# Generic predictions of plateau inflation

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Based on work with Diederik Roest and Vincent Vennin

# Outline

- Inflation as a Taylor expansion
- Inflation as a Padé approximant
- Conclusions

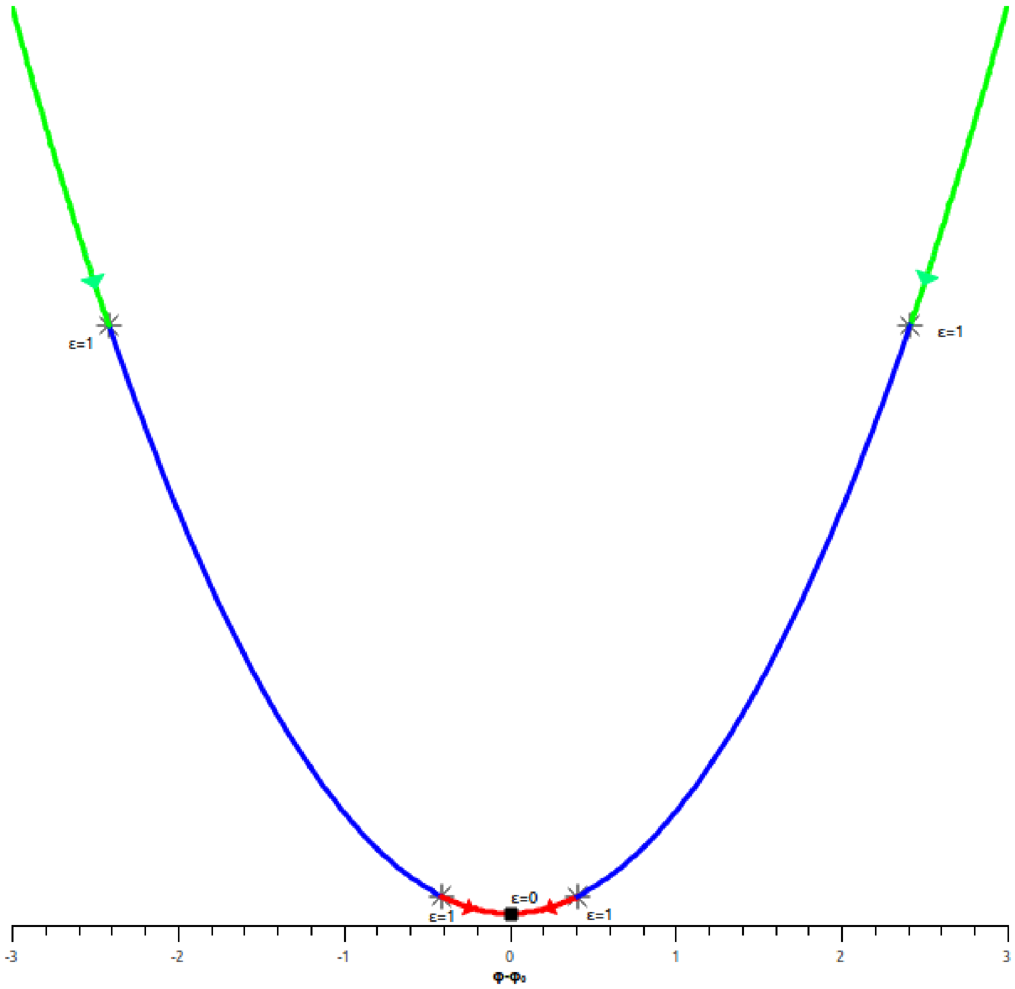
# Hubble function

- Hubble function a possibility do describe inflation
- $H(\phi) = \frac{\dot{a}}{a}$, thus describes the expansion of the universe, or an energy

