Cosmic Acceleration as an Optical Illusion

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Motivation and basics

- Motivation:
  - Do we need $\Lambda \neq 0$?
  - Why is that so hard to find out?
- What can one study?
  - Toy models vs realistic universe (here: only the latter)
  - Volume evolution or light propagation
- Basic idea:
  - Consider a large domain $\mathcal{D}$ in an irrotational dust universe,
  - divide $\mathcal{D}$ into “infinitesimal” regions — actually
    - small in cosmic terms,
    - large enough for the irrotational dust approximation,
  - follow the evolution of each such region,
  - add the contributions.

Related work

- Many proposals for alternatives to Λ (see e.g. Ellis et al for a review).
- Work on volume evolution:
  - authors such as
    - Kolb, Matarrese, (Notari,) Riotto;
    - Räsänen;
    - Li, (Seikel,) Schwarz;

    (see Buchert for a review),
  - acceleration as a real effect in an irrotational dust universe,
  - Buchert's formalism — here my approach is different!
- Light propagation: redshift-distance relation up to second order perturbation theory worked out by
  - Ben-Dayan, Gasperini, Marozzi, Nugier, Veneziano;
  - Umeh, Clarkson, Maartens.

Outline

1. Introduction
2. Evolution of Irrotational Dust Universes
   - General setup and mass-weighted average
   - Rescaled quantities and their evolution
   - Initial values from linear perturbation theory
   - Volume evolution
3. Light propagation
   - Sachs Equations and distance formulas
   - Photon path average
   - Results
4. Discussion and outlook

Irrotational dust: general setup

- Spacetime manifold ${}^{(4)}\mathcal{M} = \mathbb{R}_+ \times \mathcal{M}$
- Energy-momentum tensor ${}^{(4)}T = \mathrm{diag}(\rho, 0, 0, 0)$
- Metric $ds^2 = -dt^2 + g_{ij}(t, x)dx^i dx^j$ in synchronous gauge, any geometric quantity refers to the 3d-metric g!
- Expansion tensor $\theta^i_j = \frac{1}{2} g^{ik} \dot{g}_{kj}$
- Scalar expansion rate $\theta = \theta^i_i = \frac{\dot{\sqrt{g}}}{\sqrt{g}}$
- Shear $\sigma^i_j = \theta^i_j - \frac{\theta}{3}\delta^i_j$, hence $\theta^i_j \theta^j_i = \frac{1}{3}\theta^2 + 2\sigma^2$ with $\sigma^2 = \frac{1}{2}\sigma^i_j \sigma^j_i$.
- Ricci tensor $R^i_j = \frac{R}{3}\delta^i_j + r^i_j$ (with r^i_j … traceless part of R^i_j) obeys $\dot{R}_{ij} = \theta^k_{i|jk} + \theta^k_{j|ik} - \theta_{ij|kl} g^{kl} - \theta_{|ij}$.

Irrotational dust: dynamics

- Energy-momentum conservation $\dot{\rho} + \theta\rho = 0$
- Einstein equations

$$\begin{aligned} \frac{1}{3}\theta^2 - \sigma^2 + \frac{1}{2}R - \Lambda &= 8\pi G_N \rho, \\ -2\theta_{|i} + 3\sigma^j_{i|j} &= 0, \\ \dot{\sigma}^i_j + \theta\sigma^i_j + r^i_j &= 0. \end{aligned}$$

- $a_{\mathrm{local}}(t,x) = \left(\frac{\hat{\rho}}{\rho(t,x)}\right)^{\frac{1}{3}}$ ...the local scale factor (with $\hat{\rho}$ ... a fixed quantity of dimension mass, e.g. $M_\odot$); $a(t,x) = a_{\mathrm{local}}(t,x)$ is just the side length of a cube of mass $\hat{\rho}$ consisting of material of density ρ.

