Radiative effects in decay of metastable vacua: a Green's function approach

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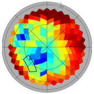

Kosmologietag 10
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8 May 2015

Introduction and motivation

Numerous phenomenological examples across high-energy physics, astro-particle physics and cosmology:

- symmetry restoration at finite temperature and phase transitions in the early Universe

  [Kirzhnits, Linde, PLB**42** (1972) 471; Dolan, Jackiw, PRD**9** (1974) 3320; Weinberg, PRD**9** (1974) 3357]

  - first-order phase transitions may produce relic gravitational waves

    [Witten, PRD**30** (1984) 272; Kosowsky, Turner, Watkins, PRD**45** (1992) 4514; Caprini, Durrer, Konstandin, Servant, PRD**79** (2009) 083519]

  - potential role in the generation of the baryon asymmetry of the Universe

    [Morrissey, Ramsey-Musolf, New J.Phys.**14** (2012) 125003; Chung, Long, Wang, PRD**87** (2013) 023509]

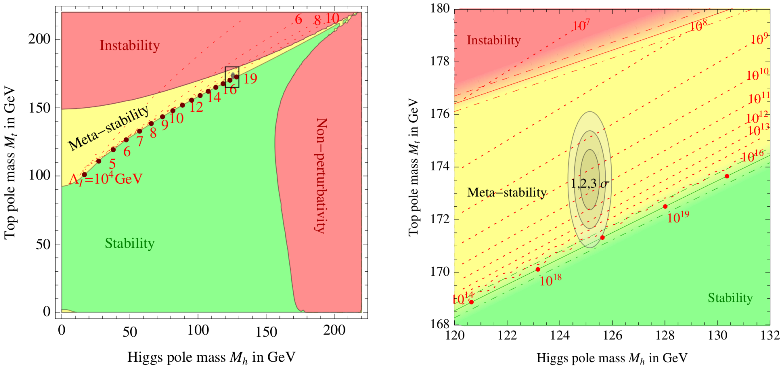
- dynamics of both topological and non-topological defects, and other non-perturbative phenomena in non-linear field theories, e.g. domain walls, Q balls, oscillons, etc ...

- vacuum stability of the SM and its extensions . . .

Introduction and motivation

The **perturbatively**-calculated SM effective potential develops an instability at a scale $\sim 10^{11}$ GeV, given a ~ 125 GeV Higgs boson.

[Cabibbo, Maiani, Parisi, Petronzio, NPB**158** (1979) 295; Sher, PR**179** (1989) 273 & PLB**317** (1993) 159; Isidori, Ridolfi, Strumia, NPB**609** (2001) 387; Elias-Moro, Espinosa, Giudice, Isodori, Riotto, Strumia, PLB**709** (2012) 222; Degrassi, Di Vita, Elias-Moro, Espinosa, Giudice, Isidori, Strumia, JHEP**1208** (2012) 098; Alekhin, Djouadi, Moch, PLB**716** (2012) 214; . . .]



[Buttazzo, Degrassi, Giardino, Giudice, Sala, Salvio, Strumia, JHEP**1312** (2013) 089]

Introduction and motivation

These predictions for the SM are subject to a number of uncertainties . . .

- **experimental:** determination of the top-quark pole mass

  [Bezrukov, Kalmykov, Kniehl, Shaposhnikov, JHEP**1210** (2012) 140 [1205.2893]; Masina, PRD**87** (2013) 053001]

- **phenomenological:** impact of higher-dimension operators

  [Branchina, Messina, PRL**111** (2013) 241801; Branchina, Messina, Platania, JHEP**1409** (2014) 182 [1407.4112]; Lalak, Lewicki, Olszewski, JHEP**1405** (2014) 119; Branchina, Messina, Sher, PRD**91** (2015) 013003; Eichhorn, Gies, Jaeckel, Plehn, Scherer, Sondenheimer, 1501.02812]]

- **theoretical:**
  - interpretation of the non-convexity of the effective potential

    [Weinberg, Wu, PRD**36** (1987) 2474; Alexandre, Farakos, JPA**41** (2008) 015401; Branchina, Faivre, Pangon, JPG**36** (2009) 015006; Einhorn, Jones, JHEP**0704** (2007) 051]
  - implementation of RG improvement

    [Gies, Sondenheimer, EPJC**75** (2015) 68]
  - incorporation of the inhomogeneity of the solitonic background

    [Garbrecht, Millington, 1501.07466, cf. Goldstone, Jackiw, PRD**11** (1975) 1486; e.g. of calculations in the homogeneous background, see Frampton, PRL**37** (1976) 1378; PRD**15** (1977) 2922; Camargo-Molina, O'Leary, Porod, Staub, EPJC**73** (2013) 2588]

It is necessary to consider methods of finding tunnelling rates that account **fully** for **radiative corrections** within the **inhomogeneous soliton background**, particular if these radiative effects are dominant.

