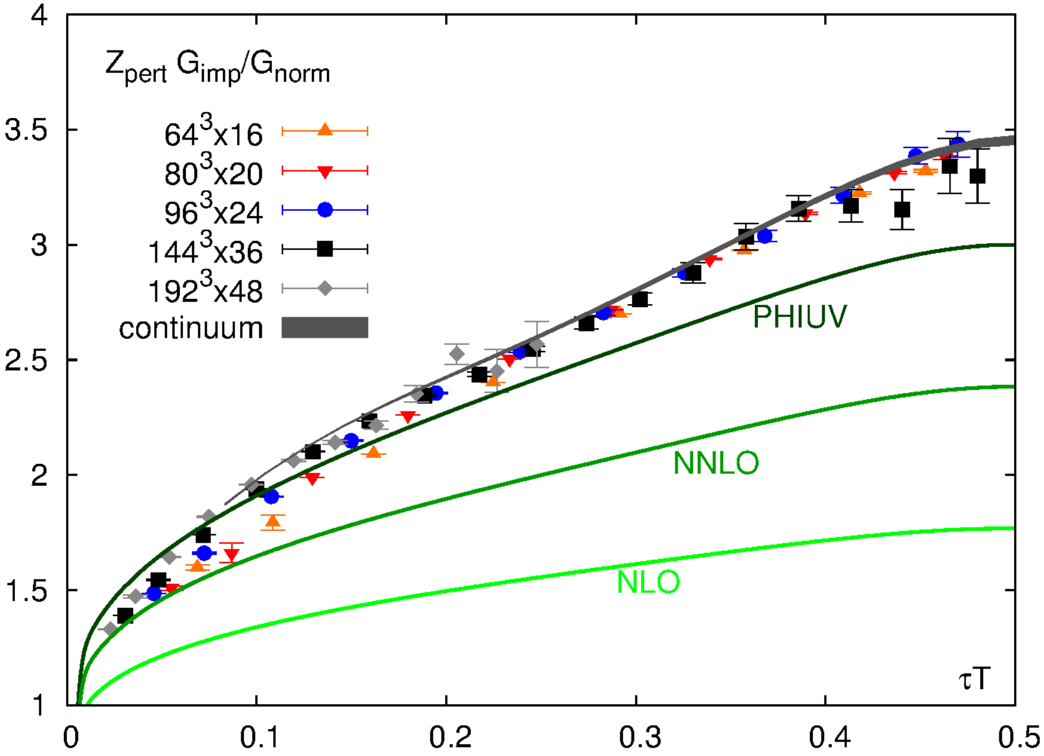
$$\Phi_{\mathrm{UV}}(\omega) = \frac{g^2(\bar{\mu}_\omega) C_F \omega^3}{6\pi} \quad , \quad \bar{\mu}_\omega \equiv \max(\omega, \pi T)$$

here we used 4-loop running of the coupling

model the spectral function using these asymptotics with two free parameters

$$\rho_{\mathrm{model}}(\omega) \equiv \max\left\{A\Phi_{\mathrm{UV}}(\omega), \frac{\omega\kappa}{2T}\right\}$$