Mass-weighted average

Energy-momentum conservation $\Rightarrow \frac{d}{dt}\left(\rho(x,t)\sqrt{g(x,t)}\right) = 0$

$\Rightarrow$ the mass content $m_{\mathcal{D}} = \int_{\mathcal{D}} \rho(x,t)\sqrt{g(x,t)}\, d^3x$ of any domain $\mathcal{D} \subset \mathcal{M}$ is time independent, $\dot{m}_{\mathcal{D}} = 0$

$\Rightarrow \langle X\rangle^{\cdot}_{\mathrm{mw}} = \langle \dot{X}\rangle_{\mathrm{mw}}$ for the mass–weighted average of a scalar X,

$$\langle X\rangle_{\mathrm{mw}}(t) = \frac{1}{m_{\mathcal{D}}} \int_{\mathcal{D}} X(x,t)\rho(x,t)\sqrt{g(x,t)}\, d^3x,$$

N.B. $\langle X\rangle^{\cdot} = \langle \dot{X}\rangle$ would not hold for a volume average $\langle X\rangle_{\mathrm{vol}}$ which would therefore require Buchert's formalism!

But $V_{\mathcal{D}} = \int_{\mathcal{D}} \sqrt{g(x,t)}\, d^3x = m_{\mathcal{D}}\langle \rho^{-1}\rangle_{\mathrm{mw}} = \frac{m_{\mathcal{D}}}{\hat{\rho}}\langle a^3\rangle_{\mathrm{mw}}$,
$\langle X\rangle_{\mathrm{vol}} = \langle Xa^3\rangle_{\mathrm{mw}}/\langle a^3\rangle_{\mathrm{mw}}$ with $a = a_{\mathrm{local}}$.

Rescaled quantities and their evolution

- $\dot{\rho} + \theta\rho = 0$ and $a = (\hat{\rho}/\rho)^{\frac{1}{3}}$ imply

$$\theta(t,x) = -\frac{\dot{\rho}(t,x)}{\rho(t,x)} = 3\frac{\dot{a}(t,x)}{a(t,x)}.$$

- Rescaled quantities

$$\hat{\rho} = a^3\rho, \quad \hat{\sigma}^i_j = a^3\sigma^i_j, \quad \hat{R} = a^2 R, \quad \hat{r}^i_j = a^2 r^i_j$$

- obey the evolution equations

$$\dot{\hat{\rho}} = 0, \qquad \dot{\hat{\sigma}}^i_j = -a\hat{r}^i_j, \qquad \dot{\hat{R}} = -2a^{-3}\hat{\sigma}^i_j\hat{r}^j_i,$$

$$\dot{\hat{r}}^i_j = a^{-3}\left(-\frac{5}{4}\hat{\sigma}^i_k\hat{r}^k_j + \frac{3}{4}\hat{\sigma}^k_j\hat{r}^i_k + \frac{1}{6}\delta^i_j\hat{\sigma}^k_l\hat{r}^l_k\right) + a^2 Y^{ki}{}_{j|k},$$

$$\text{with } Y^k{}_{ij} = \frac{3}{4}(\sigma^k_{i|j} + \sigma^k_{j|i}) - \frac{1}{2}g_{ij}\sigma^k_{m|}{}^m - \sigma_{ij|}{}^k.$$

Local Friedmann equation

The evolution equation for the local scale factor (the 0-0 Einstein equation) can be written as

$$\begin{aligned}\dot{a}^2 &= \frac{1}{3}\hat{\sigma}_{\rm in}^2 a^{-4} + \frac{8}{3}\pi G_N \hat{\rho}\, a^{-1} - \frac{1}{6}\hat{R}_{\rm in} + \frac{1}{3}\Lambda\, a^2 \\ &\quad - \frac{4}{9}\int_{t_{\rm in}}^{t} \theta(\tilde{t}) a^{-4}(\tilde{t})\, \hat{\sigma}^2(\tilde{t})\, d\tilde{t}.\end{aligned}$$

with $t_{\rm in}$ ... some initial time (typically $t_{\rm in} = 0$).

