Cosmological Braneworld Solutions

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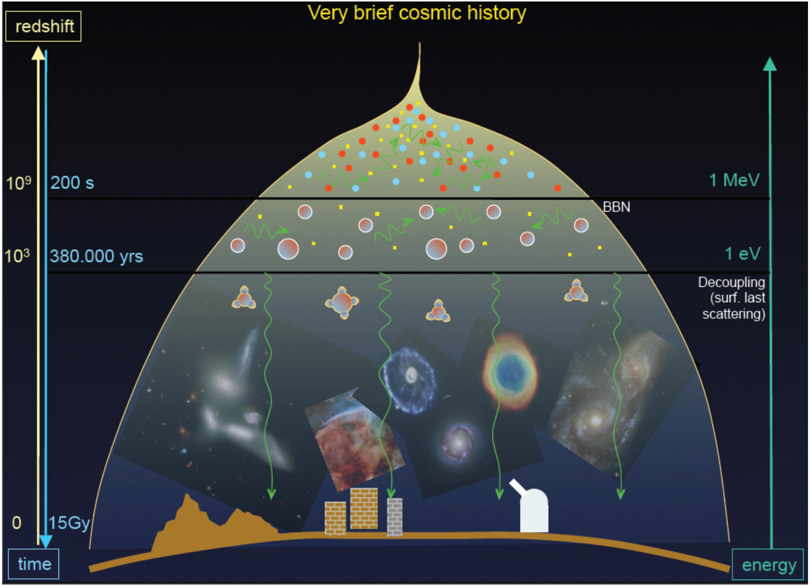
Outline

1. Motivation
2. NS5 solution
3. Time-dependent solutions
4. Future study

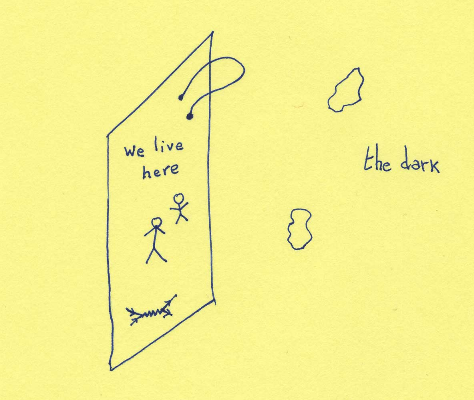
String theory, if correct, describes the very early universe:



Braneworlds

We could be living on a membrane in higher-dimensional spacetime:

- gauge and matter fields describing our universe (open string endpoints) live on the brane
- gravity (closed strings) live in the bulk
- $\Rightarrow$ natural explanation for weakness of gravity
- Possible explanation for DM (matter on another brane, with which we can only interact gravitationally)



Braneworld Localization in Hyperbolic Spacetime

Crampton, Pope and Stelle [1408.7072] gave an exact supergravity solution which localises gravity near a 5-brane worldvolume:

- take 6 D Salam-Sezgin solution (without a brane) embedded in 10D, with a hyperbolic structure $\mathcal{H}^{(2,2)}$ in the lifting dimensions

$$ds^2_{10} = \eta_{\mu\nu} dx^\mu dx^\nu + dz^2 + g_{mn} dx^m dx^n$$

$$e^\phi = (\operatorname{sech} 2\rho)^{1/2} \, ; \quad H_{3,transverse} \tag{1}$$

(2)

- include NS5 brane with (x^μ, z, ψ) worldvolume (and backreaction):

$$ds^2_{10} = \eta_{\mu\nu} dx^\mu dx^\nu + dz^2 + \mathbf{g_{mn}} dx^m dx^n \tag{3}$$

$$e^\phi = (\operatorname{sech} 2\rho - \mathbf{k} \log \tanh \rho)^{1/2} \, ; \quad \mathbf{H_{3,transverse}}$$

(4)