Heavy Quark Momentum Diffusion Constant – Model Spectral Function



including thermal corrections

$$\rho_{\rm UV}(\omega) = \frac{g^2(\bar{\mu}_\omega) C_F \omega^3}{6\pi}$$

already closer to the data

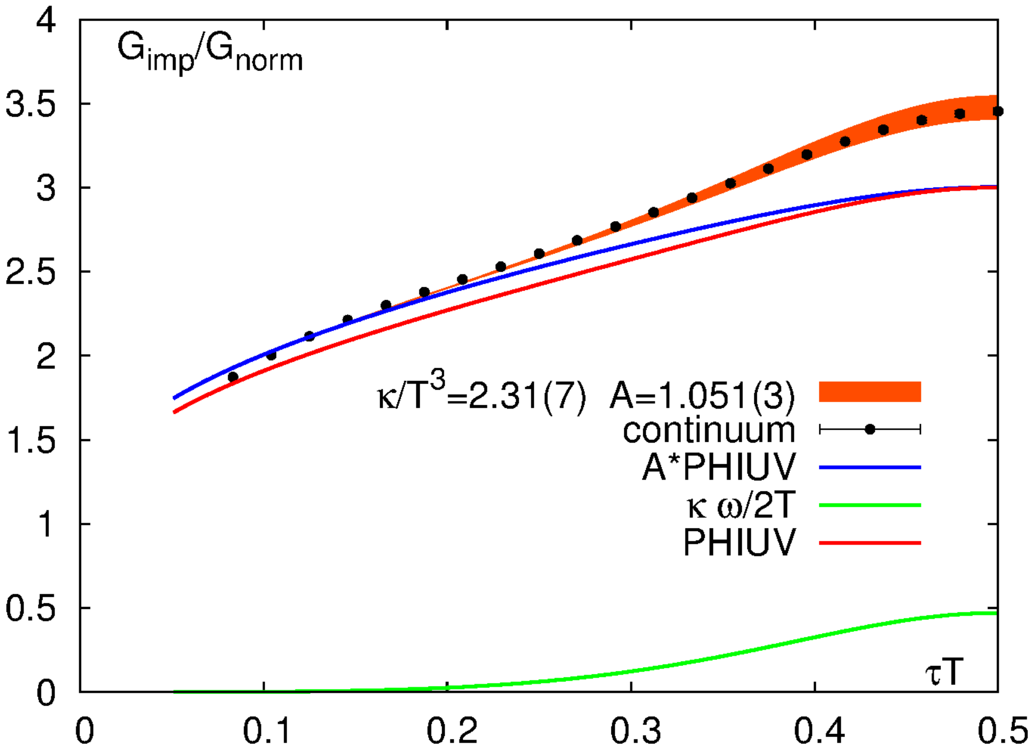
but contributions from the transport visible at large separations

Model spectral function: transport contribution + UV-asymtotics

$$\rho_{\rm model}(\omega) \equiv \max\left\{ A\rho_{\rm UV}(\omega), \frac{\omega\kappa}{2T} \right\}$$

$$G_{\rm model}(\tau) \equiv \int_0^\infty \frac{{\rm d}\omega}{\pi} \rho_{\rm model}(\omega) \frac{\cosh\left(\frac{1}{2} - \tau T\right)\frac{\omega}{T}}{\sinh\frac{\omega}{2T}}$$

Heavy Quark Momentum Diffusion Constant – Model Spectral Function



result of the fit to $\rho_{model}(\omega)$

A $\rho_{UV}(\omega)$

$\frac{\omega\kappa}{2T}$ small but relevant contribution at τT > 0.2 !

Model spectral function: transport contribution + UV-asymtotics

$$\rho_{\rm model}(\omega) \equiv \max\left\{ A\rho_{\rm UV}(\omega), \frac{\omega\kappa}{2T} \right\}$$

$$G_{\rm model}(\tau) \equiv \int_0^\infty \frac{{\rm d}\omega}{\pi}\rho_{\rm model}(\omega)\frac{\cosh\left(\frac{1}{2}-\tau T\right)\frac{\omega}{T}}{\sinh\frac{\omega}{2T}}$$

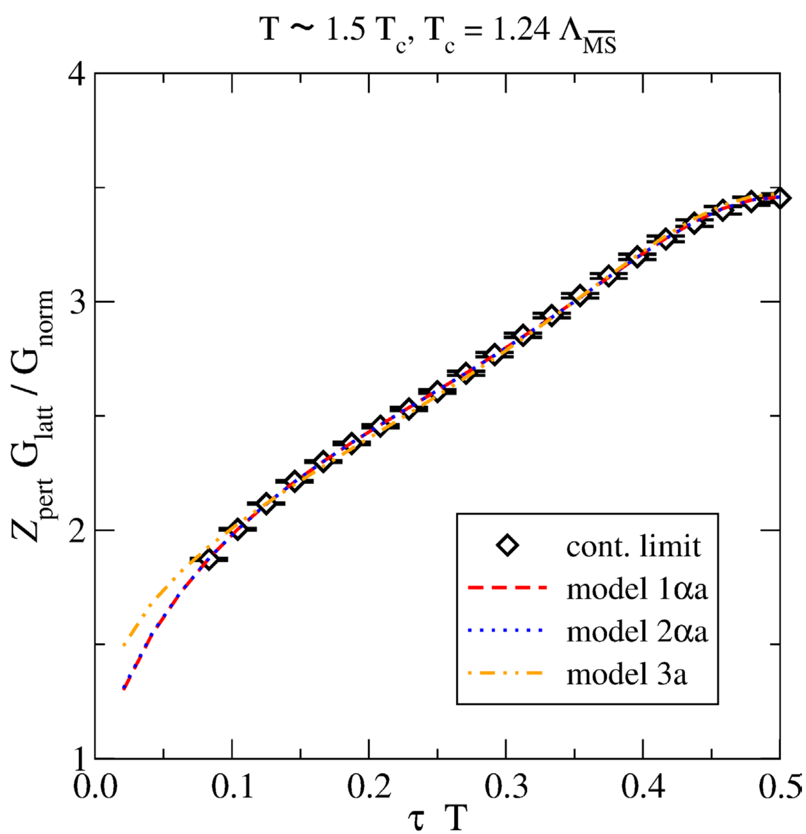
used to fit the continuum extrapolated data

