Interacting neutrinos in cosmology: exact description and constraints

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12. Kosmologietag, Bielefeld

arxiv: 1409.1577, 1705.xxxx

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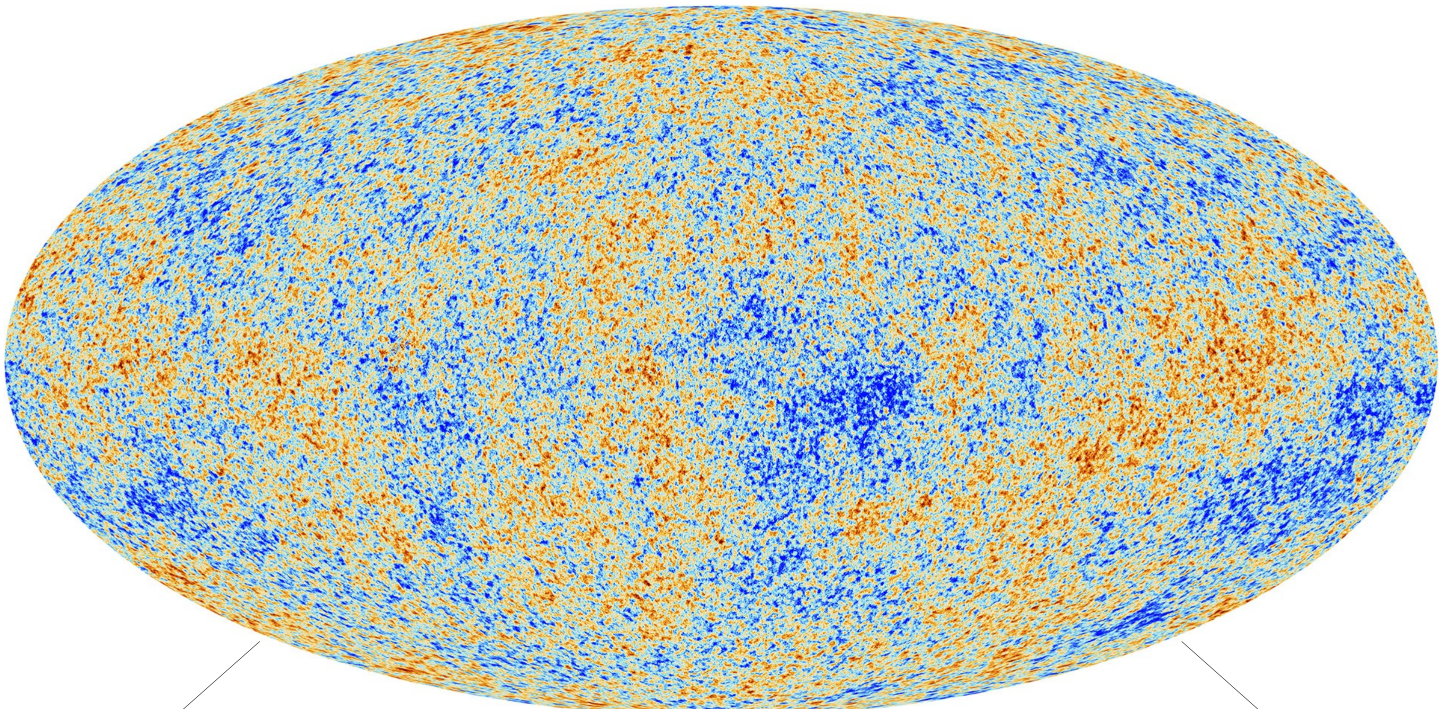
We can learn about particle physics by observing the