Linear perturbation theory: metric

- Background: Einstein – de Sitter, i.e. flat matter–only, $a_{\mathrm{EdS}} = \mathrm{const} \times t^{\frac{2}{3}}$.
- The relevant contributions all come from a single time-independent function $C(x)$:
  $g_{ij}^{(\mathrm{LPT})}(t,x) = a_{\mathrm{EdS}}^2(t)\left(\delta_{ij} + \frac{10}{9}\frac{a_{\mathrm{EdS}}^2}{t^{\frac{4}{3}}}C(x)\delta_{ij} + t^{2/3}S_{ij}(x)\right)$
  with $S_{ij}(x) = \partial_i\partial_j C(x)$.
- Corresponds to Newtonian (longitudinal) gauge metric $ds^2 = a_{\mathrm{FLRW}}^2(t)\left(-(1+2\Phi)dt^2 + (1-2\Psi)d\mathbf{x}^2\right)$
  with $\Phi = \Psi = -C/3$, $a_{\mathrm{FLRW}} = a_{\mathrm{EdS}}$.

Linear perturbation theory: $R, \theta, r, \sigma, \rho$

Straightforward calculations result in

- first order expressions (with $S_{ij} = s_{ij} + \frac{1}{3}\delta_{ij}S$)

$$R = -\frac{20}{9}t^{-\frac{4}{3}}S, \qquad \theta = 2t^{-1} + \frac{1}{3}t^{-\frac{1}{3}}S,$$

$$r^i_j = -\frac{5}{9}t^{-\frac{4}{3}}\delta^{ik}s_{kj}, \qquad \sigma^i_j = \frac{1}{3}t^{-\frac{1}{3}}\delta^{ik}s_{kj},$$

$$6\pi G_N\rho = t^{-2} - \frac{1}{2}t^{-\frac{4}{3}}S$$

- and initial conditions, at $t_{\text{in}} = 0$, of

$$\lim_{t\to 0}\frac{a}{t^{2/3}} = (6\pi G_N\hat{\rho})^{1/3}, \qquad \hat{R}_{\text{in}}(x) = -\frac{20}{9}(6\pi G_N\hat{\rho})^{\frac{2}{3}}S(x),$$

$$(\hat{\sigma}_{\text{in}})^i_j(x) = 0, \qquad (\hat{r}_{\text{in}})^i_j(x) = -\frac{5}{9}(6\pi G_N\hat{\rho})^{\frac{2}{3}}\delta^{ik}s_{kj}(x).$$

Dimensionless variables

- The entries of the matrix S_{ij} have dimensionality $t^{-2/3}$; choose constant U such that $\bar{S}_{ij} = S_{ij}/U$ is dimensionless.
- Dimensionless quantities

$$
\begin{aligned}
\bar{t} &= U^{\frac{3}{2}} t, \\
\bar{a} &= (6\pi G_N \hat{\rho})^{-\frac{1}{3}} U a, \\
\bar{R} &= (6\pi G_N \hat{\rho})^{-\frac{2}{3}} U^{-1} \hat{R}, \\
\bar{r} &= (6\pi G_N \hat{\rho})^{-\frac{2}{3}} U^{-1} \hat{r}, \\
\bar{\sigma} &= (6\pi G_N \hat{\rho})^{-1} U^{\frac{3}{2}} \hat{\sigma},
\end{aligned}
$$

- $\bar{R}_{\mathrm{in}} = -\frac{20}{9} \bar{S}$,
- $(\bar{r}_{\mathrm{in}})^i_j = -\frac{5}{9} \delta^{ik} \bar{s}_{kj}$.

The probability distribution

- $C(x) = \int (a_{\mathbf{k}} + ib_{\mathbf{k}}) e^{i\,\mathbf{k}\cdot\mathbf{x}} (2\pi)^{-3/2} d^3k$:
  - Fourier modes $a_{-\mathbf{k}} = a_{\mathbf{k}},\ \ b_{-\mathbf{k}} = -b_{\mathbf{k}}$,
  - Gaussian distribution: $a_{\mathbf{k}}, a_{\mathbf{k}'}$ independent unless $\mathbf{k} = \mathbf{k}'$.
- S_{ij} ... symmetric Gaussian random matrix,
  invariant under orthogonal conjugation
- Using the theory of such matrices one finds

  $p(\bar{S}, \bar{\delta}, \varphi) \sim e^{-\frac{1}{10}\bar{S}^2 - \frac{1}{2}\bar{\delta}^2}\, \bar{\delta}^4\, |\sin(3\varphi)|$, where
  - $\bar{\delta}, \varphi$ parametrize the eigenvalues $\bar{\delta}_k$ of $\bar{s}_{ij}$ via

    $$\bar{\delta}_k = \tfrac{2}{3}\bar{\delta}\cos(\varphi + \tfrac{2\pi k}{3});$$
  - the normalization involves the value of $\langle(\nabla^2 C)^2\rangle$:
    an integral requiring an ultraviolet cutoff
    (from now on: normalized quantities, written without bars).

