UNIVERSITÄT BIELEFELD

Electromagnetic spectral functions and charm diffusion

From correlators to spectral functions – case studies in the quenched approximation

I) Light quark vector meson spectral function

- thermal photon, dilepton rates, electrical conductivity

[J. Ghiglieri, OK, M. Laine, F. Meyer, PRD94(2016)016005]

[H-T. Ding, F. Meyer, OK, PRD94 (2016) 034504]

II) Thermal quarkonium physics in the pseudoscalar channel

[Y. Burnier, H.-T. Ding, OK, A.L. Kruse, M. Laine, JHEP11 (2017) 206]

III) Thermal quarkonium physics in the vector channel and heavy quark diffusion

[H.T. Ding, O. Kaczmarek, A.-L. Lorenz, H. Ohno, H. Sandmeyer, H.T. Shu, arXiv:2108.13693]

IV) Heavy quark momentum diffusion coefficient

[A.Francis, OK, M. Laine, T. Neuhaus, H. Ohno, PRD92(2015)116003]

V) Correlation functions under gradient flow – towards full QCD

[L. Altenkort, A.M. Eller, OK, L. Mazur, G.D. Moore, PRD103 (2021) 014511]

CRC-TR 211
Strong-interaction matter under extreme conditions

Information on **bound states and their dissociation** encoded in the spectral function

as well as

Thermal dilepton rate

$$\frac{\mathrm{d}W}{\mathrm{d}\omega \mathrm{d}^3p} = \frac{5\alpha^2}{54\pi^3} \frac{1}{\omega^2(e^{\omega/T}-1)} \rho_{\mathbf{V}}(\omega, \mathbf{T})$$

[H-T.Ding, F.Meyer, OK, PRD94(2016)034504]

Thermal photon rate

$$\omega \frac{\mathrm{d}N_\gamma}{\mathrm{d}^4x \mathrm{d}^3q} = \frac{5\alpha}{6\pi^2} \frac{1}{e^{\omega/T}-1} \rho_V(\omega = |\vec{k}|, T)$$

[J.Ghiglieri, OK, M.Laine, F.Meyer, PRD94(2016)016005]

Transport coefficients are encoded in the vector meson spectral function → Kubo formulae

Diffusion coefficients:

$$DT = \frac{T}{2\chi_q} \lim_{\omega \to 0} \frac{\rho_{\mathrm{ii}}(\omega)}{\omega}$$

On the lattice only correlation functions can be calculated

$$G(\tau, \vec{p}, T) = \int_0^\infty \frac{\mathrm{d}\omega}{2\pi} \rho(\omega, \vec{p}, T) K(\tau, \omega, T)$$

→ spectral reconstruction required

$$K(\tau, \omega, T) = \frac{\cosh\left(\omega(\tau - \frac{1}{2T}\right)}{\sinh\left(\frac{\omega}{2T}\right)}$$

In this talk: continuum extrapolated lattice correlation functions compared to perturbation theory

for a comparison of Bayesian and stochastic reconstructions of spectral functions see [H.-T. Ding, OK, S. Mukherjee, H. Ohno, H.-T. Shu, PRD97(2018)094503]

Vector-meson spectral function – hard to separate different scales

$$G(\tau,\vec{p},T) = \int_0^\infty \frac{\mathrm{d}\omega}{2\pi}\rho(\omega,\vec{p},T)K(\tau,\omega,T)$$

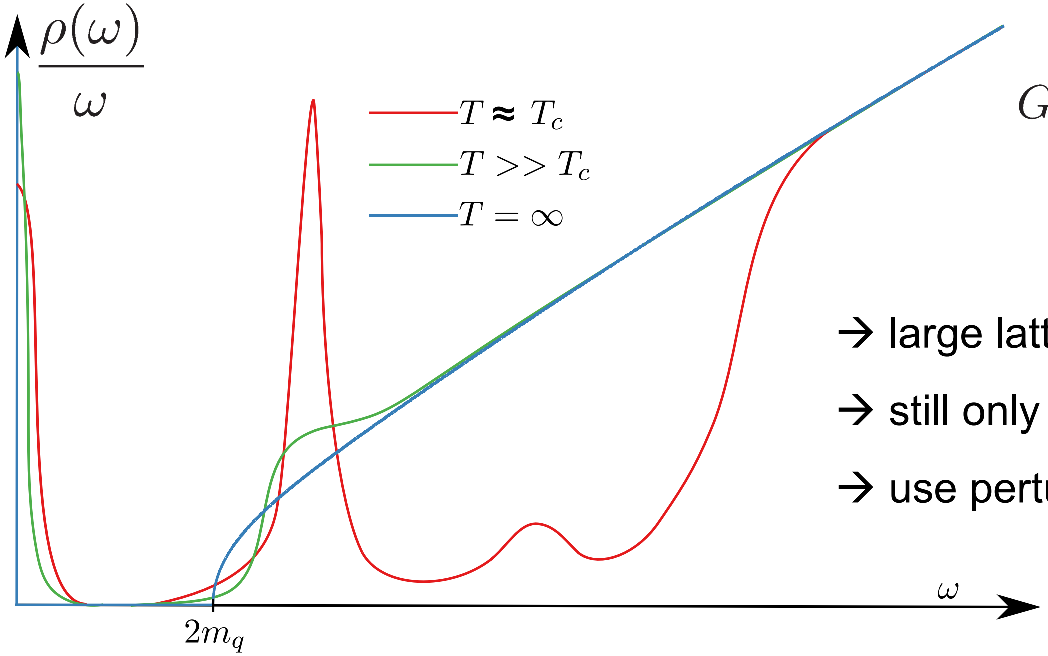
$$K(\tau,\omega,T) = \frac{\cosh\left(\omega(\tau-\frac{1}{2T})\right)}{\sinh\left(\frac{\omega}{2T}\right)}$$

Different contributions and scales enter in the spectral function

- **continuum at large frequencies**
- **possible bound states at intermediate frequencies**
- **transport contributions at small frequencies**
- **in addition cut-off effects on the lattice**

notoriously difficult to extract from correlation functions

Spectral functions in the QGP



$$G_{\mu\nu}(\tau,\vec{x}) = \langle J_\mu(\tau,\vec{x})J_\nu^\dagger(0,\vec{0})\rangle$$
$$J_\mu(\tau,\vec{x}) = 2\kappa Z_V\bar{\psi}(\tau,\vec{x})\Gamma_\mu\psi(\tau,\vec{x})$$

→ large lattices and continuum extrapolation needed

→ still only possible in the quenched approximation

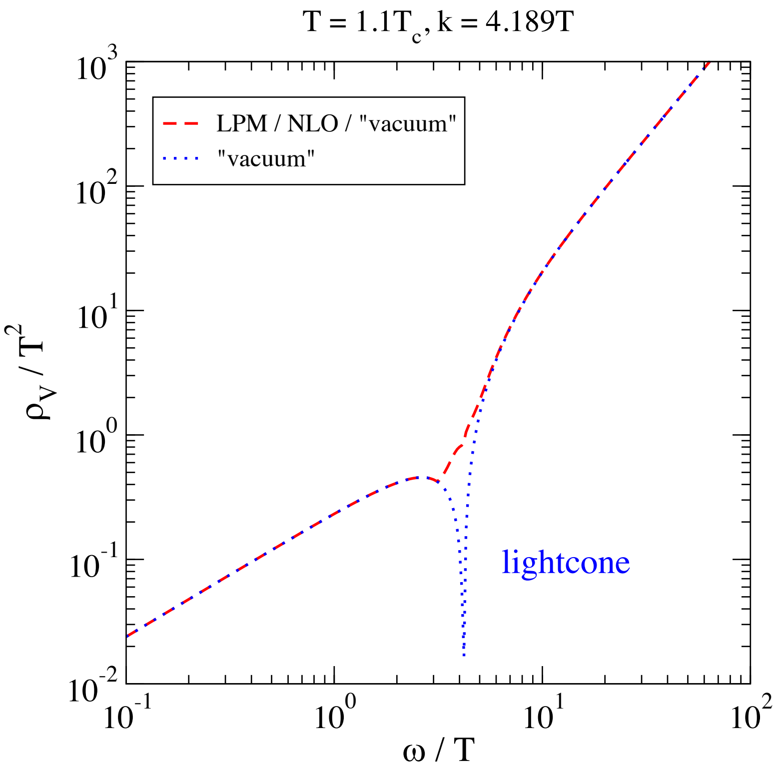
→ use perturbation theory to constrain the UV behavior

(narrow) transport peak at small ω: $\rho(\omega \ll T) \simeq 2\chi_{00}\frac{T}{M}\frac{\omega\eta}{\omega^2+\eta^2}\,, \qquad \eta = \frac{T}{MD}$

Perturbative vector meson spectral functions

Photonrate directly related to vector spectral function (at finite momentum):

$$\omega \frac{dN_\gamma}{d^4x d^3q} = \frac{5\alpha}{6\pi^2} \frac{1}{e^{\omega/T}-1} \rho_V(\omega = |\vec{k}|, T)$$



pQCD spectral function used to constrain the UV

interpolation between different (perturbative) regimes:

3T < ω < 10T: [J.Ghiglieri, G.D.Moore, JHEP 1412 (2014) 029]

ω > 10T: [I. Ghisoiu, M.Laine, JHEP 10 (2014) 84]

ω >> 10T: [M.Laine, JHEP 1311 (2013) 120]

vector spectral function in the hydrodynamic regime for $\omega, k \lesssim \alpha_s^2 T$**:**

$$\frac{\rho_V(\omega, \mathbf{k})}{\omega} = \left(\frac{\omega^2 - k^2}{\omega^2 + D^2 k^4} + 2\right)\chi_q D$$

we model the infrared behavior assuming smoothness at the light cone and fit to continuum extrapolated lattice correlators

Continuum lattice correlators vs. perturbation theory

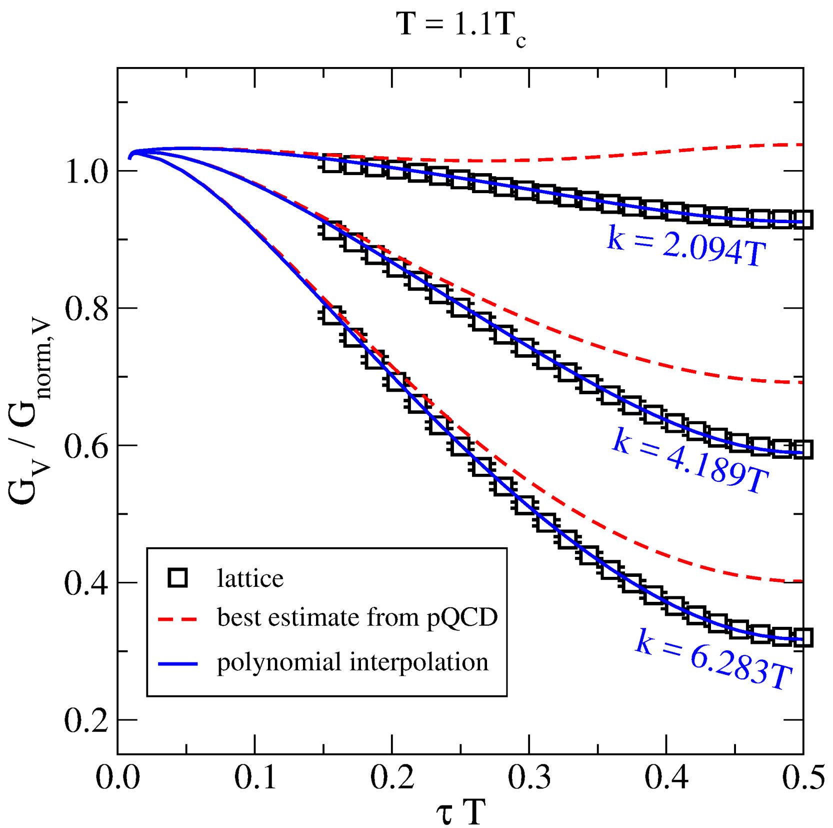
[J.Ghiglieri, OK, M.Laine, F.Meyer, PRD94(2016)016005]

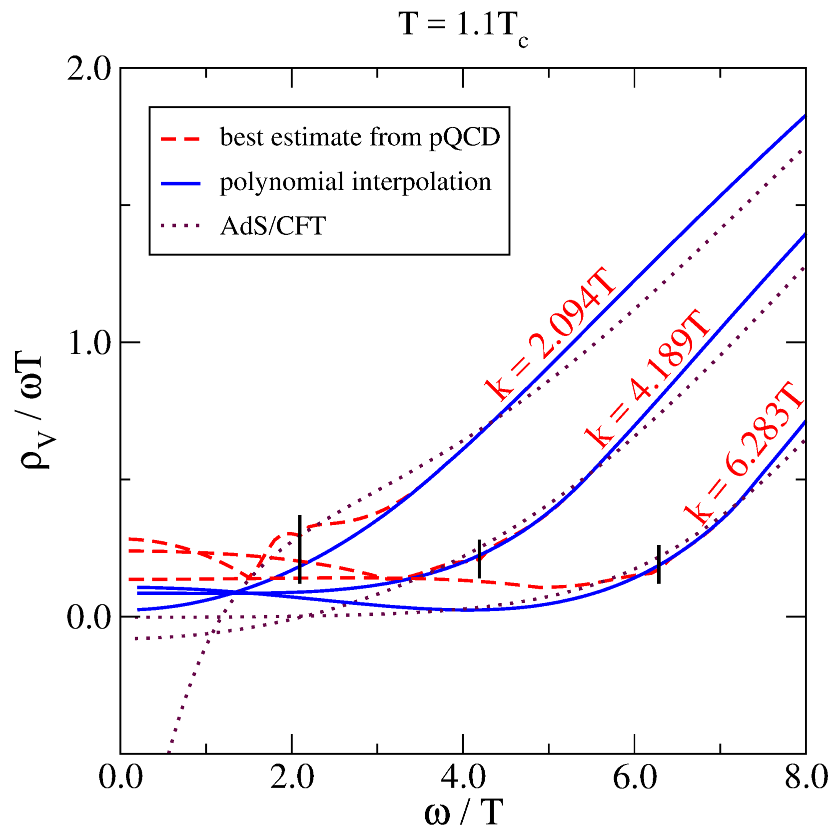
Fixed aspect ratio used to perform continuum extrapolation at finite p

$$\frac{\vec{p}}{T} = 2\pi\vec{k}\frac{N_\tau}{N_\sigma}$$

β_0	$N_s^3 \times N_\tau$	confs	$T\sqrt{t_0}$	$T/T_c\|_{t_0}$	Tr_0	$T/T_c\|_{r_0}$
7.192	$96^3 \times 32$	314	0.2796	1.12	0.816	1.09
7.544	$144^3 \times 48$	358	0.2843	1.14	0.817	1.10
7.793	$192^3 \times 64$	242	0.2862	1.15	0.813	1.09
7.192	$96^3 \times 28$	232	0.3195	1.28	0.933	1.25
7.544	$144^3 \times 42$	417	0.3249	1.31	0.934	1.25
7.793	$192^3 \times 56$	273	0.3271	1.31	0.929	1.25

use perturbation theory at large ω and fit a polynomial at small ω to extract the spectral function