Semi-classical tunnelling rate

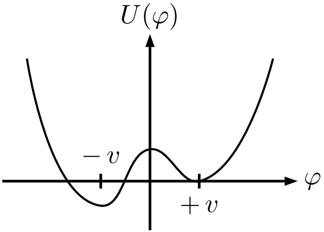
Archetype: Euclidean ϕ^4 theory with tachyonic mass $\mu^2 > 0$

$$\mathcal{L} \;=\; \frac{1}{2!}\left(\partial_\mu \phi\right)^2 \;-\; \frac{1}{2!}\,\mu^2\phi^2 \;+\; \frac{1}{3!}\,g\phi^3 \;+\; \frac{1}{4!}\,\lambda\phi^4 \;+\; U_0$$

[for self-consistent numerical studies, see Bergner, Bettencourt, PRD**69** (2004) 045002; Bergner, Bettencourt, PRD**69** (2004) 045012; Baacke, Kevlishvili, PRD**71** (2005) 025008; Baacke, Kevlishvili, PRD**75** (2007) 045001]

Non-degenerate minima:

$$\varphi \;=\; v_\pm \;\approx\; \pm\, v \;-\; \frac{3g}{2\lambda}\,, \qquad v^2 \;=\; \frac{6\mu^2}{\lambda}$$



Semi-classical tunnelling rate

The classical equation of motion

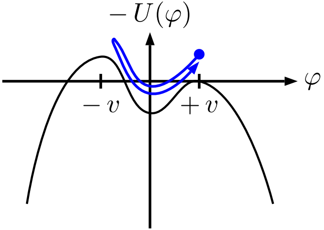
$$-\partial^2\varphi + U'(\varphi) = 0$$

is analogous to a particle moving in a potential $-U(\varphi)$.

There exists a solution — the **Coleman bounce** — satisfying

[Coleman, PRD**15** (1977) 2929; Callan, Coleman, PRD**16** (1977) 1762; Coleman Subnucl. Ser. **15** (1979) 805; Konoplich, Theor. Math. Phys. **73** (1987) 1286]

$$\varphi\Big|_{x_4 \to \pm\infty} = +v\,, \qquad \dot{\varphi}\Big|_{x_4=0} = 0\,, \qquad \varphi\Big|_{|\mathbf{x}|\to\infty} = +v$$



Semi-classical tunnelling rate

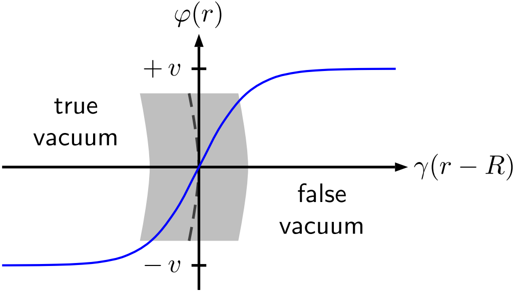
In hyperspherical coordinates, the boundary conditions are

$$\varphi\Big|_{r\to\infty} = +v\,, \qquad \mathrm{d}\varphi/\mathrm{d}r\Big|_{r=0} = 0\,,$$

with the bounce corresponding to the **kink**

[Dashen, Hasslacher & Neveu, PRD 10 (1974) 4114; *ibid.* 4130; *ibid.* 4138]

$$\varphi(r) = v\tanh\gamma(r-R)\,, \qquad \gamma = \mu/\sqrt{2}\,.$$



The bounce looks like a **bubble** of radius $R = 12\lambda/g/v$.

Semi-classical tunnelling rate

The tunnelling rate Γ is calculated from the path integral

$$Z[0] \;=\; \int [\mathrm{d}\phi]\, e^{-S[\phi]/\hbar}\,, \qquad \Gamma/V \;=\; 2\big|\mathrm{Im}\, Z[0]\big|/V/T$$

[see Callan, Coleman, PRD**16** (1977) 1762]

Expanding around the kink $\phi = \varphi + \hbar^{1/2}\hat{\phi}$, the spectrum of the operator

$$G^{-1}(\varphi) \;\equiv\; \frac{\delta^2 S[\phi]}{\delta\phi^2}\bigg|_{\phi=\varphi} \;=\; -\Delta^{(4)} \;+\; U''(\varphi)$$

contains four zero eigenvalues (translational invariance of the bounce) and one negative eigenvalue (dilatations of the bounce).