Volume evolution

$$V_{\mathcal{D}}(t; S_b, \Lambda) \sim \int a^3(t; S, \delta, \varphi, \Lambda) e^{-\frac{1}{10}(S-S_b)^2 - \frac{1}{2}\delta^2} \delta^4 \, |\sin(3\varphi)| dS \, d\delta \, d\varphi$$

can be computed numerically, using the evolution equations for $a(t; S, \delta, \varphi, \Lambda)$; implies

- volume scale factor $a_{\mathcal{D}}(t) = V_{\mathcal{D}}^{1/3}(t)$,
- Hubble rate $H_{\mathcal{D}}(t) = \dot{a}_{\mathcal{D}}(t)/a_{\mathcal{D}}(t)$,
- deceleration parameter $q_{\mathcal{D}}(t) = -\ddot{a}_{\mathcal{D}} a_{\mathcal{D}} / \dot{a}_{\mathcal{D}}^2$.

For $\Lambda = 0$ these do not agree with what we observe (even though $q_{\mathcal{D}} < 0$ is possible for positive background curvature).

Flat homogeneous case

- Hubble rate $H(t) = \frac{\dot{a}(t)}{a(t)}$,
- deceleration parameter $q = -\frac{\ddot{a}\,a}{\dot{a}^2} = \frac{d}{dt}\left(\frac{1}{H}\right) - 1$,
- redshift $1 + z = \frac{a(t_o)}{a(t)} \quad \Rightarrow \quad H(t) = -\frac{d}{dt}\ln(1+z)$,
- distance (assuming vanishing spatial curvature)

$$d = (1+z)^{\lambda} \int_0^z \frac{1}{H(z')} dz', \quad \text{with}$$

  - $\lambda = -1$ ... angular diameter distance d_A,
  - $\lambda = +1$ ... luminosity distance d_L,
  - $\lambda = \ \ 0$ ... "structure distance" (Weinberg) d_S,
- $d_L/d_A = (1+z)^2$ actually holds generally (Etherington), then we define $d_S = (1+z)d_A = (1+z)^{-1}d_L$,
- $H = \frac{dz}{dd_S}, \quad dd_S = -(1+z)dt.$

Redshift for general geometries

- x_e^μ, x_o^μ ... loci of photon emission and observation,
- s ... affine parameter for photon path (geodesic),
- $k^\mu = dx^\mu/ds$ ... tangent vector,
- u_e, u_o ... normalized tangent vectors to the worldlines of the source and the observer:

$$\Rightarrow 1 + z = \frac{(u{\cdot}k)_e}{(u{\cdot}k)_o}.$$

- If $\exists$ distinguished time coordinate t: $u_e = \frac{\partial}{\partial t}|_e$, $u_o = \frac{\partial}{\partial t}|_o$, s normalized to $ds = dt$ at the observer,
  $\Rightarrow 1 + z = \frac{dt}{ds}$, i.e. $\frac{d}{ds} = (1+z)\frac{d}{dt}$ (with $t = t_e$).
- Notation:
  - $\frac{d}{dt}$ or dot for t parametrizing the geodesic,
  - ∂_0 or $\frac{\partial}{\partial t}$ partial derivative by spacetime coordinate $t = x^0$.

