Quantum transitions through cosmological singularities

1605.02751,1701.05399

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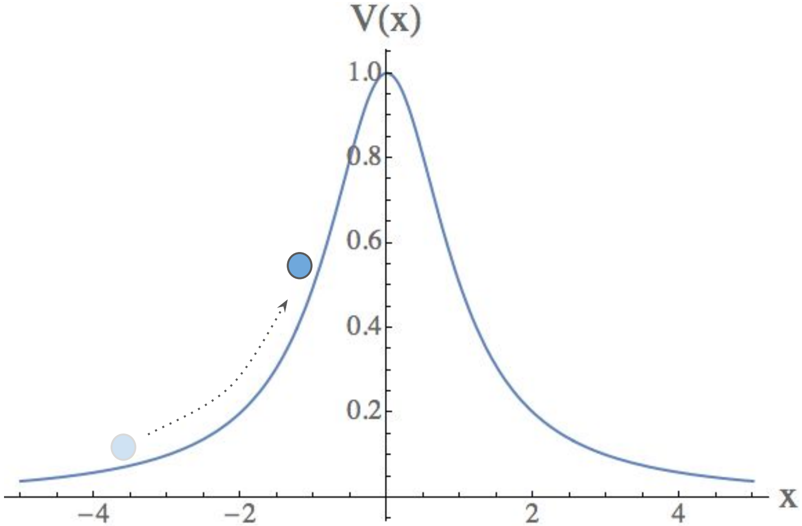
Tunneling in 1D QM

Classically: (E < V)



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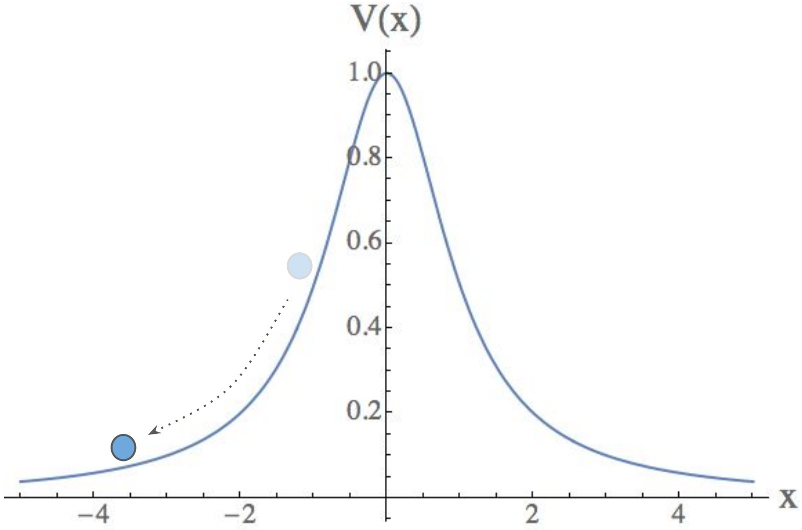

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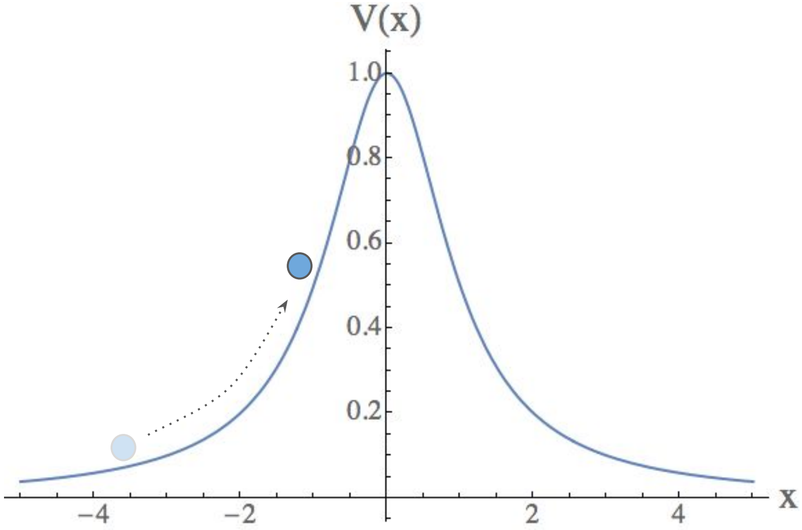