Writing $B \equiv S[\varphi]$:

$$Z[0] \;=\; -\frac{i}{2}\, e^{-B/\hbar} \left| \frac{\lambda_0 \det^{(5)}\, G^{-1}(\varphi)}{(VT)^2 \left(\frac{B}{2\pi\hbar}\right)^4 (4\gamma^2)^5 \det^{(5)}\, G^{-1}(v)} \right|^{-1/2}$$

Quantum-corrected bounce

Leading quantum corrections to both the bounce and the tunneling rate can be calculated from the **1PI effective action**

[Jackiw, PRD**9** (1974) 1686]

$$\Gamma[\phi] \;=\; -\hbar \ln Z[J] + \int \mathrm{d}^4x \; J(x)\phi(x)$$

But in anticipation of the non-positive-definite eigenspectrum of fluctuations, we want to evaluate the functional integral in $Z[J]$ by expanding around the **quantum-corrected bounce** $\varphi^{(1)}$, i.e.

$$\left.\frac{\delta\Gamma[\phi]}{\delta\phi(x)}\right|_{\phi=\varphi^{(1)}} \;=\; 0$$

$$\left.\frac{\delta S[\phi]}{\delta\phi(x)}\right|_{\phi=\varphi^{(1)}} \;=\; J(x) \;\neq\; 0$$

Note, in the usual standard evaluation of the 1PI effective action, the physical limit is $J(x) \longrightarrow 0$.

Quantum-corrected bounce

At order $\hbar$, the **quantum-corrected bounce** $\varphi^{(1)}(x)$ satisfies

$$-\partial^2\varphi^{(1)}(x) + U'(\varphi^{(1)}; x) + \hbar\Pi(\varphi; x)\varphi(x) \;=\; 0$$

including the **tadpole correction**

$$\Pi(\varphi; x) \;=\; \frac{\lambda}{2}\, G(\varphi; x, x)$$

It follows that the self-consistent choice of $J(x)$ for this method of evaluation is

$$J(x) \;=\; -\hbar\Pi(\varphi; x)\varphi(x)$$

[see Gabrecht, Millington, 1501.07466]

Expanding around $\varphi^{(1)} = \phi - \hbar\delta\varphi$, $\delta\varphi = \varphi^{(1)} - \varphi$, we find

$$\Gamma[\varphi^{(1)}] \;=\; S[\varphi^{(1)}] \;+\; \frac{i\pi\hbar}{2} \;+\; \hbar^2 B^{(2)\prime}[\varphi]$$
$$+\;\frac{\hbar}{2}\ln\left|\frac{\lambda_0 \det^{(5)}\, G^{-1}(\varphi)}{(VT)^2\left(\frac{B}{2\pi\hbar}\right)^4(4\gamma^2)^5 \det^{(5)}\, G^{-1}(v)}\right|^{-1/2}$$

Quantum-corrected bounce

The **tunnelling rate** per unit volume, at order $\hbar^2$, is

$$\Gamma/V \ = \ \left(\frac{B}{2\pi\hbar}\right)^2 (2\gamma)^5 |\lambda_0|^{-\frac{1}{2}} \exp\left[-\frac{1}{\hbar}\left(B + \hbar B^{(1)} + \hbar^2 B^{(2)} + \hbar^2 B^{(2)\prime}\right)\right]$$

Expanded in $\delta\varphi(x) \equiv \varphi^{(1)}(x) - \varphi(x)$:

- one-loop corrections captured by the **functional determinant**:

$$B^{(1)} \ = \ \frac{1}{2}\,\mathrm{tr}^{(5)}\left(\ln G^{-1}(\varphi) \ - \ \ln G^{-1}(v)\right)$$

- two-loop corrections (i) $B^{(2)}$ due to the action of the **corrected bounce** and (ii) $B^{(2)\prime}$ due to the corrections to the one-loop fluctuation determinant:

$$B^{(2)} \ = \ -\frac{1}{2}\int \mathrm{d}^4x\ \varphi(x)\Pi(\varphi;x)\delta\varphi(x) \ = \ -2B^{(2)\prime}$$

