Forecasting cosmological parameters of dark energy models using the nonlinear power spectrum

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Outline and Motivation

- Future LSS probes like Euclid, will measure positions, shapes and redshifts of ~10^7 galaxies, giving us valuable information about the small scales in their 2-point correlation function and the power spectrum (PS).
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- We can either test these signatures by using N-body simulations and fitting functions or by calculating perturbatively the power spectrum.

- **Currently working on these two approaches:**

- Coupled Quintessence for which we have high quality N-body simulations available[#].
- Horndeski theories of DE where we can use resummation techniques such as eRPT to estimate the PS at mildly nonlinear scales[&].

Short review on coupled quintessence

- Dark Matter is coupled to a scalar field through its mass

$$\mathrm{S}=\int d^4x\,\sqrt{-g}\left[\frac{1}{16\pi G}R+-\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi-V(\phi)-m(\phi)\overline{\psi}\psi+\mathcal{L}_{kin,\psi}\right]$$

- Mass of DM particles depends on value of field, this defines the coupling strength as:

$$\mathrm{Q}(\phi)=-\frac{d\ln m(\phi)}{d\phi}$$

- Baryons are not coupled, radiation is traceless. Total $T_{\mu\nu}$ is conserved:

$$\nabla_\mu T^\mu_{\nu(c)}=+Q_c(\phi)T_{(c)}\nabla_\nu\phi\ ,\ \nabla_\mu T^\mu_{\nu(b)}=0\ ,\ \nabla_\mu T^\mu_{\nu(\phi)}=-Q_c(\phi)T_{(c)}\nabla_\nu\phi$$

- We use an exponential potential and assume a constant coupling:

$$\mathrm{V}(\phi)=V_0e^{-\alpha\phi}\qquad\qquad\beta=\sqrt{3/2}Q$$

- Interesting background tracking solutions, helps to alleviate the coincidence problem and predicts characteristic features at the perturbative level.

CQ at the perturbation level

- Effective gravitational constant affecting only DM particles:

$$\tilde{G}_{CQ} = G_N(1 + 2\beta^2)$$

- Modified Hubble friction term in the Euler equation:

$$\tilde{H}\mathbf{v} = H(1 - \beta\dot{\phi}/H)\mathbf{v}$$

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This has been implemented in N-body simulations, yielding interesting results for modified structure formation:

- Gravitational bias between baryons and DM at the linear level, decreasing baryon fraction in halos.
- Increase of number density of high-mass objects compared to ΛCDM at all z.
- Lower concentration of halos, emptier voids
- Modifications of the small scale power spectrum

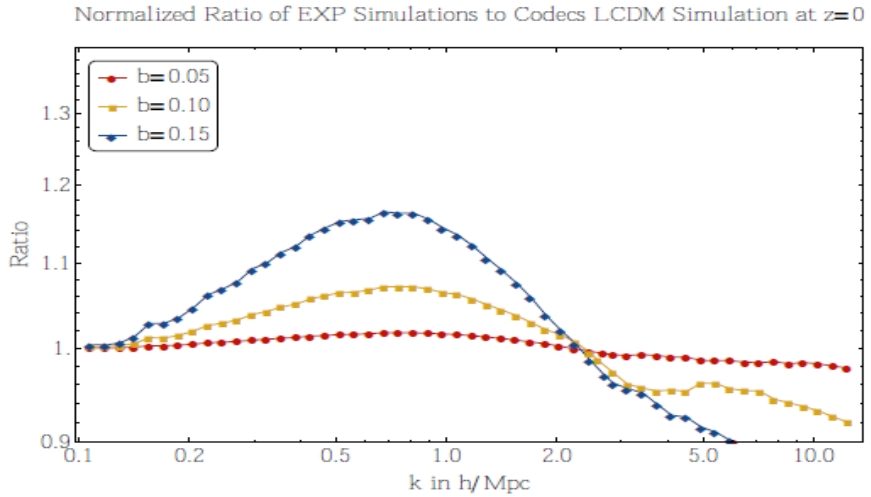
The nonlinear power spectrum in CQ

- Increasing the coupling, enhances the normalization of the Power Spectrum as shown in the CoDECS simulations.

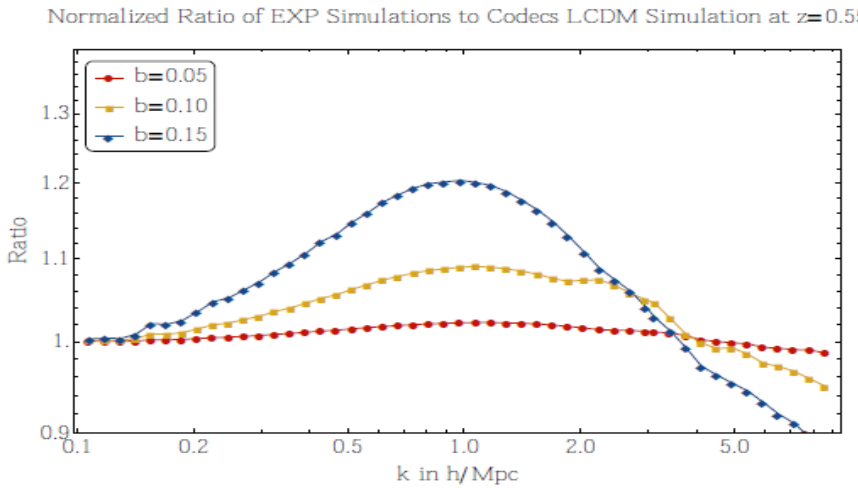
- Degenerate with an increase of the parameter $\sigma 8$, since in these models, density perturbations are normalized at the CMB.

- At nonlinear scales, a characteristic deviation breaks the degeneracy and can be explained qualitatively by the modified friction term which affects the virialization regime.

The nonlinear power spectrum in CQ



(a) Ratio of EXP simulations (red: $\beta = 0.05$, yellow: $\beta = 0.10$, blue:$\beta = 0.15$) at redshift $z = 0$.