Cosmic Microwave Background



Dark matter:

$$\Omega_c h^2 = 0.1198 \pm 0.0015$$

$$\Gamma_{\rm eff} \lesssim 10^{-26}\mathrm{s}^{-1}$$

$$p_{\rm ann} \lesssim 10^{-27}\mathrm{cm}^3\mathrm{s}^{-1}\mathrm{GeV}^{-1}$$

Neutrinos:

$$\sum m_\nu < 0.23\,\mathrm{eV}$$

$$N_{\rm eff} = 3.15 \pm 0.23$$

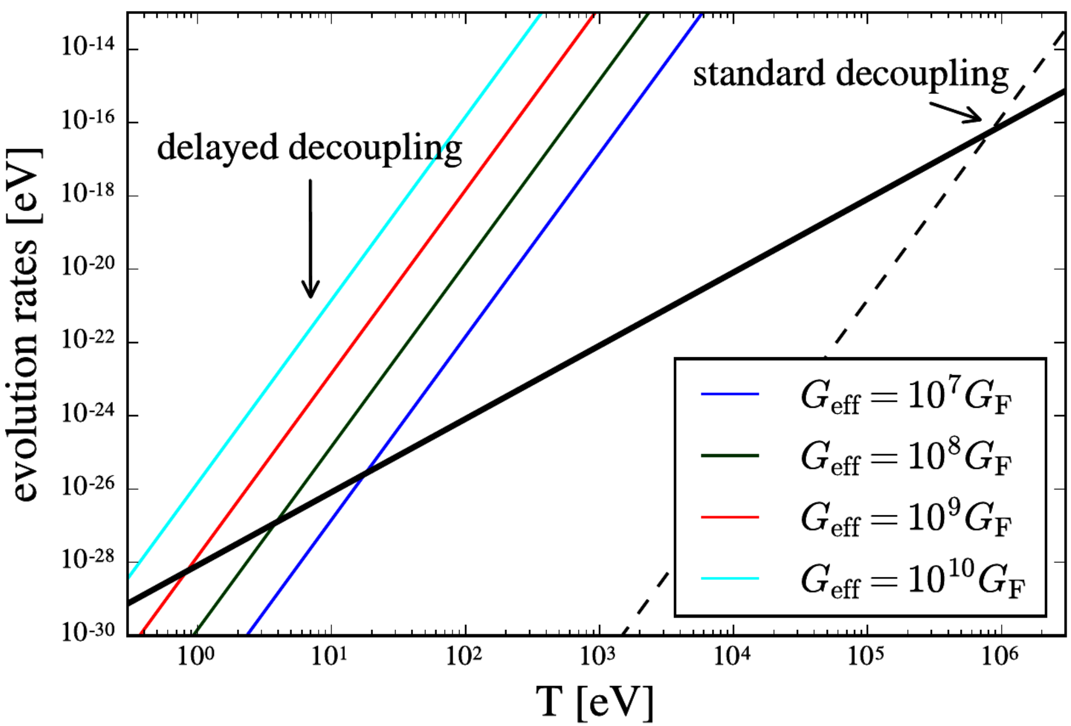
Interactions???

massless neutrinos ⚡ observation of neutrino oscillations

→ Models of neutrino mass generation, “Majoron models“

$$\mathcal{L}_{\rm int} = \mathfrak{g}_{ij}\bar{\nu}_i\nu_j\phi + \mathfrak{h}_{ij}\bar{\nu}_i\gamma_5\nu_j\phi$$

→ **non-standard neutrino interactions** $\Gamma_{\rm new} \sim G_{\rm eff}^2 T^5$ (massive scalar limit)



→ **cosmological signature?**

Impact on the CMB described by **Boltzmann hierarchy for interacting neutrinos**

$\downarrow$

… What‘s that…???

→ Cosmic perturbation theory

Small fluctuations from inflation are the seeds for the structures observed today

1.) Perturbed Einstein equation: $$\bar{G}_{\mu\nu} + \delta G_{\mu\nu} = 8\pi G(\bar{T}_{\mu\nu} + \delta T_{\mu\nu})$$

2.) Perturbed Boltzmann equations:

Perturbed phase-space density: $f(\mathbf{k}, \mathbf{q}, \eta) = \bar{f}(q)\,(1 + \Psi(\mathbf{k}, \mathbf{q}, \eta))$

$$\boxed{\dot{\Psi}_i(\mathbf{k}, \mathbf{q}, \eta) + \mathrm{i}\frac{|\mathbf{q}||\mathbf{k}|}{\epsilon}(\hat{k}\cdot\hat{q})\,\Psi_i(\mathbf{k}, \mathbf{q}, \eta) + \frac{\partial \ln \bar{f}_i(|\mathbf{q}|, \eta)}{\partial \ln |\mathbf{q}|}\left[\dot{\tilde{\eta}} - (\hat{k}\cdot\hat{q})^2\,\frac{\dot{h} + 6\dot{\tilde{\eta}}}{2}\right] = \left(\frac{\partial f_i}{\partial \eta}\right)^{(1)}_{\mathrm{coll}}}$$