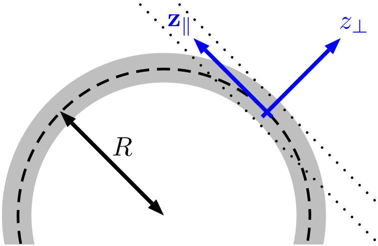
Approximations

1. **Thin-wall** approximation $\mu R \gg 1$: drop the damping term

$$\frac{1}{\mu r}\frac{\mathrm{d}}{\mathrm{d}r} \sim \frac{1}{\mu R}\frac{\mathrm{d}}{\mathrm{d}r} .$$

2. **Planar-wall** approximation: sum over discrete angular momenta replaced by an integral over linear momenta, i.e.

$$\frac{j(j+2)\hbar}{\mu^2 R^2} \longrightarrow \frac{k^2}{\mu^2} .$$



Green's function

Green's function: $u^{(\prime)} \equiv \varphi(r^{(\prime)})/v$ and $m \equiv \left(1 + k^2/4/\gamma^2\right)^{1/2}$

$$
\begin{aligned}
G(u, u', m) \;&=\; \frac{1}{2\gamma m}\left[\vartheta(u-u')\left(\frac{1-u}{1+u}\right)^{\frac{m}{2}}\left(\frac{1+u'}{1-u'}\right)^{\frac{m}{2}}\right.\\
&\times\left(1 - 3\,\frac{(1-u)(1+m+u)}{(1+m)(2+m)}\right)\\
&\times\left.\left(1 - 3\,\frac{(1-u')(1-m+u')}{(1-m)(2-m)}\right) + (u \leftrightarrow u')\right]
\end{aligned}
$$

The renormalization is performed using the effective potential of a homogeneous false-vacuum configuration.

Renormalized tadpole correction:

$$
\Pi^R(u) \;=\; \frac{3\lambda\gamma^2}{16\pi^2}\left[6 + \left(1-u^2\right)\left(5 - \frac{\pi}{\sqrt{3}}\,u^2\right)\right]
$$

Heat-kernel method

[Diakonov, Petrov and Yung, PLB**130** (1983) 385; Diakonov, Petrov, Yung, Sov.J.Nucl.Phys.**39** (1984) 150 [Yad.Fiz.**39** (1984) 240]; Konoplich, Theor.Math.Phys.**73** (1987) 1286 [Teor.Mat.Fiz.**73** (1987) 379; Vassilevich, Phys. Rept. **388** (2003) 279; Carson, McLerran, PRD**41** (1990) 647; Carson, Li, McLerran, Wang, PRD**42** (1990) 2127; Carson, PRD**42** (1990) 2853]

Functional determinant over the positive-definite modes:

$$\mathrm{tr}^{(5)} \ln G^{-1}(\varphi; x) \; = \; -\int \mathrm{d}^4 x \int_0^\infty \frac{\mathrm{d}\tau}{\tau} \, K(\varphi; x, x|\tau) \; .$$

The **heat kernel** is the solution to the **heat-flow equation**

$$\partial_\tau K(\varphi; x, x'|\tau) \; = \; G^{-1}(\varphi; x) K(\varphi; x, x'|\tau) \; ,$$

with $K(\varphi; x, x'|0) = \delta^{(4)}(x - x')$.

But it's **Laplace transform**

$$\mathcal{K}(\varphi; x, x'|s) \; = \; \int_0^\infty \mathrm{d}\tau \; e^{s\tau} \, K(\varphi; x, x'|\tau)$$

is just the **Green's function** with $k^2 \to k^2 + s$.

[cf. direct integration of the Green's function: Baacke, Junker, MPLA**8** (1993) 2869; PRD**49** (1994) 2055; **50** (1994) 4227; Baacke, Daiber, PRD**51** (1995) 795; Baacke, PRD**78** (2008) 065039; Gel'fand-Yaglom thoerem JMP**1** (1960) 48: Baacke, Kiselev PRD**48** (1993); Dunne, Kirsten, JPA**39** (2006) 11915; Dunne, JPA**41** (2008) 304006]

Example

To enhance the radiative effects, while remaining in a perturbative regime, we consider an N**-field model**:

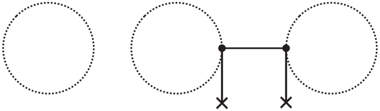
[see 't Hooft, NPB**72** (1974) 461]

$$\mathcal{L} \supset \sum_{i=1}^{N} \left[\frac{1}{2!} \left(\partial_\mu \chi_i\right)^2 + \frac{1}{2!} m_\chi^2 \chi_i^2 + \frac{\lambda}{4} \phi^2 \chi_i^2 \right].$$