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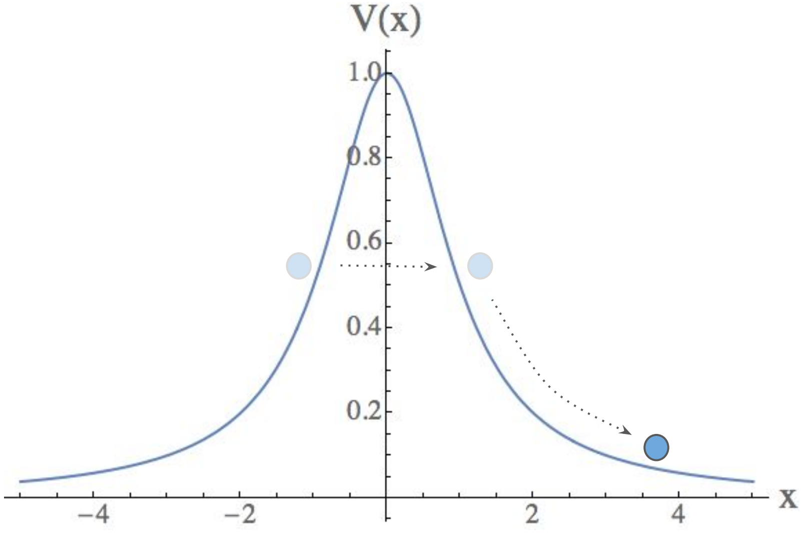
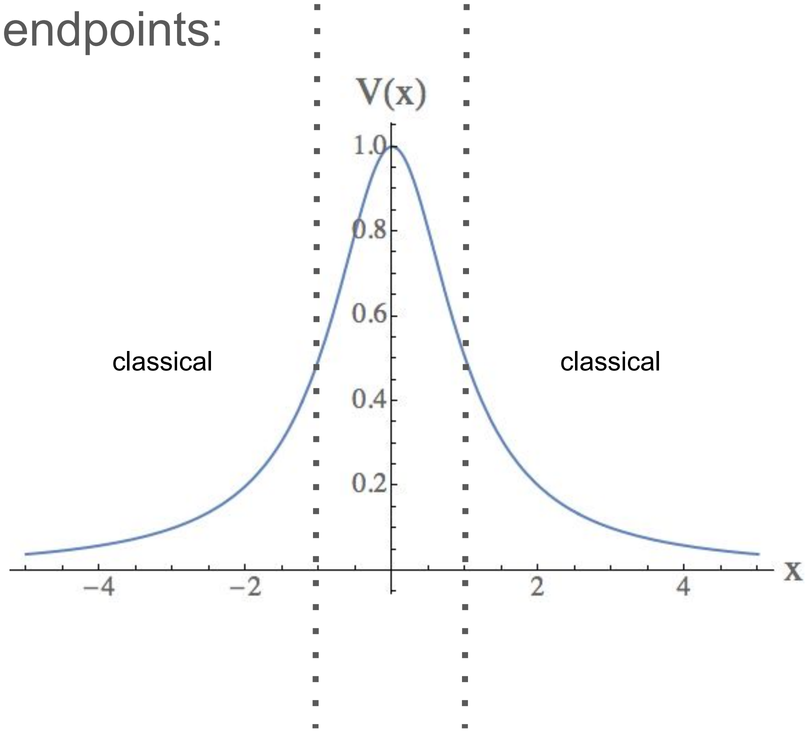