  Hamilton-Jacobi equations
    - $V = 3H(\phi)^2 - 2H'(\phi)^2$
    - $\dot{\phi} = -2H'(\phi)$
- During inflation $H$ decreases, as does $V$, until $\epsilon \equiv 2\left(\frac{H'(\phi)}{H(\phi)}\right)^2 = 1$
- Advantage: No need for slow roll approximation, except for calculating $n_s, r$

# Inflation as a Taylor expansion

- Parameterize $H(\phi) = H_0(1 + \sum_{k=1}^{M} \frac{a_k}{k!} \phi^k)$
- How to proceed:
  1. Take $a_k \in [-1,1]$ for all $k$
     such that $H(0) > 0$ and $0 < \epsilon(0) < 1$
  2. Search where $\epsilon = 1$ (end inflation)
  3. Search where $\epsilon = 0$ (eternal inflation)
  4. If flow to $\epsilon = 0$, discard model
  5. At point where $\Delta N = 60$, calculate $n_s, r$

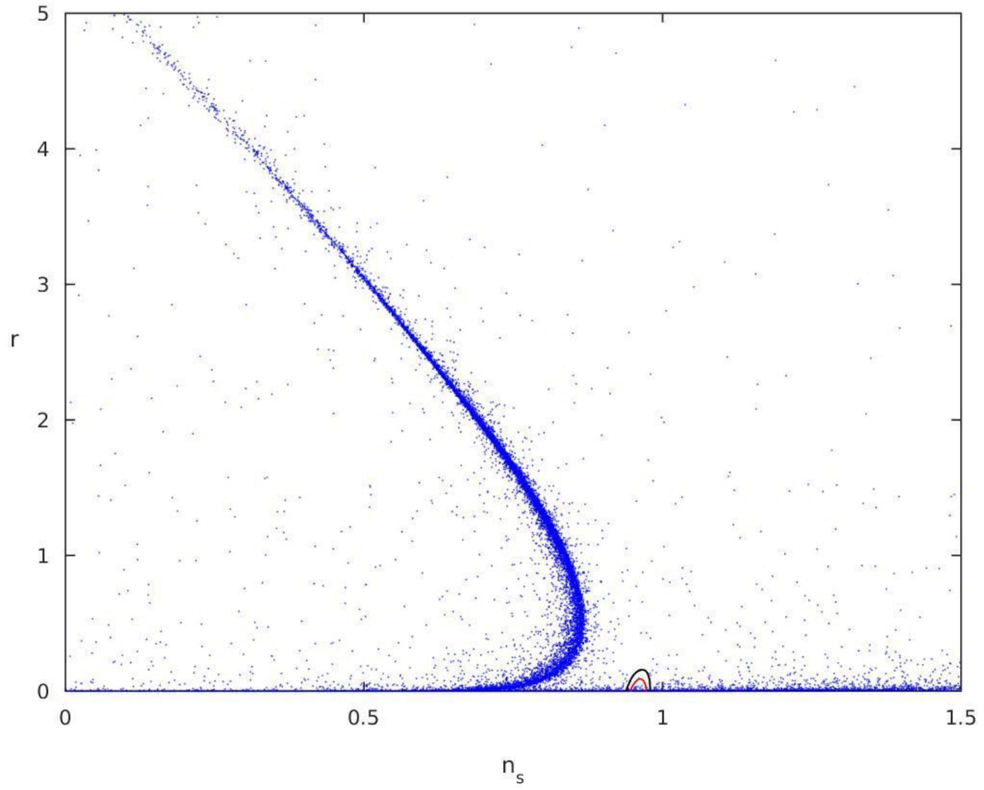



Ref. Hoffman & Turner, astro-ph/0006321
Kinney, astro-ph/0206032

# Inflation as a Taylor expansion

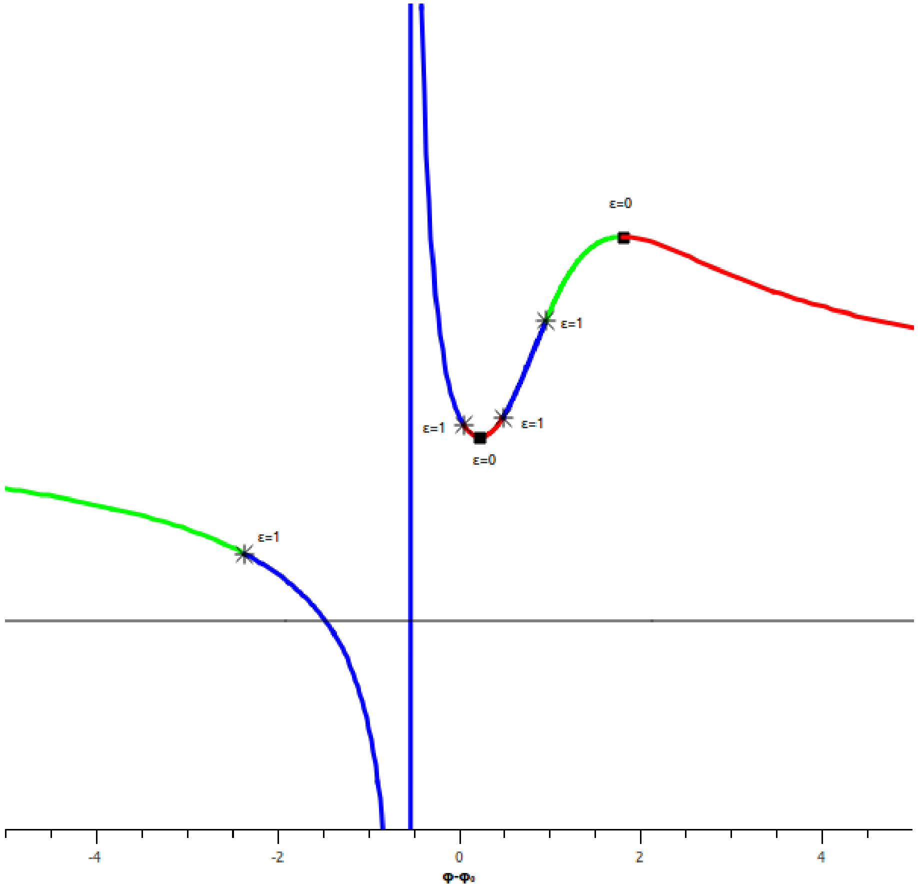
- Parameterize $H(\phi) = 1 + \frac{a_1}{1!}\phi + \frac{a_2}{2!}\phi^2 + \ldots$
  Where $a_i \in [-1,1]$ for all $i$.

- Data far from Planck contours
  (2.1% in Planck 2σ contour)