For $m_\chi^2 \gg \gamma^2$, the χ **renormalized tadpole correction** is

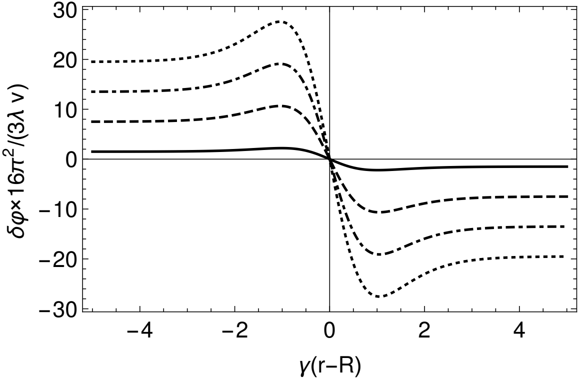
$$\Sigma^R(u) = \frac{\lambda \gamma^2}{8\pi^2} \frac{\gamma^2}{m_\chi^2} \left[72 + \left(1 - u^2\right)\left(40 - 3u^2\right)\right].$$

Dominant $\hbar$ and $\hbar^2$ corrections:



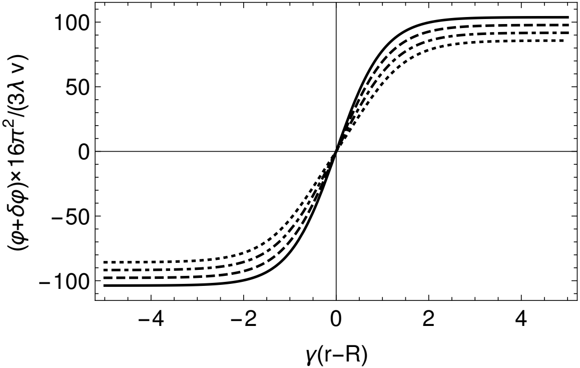
Quantum-corrected bounce

$$\delta\varphi(u) \;=\; -\frac{v}{\gamma}\int_{-1}^{1} \mathrm{d}u' \, \frac{u' \, G(u,u',m)\big|_{k=0}}{1-u'^2} \left(\Pi^R(u') + N\Sigma^R(u')\right)$$



$N\gamma^2/m_\chi^2$: 0 (solid), 0.5 (dashed), 1 (dash-dotted) and 1.5 (dotted)

Quantum-corrected bounce



$N\gamma^2/m_\chi^2$: 0 (solid), 0.5 (dashed), 1 (dash-dotted) and 1.5 (dotted)

[Qualitative agreement with Bergner, Bettencourt, PRD**69** (2004) 045002]

$\longrightarrow$ reduction in bounce action $\longrightarrow$ increase in tunneling rate.

Concluding remarks

- Green's function approach to the calculation of radiative corrections to tunnelling rates from false vacua, including the
  - functional determinant and
  - corrections to the bounce itself.
- In the scalar toy model considered, this calculation could be performed **analytically**, by employing the
  - thin-wall approximation and
  - planar-wall approximation.
- The method is well-suited to numerical analysis and may be of **particular use for radiatively-generated minima**, e.g. for the
  - instability of the SM,
  - massless Coleman-Weinberg model of SSB or

    [Coleman & Weinberg, PRD 7 (1973) 1888]
  - symmetry restoration at finite temperature.

Back-up slides

Tunnelling rate

$$B = \frac{8\pi^2 R^3 \gamma^3}{\lambda}$$

$$B^{(1)} = -B\left(\frac{3\lambda}{16\pi^2}\right)\left[\frac{\pi}{3\sqrt{3}} + 21 + \frac{2542}{15}\frac{\gamma^2}{m_\chi^2}N\right]$$

$$\begin{aligned} B^{(2)} + B^{(2)\prime} &= \frac{1}{2}\int \mathrm{d}^4x\, \varphi(u)\left(\Pi^R(u) + N\Sigma^R(u)\right)\delta\varphi(u) \\ &= -\frac{B}{3}\left(\frac{3\lambda}{16\pi^2}\right)^2\left[\frac{291}{8} - \frac{37}{4}\frac{\pi}{\sqrt{3}} + \frac{5}{56}\frac{\pi^2}{3}\right. \\ &\qquad \left. + \left(\frac{667}{2} - \frac{2897}{42}\frac{\pi}{\sqrt{3}}\right)\frac{\gamma^2}{m_\chi^2}N + \frac{5829}{14}\frac{\gamma^4}{m_\chi^4}N^2\right] \end{aligned}$$