Continuum lattice correlators vs. perturbation theory

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[J.Ghiglieri, OK, M.Laine, F.Meyer, PRD94(2016)016005]

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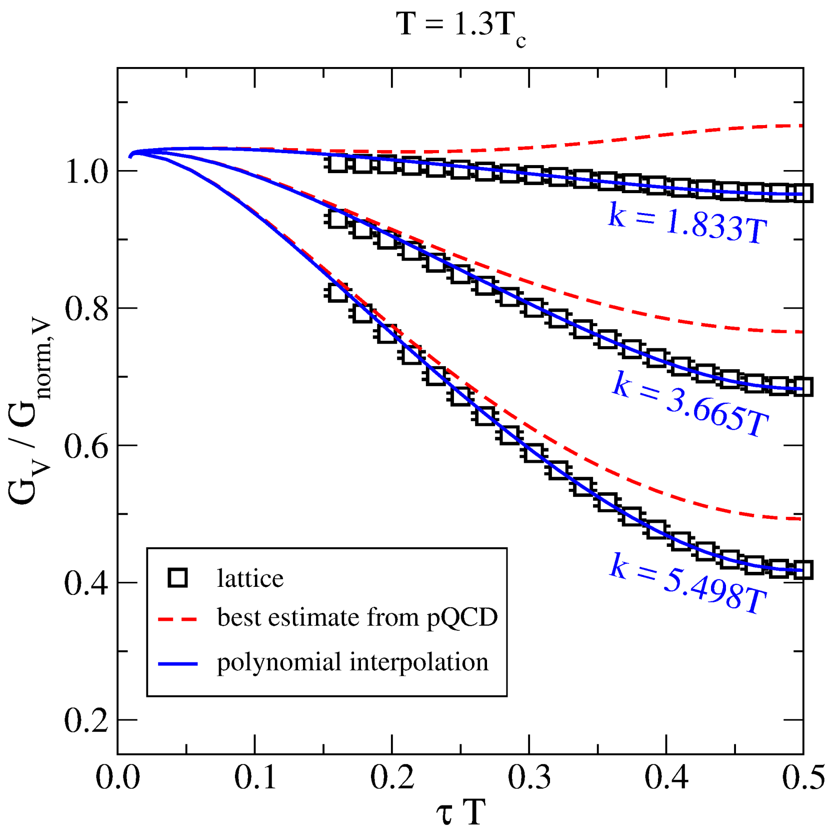
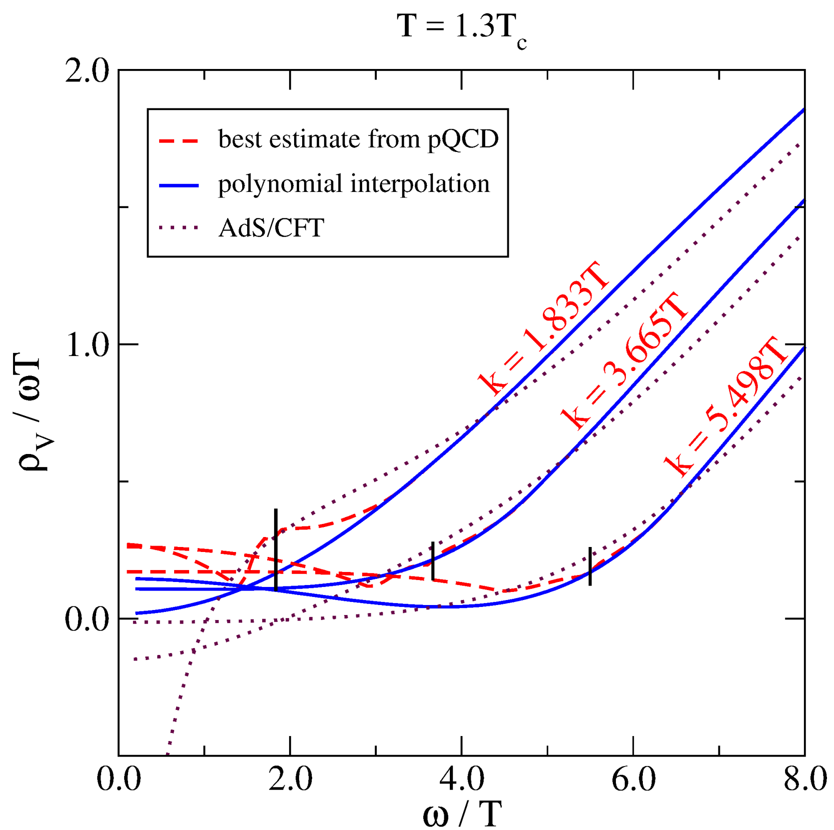
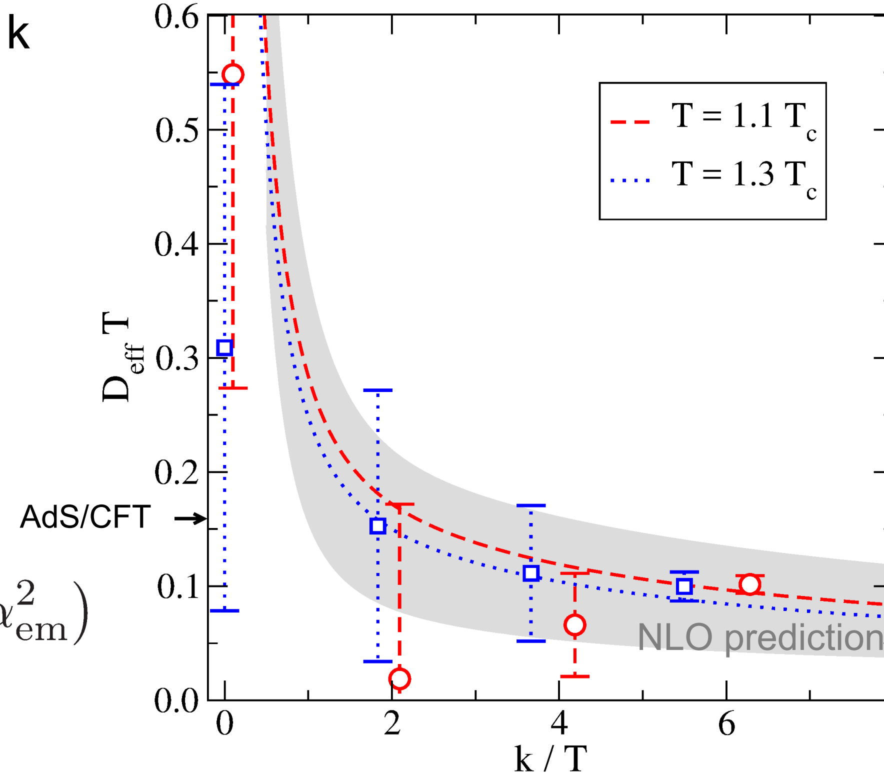




[J.Ghiglieri, OK, M.Laine, F.Meyer, PRD94(2016)016005]

The spectral function at the photon point ω = k

$$D_{\rm eff}(k) \equiv \begin{cases} \dfrac{\rho_{\rm V}(k,\mathbf{k})}{2\chi_{\rm q} k} & , \quad k>0 \\ \lim\limits_{\omega\to 0^+} \dfrac{\rho^{ii}(\omega,\mathbf{0})}{3\chi_{\rm q}\omega} & , \quad k=0 \end{cases} .$$



can be used to calculate the photon rate

$$\frac{{\rm d}\Gamma_\gamma(\mathbf{k})}{{\rm d}^3\mathbf{k}} = \frac{2\alpha_{\rm em}\chi_{\rm q}}{3\pi^2} n_{\rm B}(k) D_{\rm eff}(k) + \mathcal{O}\left(\alpha_{\rm em}^2\right)$$

becomes more perturbative at larger k, approaching the NLO prediction (valid for k>>gT)

[J. Ghiglieri, G.D. Moore, JHEP12 (2014) 029]

but non-perturbative for k/T < 3

Electrical conductivity obtained in the limit k→0 between the results from

AdS/CFT: $DT = \dfrac{1}{2\pi}$

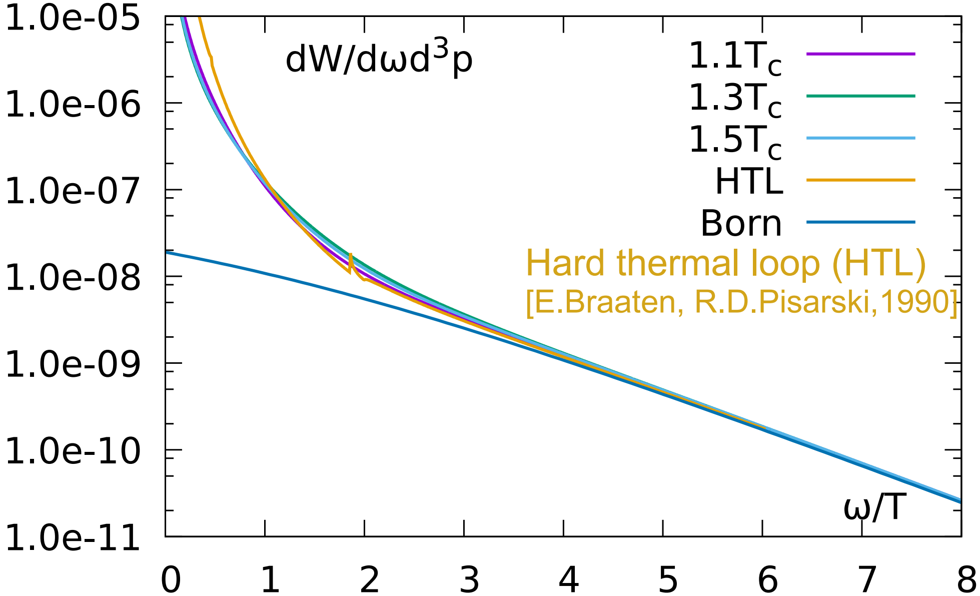
[G.Policastro, D.T.Son,A.O.Starinets, JHEP09(2002)043]

LO perturbation theory [Arnold, Moore Yaffe, JHEP 05 (2003)]

using lattice value for χ_q/T^2: $DT = 2.9 - 3.1$

[H-T.Ding, F.Meyer, OK, PRD94(2016)034504]

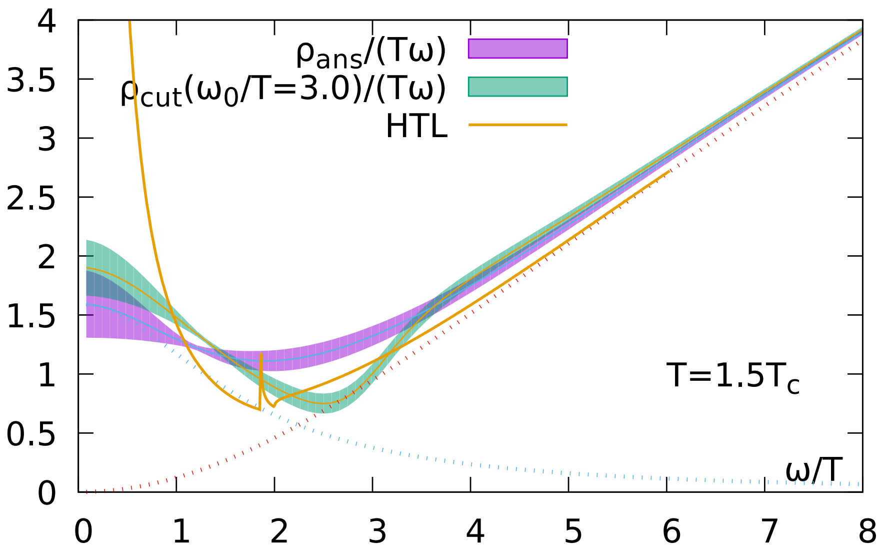
Dileptonrate directly related to vector spectral function:

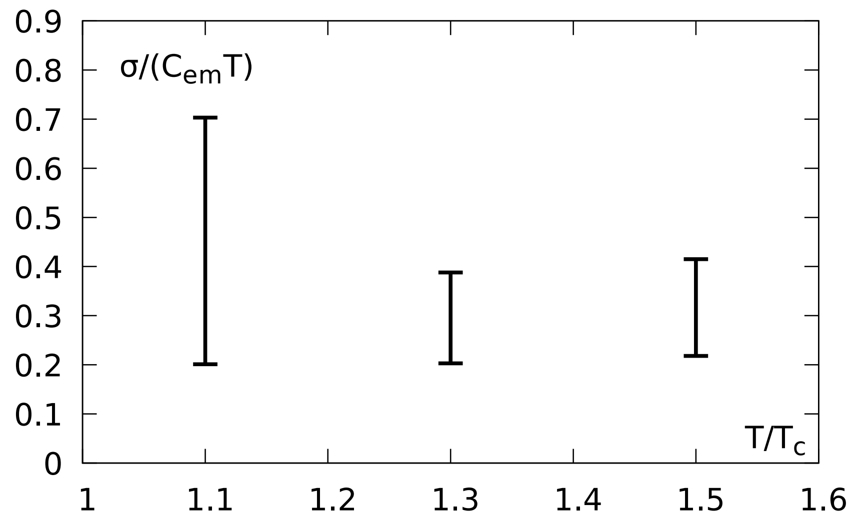


$$\frac{dW}{d\omega d^3p} = \frac{5\alpha^2}{54\pi^3} \frac{1}{\omega^2 (e^{\omega/T} - 1)} \rho_{\mathbf{V}}(\omega, \mathbf{T})$$

continuum estimate for the of the electrical conductivity

$$\frac{\sigma_{el}}{C_{em}T} = \frac{1}{6} \lim_{\omega \to 0} \frac{\rho_{ii}(\omega, \vec{p} = 0, T)}{\omega T}$$





