Topological Inflation and Graceful Exit

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A. M. and T.Prokopec, arxiv[1505.xxxxx]
D. Glavan, A. M., T. Prokopec, arxiv[1504.07782]

Basic idea

- Vilenkin and Linde (1994): topological defects can serve as sites for inflation if size of a defect is comparable or greater than the Hubble radius
- Monopole size δ_0 (gradient energy proportional to the potential en.):

$$\left(\frac{\phi_0}{\delta_0}\right)^2 \propto V(0)\,,$$

- The horizon size corresponding to the vacuum energy $V(0)$:

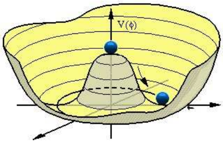
$$H^{-1} = M_P\sqrt{3/V(0)}$$

  - For $\delta_0 > H^{-1}$ ($\phi_0 > M_P$): inflation will start in the false vacuum (monopole center)
  - For $\delta_0 \ll H^{-1}$ ($\phi_0 \ll M_P$): gravity does not considerably affect monopole's structure

- *Advantages*: inflation with generic initial conditions
- *Disadvantages*: eternal inflation (once started will never end)
- *Solution*: introduce another scalar field

Topological defects

- Topological defects, i.e. global monopoles are created during phase transitions by the Kibble mechanism; symmetry is spontaneously broken from $O(3)$ to $O(2)$ by VEV ϕ_0



- Action that governs dynamics of global monopoles

$$S_\phi = \int d^4x\sqrt{-g}\left(-\frac{1}{2}g^{\mu\nu}(\partial_\mu\phi^a)(\partial_\nu\phi^a) - V(\phi^a)\right),$$

- Higgs type of $O(3)$ symmetric potential

$$V(\phi^a) = \frac{\lambda}{4}\left(\phi^a\phi^a - \phi_0^2\right)^2 ,$$

- The simplest solution is a global monopole

$$\vec{\phi}(t,\vec{r}\,) = \phi(r)\left(\sin\theta\,\cos\varphi, \sin\theta\,\sin\varphi, \cos\theta\right)^T ,$$

- Inflation starts if:

$$\delta_0 > H^{-1}, \quad \text{with} \quad \delta_0 \propto \frac{2}{\sqrt{\lambda}\phi_0}$$

Nonminimal coupling - Jordan to Einstein frame

- Action for the scalar field in Jordan frame:

$$S_\psi = \int d^4x\sqrt{-g}\left(\frac{1}{2}RF(\psi) - \frac{1}{2}(\partial\psi)^2\right) , \quad F(\psi) = M_P^2 - \xi_2\psi^2 - \xi_4\psi^4/M_P^2 .$$

- For studying inflation, useful to transform to Einstein frame:

$$g_{E\mu\nu} = \frac{F(\psi)}{M_P^2} g_{\mu\nu} , \quad d\psi_E = \frac{M_P}{F(\psi)}\sqrt{F(\psi) + \frac{3}{2}\left(\frac{dF(\psi)}{d\psi}\right)^2} d\psi , \quad \phi_E = \frac{\phi}{F(\psi)/M_P} .$$

- Action in Einstein frame:

$$S_E = \int d^4x\sqrt{-g_E}\Big(\frac{1}{2}M_P^2 R_E - \frac{1}{2}(\partial\phi_E) - \frac{1}{2}(\partial\psi_E) - V_E(\phi_E, \psi_E)\Big) .$$

- Potential in Einstein frame:

$$\boxed{V_E(\phi_E, \psi_E) = \frac{\lambda}{4}\left(\phi_E^2 - \frac{\phi_0^2}{(F(\psi)/M_P^2)^2}\right)^2 .}$$

- We study homogeneous case in which the monopole size is greater than the Hubble radius:

$$V_E(\phi_E, \psi_E) \simeq V_E(\psi) \simeq \frac{\lambda\phi_0^4}{4} \frac{1}{(F(\psi)/M_P^2)^4} .$$