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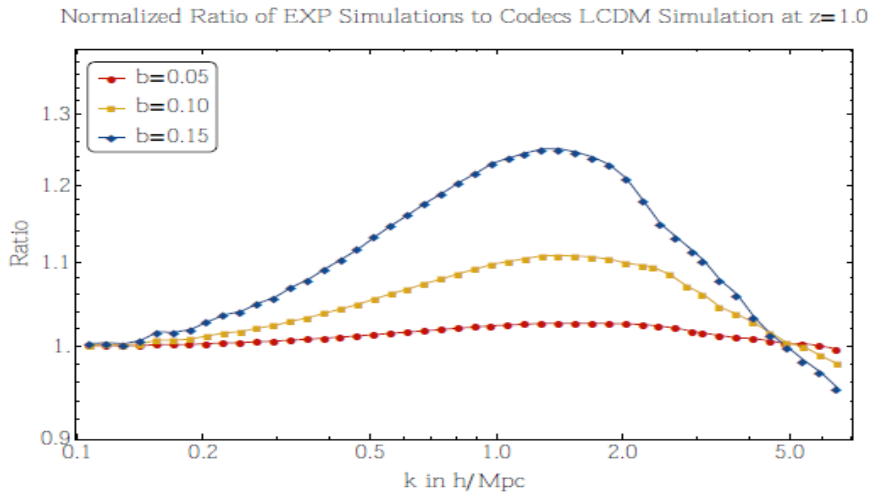
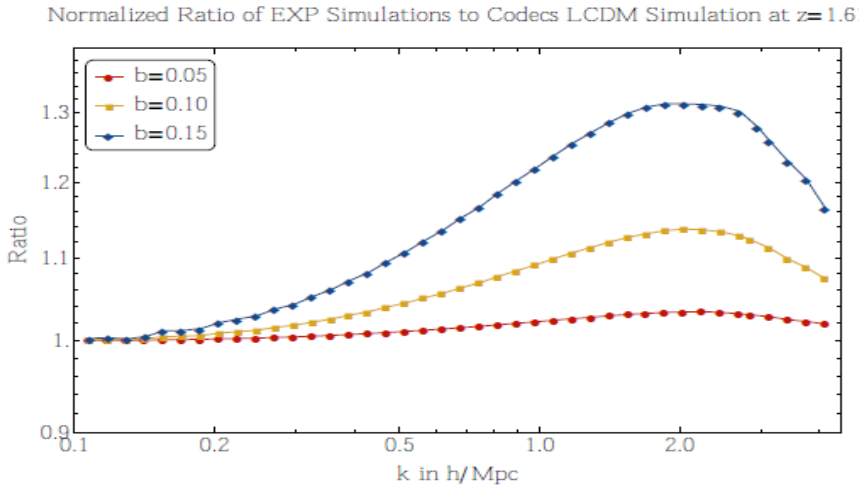


(c) Ratio of EXP simulations (red: $\beta = 0.05$, yellow: $\beta = 0.10$, blue:$\beta = 0.15$) at redshift $z = 1.0$.

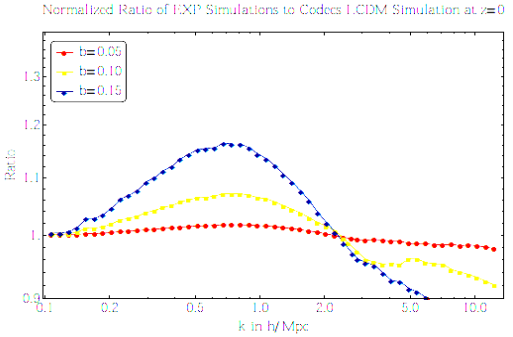


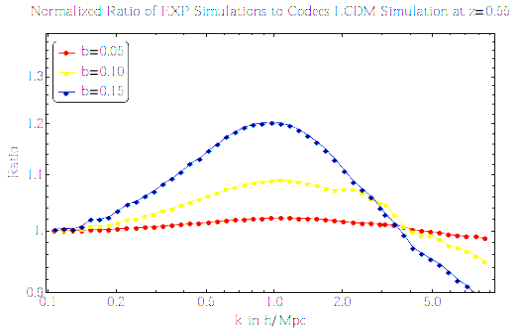
(d) Ratio of EXP simulations (blue: $\beta = 0.05$, magenta: $\beta = 0.10$, yellow:$\beta = 0.15$) at redshift $z = 1.61$.

The nonlinear power spectrum in CQ

- After normalizing to the same $\sigma 8$, compared to $\Lambda\ CDM$ there is a “bump” that increases its amplitude with increasing coupling while its maximum locates at smaller scales for higher redshifts.

We want to use this characteristic feature to test how well a future mission like Euclid is able to measure it and distinguish this particular CQ from other classes of models.





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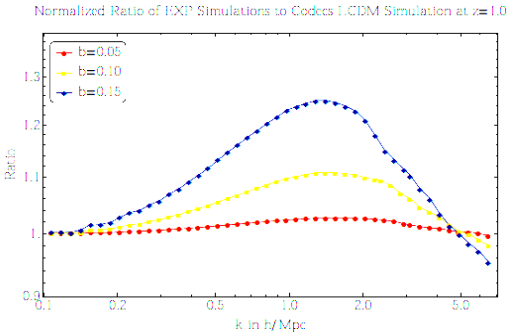

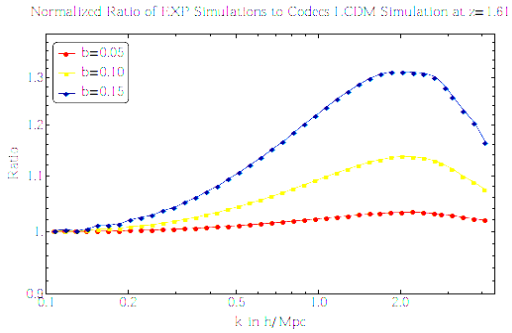

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What do we need to forecast using the nonlinear power spectrum?

- We cannot run N-body simulations varying all interesting cosmological parameters.
- Perturbation Theory or Effective Theories of LSS do not reach yet the interesting range in k for this particular case. Furthermore it is complicated to include non-ΛCDM models. (see later in talk)
- We need fitting functions that can be varied w.r.t. cosmological parameters.

Fitting functions

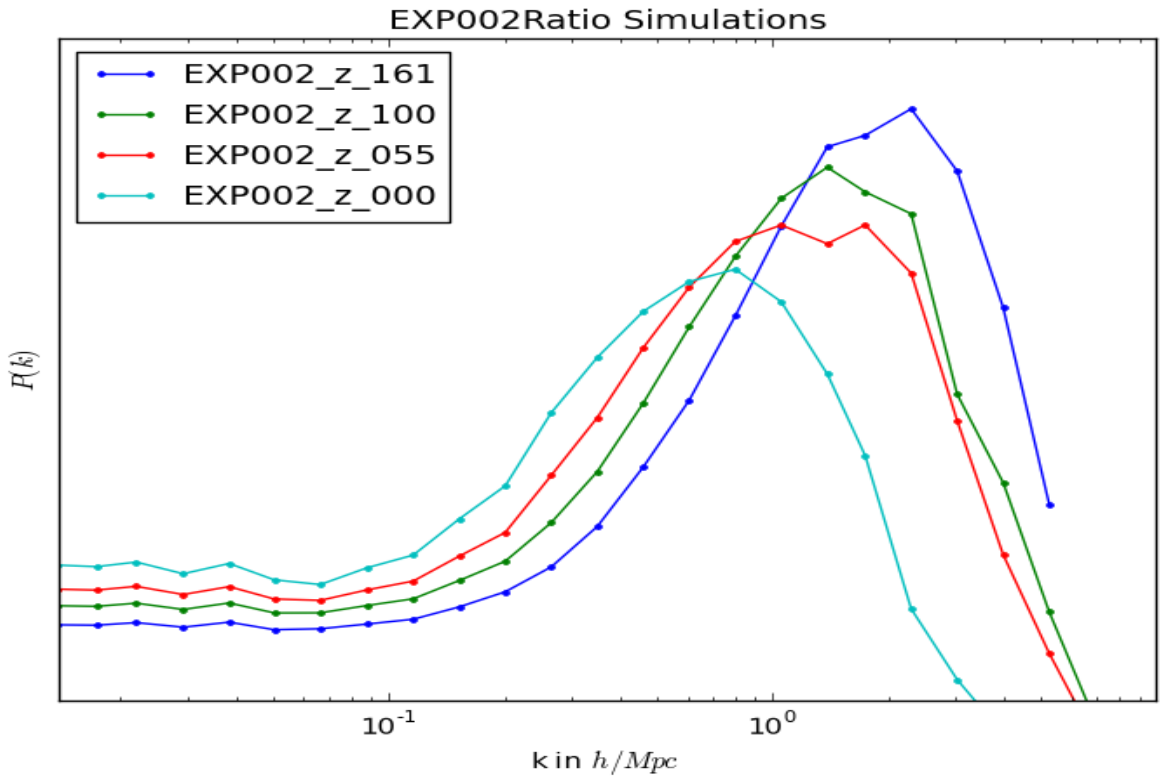
- Use CoDECS[1] EXP simulations with three different couplings.