The wavefunction has non-zero support on the right-hand side

Tunneling in 1D QM

Interested in classical endpoints:



Path-Integral Framework

$$\langle x_f, t_f \mid x_i, t_i \rangle = \mathcal{N} \int_{x_i, t_i}^{x_f, t_f} D[x(t)]\, e^{iS}$$

“Transition amplitude = Sum over all paths (even crazy ones) weighted by the action”

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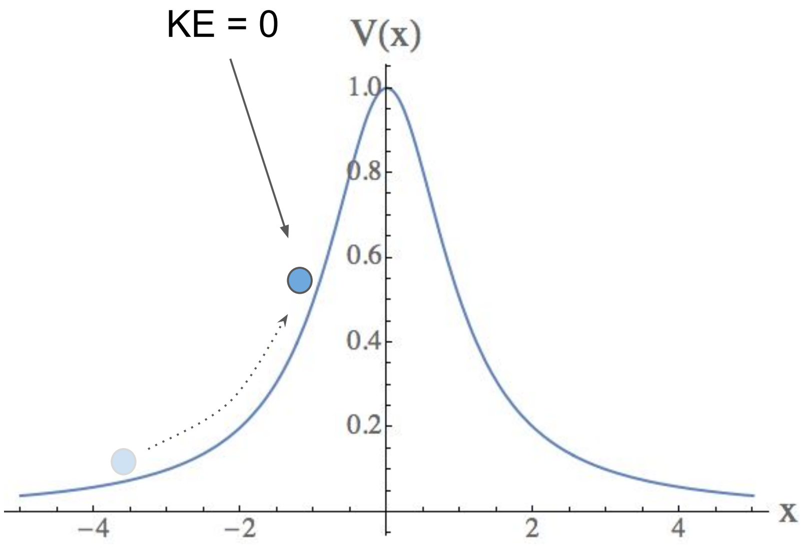
Coleman’s “Instanton” Method



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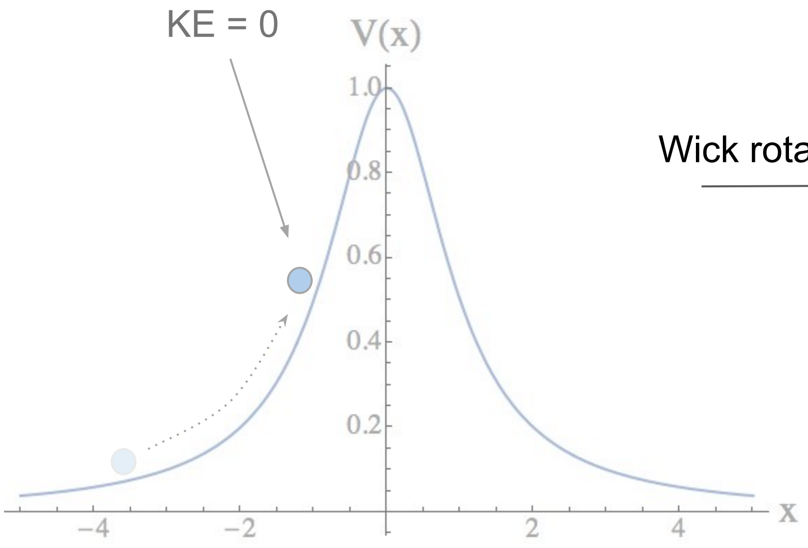