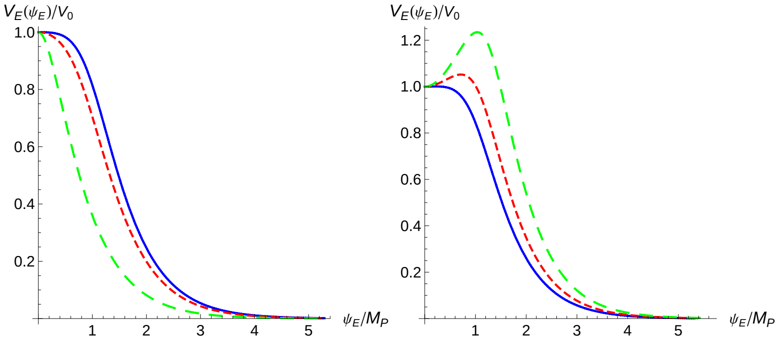


Figure: $\xi_4 = -0.1$; *Left panel:* $\xi_2 = -0.01$ (blue solid), $\xi_2 = -0.1$ (red dashes) and $\xi_2 = -1$ (long green dashes). *Right panel:* $\xi_2 = 0.01$ (blue solid), $\xi_2 = 0.1$ (red dashes) and $\xi_2 = 0.2$ (long green dashes). Note that when $\xi_2 < 0$, V_E has a local maximum at $\phi_E = 0$ ($\phi = 0$), while for $\xi_2 > 0$, V_E has a local minimum at $\phi_E = 0$ and two local maxima at some $\phi_E = \pm\phi_{E0} \neq 0$.

Slow roll approximation

- Equation of motion and the Einstein equations in slow-roll approximation ($\ddot{\psi}_E \ll 3H_E\dot{\psi}_E$, $\dot{\psi}_E^2 \ll V(\psi_E)$) are:

$$3H_E\dot{\psi}_E = -V'(\psi_E)\,, \quad H_E^2 = \frac{1}{3M_P^2}\,V_E(\psi_E)\,, \quad \dot{H}_E = -\frac{\dot{\psi}_E^2}{2M_P^2}\,.$$

- The slow-roll parameters:

$$\epsilon_E(\psi) = -\dot{H}_E/H_E^2\,, \quad \eta_E(\psi) = \dot{\epsilon}_E/(\epsilon_E H_E)\,, \quad \xi_E(\psi) = \dot{\eta}_E/(\eta_E H_E)$$

- The number of e-folds is:

$$N(\psi) = \int_t^{t_e} H_E(\tilde{t}\,)d\tilde{t} = \frac{3}{4}\ln\left(\frac{F(\tilde{\psi})}{M_{\rm P}^2}\right) + \frac{1}{8\xi_2}\ln\left(\frac{M_{\rm P}^2 F'(\tilde{\psi})}{\tilde{\psi}^3}\right)\Bigg|_{\psi}^{\psi_e}\Bigg|\,,$$

ψ_e: end of inflation $\epsilon_E(\psi_e) = 1$.

- Why do we need $\xi_4 \neq 0$?
  - for $\xi_4 = 0$:

$$\boxed{\epsilon_E = \frac{-8\xi_2}{1 - 6\xi_2}}$$

  - During inflation: $\epsilon_E \ll 1 \rightarrow |\xi_2| \ll 1$,
  - After inflation: $\epsilon_E \rightarrow 4/3 \rightarrow \xi_2 \rightarrow -\infty$
- Important $\boxed{\epsilon_E \rightarrow 4/3}$ regardless of the shape of $F(\psi)$

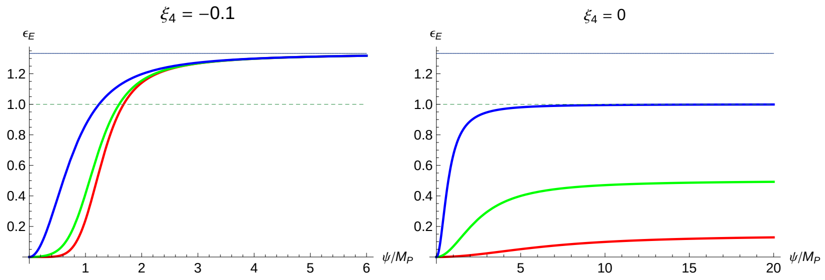


Figure: Red: $\xi_2 = -0.002$; Green: $\xi_2 = -0.1$; Blue: $\xi_2 = -0.5$.