Model	Potential	α	β_0	β_1	Scalar field normalization	Potential normalization	$w_\phi(z=0)$	$\mathcal{A}_s(z_{\rm CMB})$	$\sigma_8(z=0)$
ΛCDM	$V(\phi) = A$	–	–	–	–	$A = 0.0219$	-1.0	2.42×10^{-9}	0.809
EXP001	$V(\phi) = Ae^{-\alpha\phi}$	0.08	0.05	0	$\phi(z=0) = 0$	$A = 0.0218$	-0.997	2.42×10^{-9}	0.825
EXP002	$V(\phi) = Ae^{-\alpha\phi}$	0.08	0.1	0	$\phi(z=0) = 0$	$A = 0.0218$	-0.995	2.42×10^{-9}	0.875
EXP003	$V(\phi) = Ae^{-\alpha\phi}$	0.08	0.15	0	$\phi(z=0) = 0$	$A = 0.0218$	-0.992	2.42×10^{-9}	0.967
EXP008e3	$V(\phi) = Ae^{-\alpha\phi}$	0.08	0.4	3	$\phi(z=0) = 0$	$A = 0.0217$	-0.982	2.42×10^{-9}	0.895
SUGRA003	$V(\phi) = A\phi^{-\alpha}e^{\phi^2/2}$	2.15	-0.15	0	$\phi(z\to\infty) = \sqrt{\alpha}$	$A = 0.0202$	-0.901	2.42×10^{-9}	0.806

1: Baldi (2011) The CoDECS project

Fitting functions

- Use CoDECS[1] EXP simulations with three different couplings.
- We need to estimate the nonlinear PS with much more accuracy than previously. We developed an automatic method that corrects numerical anomalies beyond the Nyquist frequency.



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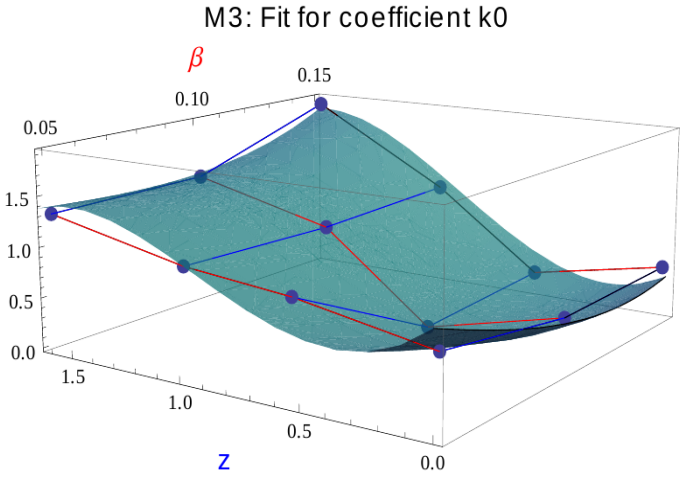
- Multidimensional nonlinear fit: Tested 8 models for goodness of fit, each with 5 coefficients depending polynomially on the parameters.

Model Name	Analytical expression	N_p
M1	$f(k) = 1 + a_0 + a_1 \cdot k + c \cdot k \cdot \tanh((k - k_0) \cdot b)$	5
M2	$f(k) = 1 + a_0 + a_1 \cdot k + c \cdot k \cdot \arctan((k - k_0) \cdot b)$	5
M3	$f(k) = 1 + a_0 + a_1 \cdot k + c \cdot k \cdot \mathrm{Gudermannian}((k - k_0) \cdot b)$	5
M4	$f(k) = 1 + a_0 + a_1 \cdot k + c \cdot k \cdot \mathrm{erf}((k - k_0) \cdot b)$	5
M5	$f(k) = 1 + a_0 + a_1 \cdot k + \frac{c \cdot k}{1 + e^{-(k-k_0) \cdot b}}$	5
M6	$f(k) = 1 + a_0 + a_1 \cdot k + c \cdot k \cdot (\frac{1}{4} + e^{(-b+(k-k_0))^2})$	5
M7	$f(k) = 1 + a_0 + a_1 \cdot k + c \cdot k \cdot \frac{b \cdot (k-k_0)}{\sqrt{1 + b^2 \cdot (k-k_0)^2}}$	5
M8	$f(k) = h - a_1 \cdot k + c \cdot \left(A + \frac{(\mathcal{K} - A) \cdot k}{\left(1 + Q \cdot e^{-B(k-M)}\right)^{1/\nu}} \right)$	9