Sachs Equations

$$
\begin{aligned}
-\frac{d\theta_{\rm opt}}{ds} + {\theta_{\rm opt}}^2 + |\sigma_{\rm opt}|^2 &= -\frac{1}{2} R_{\alpha\beta} k^\alpha k^\beta , \\
-\frac{d\sigma_{\rm opt}}{ds} + 2\theta_{\rm opt}\sigma_{\rm opt} &= -\frac{1}{2} R_{\alpha\beta\mu\nu} \varepsilon^\alpha k^\beta \varepsilon^\mu k^\nu ,
\end{aligned}
$$

- $\theta_{\rm opt}, \sigma_{\rm opt}$ ... expansion rate and shear of the null bundle,
- $\varepsilon = \varepsilon_{(1)} + \sqrt{-1}\,\varepsilon_{(2)}$ where $\varepsilon_{(1)}$, $\varepsilon_{(2)}$ are spacelike unit vectors with $\varepsilon_{(i)} \cdot k = \varepsilon_{(i)} \cdot u_o = 0$,
- $R_{\alpha\beta\mu\nu}$ (2nd Eq.) can be replaced by the Weyl tensor $C_{\alpha\beta\mu\nu}$ $\rightarrow$ "Weyl focusing".

Angular diameter and structure distance

- d_A is determined by $-\frac{d}{ds}\ln d_A = \theta_{\rm opt}$,
- the Sachs equations become

$$\frac{d^2 d_A}{ds^2} = -(|\sigma_{\rm opt}|^2 + \frac{1}{2}R_{\alpha\beta}k^\alpha k^\beta)d_A,$$
$$\frac{d}{ds}(\sigma_{\rm opt}d_A^2) = \frac{1}{2}R_{\alpha\beta\mu\nu}\varepsilon^\alpha k^\beta \varepsilon^\mu k^\nu d_A^2.$$

- $d_A = (1+z)^{-1}d_S$, $\frac{d}{ds} = (1+z)\frac{d}{dt}$ gives

$$\ddot{d}_S - [\ln(1+z)]^{\cdot}\,\dot{d}_S + i d_S = 0 \qquad \text{with}$$

$$i = (1+z)^{-2}(|\sigma_{\rm opt}|^2 + \frac{1}{2}R_{\alpha\beta}k^\alpha k^\beta) - \frac{d^2}{dt^2}\ln(1+z).$$

- $d_{S\sharp} = \int_{t_e}^{t_o}(1+z)dt = \int_0^z \frac{1}{-[\ln(1+z)]^{\cdot}}dz$ is a solution for $i = 0$ (holds e.g. for flat homogeneous universes!); useful because $\frac{d^2 d_S}{dd_{S\sharp}^2} = \frac{-i}{(1+z)^2}d_S$.

Inferred Hubble rate and deceleration

- "Hubble rates" $H_{\rm inf} = \frac{dz}{dd_S}$, $H_\sharp = \frac{dz}{dd_{S\sharp}} = -\frac{d}{dt}\ln(1+z)$,
  - $\frac{H_\sharp}{H_{\rm inf}} = 1 - \int_t^{t_o} \frac{i}{(1+z)} d_S\, dt'$,
  - $i > 0$ ($i < 0$): observations over-(under-)estimate expansion rates in previous epochs,
- inferred time parameter $dt_{\rm inf} = -\frac{dd_S}{1+z} = -\frac{\dot{d}_S}{1+z}dt$,
  - $H_{\rm inf}\, dt_{\rm inf} = H_\sharp\, dt = -d\ln(1+z)$,
- deceleration $q_{\rm inf} = \frac{d}{dt_{\rm inf}}\left(\frac{1}{H_{\rm inf}}\right) - 1$, $\quad q_\sharp = \frac{d}{dt}\left(\frac{1}{H_\sharp}\right) - 1$
  - $q_{\rm inf} = q_\sharp + i\,\frac{d_S(1+z)}{\dot{d}_S\ \dot{z}}$,
  - $i < 0$ can simulate acceleration.

Homogeneous and irrotational dust universes

Assume

- Metric $ds^2 = g_{\alpha\beta}dx^\alpha dx^\beta = -dt^2 + g_{ij}(t,x)dx^i dx^j$,
- energy-momentum tensor with $T_{ij} = g_{ij}T^k_k/3$ and $T_{0i} = 0$.