Spectral index and tensor to scalar ratio

- The spectral indices n_s and n_t: variation of the scalar and tensor perturbations $\Delta_s^2(k)$ and $\Delta_t^2(k)$ with respect to k (at the first horizon crossing during inflation $k = k_* = Ha$):

$$\begin{aligned} \Delta_s^2(k) &= \Delta_s^2(k_*)\left(\frac{k}{k_*}\right)^{n_s-1}, \qquad \Delta_s^2(k_*) = \frac{H_E^2}{8\pi^2\epsilon_E M_{\rm P}^2}, \\ \Delta_t^2(k) &= \Delta_t^2(k_*)\left(\frac{k}{k_*}\right)^{n_t}, \qquad \Delta_t^2(k_*) = \frac{2H_E^2}{\pi^2 M_{\rm P}^2}. \end{aligned} \tag{1}$$

- To the leading order in slow roll approximation, spectral indices are

$$n_s \simeq -2\epsilon_E - \eta_E\,, \quad n_t \simeq -2\epsilon_E\,,$$

and tensor to scalar ratio r and running of the spectral index n_s are:

$$r(k_*) = 16\epsilon_E\,, \quad \alpha(k_*) = -(2\epsilon_E + \xi_E)\eta_E\,.$$

Results

- n_s sensitive on ξ_2:

$$\boxed{n_s \simeq 0.955} \quad \text{for} \quad \xi_2 \simeq -0.002 \quad \text{and} \quad \xi_4 \simeq -0.1$$

- r sensitive on ξ_4:

$$\boxed{r \simeq \frac{10^{-6}}{|\xi_4|}}, \quad \text{for} \quad r \simeq 10^{-2} \quad \text{we need} \quad \xi_4 \simeq -10^{-4} \quad \text{price: lower } n_s$$

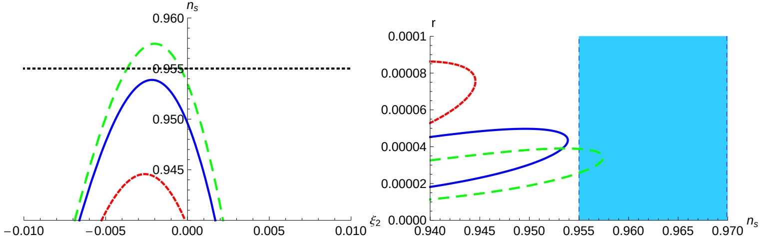


Figure: Left: $\xi_4 = -0.1$; Right: $\xi_4 = -0.02$. Red: $N = 50$; Blue: $N = 60$; Green: $N = 65$.

- Running of the spectral index $\alpha = dn_s/d\ln k$ is negative and $\alpha \simeq -10^{-3}$
- Number of e-folds for $\psi = 10^{-6}$ is always $N_0 > 60$

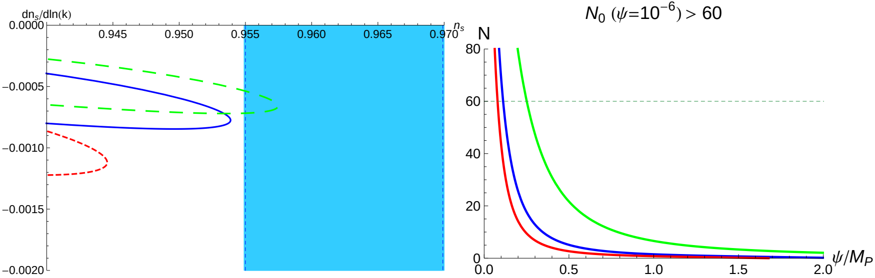


Figure: **Left**: $\xi_4 = -0.02$; Red: $N = 50$; Blue: $N = 60$; Green: $N = 65$. **Right**: $\xi_2 = -0.002$; Red: $\xi_4 = -0.1$, Green: $\xi_4 = -0.05$, Blue: $\xi_4 = -0.01$