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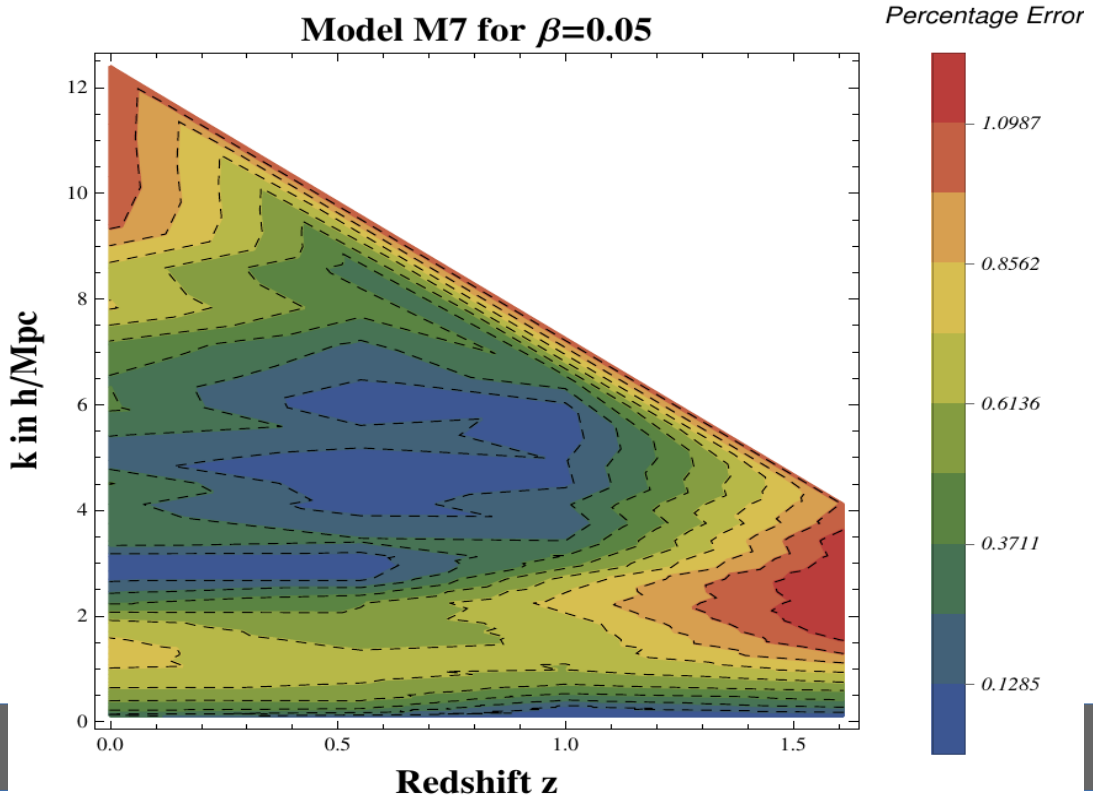
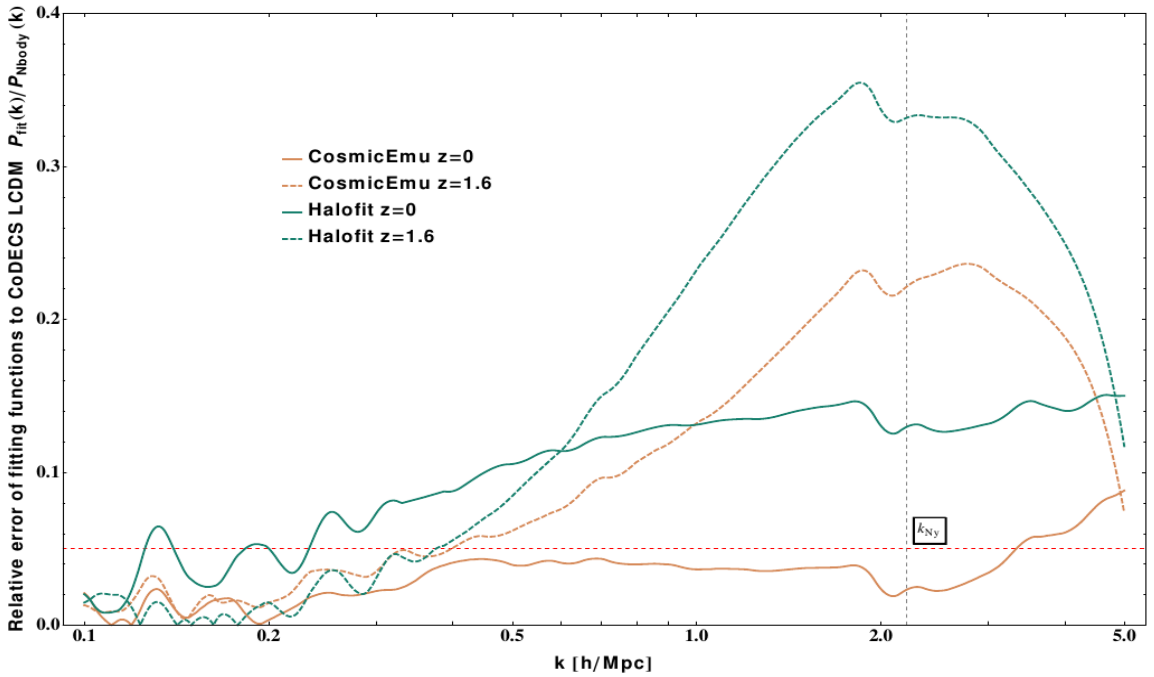
- Multidimensional nonlinear fit: Tested 8 models for goodness of fit, each with 5 coefficients depending polynomially on the parameters.
- Accuracy goal: 1-2%



Implement into Fisher Forecast

- We have fitting functions that describe the effect of the DM-DE coupling in CQ on the nonlinear power spectrum.
- Since we want to extract information at high-k there is no better option than using fitting functions that describe the nonlinear PS for ΛCDM.
- Halofit[2] introduces errors higher than 10% at the scales of interest, therefore we use the FrankenEmu from the Cosmic Emulator project [1].



[1] Heitmann et al. (2014), [2] Takahashi et al. (2012)

Fisher Forecast

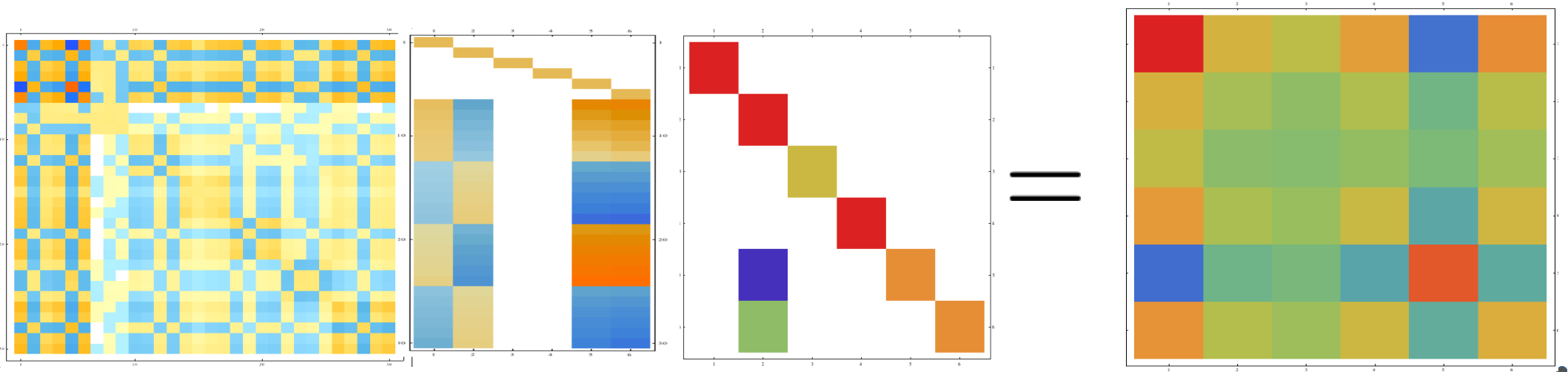
- We use 6 cosmological parameters:

$$\{\beta^2,\ h,\ \log A,\ n_s, \omega_b,\ \omega_c\}$$

- The observed power spectrum for galaxy clustering:

$$P_{\rm obs}\left(z,k,\mu;\theta\right) = P_{\rm s}(z) + \frac{D_A^2(z)_{ref}H(z)}{D_A^2(z)H(z)_{ref}} b^2(z)\left(1+\beta(z)\mu^2\right)^2 P(k,z) e^{-k^2\mu^2(\sigma_z^2/H(z)+\sigma_v^2(z))}$$