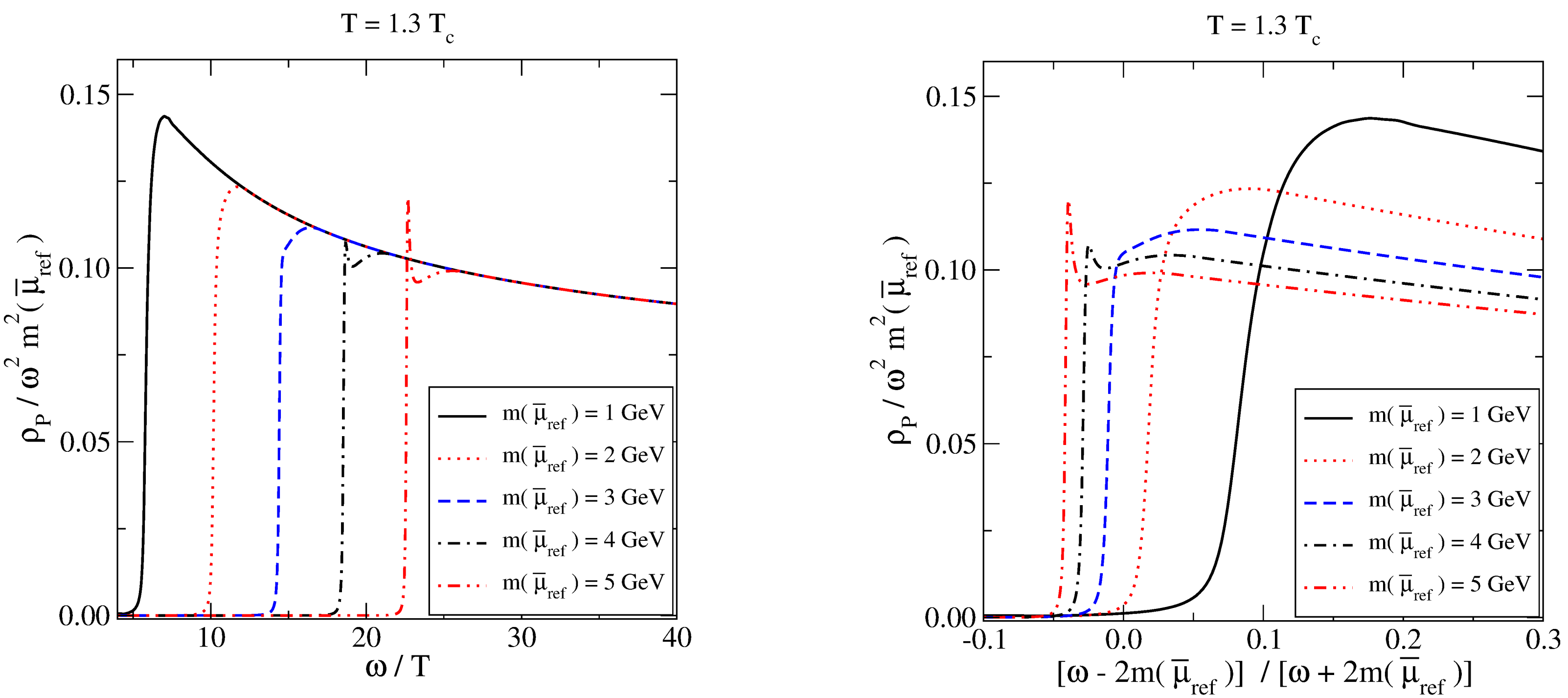
[Y. Burnier, H.-T. Ding, OK et al. JHEP11 (2017) 206]

Using **continuum extrapolated Euclidean correlation functions** from Lattice QCD

$$G_{\rm P}(\tau) \equiv M_{\rm B}^2 \int_{\vec{x}} \Big\langle (\bar\psi i \gamma_5 \psi)(\tau,\vec{x}) \; (\bar\psi i \gamma_5 \psi)(0,\vec{0}) \Big\rangle_{\rm c} \,, \quad 0 < \tau < \frac{1}{T} \,,$$

and best knowledge on the spectral function from **perturbation theory and pNRQCD**

interpolated between different regimes



no transport contribution in pseudo-scalar channel channel quenched SU(3) gauge configurations (separated by 500 updates)

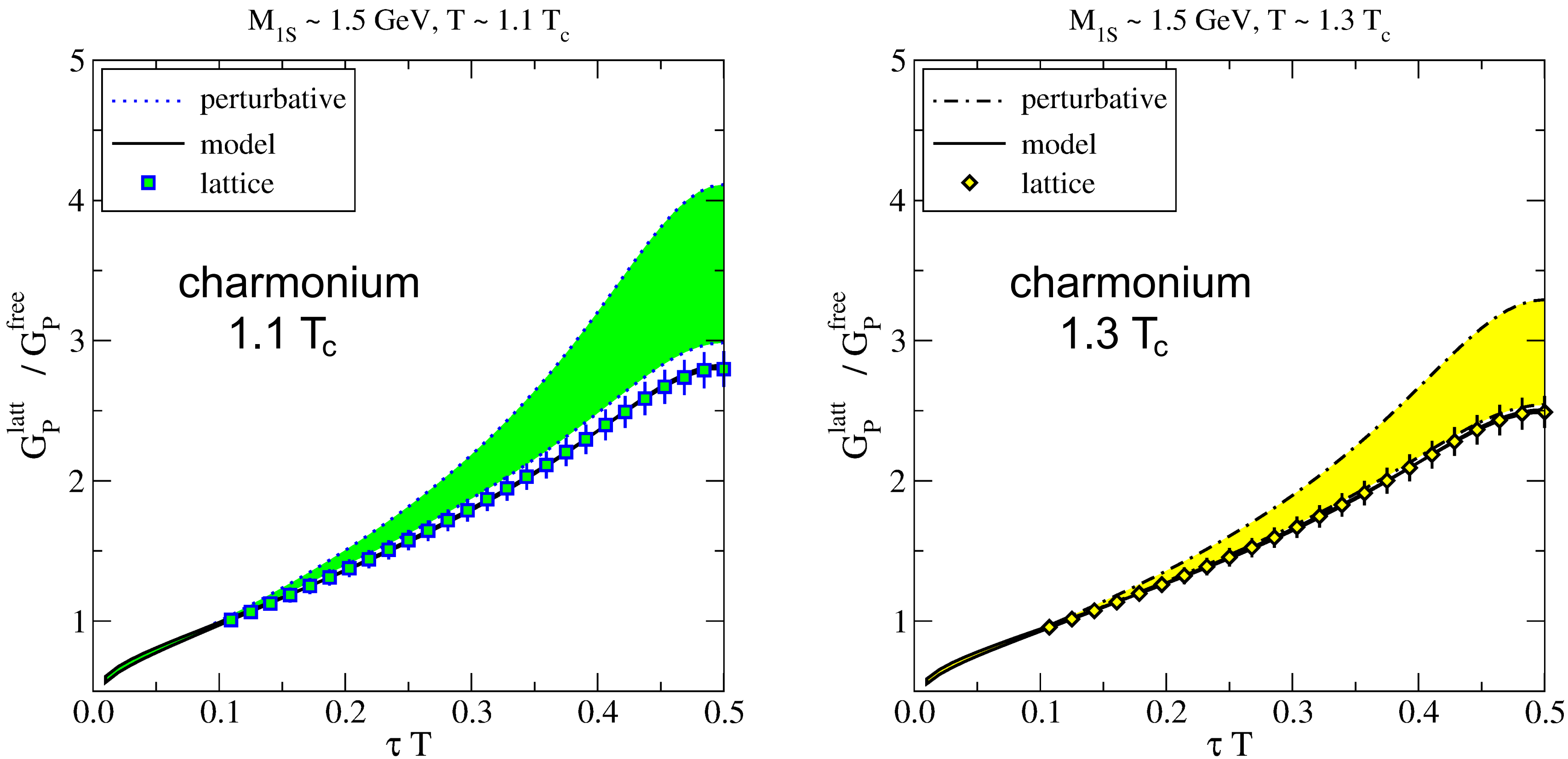
non-perturbatively O(a) clover improved Wilson fermion valence quarks

6 quark masses between charm and bottom → interpolate to physical c and b mass

β	N_s	N_τ	confs	r_0/a	T/T_c	c_{SW}	κ_c	κ	$\frac{m^2(1/a)}{m^2(\bar{\mu}_{ref})}$
7.192	96	48	237	26.6	0.74	1.367261	0.13442	0.12257, 0.12800, 0.13000, 0.13100, 0.13150, 0.13194	0.6442
		32	476		1.12				
		28	336		1.27				
		24	336		1.49				
		16	237		2.23				
7.394	120	60	171	33.8	0.76	1.345109	0.13408	0.124772, 0.12900, 0.13100, 0.13150, 0.132008, 0.132245	0.6172
		40	141		1.13				
		30	247		1.51				
		20	226		2.27				
7.544	144	72	221	40.4	0.75	1.330868	0.13384	0.12641, 0.12950, 0.13100, 0.13180, 0.13220, 0.13236	0.5988
		48	462		1.13				
		42	660		1.29				
		36	288		1.51				
		24	237		2.26				
7.793	192	96	224	54.1	0.76	1.310381	0.13347	0.12798, 0.13019, 0.13125, 0.13181, 0.13209, 0.13221	0.5715
		64	249		1.13				
		56	190		1.30				
		48	210		1.51				
		32	235		2.27				

In this talk: only results based on continuum extrapolated correlation functions

See [Y. Burnier, H.-T. Ding, OK et al. JHEP11 (2017) 206] for more details

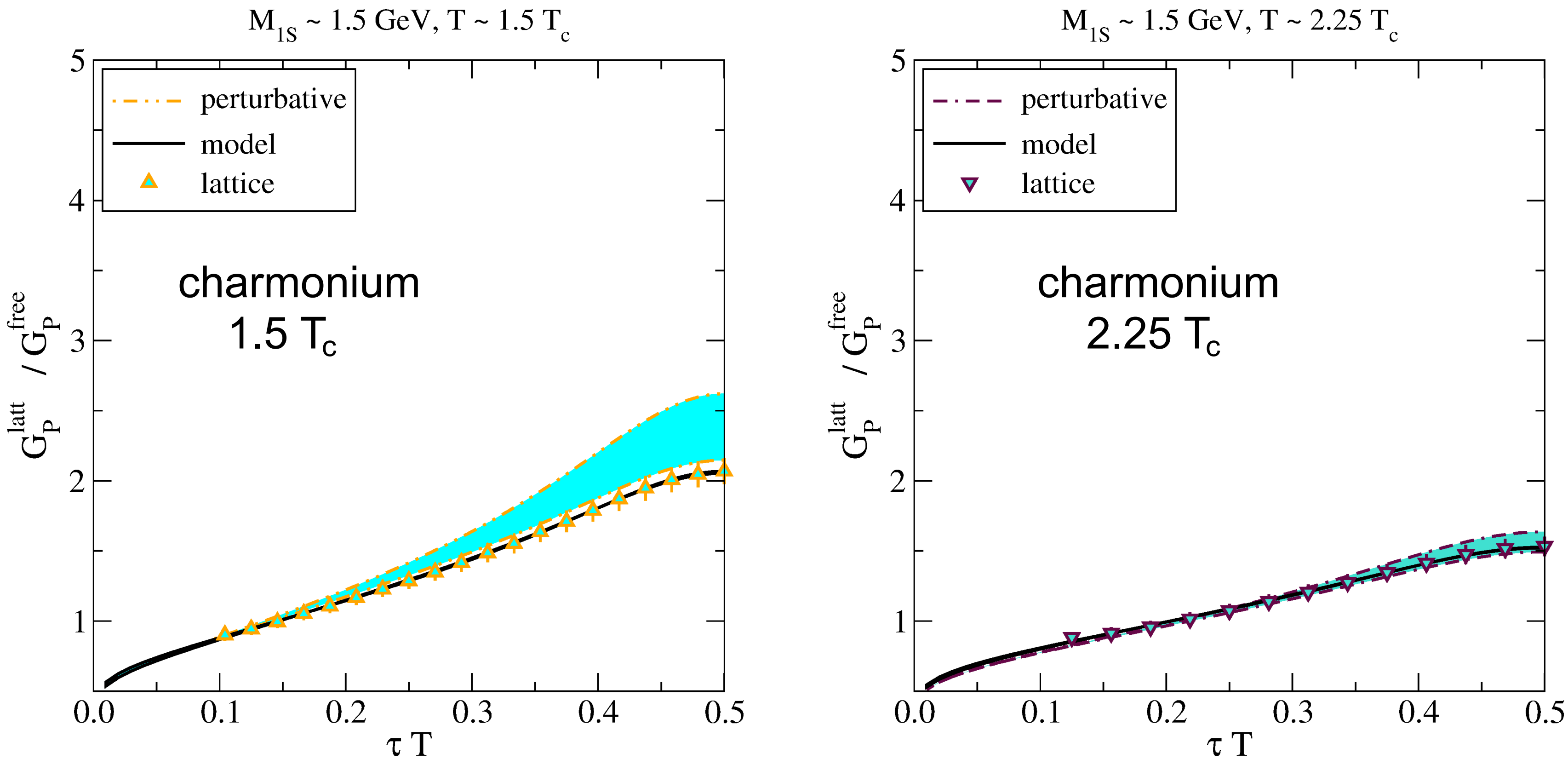


differences between lattice and perturbation theory may have a simple explanation

A: uncertainties related to the perturbative renormalization factors
B: non-perturbative mass shifts

$$\rho_{\mathrm{P}}^{\mathrm{model}}(\omega) \;\equiv\; A\,\rho_{\mathrm{P}}^{\mathrm{pert}}(\omega - B)\;.$$

→ continuum lattice data well described by this model with χ^2/d.o.f < 1



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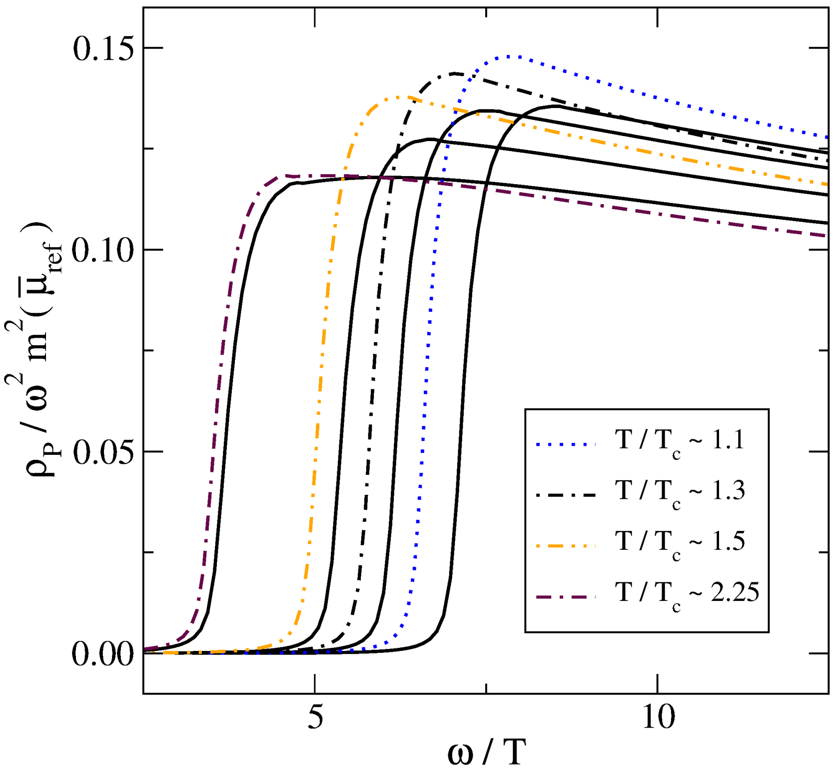
→ continuum lattice data well described by this model with χ^2/d.o.f < 1