Apply on all relevant particle species:

	interacting	non-interacting
relativistic	photons	**neutrinos ???**
non-relativistic	baryons	CDM

Decompose phase-space perturbation into Legendre polynomials:

$$\Psi(|\mathbf{k}|, |\mathbf{q}|, \hat{k}\cdot\hat{q}) = \sum_{\ell=0}(-i)^\ell(2\ell+1)\Psi_\ell(|\mathbf{k}|, |\mathbf{q}|)P_\ell(\hat{k}\cdot\hat{q})$$

→ Taking moments: $\int_{-1}^{1} \mathrm{d}(\hat{k}\cdot\hat{q})P_\ell(\hat{k}\cdot\hat{q})$ [Boltzmann eq.]

→ Neutrino Boltzmann hierarchy:

ultra-relativistic

$$\dot{\delta}_\nu = -\frac{4}{3}\theta - \frac{2}{3}\dot{h},$$
$$\dot{\theta} = k^2\left(\frac{1}{4}\delta - \sigma\right),$$
$$\dot{F}_2 = 2\dot{\sigma} = \frac{8}{15}\theta - \frac{3}{5}kF_3 + \frac{4}{15}\dot{h} + \frac{8}{5}\dot{\eta},$$
$$\dot{F}_{\ell\geq 3} = \frac{k}{2\ell+1}\left[lF_{\ell-1} - (\ell+1)F_{\ell+1}\right]$$

massive

$$\dot{\Psi}_0 = -\frac{qk}{\epsilon}\Psi_1 + \frac{1}{6}\dot{h}\frac{d\ln f_0}{d\ln q},$$
$$\dot{\Psi}_1 = \frac{qk}{3\epsilon}(\Psi_0 - 2\Psi_2),$$
$$\dot{\Psi}_2 = \frac{qk}{5\epsilon}(2\Psi_1 - 3\Psi_3) - \left(\frac{1}{15}\dot{h} + \frac{2}{5}\dot{\eta}\right)\frac{d\ln f_0}{d\ln q},$$
$$\dot{\Psi}_{\ell\geq 3} = \frac{qk}{(2\ell+1)\epsilon}\left[\ell\Psi_{\ell-1} - (\ell+1)\Psi_{\ell+1}\right]$$

How to include neutrino interactions?

1.) Relaxation time approximation:

$$\dot{\mathcal{F}}_{\nu 2} = \frac{8}{15}\theta_\nu - \frac{3}{5}k\mathcal{F}_{\nu 3} + \frac{4}{15}\dot{h} + \frac{8}{5}\dot{\tilde{\eta}} + \frac{9}{10}\alpha_2\dot{\tau}_\nu\mathcal{F}_{\nu 2} \ ,$$

$$\dot{\mathcal{F}}_{\nu\ell} = \frac{k}{2\ell+1}\left[\ell\mathcal{F}_{\nu(\ell-1)} - (\ell+1)\mathcal{F}_{\nu(\ell+1)}\right] + \alpha_\ell\dot{\tau}_\nu\mathcal{F}_{\nu\ell} \ , \quad \ell \geq 3$$

→ motivated from the photon hierachy

*Cyr-Racine, Sigurdson, arxiv:1306.1536

2.) Parameterisation used to fit cosmologocal data:

$$\dot{\delta}_\nu = -\frac{4}{3}\theta_\nu - \frac{2}{3}\dot{h} + \frac{\dot{a}}{a}(1-3c^2_{\text{eff}})\left(\delta_\nu + 4\frac{\dot{a}}{a}\frac{\theta_\nu}{k^2}\right) \ ,$$

$$\dot{\theta}_\nu = k^2\left(\frac{1}{4}\delta_\nu - \sigma_\nu\right) - \frac{k^2}{4}(1-3c^2_{\text{eff}})\left(\delta_\nu + 4\frac{\dot{a}}{a}\frac{\theta_\nu}{k^2}\right) \ ,$$