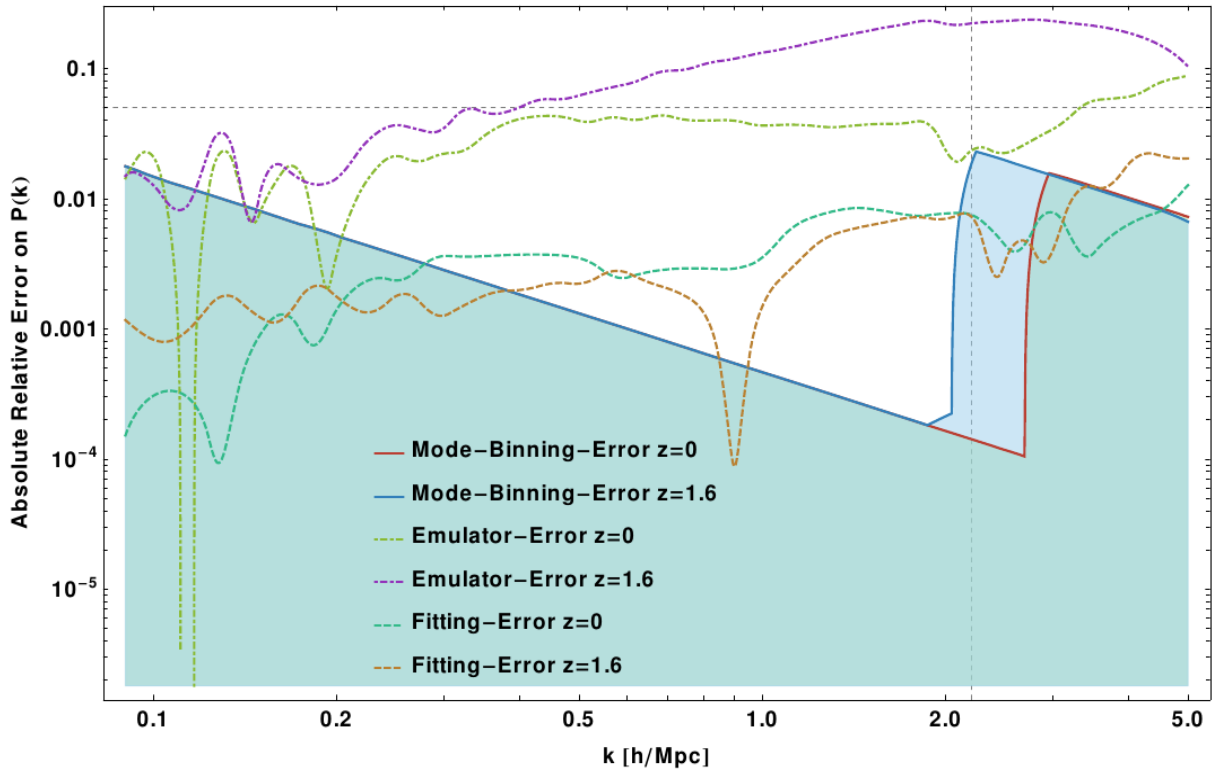
- Using information on the Growth, the Hubble function, the Angular Diameter Distance and the Growth Rate at 6 redshift bins, using Euclid specifications.



Inclusion of errors

- In order to be as realistic as possible we include in our Fisher analysis the errors on the PS coming from the fitting functions, the cosmic emulator and the binning and extraction of the N-body PS.
- These errors add noise to the Fisher matrix in the form of an effective galaxy number density:

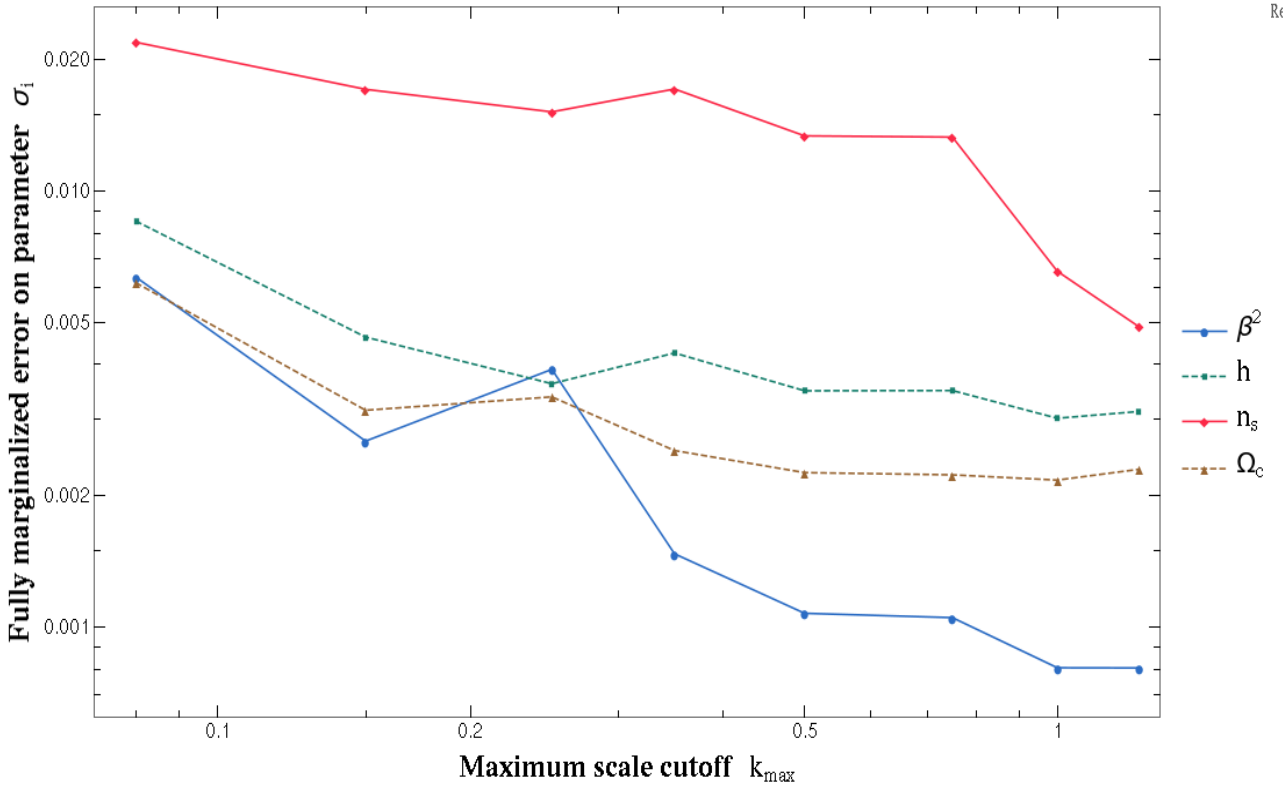
$$\tilde{n}_{eff} = \frac{n_d}{1+n_d\sigma^2}$$