Monopol during inflation

- EOM for monopole if size greater than Hubble radius (neglect gradient terms)

$$V(\phi) \simeq V(0) - \frac{1}{2}\mu^2\phi^2\,, \quad \mu^2 = \lambda\phi_0^2 \propto \frac{1}{\delta_0^2}\,,$$

$$\left(\frac{d^2}{dt^2} + 3H\frac{d}{dt} - \mu^2\right)\phi(t) \simeq 0\,, \tag{2}$$

$$\phi \propto \mathrm{e}^{\lambda t} \to \lambda^2 + 3H_I\lambda - \mu^2 = 0\,. \tag{3}$$

- The positive solution is the relevant solution, since it gives the growing solution for ϕ that eventually dominates:

$$\lambda = \lambda_+ = \frac{3H_I}{2}\left[\sqrt{1 + \frac{4\mu^2}{9H_I^2}} - 1\right] \simeq \frac{\mu^2}{3H_I} \ll H_I \tag{4}$$

$$\boxed{\frac{R_c}{R_H} \sim \frac{H_I}{\mu}\exp\left(\frac{\mu^2}{3H_I}t\right)}\,, \tag{5}$$

where $N_I = H_I t$ is the number of e-folds and $R_H = 1/H_I$ is the Hubble radius.

Monopole during radiation

- After inflation, due to preheating, the Universe gets filled with relativistic particles, and the Hubble rate becomes, $H \simeq 1/(2t)$, corresponding to the scale factor, $a(t) \propto t^{1/2}$.

$$\left(\frac{d^2}{dz^2} + \frac{3}{2z}\frac{d}{dz} - \frac{1}{4}\right)\phi(z) \simeq 0\,, \quad \frac{z}{2} = \mu t = \frac{\mu}{2H(t)}$$

- The general solution is a linear combination of two modified Bessel functions of the first kind, $I_{1/4}(z/2)/z^{1/4}$ and $I_{-1/4}(z/2)/z^{1/4}$; for $\mu \ll H$ (small z expansion):

$$\phi_1(z) = \frac{1}{\Gamma(5/4)}[1 + \mathcal{O}(z^2)]\,, \quad \phi_2(z) = \frac{1}{\Gamma(3/4)\sqrt{z/2}}[1 + \mathcal{O}(z^2)]\,.$$

Even when the second solution dominates, when compared with the Hubble radius, the monopole core contracts during radiation

$$\boxed{\frac{R_c}{R_H} \propto \mathrm{e}^{-2N_R}}\,, \quad N_R \simeq \frac{\mu^2}{6H_I^2}N_I + \frac{1}{2}\ln\left(\frac{H_I}{\mu}\right)\,, \qquad (6)$$

N_R is the number of e-folds of radiation, after which the monopole will re-enter the Hubble radius, and start rapidly collapsing to its Minkowski space physical core size.

Summary

- Quite generically the model produces a red spectrum of scalar cosmological perturbations and a small amount of gravitational radiation. With a suitable choice of the nonminimal couplings, the spectral slope can be as large as $n_s \simeq 0.955$, which is about one standard deviation away from the central value measured by the Planck satellite.
- The model can be ruled out by future measurements if any of the following is observed:
  - (a) the spectral index of scalar perturbations is $n_s > 0.960$;
  - (b) the amplitude of tensor perturbations is above about $r \sim 10^{-2}$;
  - (c) the running of the spectral index of scalar perturbations is positive.
- Graceful exit: monopole expands during inflation and shrinks during radiation. Such a monopole represents today a tiny contribution to the total energy density, and hence its effect on the Universe's expansion can be neglected.
- Future work: investigate whether the monopole can induce any inhomogeneous perturbations that are observable today

Thank you for your attention !