Then (using $1+z = dt/ds$ and geodesic equations) one finds

$$
\begin{aligned}
H_\sharp &= -\frac{d}{dt}\ln(1+z) = -\frac{\theta}{3} + \sigma_{ij}\dot{x}^i\dot{x}^j, \\
i &= (1+z)^{-2}|\sigma_{\rm opt}|^2 + R/6 - \sigma^2 \\
&\quad + (-\sigma_{ij}\theta - 2\sigma_i^k\sigma_{kj} - r_{ij} + 2\sigma_{ij}\sigma_{kl}\dot{x}^k\dot{x}^l)\dot{x}^i\dot{x}^j \\
&\quad + \dot{x}^i\partial_i\theta/3 + \dot{x}^k(\partial_k\sigma_{ij})\dot{x}^i\dot{x}^j - 2\sigma_{ij}\Gamma^i_{kl}\dot{x}^k\dot{x}^l\dot{x}^j, \\
R_{\alpha\beta\mu\nu}\varepsilon^\alpha k^\beta\varepsilon^\mu k^\nu &= (1+z)^2\Big(\frac{2}{3}\theta\sigma_{ij} - \sigma_{ik}\sigma_j^k + 2r_{ij} + \dot{x}^l\sigma_{lm}\dot{x}^m\sigma_{ij} \\
&\quad - \dot{x}^l\sigma_{li}\dot{x}^m\sigma_{mj} - 4\dot{x}^k\sigma_{i[j|k]}\Big)\varepsilon^i\varepsilon^j.
\end{aligned}
$$

Photon path average: basic ideas

Idea: replace $H_\sharp$ and i by suitably defined averages; $\langle X \rangle_{\mathrm{pp}}(t)$ should be the average of X over all

- spatial positions $\mathbf{x}$ occupied by photons,
- directions $\mathbf{v}$ of propagation.

Photon paths correspond to curves in $\mathbf{x}$–space.

- Flat homogeneus case: straight lines,
- if the shapes were not altered by inhomogeneities, then $p(\dots)$ would give the distribution of the basic parameters with respect to the euclidean metric $dl^2 = \delta_{ij} dx^i dx^j$ along such a curve;
- approximation: same distribution even in the general case.

Photon path average: definition

- From euclidean length to physical time:
  $v^i = \frac{dx^i}{dl} = \dot{x}^i \frac{dt}{dl}$ ... tangent vector with $\delta_{ij} v^i v^j = 1$,
  g-norm $\sqrt{g_{ij} v^i v^j}$, use $g_{ij}\dot{x}^i\dot{x}^j = 1 \Rightarrow dt = \sqrt{g_{ij} v^i v^j}\, dl$.
- For every euclidean path segment dl
  - average over the parameters of the model ($\langle \ldots \rangle_{\mathrm{mw}}$),
  - average over all directions $\mathbf{v}$ ($\int_{S^2} \ldots d^2 v$),
  - weight by the time $dt = \sqrt{g_{ij} v^i v^j}\, dl$ spent in the segment

$$\Rightarrow \langle X \rangle_{\mathrm{pp}} = \frac{\langle \int_{S^2} X \sqrt{g_{ij} v^i v^j} d^2 v \rangle_{\mathrm{mw}}}{\langle \int_{S^2} \sqrt{g_{ij} v^i v^j} d^2 v \rangle_{\mathrm{mw}}}.$$

If X depends on $\dot{x}^i$ explicitly, use $\dot{x}^i = \frac{v^i}{\sqrt{g_{ij} v^i v^j}}$.

Some approximations

We want $\langle \int_{S^2} X \sqrt{g_{ij} v^i v^j} d^2 v \rangle_{\rm mw}$ for $X = 1$, $X = H_\sharp$, $X = i$.

- Integrands: polynomial in v except for $\sqrt{g_{ij} v^i v^j}$.
- Diagonal basis, $g_{ij} = \bar{g}(\delta_{ij} + \gamma_{ij})$, $\bar{g} = \frac{g_{11}+g_{22}+g_{33}}{3}$
  $\Rightarrow \left(\sqrt{g_{ij} v^i v^j}\right)^\lambda = \bar{g}^{\lambda/2}(1 + \frac{\lambda}{2}\, \gamma_{ij} v^i v^j + \ldots)$,
- omit anything quadratic or higher in γ_{ij},
- omit $\sigma_{\rm opt}$-term in the expression for i, then