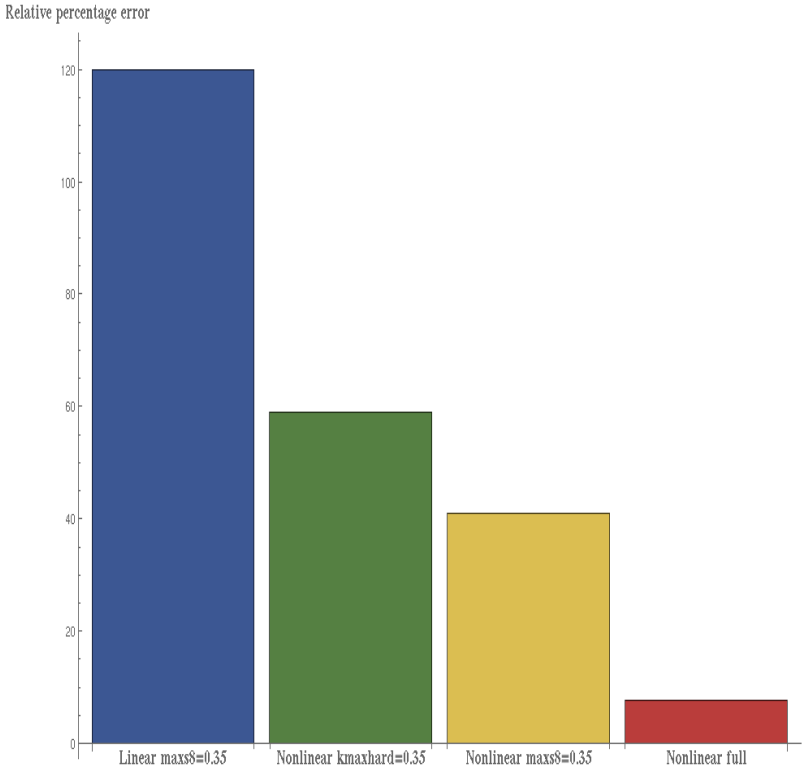


* See analogous plot by Fosalba, Crocce, et al. (2013) MICE simulations

Results

- Including more of the nonlinear scales, improves considerably the constraints on the cosmological parameters, especially on β^2
- At small scales, information from initial conditions is lost due to mode-mode coupling, but effect of the DE coupling starts being important.
- Compared to previous works[1], the error on beta is one order of magnitude smaller.





[1] Amendola, Pettorino, Quercellini, Vollmer (2012).

Results

BAO fully nonlinear

	β^2	h	$A_s \times 10^9$	n_s	Ω_b	Ω_c
fiducial	0.0025	0.703616	2.42	0.966	0.0450275	0.225622
abs. error	0.000196997	0.00169162	0.0205873	0.00228037	0.000276015	0.000675909
rel. error	7.9%	0.24%	0.85%	0.24%	0.61%	0.3%

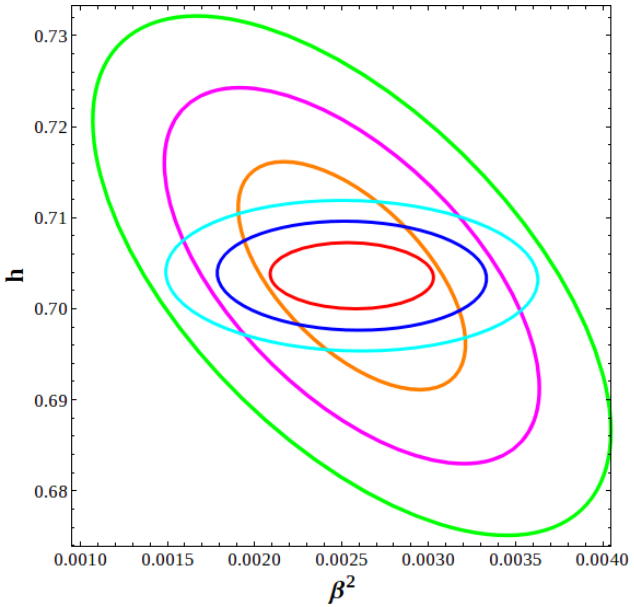
Weak Lensing fully nonlinear

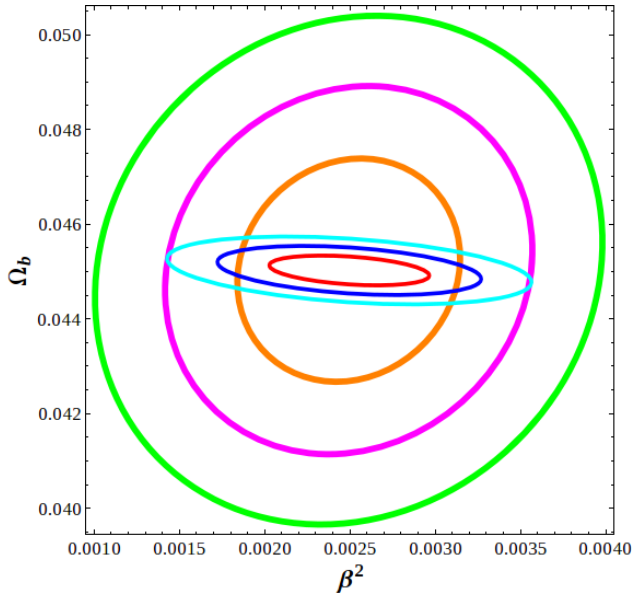
	β^2	h	$A \times 10^9$	n_s	Ω_b	Ω_c
fiducial	0.0025	0.703616	2.42	0.966	0.0450275	0.225622
abs. error	0.000335166	0.00982287	0.105293	0.00952912	0.000998522	0.0027286
rel. error	13.%	1.4%	4.4%	0.99%	2.2%	1.2%