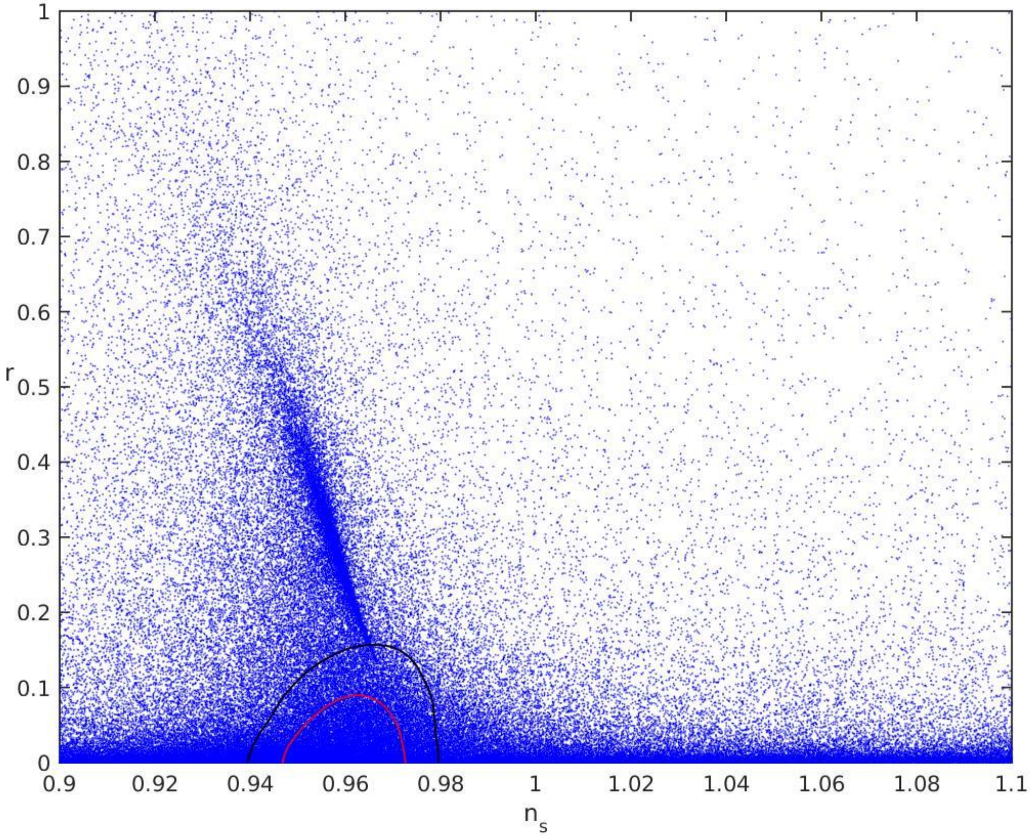
# Inflation as a Padé approximant

- Padé approximant natural expansion around $\phi = 0$ and $\phi = \infty$
- $H(\phi) = \frac{\sum_{n=0}^{N} a_n \phi^n}{1+\sum_{m=1}^{M} b_m \phi^m}$.
- For a plateau, choose $N = M$.
- 2 inflation domains:
  a) Around $\phi = 0$, chose $a_n, b_n$ such that derivatives of $H(\phi)$ maximally 1 at $\phi = 0$
  b) Around $\phi = \infty$, chose $\{a_i, b_i\} \in [-1,1]$