$$\dot{\mathcal{F}}_{\nu 2} = 2\dot{\sigma}_\nu = \frac{8}{15}\theta_\nu - \frac{3}{5}k\mathcal{F}_{\nu 3} + \frac{4}{15}\dot{h} + \frac{8}{5}\dot{\tilde{\eta}} - (1-3c^2_{\text{vis}})\left(\frac{8}{15}\theta_\nu + \frac{4}{15}\dot{h} + \frac{8}{5}\dot{\tilde{\eta}}\right) \ ,$$

$$\dot{\mathcal{F}}_{\nu\ell} = \frac{k}{2\ell+1}\left[\ell\mathcal{F}_{\nu(\ell-1)} - (\ell+1)\mathcal{F}_{\nu(\ell+1)}\right] \ , \quad \ell \geq 3 \ ,$$

$$(c^2_{\text{eff}}, c^2_{\text{vis}}) = \left(\tfrac{1}{3}, \tfrac{1}{3}\right)$$

→ standard case

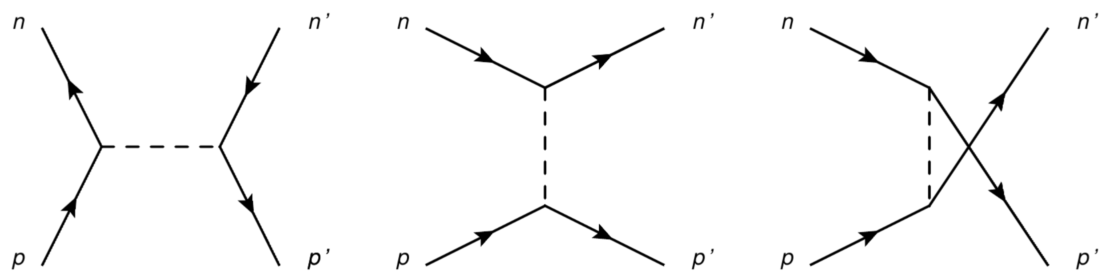
$c^2_{\text{vis}} = 0 \quad c^2_{\text{eff}} = \frac{1}{3}$ → tightly coupled limit

*e.g. Melchiorri, arxiv:1109.2767, ...

Exact description of interacting neutrinos needs calculation of the

collision integral.

$$\rightarrow \dot{F}(\boldsymbol{k},\boldsymbol{q},\eta) + \mathrm{i}\frac{|\boldsymbol{q}||\boldsymbol{k}|}{\epsilon}(\hat{k}\cdot\hat{q})\,F(\boldsymbol{k},\boldsymbol{q},\eta) + \frac{\partial\bar{f}(|\boldsymbol{q}|)}{\partial\ln|\boldsymbol{q}|}\left[\dot{\tilde{\eta}} - (\hat{k}\cdot\hat{q})^2\,\frac{\dot{h}+6\dot{\tilde{\eta}}}{2}\right] = \boxed{\left(\frac{\partial f}{\partial\eta}\right)^{(1)}_{\mathrm{coll}}}$$



difference to photon case: Thomson scattering = low energy transfer

$$\left(\frac{\partial f_i}{\partial\eta}\right)^{(1)}_{ij\leftrightarrow kl}(\boldsymbol{k},\boldsymbol{q},\eta) = \frac{g_j g_k g_l}{2|\boldsymbol{q}|(2\pi)^5}\int\frac{\mathrm{d}^3\boldsymbol{q}'}{2|\boldsymbol{q}'|}\int\frac{\mathrm{d}^3\boldsymbol{l}}{2|\boldsymbol{l}|}\int\frac{\mathrm{d}^3\boldsymbol{l}'}{2|\boldsymbol{l}'|}\,\delta^{(4)}_{\mathrm{D}}(q+l-q'-l')$$
$$\times\,|\mathcal{M}_{ij\leftrightarrow kl}|^2\Big(\bar{f}_k(|\boldsymbol{q}'|,\eta)\,F_l(\boldsymbol{k},\boldsymbol{l}',\eta) + \bar{f}_l(|\boldsymbol{l}'|,\eta)\,F_k(\boldsymbol{k},\boldsymbol{q}',\eta)$$
$$-\,\bar{f}_i(|\boldsymbol{q}|,\eta)\,F_j(\boldsymbol{k},\boldsymbol{l},\eta) + \bar{f}_j(|\boldsymbol{l}|,\eta)\,F_i(\boldsymbol{k},\boldsymbol{q},\eta)\Big)$$