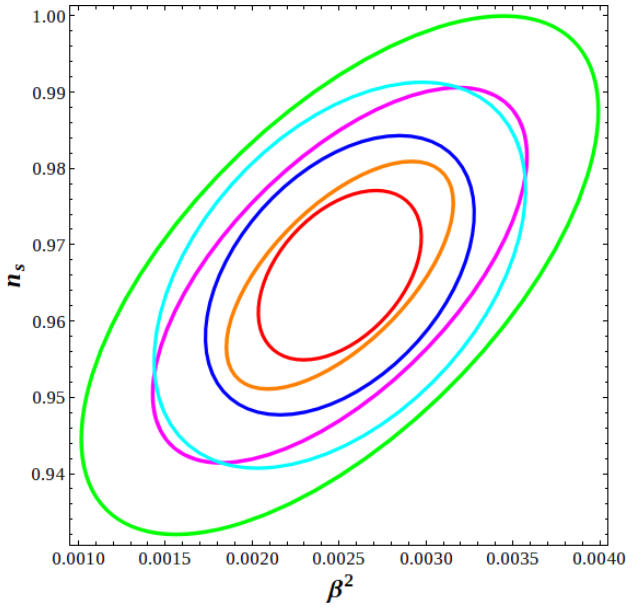
Old results[1], linear

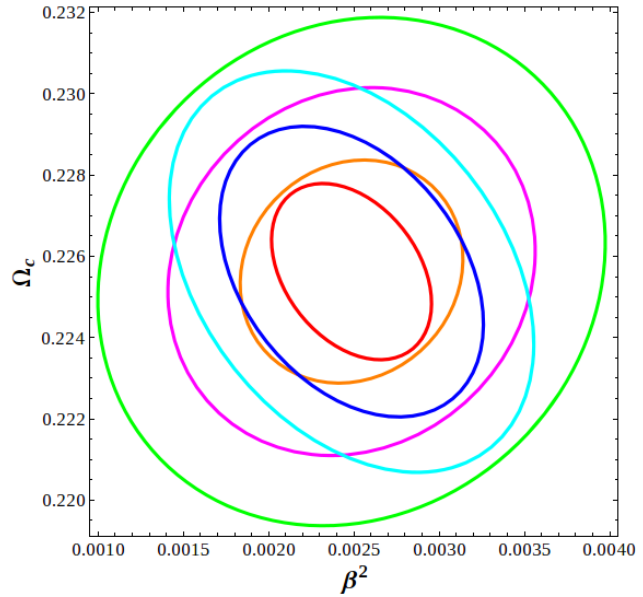
Parameter	σ_i CMB	σ_i $P(k)$	σ_i WL
β^2	0.0094	0.0015	0.012
α	0.55	0.12	0.083
Ω_c	0.022	0.010	0.012
h	0.15	0.036	0.039
Ω_b	0.00087	0.0022	0.010
n_s	0.014	0.034	0.026
σ_8	-	0.0084	0.024
$\log A$	0.0077	-	-

[1] Amendola, Pettorino, Quercellini, Vollmer (2012).









Weak Lensing and BAO are complimentary and break some parameter degeneracies.

If we manage to understand the nonlinear power spectrum accurately, Euclid will be a really powerful tool.

Change of topic

Horndeski in the “quasistatic” limit

- In general Horndeski models, assuming the quasistatic limit, the anisotropic stress and the “effective gravitational constant” can be expressed as:

$$\eta(k,a) = -\frac{\Phi}{\Psi} = h_2\left(\frac{1+k^2h_4}{1+k^2h_5}\right) \qquad Y(k,a) = -\frac{2k^2\Psi}{3\Omega_m\delta_m} = h_1\left(\frac{1+k^2h_5}{1+k^2h_3}\right)$$

- This modified Poisson's equation will have an effect on structure formation of DM that we would like to study at the nonlinear scale.

- We will just take general functions of time $h_i(t)$ which given a specific model, can always be specified.

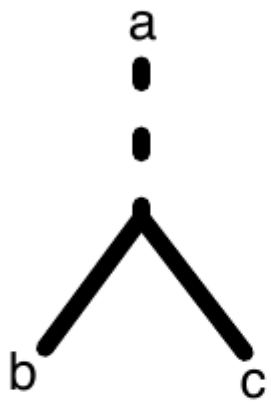
eRPT Resummation

- Resummation method developed by Stefano Anselmi and Massimo Pietroni, “...A leap beyond the BAO scale” (2012)
- We start with the evolution equation for the PS in this method, but add scale dependence and a “Horndeski modified” propagator:

$$\begin{aligned}\partial_{\mathcal{X}}\tilde{P}_{ab}(k;\mathcal{X}) = &-\tilde{\Omega}_{ac}(\mathbf{k};\mathcal{X})\tilde{P}_{cb}(\mathbf{k};\mathcal{X}) - \tilde{\Omega}_{bc}(\mathbf{k};\mathcal{X})\tilde{P}_{ac}(\mathbf{k};\mathcal{X}) \\ &+ H_{\mathbf{a}}(k;\mathcal{X},\mathcal{X}_{in})\tilde{P}_{\mathbf{a}b}(\mathbf{k};\mathcal{X}) + H_{\mathbf{b}}(k;\mathcal{X},\mathcal{X}_{in})\tilde{P}_{a\mathbf{b}}(\mathbf{k};\mathcal{X}) \\ &+ \int \mathrm{d}s \left[\tilde{\Phi}_{ad}(k;\mathcal{X},s)G^{eik}_{bd}(k;\mathcal{X},s) + G^{eik}_{ad}(k;\mathcal{X},s)\tilde{\Phi}_{db}(k;\mathcal{X},s)\right]\end{aligned}$$

propagator tree-level: $-iG^{eik}_{ab}(\mathbf{k};\eta_a,\eta_b)$

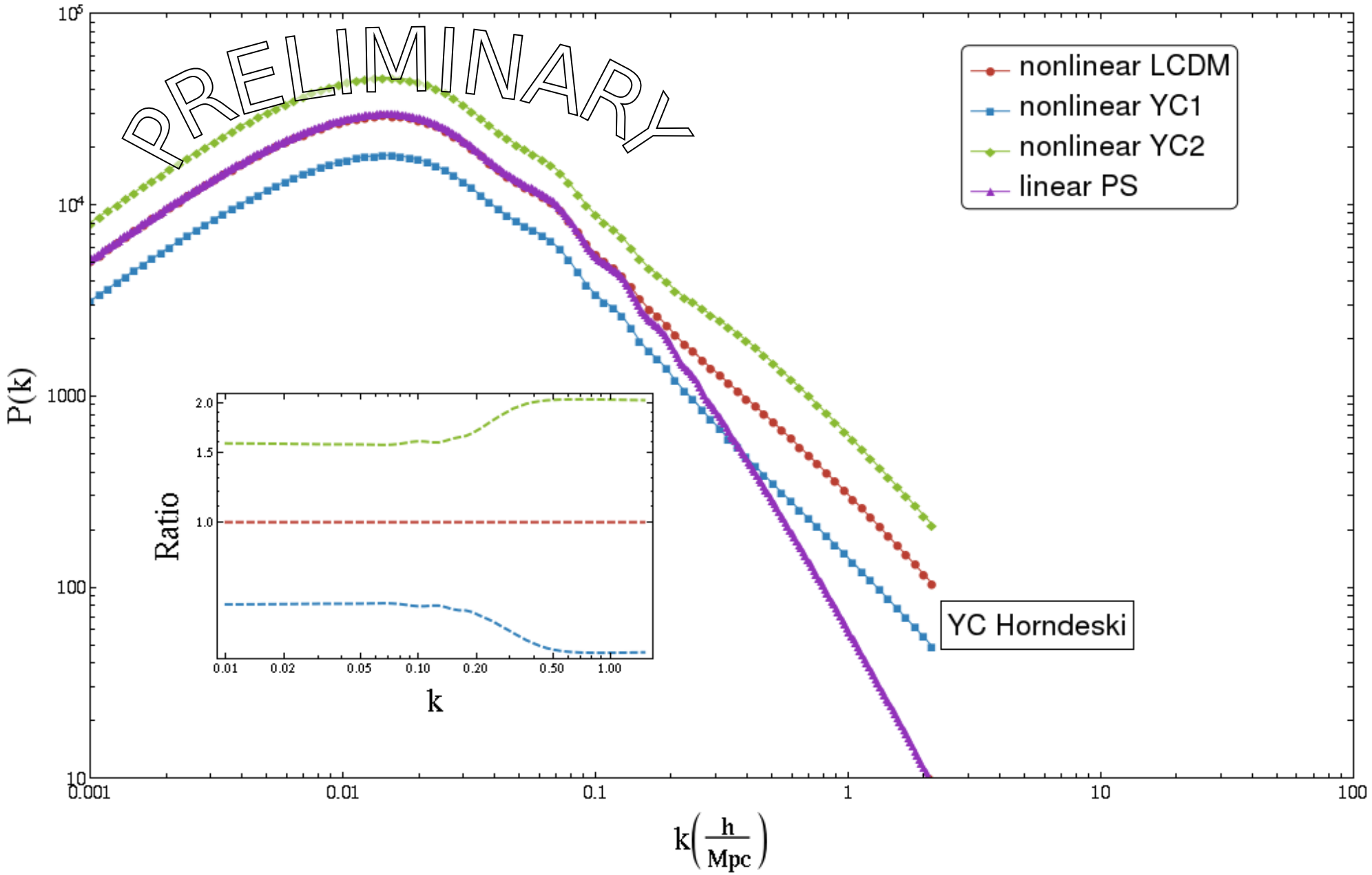
power spectrum tree-level: $P^{eik}_{ab}(\mathbf{k};\eta_a,\eta_b)$