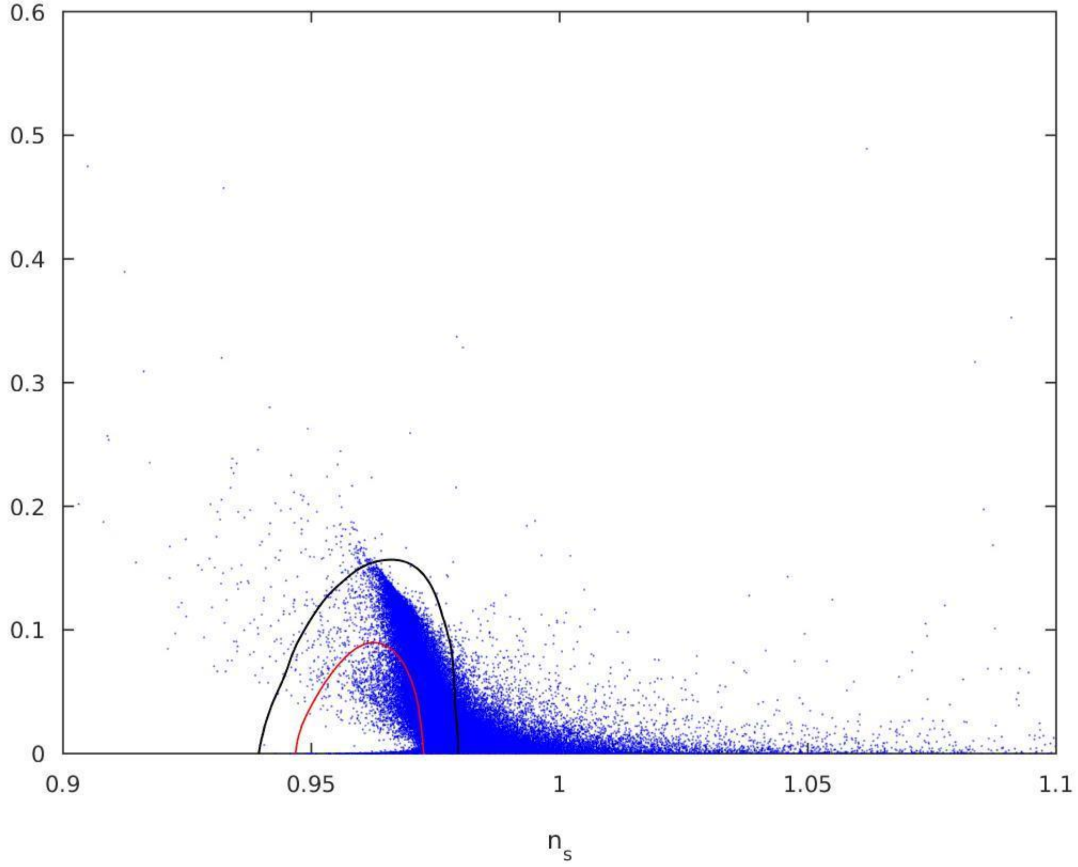
# Inflation as a Padé approximant

$\phi \approx 0$




9.4% in Planck 2σ contour

$\phi \approx \infty$




96% in Planck 2σ contour

# Conclusions

- Old approach using Taylor is not recomended by Planck data
- Padé approximants do much better.

# Parameterizing inflation

- 4 ways to parameterize inflation:
  1. $H(\phi)$ Advantage: exact trajectory
  2. $V(\phi) = 3H(\phi)^2 - 2H'(\phi)^2$, closer to standard model, but slow roll
  3. $\epsilon(\phi) = 2\left(\frac{H'(\phi)}{H(\phi)}\right)^2$, does not encode direction
  4. Truncation of flow parameters

     ${}^{l}\lambda = \frac{\left(H'(\phi)\right)^{n-1} H^{(n+1)}(\phi)}{H(\phi)^n}$, ${}^{M+1}\lambda(\epsilon_1,\ {}^{1}\lambda,\ \ldots,\ {}^{M}\lambda)$ parameterizes inflation.

# Exact results

- Find the exact inflation trajectory for a group of Hubble fuctions
- From a Hubble function with 2 parameters and a shift symmetry in $\phi$.
- Example: $H(\phi) = H_0 \frac{1+a\phi}{1+b\,\phi}$ where $a, b, H_0$ are constants w.r.t. $\phi$

# Exact results

- $H(\phi) = H_0 \frac{1+a\phi}{1+b\,\phi}$
- H invariant under $\phi \to \phi + \Delta\phi$

$$H_0 \to H_0 \frac{1+a\Delta\phi}{1+b\Delta\phi}$$

$$a \to \frac{a}{1+a\Delta\phi}$$

$$b \to \frac{b}{1+b\Delta\phi}$$

- Thus there is an invariant: $\frac{ab}{a-b}$

# Exact results

- $H(\phi) = H_0 \frac{1+a\phi}{1+b\,\phi}$
- Calculate $\epsilon_1, \epsilon_2$
- Rewrite invariant as function of$\epsilon_1(\phi = 0), \epsilon_2(\phi = 0)$, relation is true for all $\phi$
- $\gamma = \frac{\epsilon_2^2 - 4\epsilon_1^2}{\epsilon_1^{3/2}}$

# Exact results

- $H(\phi) = H_0 \frac{1+a\phi}{1+b\ \phi}, \gamma = \frac{\epsilon_2^2 - 4\epsilon_1^2}{\epsilon_1^{3/2}}$ invariant
- Calculate $\epsilon_2(\epsilon_1, \gamma)$
- From $\frac{d\epsilon_1}{dN} = \epsilon_1 \epsilon_2$ followes $N(\epsilon_1)$