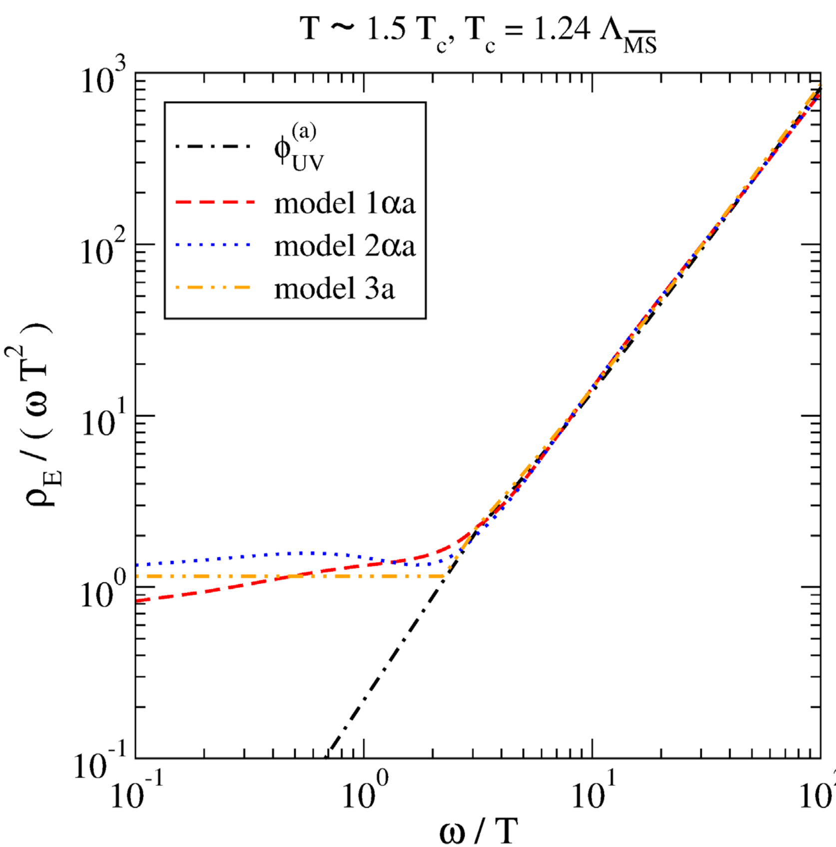
→ **first continuum estimate of κ :**

$$\kappa/T^3 = \lim_{\omega\to 0}\frac{2T\rho_{\rm E}(\omega)}{\omega} \simeq 2.31(7)$$