$$\int_{S^2} \sqrt{g_{ij} v^i v^j} d^2 v \approx 4\pi \sqrt{\bar{g}},$$

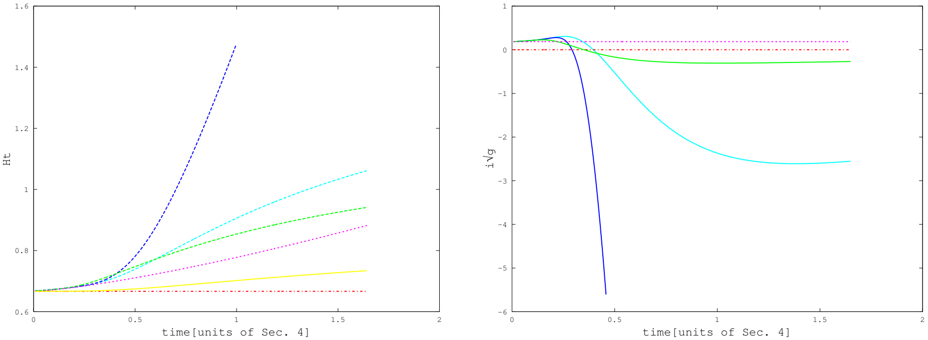
$$\frac{1}{4\pi\sqrt{\bar{g}}} \int_{S^2} H_\sharp \sqrt{g_{ij} v^i v^j} d^2 v \approx \frac{5\theta + 4\sigma^1_1\gamma_{11} + 4\sigma^2_2\gamma_{22} + 4\sigma^3_3\gamma_{33}}{15},$$

$$\frac{1}{4\pi\sqrt{\bar{g}}} \int_{S^2} i \sqrt{g_{ij} v^i v^j} d^2 v \approx \frac{\hat{R}_{\rm in}}{6a^2} + \frac{4}{9a^2} \int_0^t \theta(\tilde{t}) a^2 \sigma^2 d\tilde{t} - \frac{22}{15}\sigma^2$$
$$- \frac{4}{105}[(7\sigma^1_1\theta + 7r^1_1 + 6(\sigma^1_1)^2)\gamma_{11} + \ldots].$$

- Second order perturbation theory computations of $\langle H_\sharp \rangle_{\rm pp}$, $\langle i \rangle_{\rm pp}$ straightforward, but not all of the previous simplifications and approximations respect $2^{\rm nd}$ order PT.
- Non-perturbative results with GNU octave:
  - Euler method with logarithmic time steps,
  - constant $\hat{r} = \hat{r}_{\rm in}$,
- three scenarios for modelling collapse:
  1. collapse to half of maximal size, quantities retain values,
  2. collapse to half of maximal size, then ignore,
  3. ignore any region from the moment it starts collapsing

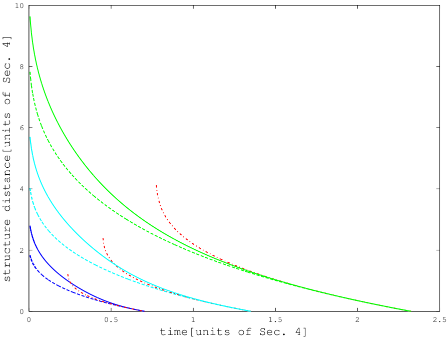
  (motivated by virial theorem, second one most realistic).
- $\langle \sqrt{\bar{g}} \rangle_{\rm mw} = \langle \sqrt{(g_{11} + g_{22} + g_{33})/3} \rangle_{\rm mw}$ close to its EdS equivalent $t^{2/3}$.
- Coding in plots: dashed lines for $H_\sharp$, $q_\sharp$, $d_{S\sharp}$, solid lines for other quantities from scenarios 1/2/3.