$$\dot{\Psi}_0(q) = -\,k\Psi_1(q) + \frac{1}{6}\frac{\partial \ln \bar{f}}{\partial \ln q}\dot{h} - \frac{40}{3}G^{\rm m} q\, T_{\nu,0}^4\, \Psi_0(q)$$
$$+\, G^{\rm m}\int q'\, \frac{q'}{q\bar{f}(q)}\left[2K_0^{\rm m}(q,q') - \frac{20}{9}q^2\, {q'}^2 e^{-q/T_{\nu,0}}\right]\, \bar{f}_\nu(q')\, \Psi_0(q')\,,$$
$$\dot{\Psi}_1(q) = -\,\frac{2}{3}k\Psi_2(q) + \frac{1}{3}k\Psi_0(q) - \frac{40}{3}G^{\rm m} q\, T_{\nu,0}^4\, \Psi_1(q)$$
$$+\, G^{\rm m}\int q'\, \frac{q'}{q\bar{f}(q)}\left[2K_1^{\rm m}(q,q') + \frac{10}{9}q^2\, {q'}^2 e^{-q/T_{\nu,0}}\right]\, \bar{f}(q')\, \Psi_1(q')\,,$$
$$\dot{\Psi}_2(q) = -\,\frac{3}{5}k\Psi_3(q) + \frac{2}{5}k\Psi_1(q) - \frac{\partial \ln \bar{f}}{\partial \ln q}\left(\frac{2}{5}\dot{\tilde{\eta}} + \frac{1}{15}\dot{h}\right) - \frac{40}{3}G^{\rm m} q\, T_{\nu,0}^4\, \Psi_2(q)$$
$$+\, G^{\rm m}\int q'\, \frac{q'}{q\bar{f}(q)}\left[2K_2^{\rm m}(q,q') - \frac{2}{9}q^2\, {q'}^2 e^{-q/T_{\nu,0}}\right]\, \bar{f}(q')\, \Psi_2(q')\,,$$
$$\dot{\Psi}_{\ell>2}(q) = \frac{k}{2\ell+1}\left[\ell\Psi_{\ell-1}(q) - (\ell+1)\Psi_{\ell+1}(q)\right] - \frac{40}{3}G^{\rm m} q\, T_{\nu,0}^4\, \Psi_\ell(q)$$
$$+\, G^{\rm m}\int q'\, 2\frac{q'}{q\bar{f}(q)}K_\ell^{\rm m}(q,q')\, \bar{f}(q')\, \Psi_\ell(q')\,.$$

IMO, Rampf, Wong, arXiv:1409.1577

- **momentum-dependence reflects non-negligible energy transfer**
- formally very different from other approaches

  → implement in Boltzmann code CLASS (Lesgourgues, Tram, Blas)

Numerical problems...

$$\dot{\Psi}_{\ell>2}(q) = \frac{k}{2\ell+1}\left[\ell\Psi_{\ell-1}(q) - (\ell+1)\Psi_{\ell+1}(q)\right] - \frac{40}{3}G^{\mathrm{m}} q\, T_{\nu,0}^4\, \Psi_\ell(q) + G^{\mathrm{m}} \int q'\,2\frac{q'}{q\bar{f}(q)} K_\ell^{\mathrm{m}}(q,q')\,\bar{f}(q')\,\Psi_\ell(q')$$