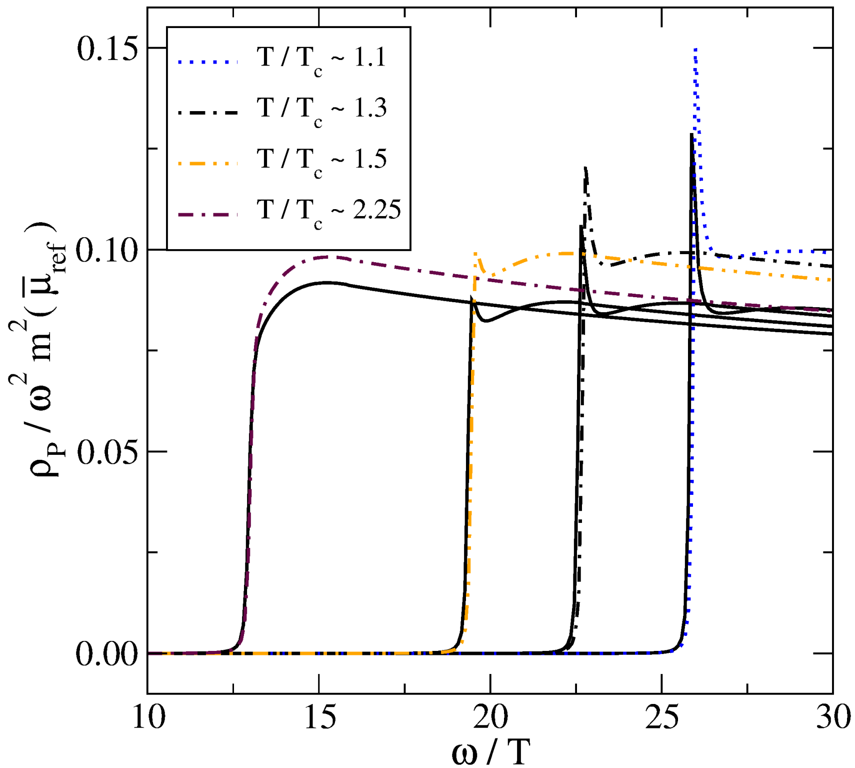
Pseudo-scalar spectral functions

[Y. Burnier, H.-T. Ding, OK et al. JHEP11 (2017) 206]

charmonium: $m(\bar{\mu}_{ref}) = 1$ GeV



bottomonium: $m(\bar{\mu}_{ref}) = 5$ GeV



charmonium:

no resonance peaks are needed for representing the lattice data even for 1.1 T_c

modest threshold enhancement sufficient in the analyzed temperature region

bottomonium:

thermally broadened resonance peak

present up to temperatures around 1.5 T_c

[H.T. Ding, O. Kaczmarek, A.-L. Lorenz, H. Ohno, H. Sandmeyer, H.-T. Shu, arXiv:2108.13693]

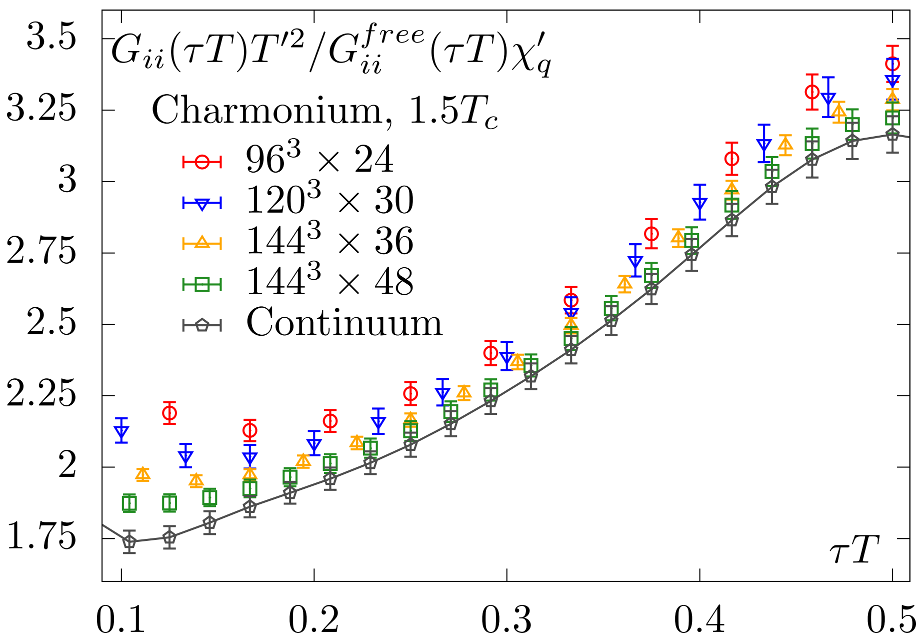
quenched SU(3) gauge configurations (separated by 500 updates)

non-perturbatively O(a) clover improved Wilson fermion valence quarks

6 quark masses between charm and bottom

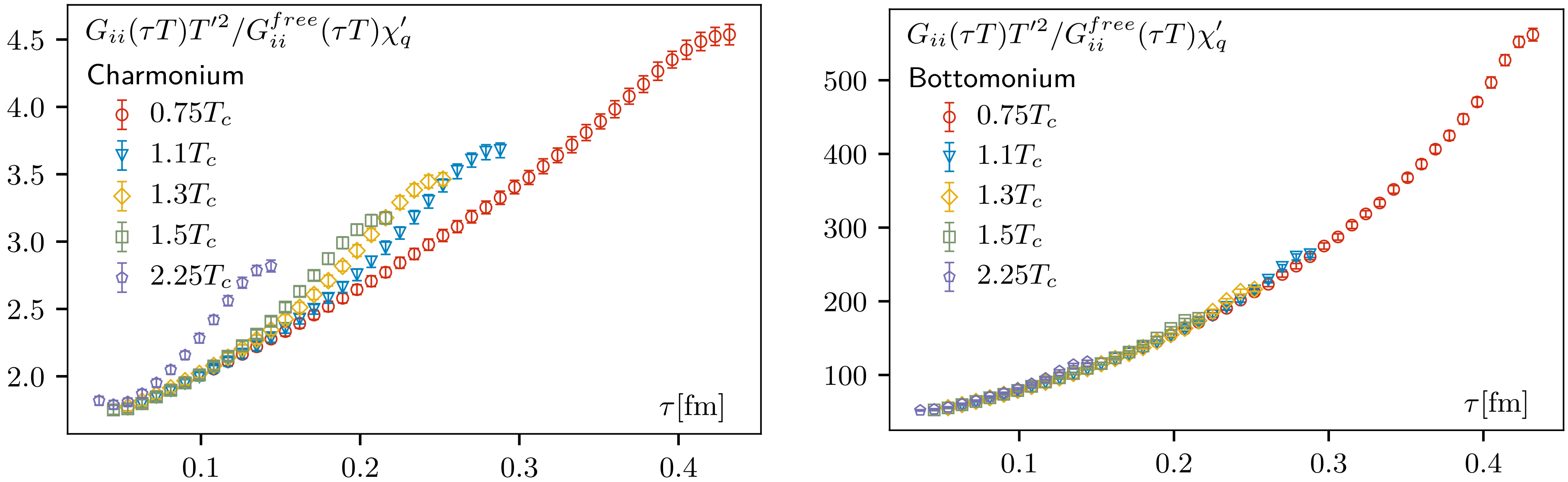
→ interpolate to physical c and b mass

well controlled continuum extrapolation:



β	r_0/a	a[fm](a^{-1}[GeV])	N_σ	N_τ	T/T_c	# confs
7.192	26.6	0.018(11.19)	96	48	0.75	237
				32	1.1	476
				28	1.3	336
				24	1.5	336
				16	2.25	237
7.394	33.8	0.014(14.24)	120	60	0.75	171
				40	1.1	141
				30	1.5	247
				20	2.25	226
7.544	40.4	0.012(17.01)	144	72	0.75	221
				48	1.1	462
				42	1.3	660
				36	1.5	288
				24	2.25	237
7.793	54.1	0.009(22.78)	192	96	0.75	224
				64	1.1	291
				56	1.3	291
				48	1.5	348
				32	2.25	235

[H.T. Ding, O. Kaczmarek, A.-L. Lorenz, H. Ohno, H. Sandmeyer, H.-T. Shu, arXiv:2108.13693]



we model the UV part of the spectral function again by a perturbative model

$$\rho_{ii}^{mod}(\omega) = A\rho_V^{pert}(\omega - B)$$

in the vector channel an additional IR contribution appears as a transport peak

$$\rho_{ii}^{trans}(\omega) = 3\chi_q \frac{T}{M}\frac{\omega\eta}{\omega^2+\eta^2} \qquad D = \frac{1}{3\chi_q}\lim_{\omega\to 0}\frac{\rho_{ii}^{trans}(\omega)}{\omega} \qquad \eta = \frac{T}{MD}$$

with an almost constant contribution to the correlator at large distances for small η

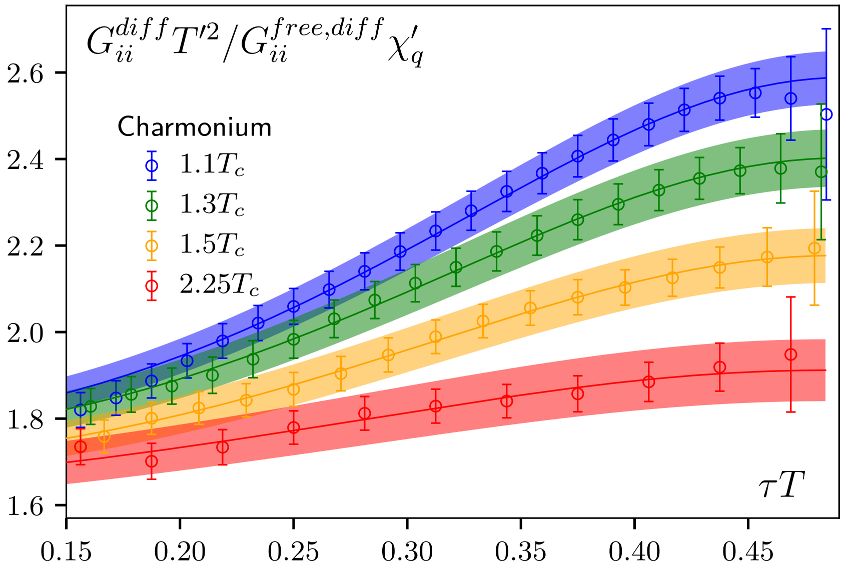
$$\rho_{ii}(\omega) = \rho_{ii}^{trans}(\omega) + \rho_{ii}^{mod}(\omega) \qquad G_{ii}(\tau T) = G_{ii}^{trans}(\tau T) + G_{ii}^{mod}(\tau T)$$

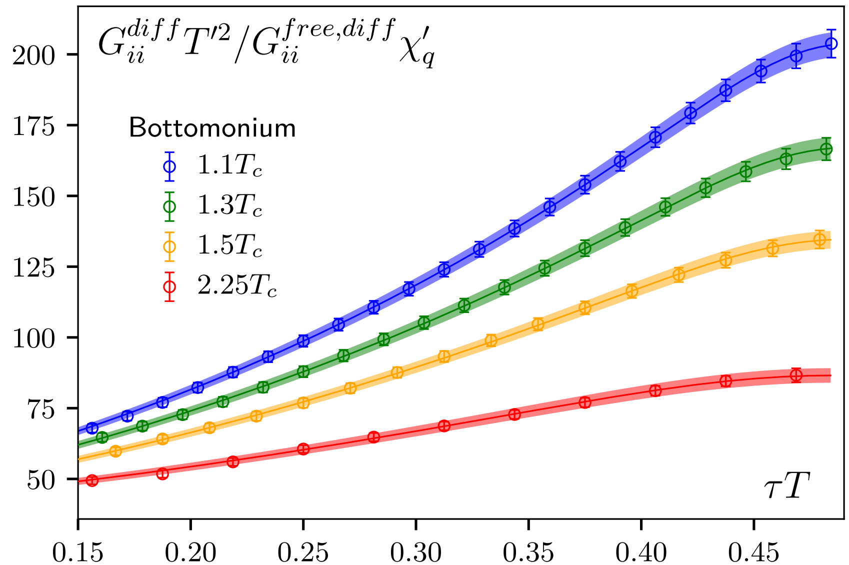
an almost constant contribution of the transport peak is effectively removed in

$$G_{ii}^{diff}(\tau/a) = G_{ii}(\tau/a+1) - G_{ii}(\tau/a)$$

and the UV part of the spectral function well described by the perturbative model

$$\rho_{ii}^{mod}(\omega) = A\rho_V^{pert}(\omega - B)$$



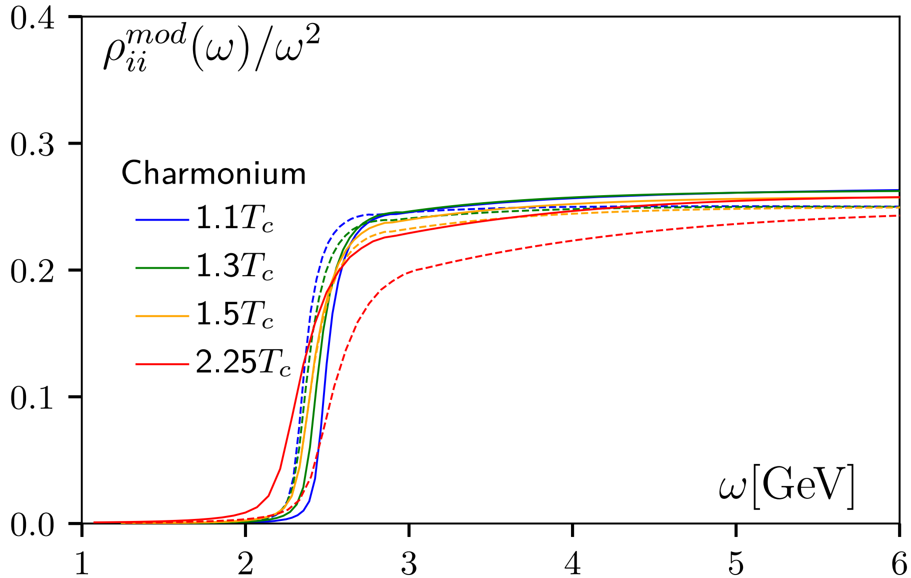


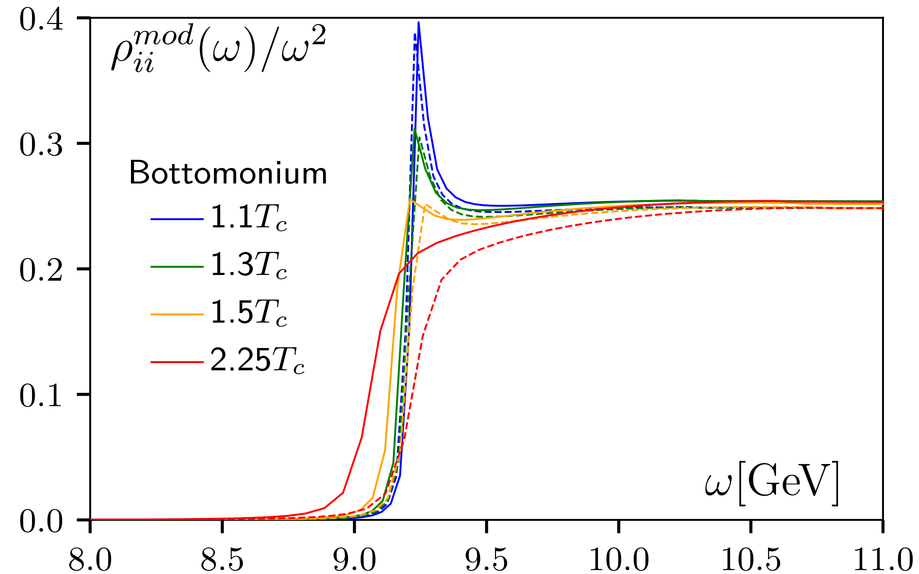
differences between lattice and perturbation theory may have a simple explanation