Heavy Quark Momentum Diffusion Constant – systematic uncertainties

model corrections to ρ_{IR} by a power series in ω



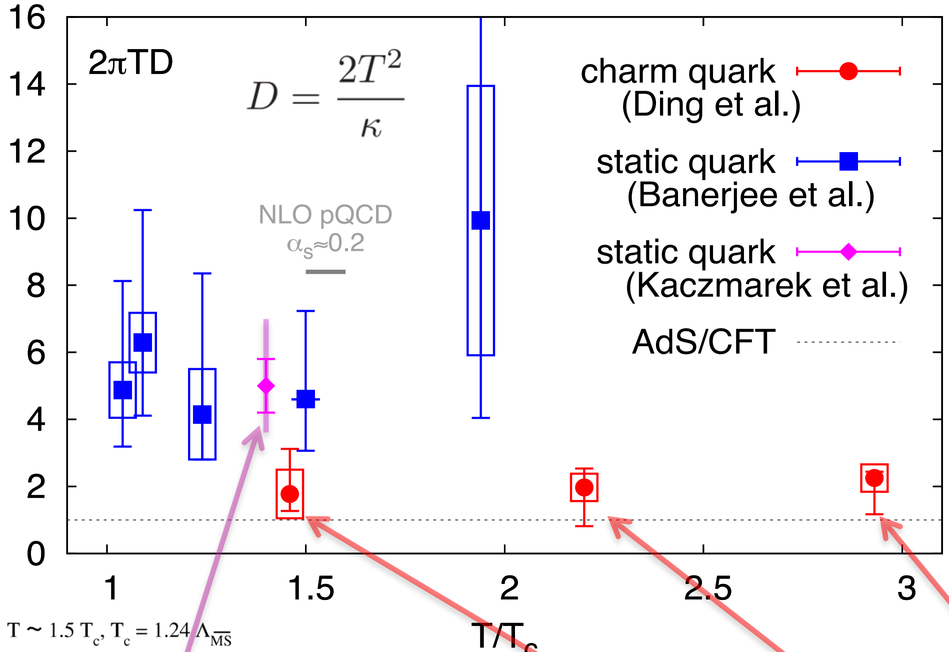


analysis of the systematic uncertainties

→ continuum estimate of κ :

$$\kappa/T^3 = \lim_{\omega\to 0} \frac{2T\rho_{\mathrm{E}}(\omega)}{\omega} = 1.8\ldots 3.4$$

Lattice QCD results on heavy quark diffusion coefficients

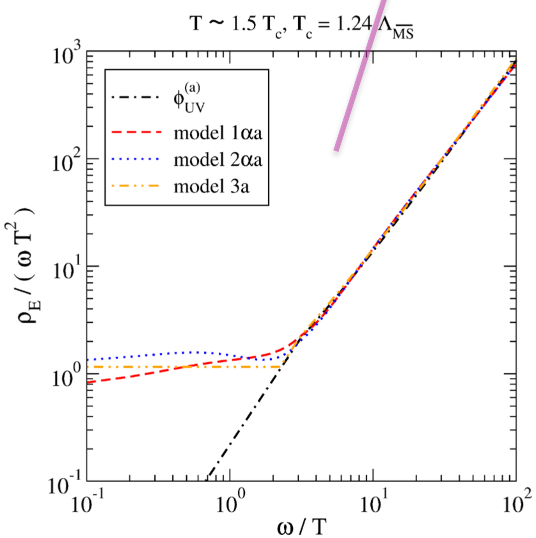


[H.T.Ding, OK et al., PRD86(2012)014509]

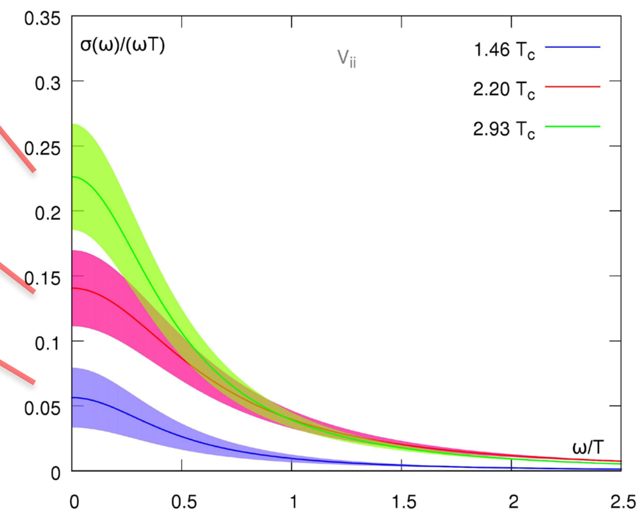
[Banerjee et al., PRD85(2012) 014510]

[A.Francis, OK et al., 2015]

charm quark mass
quenched approximation
no continuum yet



heavy quark mass limit
quenched approximation
continuum extrapolated



next goals: continuum extrapolation for charm and bottom + including dynamical quarks

→ quark mass dependence of diffusion coefficients + sequential melting of quarkonia