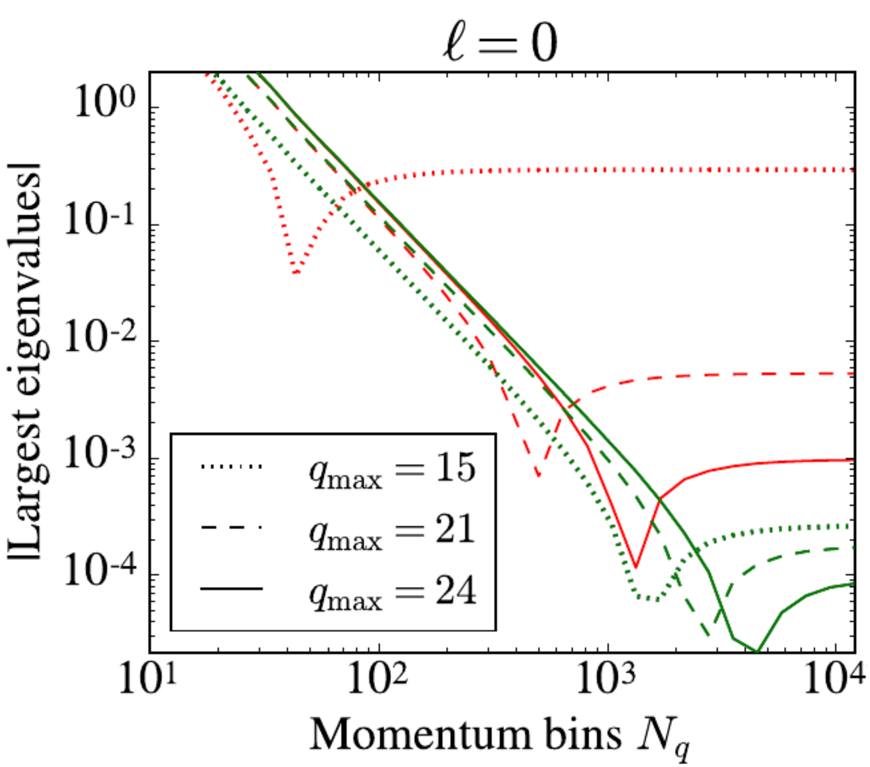
Discretized Boltzmann hierarchy

$$\dot{\Psi}_{\ell,i} = G_{\ell,i} + \sum_j M_{\ell,ij}\Psi_{\ell,j}$$

Homogenous solution: $\boxed{\mathbf{\Psi}_\ell^h = \sum_k c_k\,\mathbf{v}_k e^{\lambda_k \eta}}$

Exponential grow for positive eigenvalues!

Finite momentum-grid size → (small) positive eigenvalues...



Solution:

1) Calculate eigenvalues

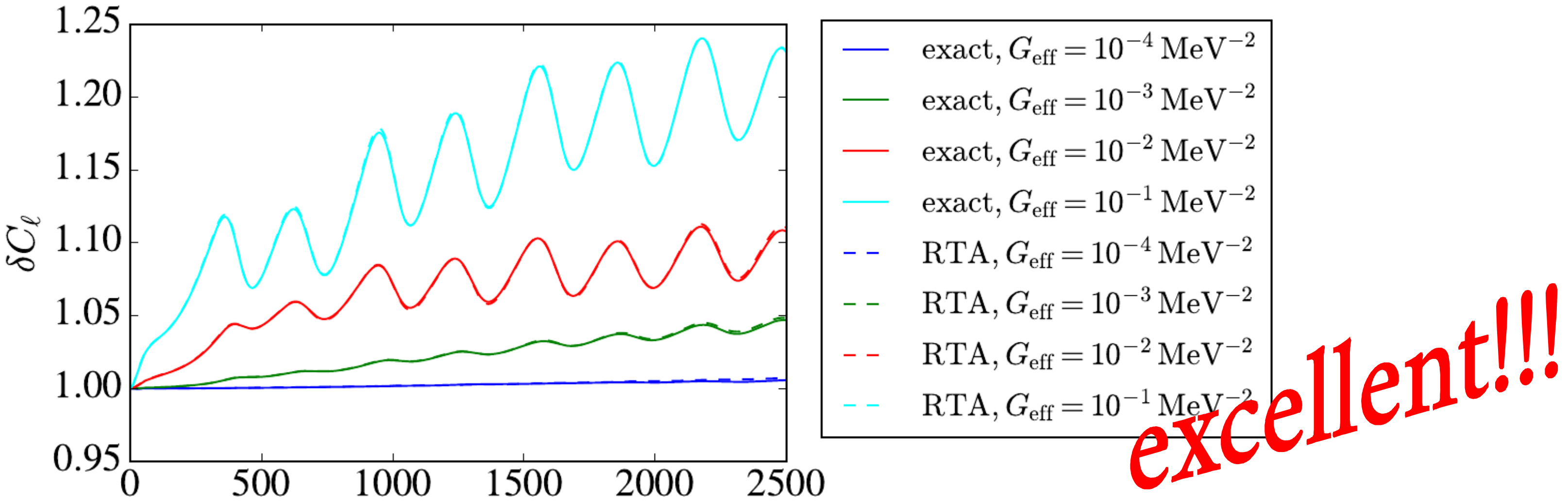
2) Set positive eigenvalues to zero

3) Obtain corrected scattering matrix

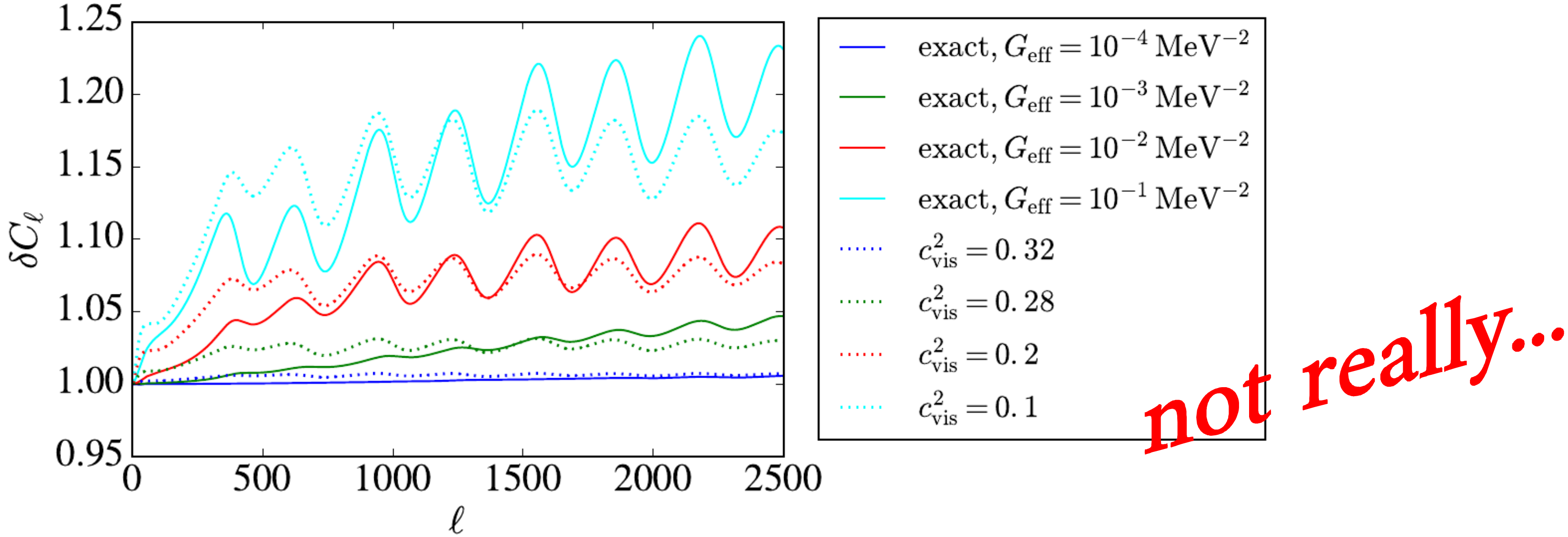
4) Run code only for sufficiently large $q_{\max}$