A: uncertainties related to renormalization
B: non-perturbative mass shifts

→ continuum lattice data well described by this model with χ^2/d.o.f < 1

UV part well described by the perturbative model: $\rho_{ii}^{mod}(\omega) = A\rho_V^{pert}(\omega - B)$





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no resonance peaks are needed for representing the lattice data even for 1.1 T_c

modest threshold enhancement sufficient in the analyzed temperature region

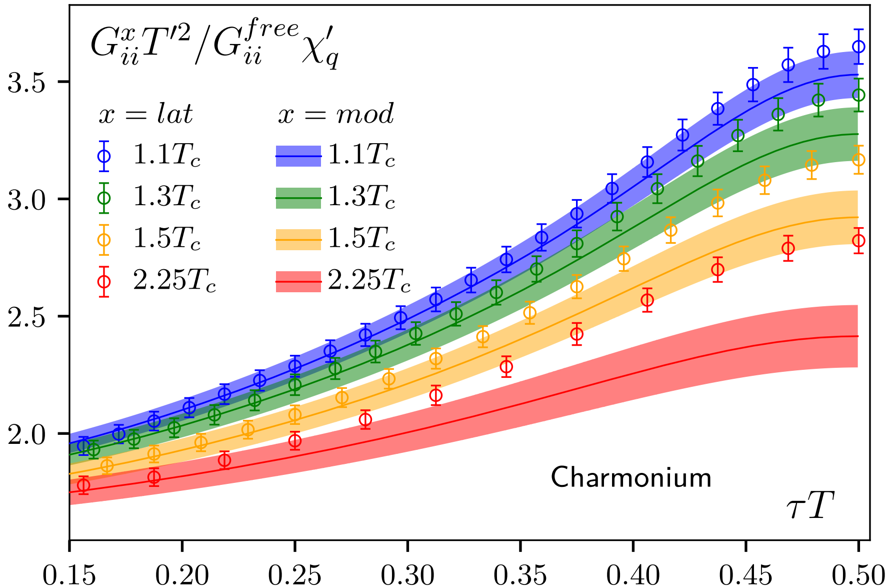
bottomonium:

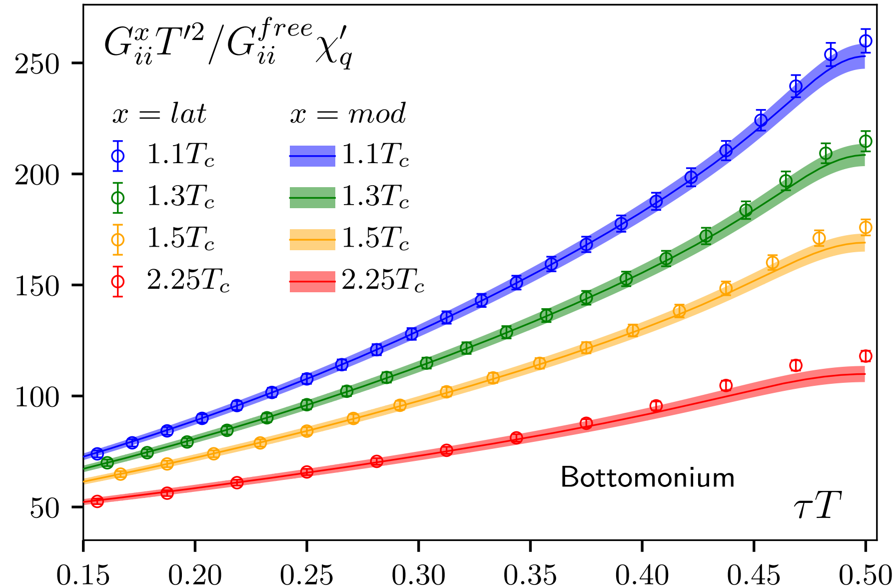
thermally broadened resonance peak

present up to temperatures around 1.5 T_c

comparison of the full correlator to the model

$$\rho_{ii}(\omega) = \rho_{ii}^{trans}(\omega) + \rho_{ii}^{mod}(\omega) \qquad G_{ii}(\tau T) = G_{ii}^{trans}(\tau T) + G_{ii}^{mod}(\tau T)$$



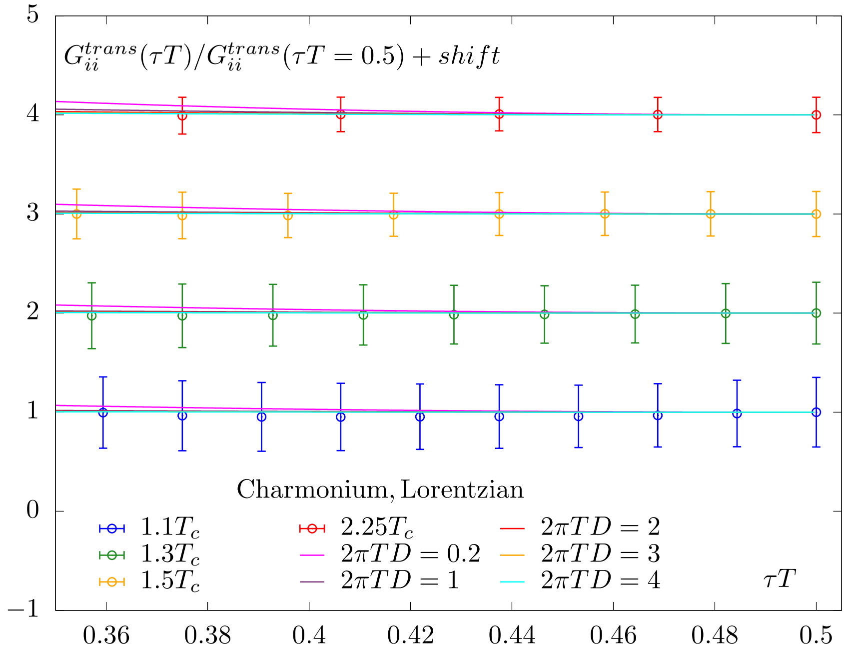


comparison of the full correlator to the model → difference due to transport contribution

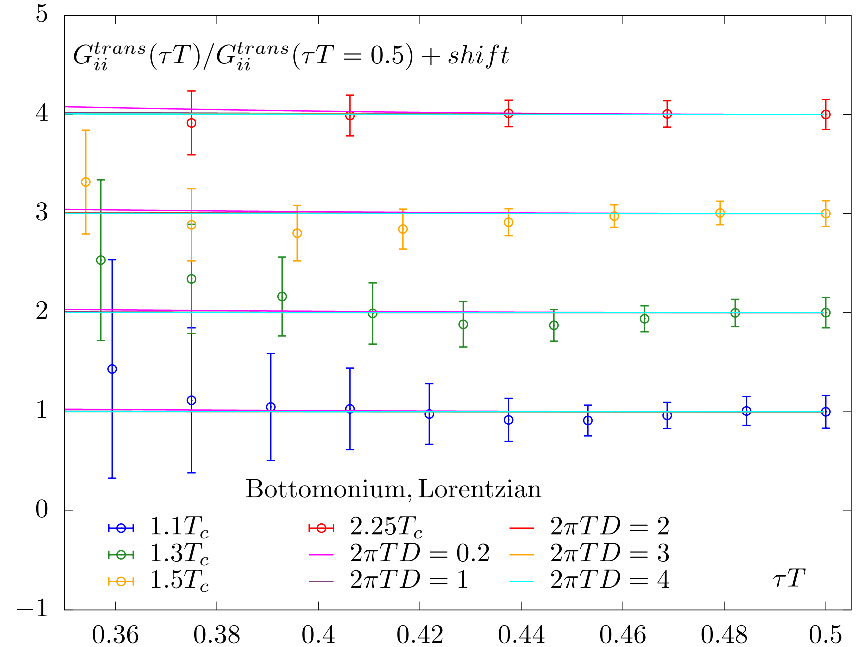
$$\rho_{ii}^{trans}(\omega) = 3\chi_q \, \frac{T}{M} \frac{\omega\eta}{\omega^2 + \eta^2} \qquad D = \frac{1}{3\chi_q} \lim_{\omega \to 0} \frac{\rho_{ii}^{trans}(\omega)}{\omega} \qquad \eta = \frac{T}{MD}$$

transport contribution in the vector channel: $\rho(\omega \ll T) \simeq 2\chi_{00} \frac{T}{M} \frac{\omega\eta}{\omega^2 + \eta^2}$, $\eta = \frac{T}{MD}$

charmonium:



bottomonium:



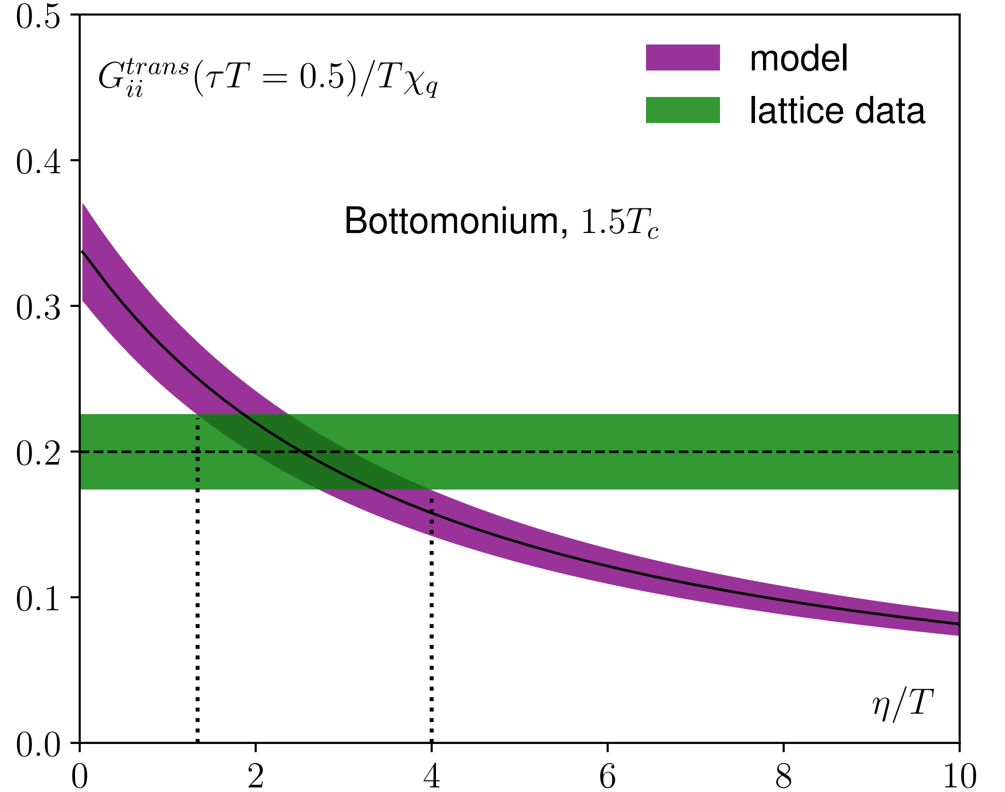
varying $2\pi DT$ between 0.2 and 4

→ only small curvature due to transport contribution within our errors

→ hard to determine transport coefficients w/o additional constraints transport contribution in the vector channel: $\rho(\omega \ll T) \simeq 2\chi_{00} \frac{T}{M} \frac{\omega\eta}{\omega^2+\eta^2}\,, \qquad \eta = \frac{T}{MD}$

correlator at the midpoint most sensitive to the transport contribution:

allows to estimate η/T for charm and bottom



Taylor expansion of the correlator around the midpoint → thermal moments:

$$G(\tau,T) = G^{(0)} \sum_{n=0}^{\infty} R_{2n,0} \left(\tau T - \frac{1}{2}\right)^{2n}, \quad R_{n,m} \equiv \frac{G^{(n)}}{G^{(m)}}$$