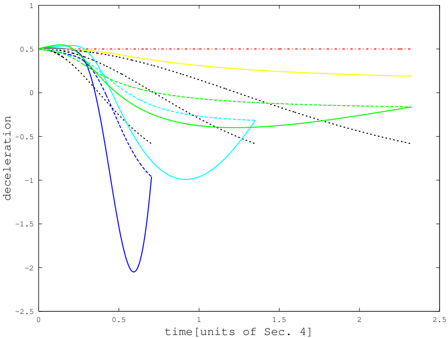
Time evolution of Ht and $i\sqrt{\bar{g}}$



- Yellow line: result from volume averaging
- Strong deviations from homogeneous case: mainly from local anisotropy

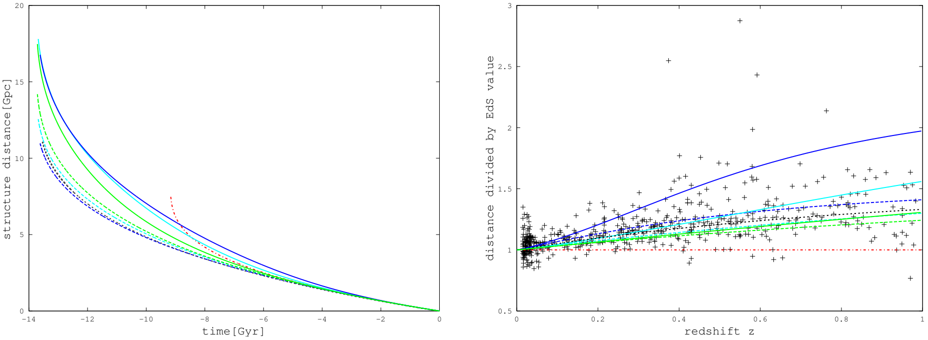
Time evolution of distance and deceleration





- t_o chosen such that $H_o t_o = 1$
- dotted black lines: ΛCDM

Distance in physical units



- Black crosses: observed supernovae
- Better agreement for smaller redshift

Last scattering

- A combination of our programs and linear perturbation theory gives $t_{\text{ls}} \approx 5.3 \times 10^{-5}$ (the time at which $z = 1090$).
- Distance to CMB:
  - $d_S(t) = d_S(0) + d_S^{(1)} t^{1/3} + \mathcal{O}(t^{2/3})$ near $t = 0$
  - results in $d_S(t_{\text{ls}}) \approx 20.7/20.9/19.8$ Gpc for scenarios 1/2/3;
  - overestimation by 50% (compared to Planck data): σ_{opt}-term or measured vs computed d_A?
- Density perturbations (total):

$$\left(\frac{\Delta\rho}{\rho}\right)_{\text{ls}} = \frac{1}{2} t_{\text{ls}}^{\frac{2}{3}} \Delta S = \frac{1}{2} \times (5.3 \times 10^{-5})^{\frac{2}{3}} \times \sqrt{5} \approx 1.6 \times 10^{-3}$$

  - Baryonic ones should satisfy $\Delta\rho_B/\rho_B = 3\Delta T/T$ (T ... temperature),
  - fits well with $\Delta T/T \approx 10^{-5}$, $\Delta\rho/\rho \approx 50\Delta\rho_B/\rho_B$ (structure formation).

Summary of results

Considering a universe that

- is matter dominated and obeys the Einstein equations,
- in its early stages was very close to being spatially flat and homogeneous, with only Gaussian perturbations, and
- has vanishing cosmological constant, $\Lambda = 0$,

there is a time t_o when observervations would suggest

- an inferred Hubble rate H_{inf} such that $H_{\text{inf}} t_o \approx 1$,
- an inferred deceleration parameter of $q_{\text{inf}} \approx -0.5$, and
- density perturbations at a redshift of 1090 that fit well with structure formation.

Approximations used

- Irrotational dust,
- statistical quantities $\rightarrow$ expectation values,
- distribution as if photon paths were straight lines,
- simplified evolution for r_{ij},
- neglect of quadratic or higher terms in $\gamma_{ij} = \bar{g}^{-1} g_{ij} - \delta_{ij}$,
- neglect of σ_{opt}, i.e. Weyl focusing,
- numerical errors from discretization.

The second and third approximation violate 2^{nd} order perturbation theory $\rightarrow$ serious discrepancy with literature.

Outlook

- Work on all points in the approximation list, particular emphasis on those that violate 2^{nd} order PT
- Why do cancellations occur in 2^{nd} order PT?
- Better understanding of CMB
- Stochastic model: Joint distribution for (d_S, z)?