interaction vertex: $-ie^{\eta}\gamma_{abc}(\mathbf{k}_a,\mathbf{k}_b,\mathbf{k}_c)$

counter term propagator: $i\Sigma^{eik}_{ab}(\mathbf{k};\eta_a,\eta_b)$

counter term power spectrum: $-\Phi^{eik}_{ab}(\mathbf{k};\eta_a,\eta_b)$


PRELIMINARY
nonlinear LCDM
nonlinear YC1
nonlinear YC2
linear PS
YC Horndeski
P(k)
k(h/Mpc)
Ratio
k
10^5
10^4
1000
100
10
0.001
0.01
0.1
1
10
100
2.0
1.5
1.0
0.01
0.02
0.05
0.10
0.20
0.50
1.00

Main Conclusions:

- N-body simulations together with semi-analytical tools are a powerful way of estimating the predictions of DE theories in a nonlinear regime.
- Using nonlinear information from the Power Spectrum where there is a characteristic feature coming from MG improves strongly the estimation of parameters using future surveys.
- We need to calculate this nonlinear corrections analytically for more complicated models, where there are no simulations. We show that it is possible for “Horndeski in the QS limit”.

Main Conclusions:

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Future things to do:

- Use higher correlations to compute covariances and more realistic forecasts. Test the fitting functions using perturbation theory.

- For nonlinear Horndeski there is still much theoretical work to do, but it would be great to test eRPT against some realistic simulations of a specific Horndeski model.

Main Conclusions:

- Using nonlinear information from Power Spectrum where there is a characteristic feature coming from MG(DE) improves strongly the estimation of parameters.

- We need to calculate this nonlinear corrections analytically for more complicated models, where there are no simulations. We show that it is possible for “Horndeski in the QS limit”.

THANKS!

Backup slides

Fisher Matrix

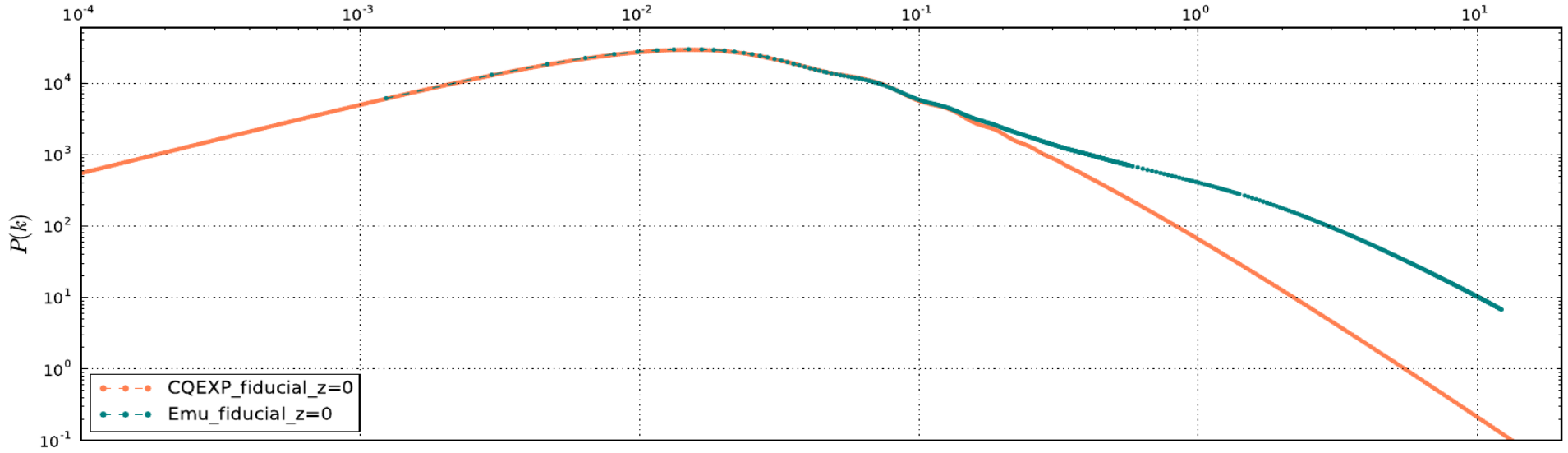
$$P_{obs}(z,k_r) = \frac{D_{Ar}^2(z)H(z)}{D_A^2(z)H_r(z)}G^2(z)b(z)^2\left(1+\beta\mu^2\right)^2 P_{0r}(k) + P_{shot}(z) \quad (8)$$

$$F_{ij} = 2\pi \int_{k_{min}}^{k_{max}} \frac{\partial \log P(k_n)}{\partial \theta_i}\frac{\partial \log P(k_n)}{\partial \theta_j}\cdot V_{eff}\cdot\frac{k^2}{8\pi^3}\cdot dk \quad (11)$$

$$V_{eff} = \int \left[\frac{n(\vec{r})P(k,\mu)}{n(\vec{r})P(k,\mu)+1}\right]^2 d\vec{r} = \left[\frac{n(\vec{r})P(k,\mu)}{n(\vec{r})P(k,\mu)+1}\right]^2 V_{survey}$$

Implement into Fisher Forecast

- We have fitting functions that describe the effect of the DM-DE coupling in CQ on the nonlinear power spectrum.
- Since we want to extract information at high-k there is no better option than using fitting functions that describe the nonlinear PS for ΛCDM.
- Halofit[2] introduces errors higher than 10% at the scales of interest, therefore we use the FrankenEmu from the Cosmic Emulator project [1].
- For linear scales $k \lesssim 0.1$ we use CQCAMB[3], a modified version of CAMB.



[1] Heitmann et al. (2014), [2] Takahashi et al. (2012), [3] Pettorino et al. (2012)