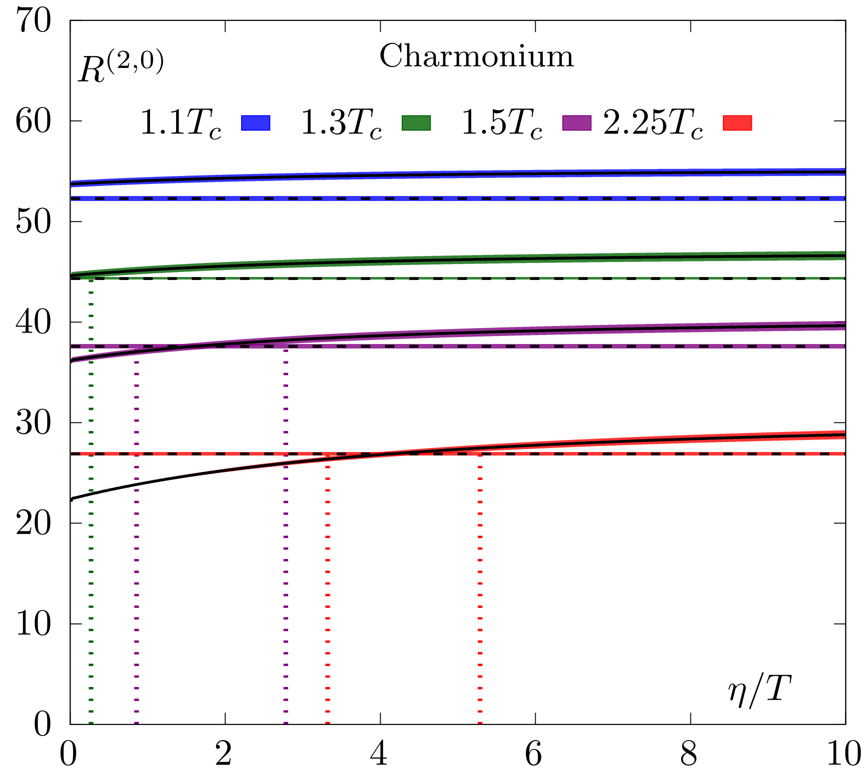
$$G^{(n)} = \frac{1}{n!} \left.\frac{\mathrm{d}^n G(\tau,T)}{\mathrm{d}(\tau T)^n}\right|_{\tau T=1/2} = \frac{1}{n!} \int_0^\infty \frac{\mathrm{d}\omega}{2\pi} \left(\frac{\omega}{T}\right)^n \frac{\rho(\omega)}{\sinh(\omega/2T)}$$

transport contribution in the vector channel: $\rho(\omega \ll T) \simeq 2\chi_{00} \frac{T}{M} \frac{\omega\eta}{\omega^2 + \eta^2}\,, \qquad \eta = \frac{T}{MD}$

correlator at the midpoint most sensitive to the transport contribution:

allows to estimate η/T for charm

Second moment almost constant for bottom



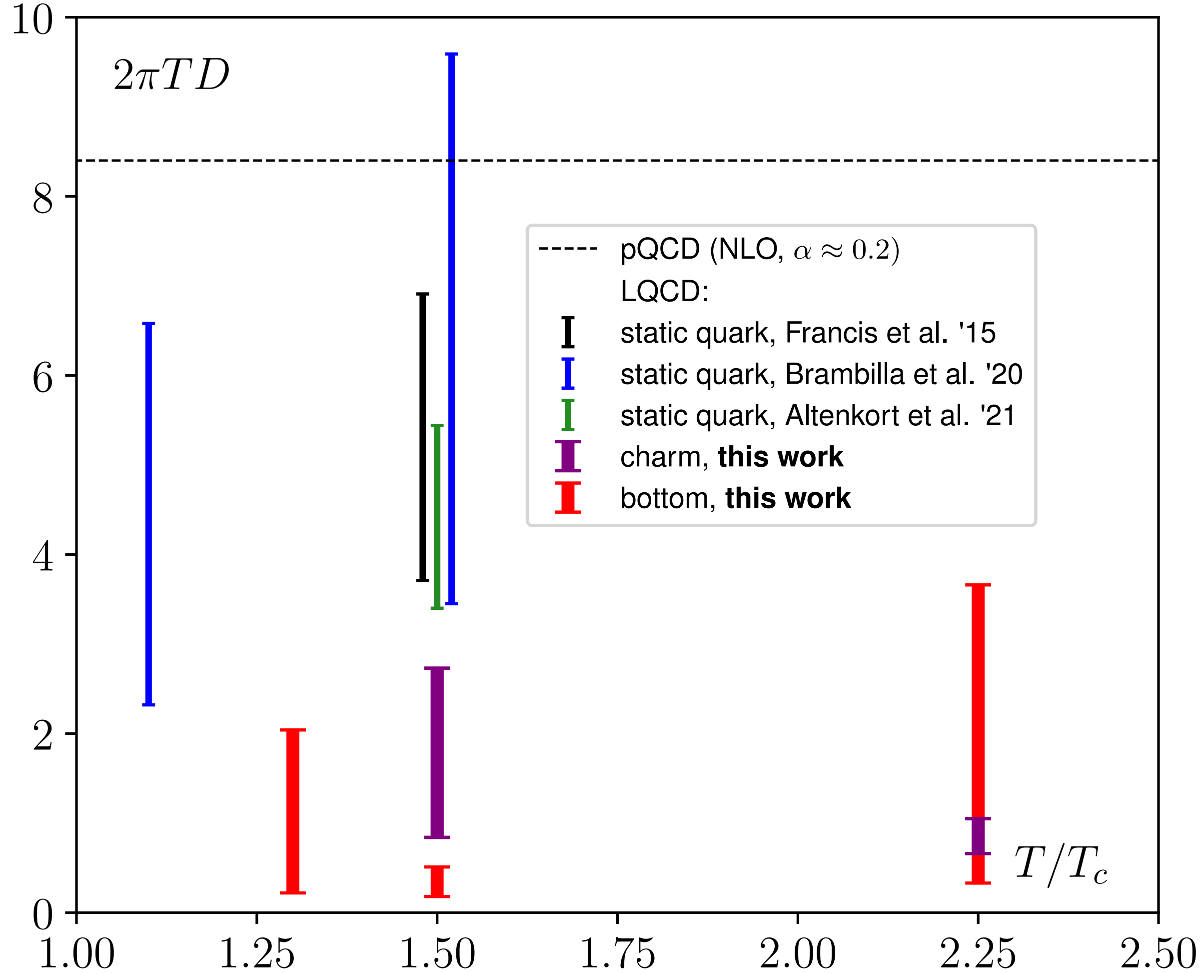
Taylor expansion of the correlator around the midpoint → thermal moments:

$$G(\tau, T) = G^{(0)} \sum_{n=0}^{\infty} R_{2n,0} \left(\tau T - \frac{1}{2}\right)^{2n}, \quad R_{n,m} \equiv \frac{G^{(n)}}{G^{(m)}}$$

$$G^{(n)} = \frac{1}{n!} \left. \frac{\mathrm{d}^n G(\tau, T)}{\mathrm{d}(\tau T)^n} \right|_{\tau T = 1/2} = \frac{1}{n!} \int_0^{\infty} \frac{\mathrm{d}\omega}{2\pi} \left(\frac{\omega}{T}\right)^n \frac{\rho(\omega)}{\sinh(\omega/2T)}$$

Using these estimates for η/T and $2\pi TD = 2\pi \frac{T}{\eta}\frac{T}{M}$

with quark masses M_c = 1.28 ($\pm$10%) GeV and M_b = 4.18 (($\pm$10%) GeV



[H.T. Ding, O. Kaczmarek, A.-L. Lorenz, H. Ohno, H. Sandmeyer, H.T. Shu, arXiv:2108.13693]

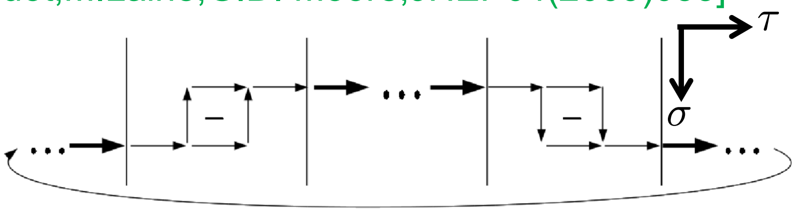
Heavy Quark Effective Theory (HQET) in the large quark mass limit

for a single quark in medium

leads to a (pure gluonic) “color-electric correlator”

[J.Casalderrey-Solana, D.Teaney, PRD74(2006)085012, S.Caron-Huot,M.Laine,G.D. Moore,JHEP04(2009)053]

$$G_E(\tau) \equiv -\frac{1}{3}\sum_{i=1}^{3} \frac{\left\langle \mathrm{Re\,Tr}\left[U(\tfrac{1}{T};\tau)\, g\, E_i(\tau,\mathbf{0})\, U(\tau;0)\, g\, E_i(0,\mathbf{0})\right]\right\rangle}{\left\langle \mathrm{Re\,Tr}[U(\tfrac{1}{T};0)]\right\rangle}$$



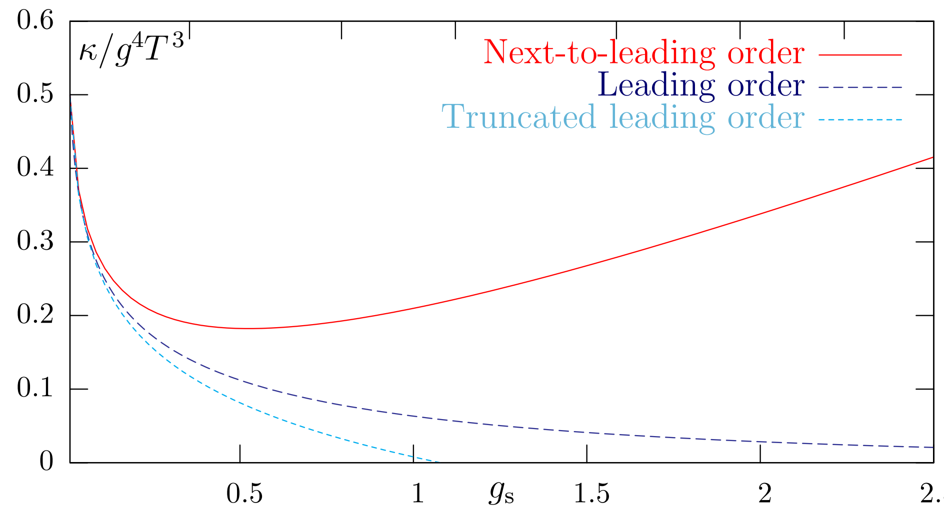
Smooth limit expected from NLO PT

[Caron-Huot, M.Laine, G.Moore, JHEP 0904 (2009) 053]

$$\kappa = \lim_{\omega\to 0} \frac{2T\rho_E(\omega)}{\omega}$$

NLO perturbative calculation:

[Caron-Huot, G. Moore, JHEP 0802 (2008) 081]



→ large correction towards strong interactions

→ non-perturbative lattice methods required

[L. Altenkort, A.M. Eller, OK, L. Mazur, G.D. Moore, PRD103 (2021) 014511]

Quenched Lattice QCD on large and fine isotropic lattices at $T \simeq 1.5\,T_c$

- standard Wilson gauge action
- fixed aspect ration $N_s/N_t = 4$, i.e. fixed physical volume $(2\text{fm})^3$

a (fm)	a^{-1} (GeV)	N_σ	N_τ	β	T/T_c	#conf.
0.0262	7.534	64	16	6.8736	1.51	10000
0.0215	9.187	80	20	7.0350	1.47	10000
0.0178	11.11	96	24	7.1920	1.48	10000
0.0140	14.14	120	30	7.3940	1.51	10000
0.0117	16.88	144	36	7.5440	1.50	10000

- perform the continuum limit, $a \to 0 \;\leftrightarrow\; N_t \to \infty$
- determine κ in the continuum using an Ansatz for the spectral fct. $\rho(\omega)$
- scale setting using r_0 and t_0 scale [A.Francis,OK,M.Laine, T.Neuhaus, H.Ohno, PRD91(2015)096002]
- previous study using multilevel with link-integration techniques to improve the signal [A.Francis, OK et al., PRD92(2015)116003]
- new study using gradient flow method to improve the signal [L. Altenkort, A.M. Eller, OK, L. Mazur, G.D. Moore, PRD103 (2021) 014511]

Gradient Flow as a noise reduction method

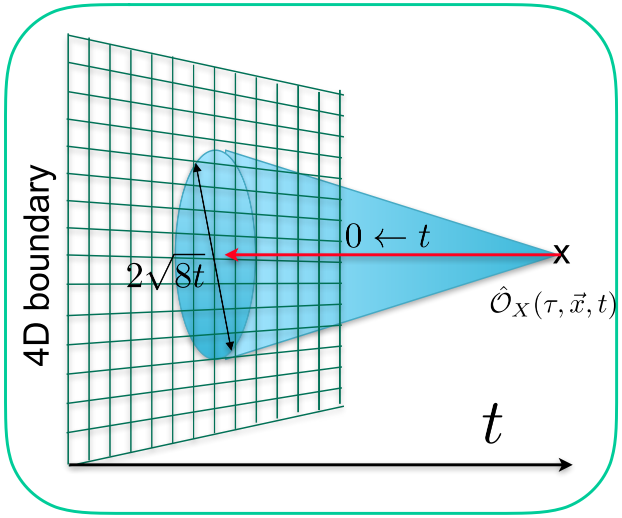
Noise reduction methods used in the quenched approximation not applicable in full QCD

Gradient flow - *diffusion* equation for the gauge fields along extra dimension, *flow-time t*

[M. Lüscher, 2010]

$$\frac{\partial}{\partial t} A_\mu(t,x) = -\frac{\partial S_{\rm YM}}{\partial A_\mu}$$

$$A_\mu(t=0,x) = A_\mu(x)$$



- continuous smearing of the gauge fields, effective smearing radius: $r_{\rm smear} \sim \sqrt{8t}$
- gauge fields become smooth and renormalized
- no UV divergences at finite flow-time *t* → operators of flowed fields are renormalized
- UV fluctuations effectively reduces → noise reduction technique
- Applicable in quenched and full QCD → first case study in quenched

What is the flow time dependence of correlation functions?

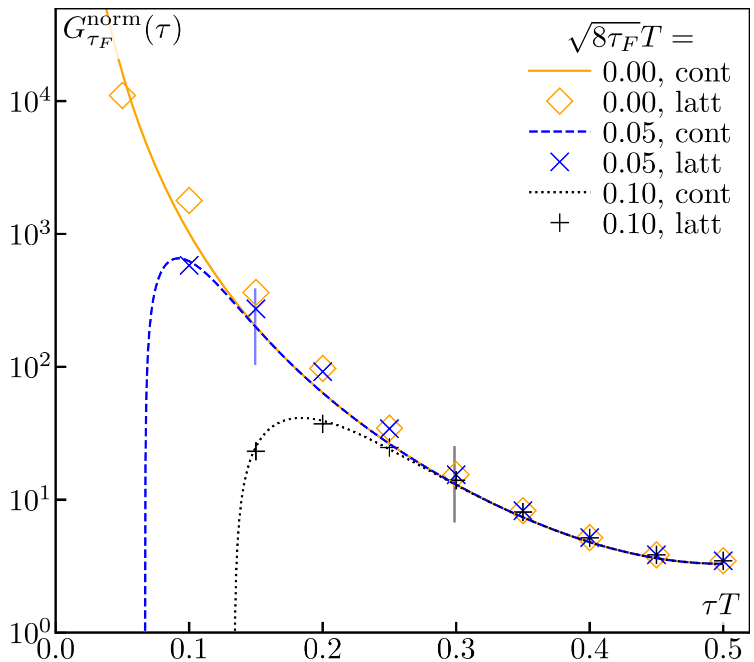
How to perform the continuum and t→0 limit correctly?