Comparison with other approaches

1.) Relaxation time approximation

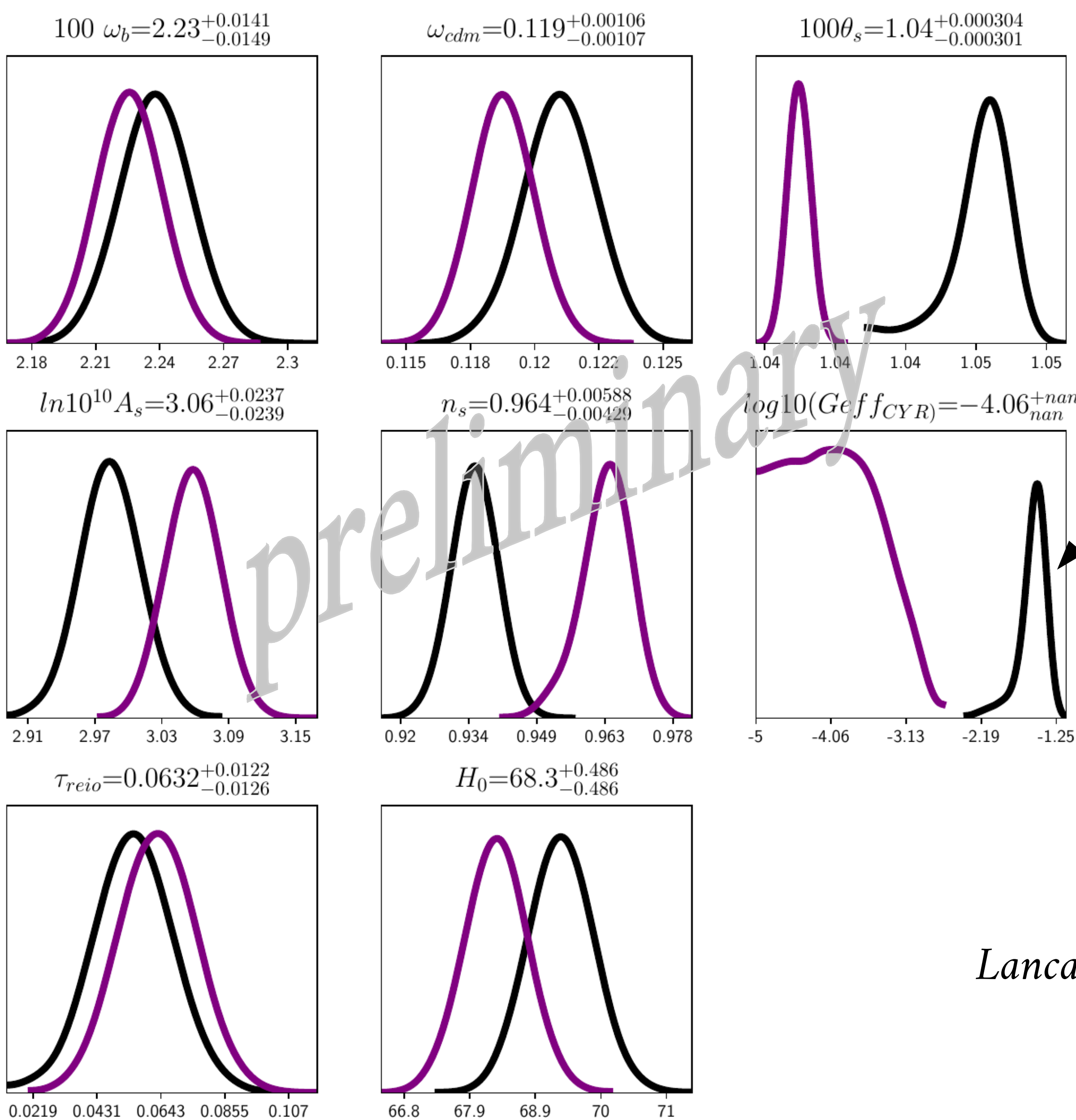


2.) $(c^2_{\rm eff}, c^2_{\rm vis})$



- Relaxation time approximation entirely sufficient.
- $(c^2_{\rm eff}, c^2_{\rm vis})$ -parameterisation should not be used!!!

MCMC results (using the relaxation time approximation)



Interacting neutrino mode!!!

Compare with

Lancaster, Cyr-Racine, arxiv: 1704.06657

Summary:

Majoron models → non-standard neutrino interactions → impact on the CMB?

➔ Calculated the Boltzmann hierarchy for interacting neutrinos

➔ Implemented it in CLASS

Conclusions:

- Boltzmann hierarchy has **formally** a much richer structure than approximations by others
- … but **relaxation time approximation** is an **excellent** effective description
- $\left(c_{\mathrm{eff}}^2, c_{\mathrm{vis}}^2\right)$ -parameterisation does not describe neutrino interactions
- MCMC: there is an interacting neutrino mode!