Braneworld Localization in Hyperbolic Spacetime

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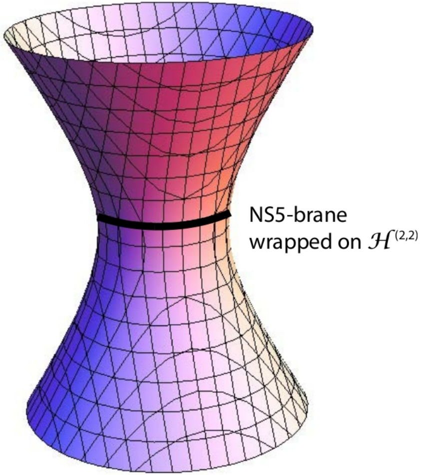

Gravitational fluctuations about the solution

We can study gravitational fluctuations about the background [1408.7072]:

$$\eta_{\mu\nu} \quad \to \eta_{\mu\nu} + h_{\mu\nu}(x, \rho) \tag{5}$$

where ρ is the non-compact radial coordinate. These must solve the wave equations

$$\begin{aligned} \Box_{10} h_{\mu\nu} &= 0 \\ \Box_4 h^{\lambda}_{\mu\nu} &= m^2 h^{\lambda}_{\mu\nu} \end{aligned}$$

where λ labels the eigenvalue of a given fluctuation. The unique bound state wavefunction is found to be the zero mode with $\lambda = 0$, which corresponds to massless gravity in 4 dimensions and a gap between the massless states and the continuum of massive fluctuations.

Equations of motion

The solution in Crampton, Pope and Stelle [1408.7072] is an exact supergravity solution, which satisfies the EOM:

$$
\begin{aligned}
R_{MN} - \frac{1}{4} H_{MRS} H_N^{RS} + 2\nabla_M \nabla_N \phi &= 0 \\
\nabla^2 (e^{-2\phi}) - \frac{1}{6} e^{-2\phi} H_{MNR} H^{MNR} &= 0 \\
\nabla_M (e^{-2\phi} H^{MNR}) &= 0
\end{aligned}
$$

Time-dependent braneworld solution

We asked: are there time-dependent (i.e. cosmological) solutions with FRW-type metrics in the 4D universe? In other words, can we *add time dependence* as below?

$$ds^2_{10} = -dt^2 + \alpha_1(t)^2 dx_i dx^i + dz^2 + g_{mn} dx^m dx^n$$

This leads to nonzero equations of motion, specifically in the μ, ν quadrant of $R_{MN} - \frac{1}{4} H_{MRS} H_N^{RS} + 2\nabla_M \nabla_N \phi$. To find a solution, these terms must be cancelled by contributions of the form:

$$R_{ii}, R_{00}, H_{0rs} H_0^{rs}, H_{irs} H_i^{rs}, \nabla_i \nabla_i \phi, \nabla_0 \nabla_0 \phi$$

or combinations of the above.

Time-dependent braneworld solution

Eventually, we found a family of solutions for $e^{\phi} \to \alpha_2(t) e^{\phi}, dz \to \alpha_3(t) dz$, where the solutions are of the form

$$\begin{aligned} ds^2_{10} &= -dt^2 + \alpha_1(t)^2 dx_i dx^i + \alpha_3(t)^2 dz^2 + g_{mn} dx^m dx^n \\ e^{\phi} &= \alpha_2(t) \left(\operatorname{sech} 2\rho - \mathbf{k} \log \tanh \rho\right)^{1/2} \end{aligned}$$

$$\begin{aligned} \alpha_1(t) &\sim t^n \\ \alpha_2(t) &\sim t^{cn} \\ \alpha_3(t) &\sim t^{\pm\sqrt{1-3n^2}} \end{aligned}$$

where $c = 3 - \frac{1}{n} \pm \frac{\sqrt{1-3n^2}}{n}$.

Future study

The next steps are:

- to understand these solutions and their dynamics physically
- to analyse the 4 D gravitational fluctuations $h_{\mu\nu}$ about these time-dependent backgrounds, to see if the massless bound state which localizes gravity on the brane remains.
- to examine the corrections to 4D Newtonian gravity from the continuum of massive gravitational modes.