Gradient Flow method – perturbation theory vs. lattice QCD

LO perturbative limits

for the flow-time dependence:

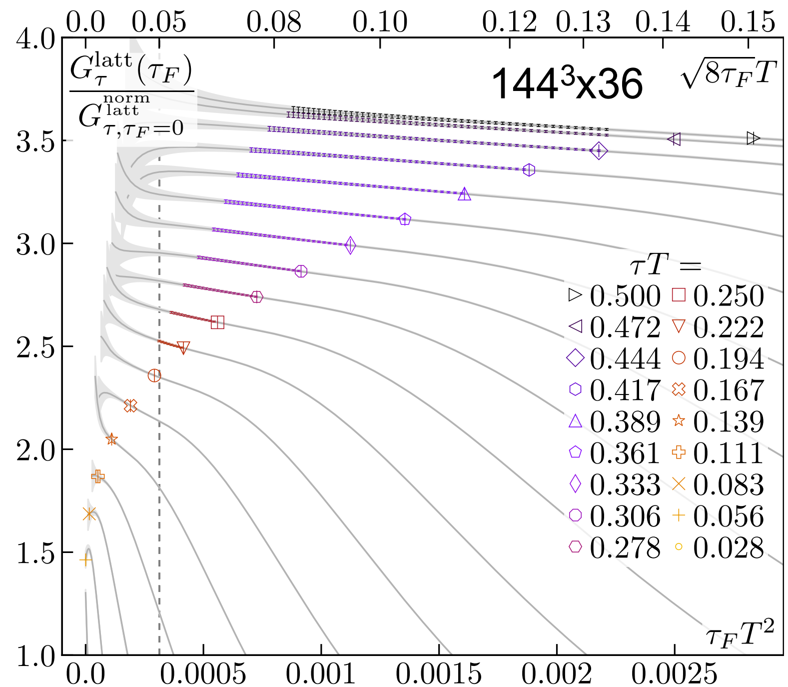
$$\tilde{\tau}_f < 0.1136(\tau T)^2$$



[A.M Eller, G.D. Moore, PRD97 (2018) 114507]

First lattice QCD results on the flow

dependence of the color-electric correlator:



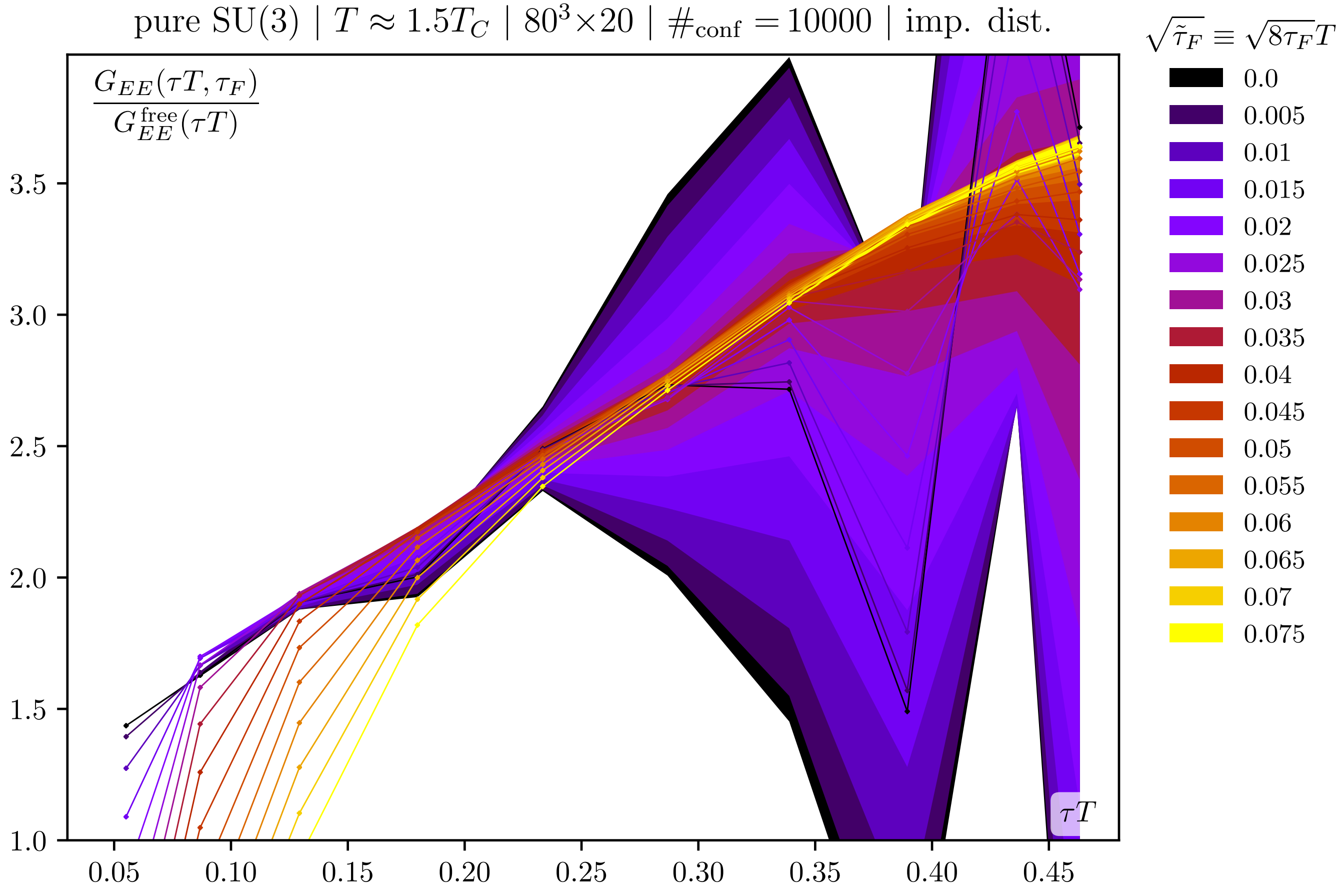
[L. Altenkort, A.M. Eller, OK, L. Mazur, G.D. Moore, PRD103 (2021) 014511]

Effective reduction of UV fluctuations → good noise reduction technique

Signal gets destroyed at flow times above the perturbative estimate

Linear behavior at intermediate flow times

Gradient Flow method – EE correlator

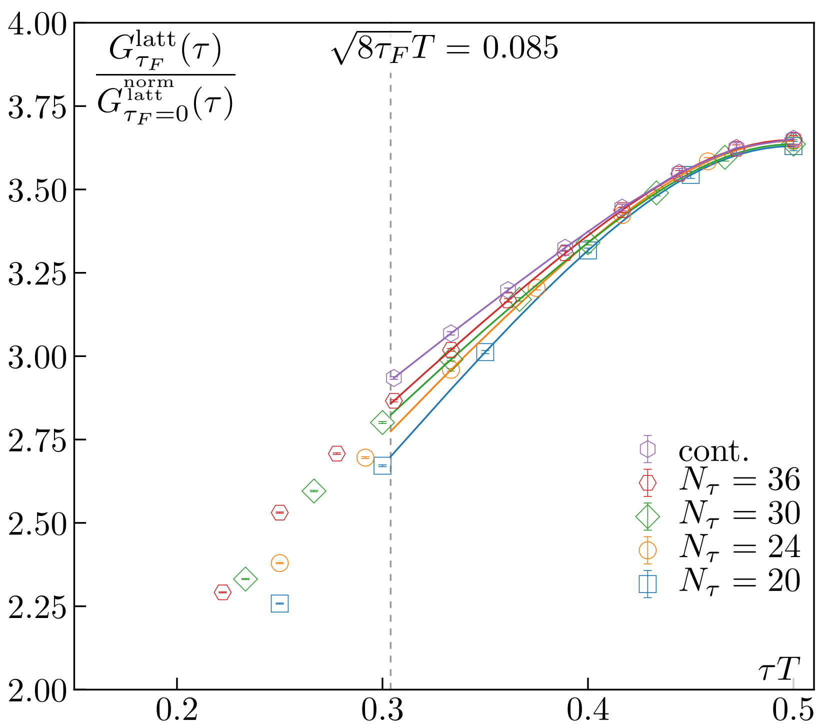


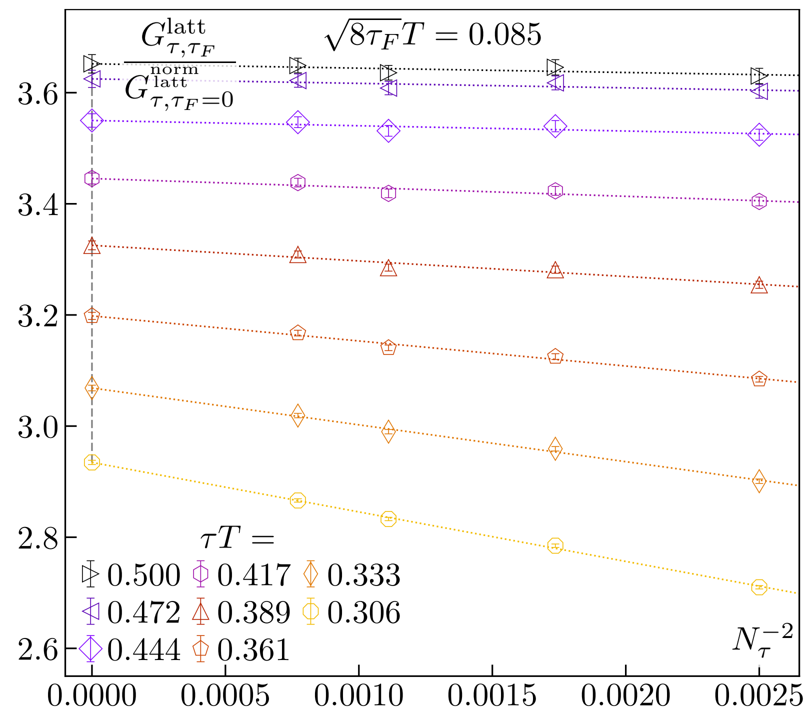
Effective reduction of UV fluctuations → good noise reduction technique

Signal gets destroyed at small distances → large-ω part of the spectral function modified

Strategy: 1) continuum limit at fixed physical flow times 2) flow time limit t→0

- signal to noise ratio strongly improves with flow
- operators become renormalized after some amount of flow
- cut-off effects get reduced with increasing flow time
- continuum limit, *a→0 (N_t→ ∞)*, at fixed physical flow time:





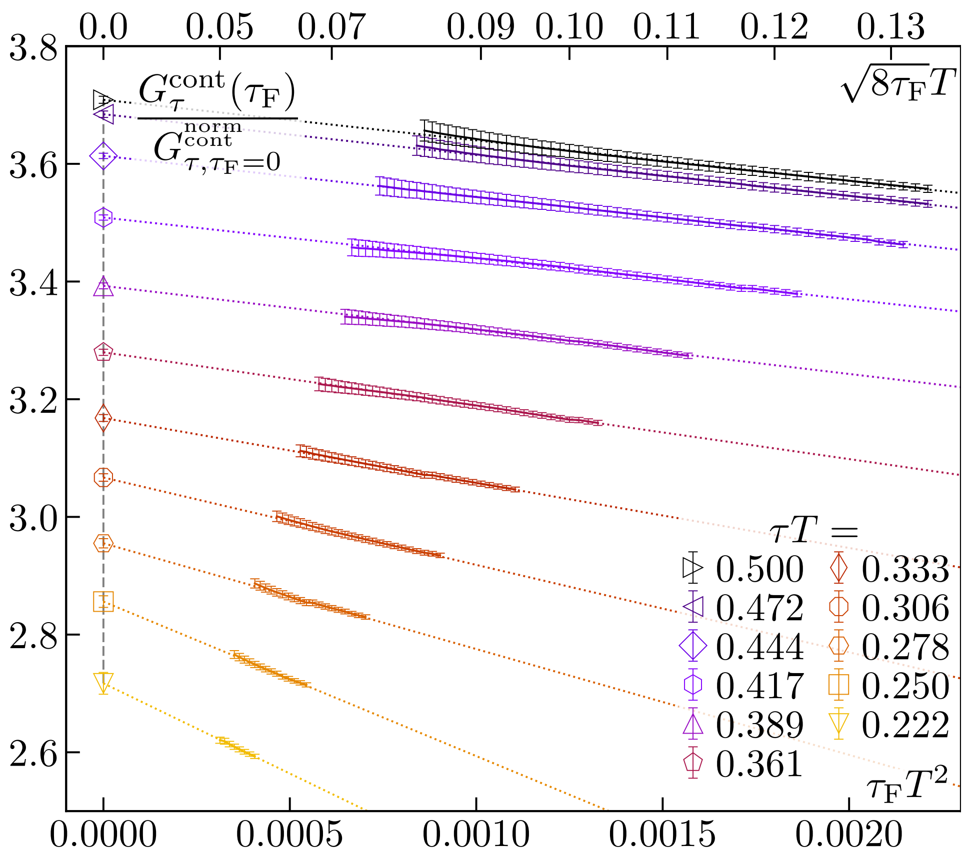
- well defined continuum correlators for different finite flow times

next step: flow time to zero extrapolation of continuum correlators

Continuum limit, *a→0 (N_t→ ∞),*
at fixed physical flow time:

followed by

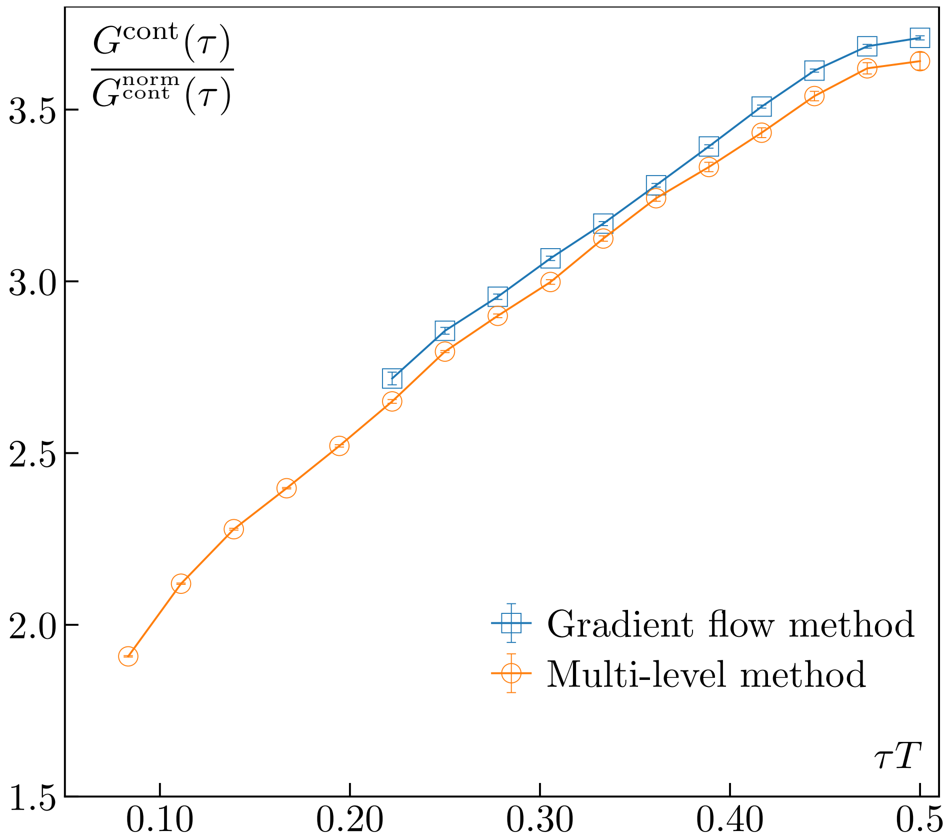
Flow time limit, *t→0,*
for each distance:



→ well defined continuum and flow time extrapolation

→ well defined renormalized correlation function

Comparison of gradient flow and multi-level method:



gradient flow: N_t=16,20,24,30,36

[L. Altenkort, A.M. Eller, OK, L. Mazur, G.D. Moore, PRD103 (2021) 014511]

multi-level: N_t=20,24,36,48

with an update in the analysis and perturbative renormalization of data from [A.Francis, OK et al., PRD92(2015)116003]

Comparable τT dependence at large distances

Uncertainty in the renormalization resolved by gradient flow

Heavy quark momentum diffusion coefficient expected to be slightly larger

Extension to full QCD possible using gradient flow method

[L. Altenkort, A.M. Eller, OK, L. Mazur, G.D. Moore, PRD103 (2021) 014511 and arXiv:2109.11303]

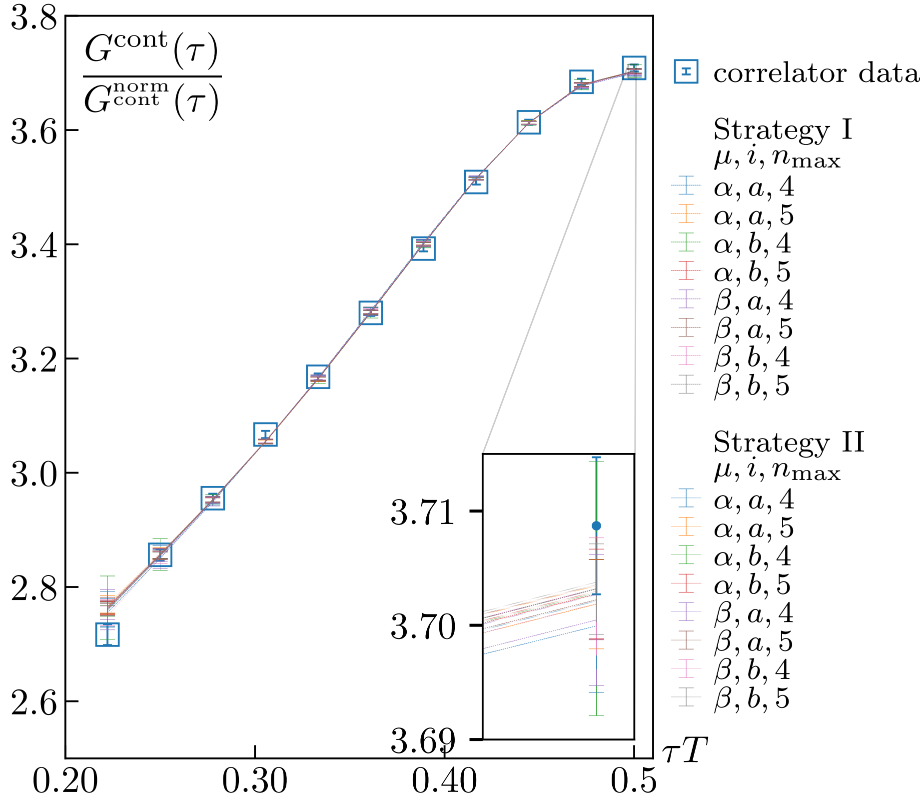
Using continuum extrapolated lattice correlators

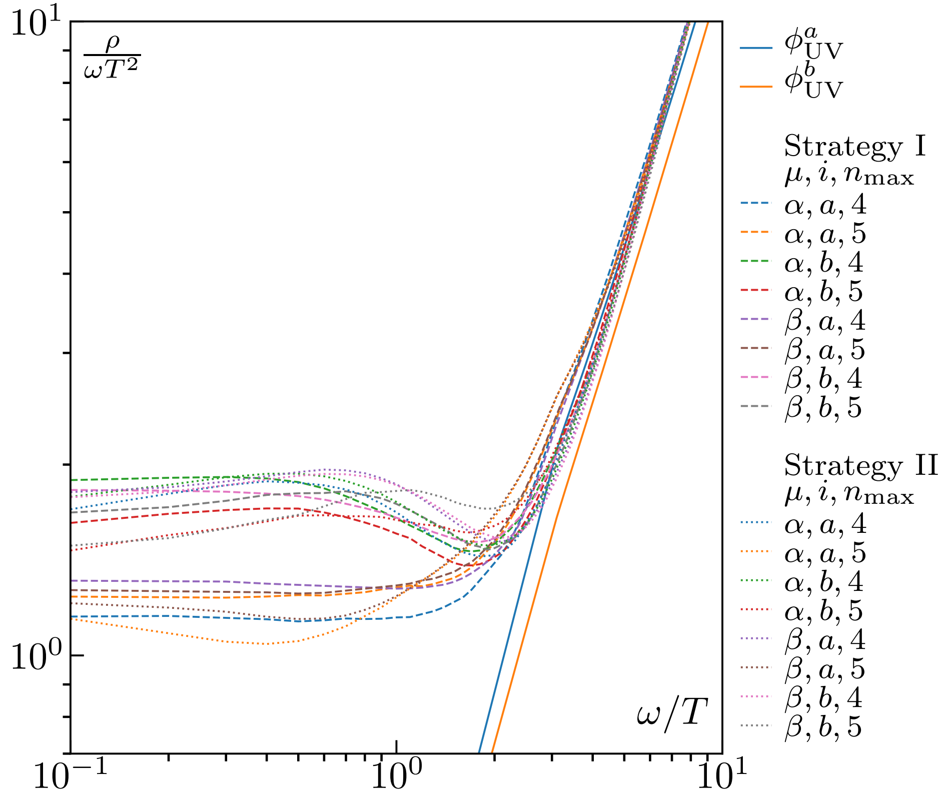
Spectral function models with correct asymptotic behavior

$$\rho_{\mathrm{UV}}(\omega) = \frac{g^2(\bar{\mu}_\omega) C_F \omega^3}{6\pi}$$

modeling corrections to ρ_{IR} by a power series in ω

$$\rho_{\mathrm{IR}}(\omega) = \frac{\kappa\omega}{2T}$$

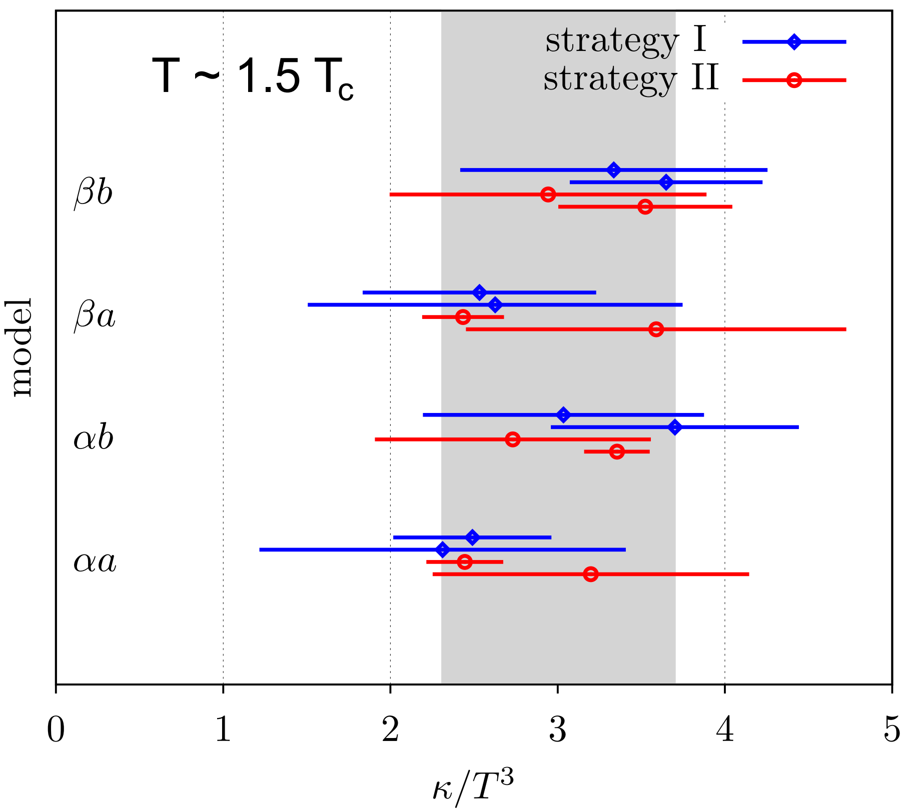




$$G_{\mathrm{model}}(\tau) \equiv \int_0^\infty \frac{\mathrm{d}\omega}{\pi} \rho_{\mathrm{model}}(\omega) \frac{\cosh\left(\frac{1}{2} - \tau T\right)\frac{\omega}{T}}{\sinh\frac{\omega}{2T}}$$

$$\kappa/T^3 = \lim_{\omega\to 0} \frac{2T\rho_{\mathrm{E}}(\omega)}{\omega}$$

[L. Altenkort, A.M. Eller, OK, L. Mazur, G.D. Moore, PRD103 (2021) 014511]



Detailed analysis of systematic uncertainties

→ **continuum estimate of κ :**

$$\kappa/T^3 = \lim_{\omega\to 0} \frac{2T\rho_E(\omega)}{\omega} = 2.31\ldots 3.70$$

Related to diffusion coefficient D and drag coefficient η_D (in the non-relativistic limit)

$$2\pi T D = 4\pi \frac{T^3}{\kappa} = 3.40\ldots 5.44$$

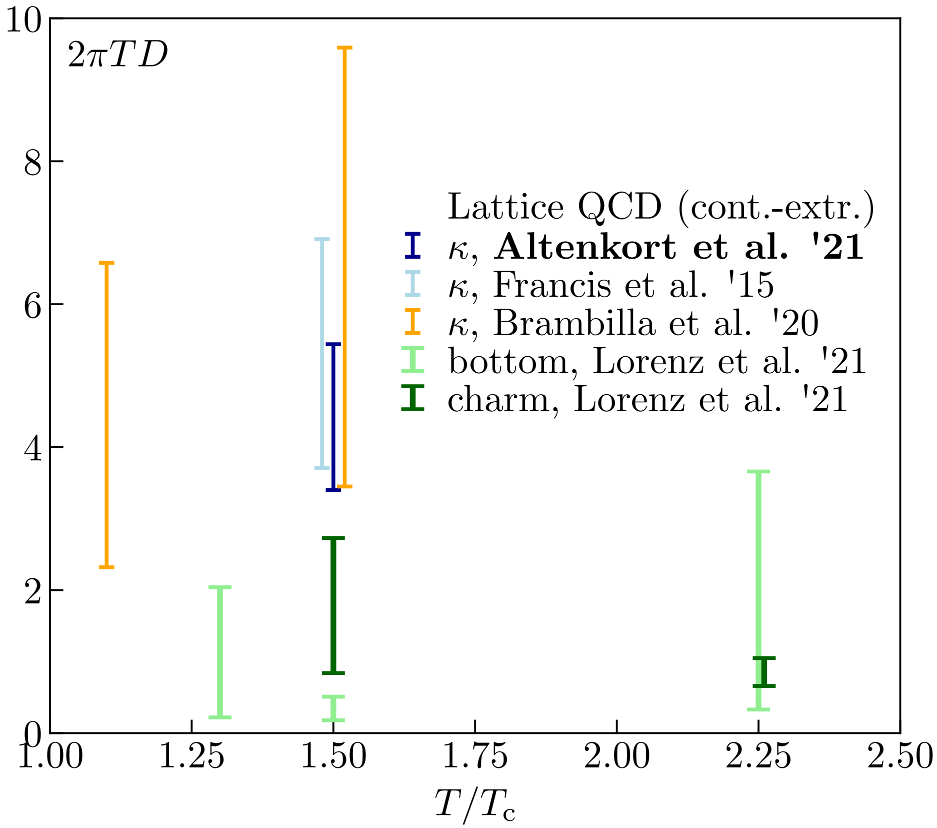
$$\eta_D = \frac{\kappa}{2M_{kin}T}\left(1 + O\left(\frac{\alpha_s^{3/2}T}{M_{kin}}\right)\right)$$

time scale associated with the kinetic equilibration of heavy quarks:

$$\tau_{\rm kin} = \eta_D^{-1} = (1.63\ \ldots\ 2.61)\left(\frac{T_c}{T}\right)^2\left(\frac{M}{1.5\rm GeV}\right)\ \rm fm/c$$

→ **close to T_c, $\tau_{kin} \simeq$ 1fm/c and therefore charm quark kinetic equilibration appears to be almost as fast as that of light partons.**

Recent results for $2\pi TD$ based on continuum extrapolated lattice data



heavy quark mass limit

[L. Altenkort, A.M. Eller, OK, L. Mazur, G.D. Moore, PRD103 (2021) 014511]

[A.Francis, OK, M. Laine, T. Neuhaus, H. Ohno, PRD92(2015)116003]

[N. Brambilla, V. Leino, P. Petreczky, A. Vairo, PRD102 (2020)]

finite quark mass: bottom and charm

[H.T. Ding, O. Kaczmarek, A.-L. Lorenz, H. Ohno, H. Sandmeyer, H.T. Shu, arXiv:2108.13693]

Next steps: understand the quark mass dependence

- finite mass correction to κ is encoded in a color-magnetic correlator
  [A. Bouttefeux, M. Laine, JHEP12 (2020) 150]
- but renormalization of the color-magnetic correlator needs to be solved
  [M. Laine, JHEP06 (2021) 139]

Conclusion – Disclaimer - Outlook

Well defined methodology to extract spectral and transport properties from lattice QCD

Continuum extrapolated correlators from quenched lattice QCD are

well described by perturbative model spectral functions down to T $\approx$ T_c

for observable with an external scale (mass, momentum) $\gtrsim \pi$T

All results in this talk were obtained in the quenched approximation

What may change when going to full QCD?

$$\Lambda_{\overline{\mathrm{MS}}}|_{N_f=0} \approx 255\mathrm{MeV} \qquad \Lambda_{\overline{\mathrm{MS}}}|_{N_f=3} \approx 340\mathrm{MeV}$$

$$T_c|_{N_f=0} \approx 1.24\Lambda_{\overline{\mathrm{MS}}}|_{N_f=0} \qquad T_c|_{N_f=3} \approx 0.45\Lambda_{\overline{\mathrm{MS}}}|_{N_f=3}$$

$$\alpha_s^{EQCD}|_{T\simeq T_c} \simeq 0.2 \qquad \alpha_s^{EQCD}|_{T\simeq T_c} > 0.3$$

1^{st} order deconfinement transition **chiral crossover transition**

Physics may become more non-perturbative, more interesting, more complicated…

Quenched theory is a nice playground but full QCD studies crucial!

Stay tuned in the next years for the first results in full QCD...