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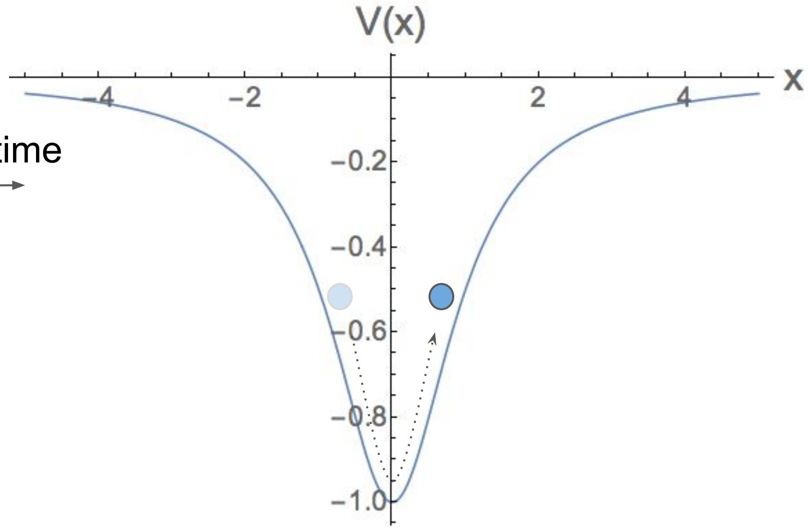
Wick rotation to Eucl. time



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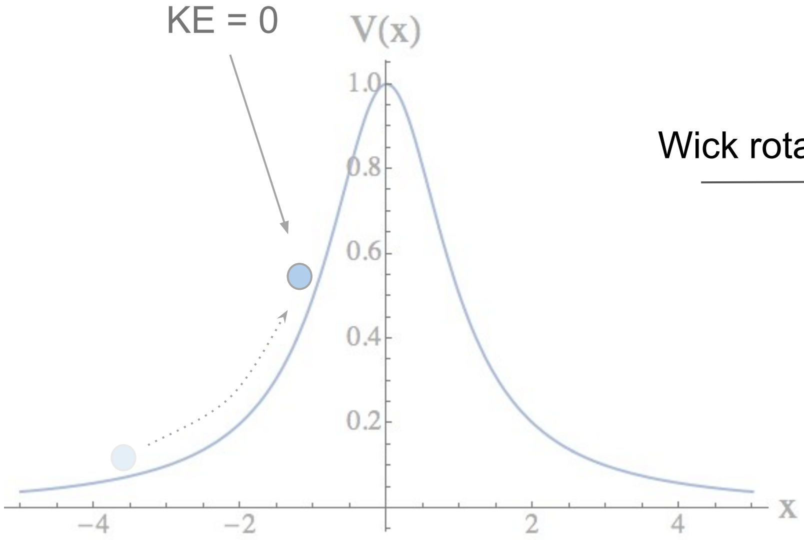

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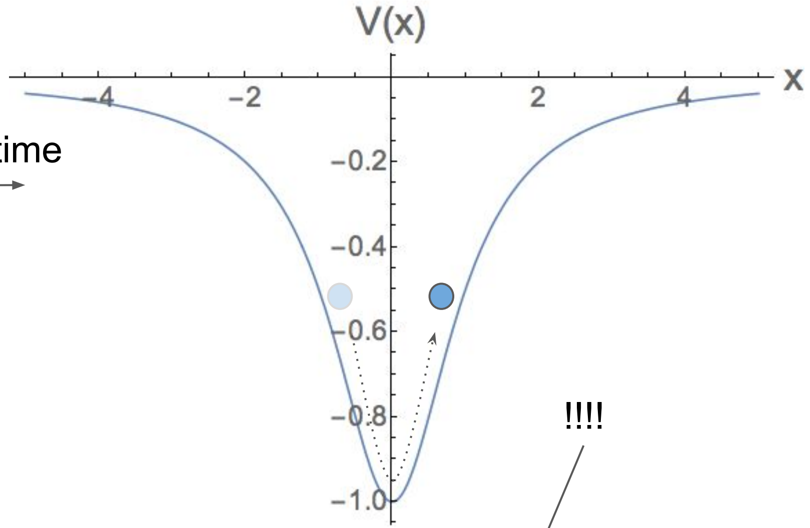

!!!!

$$S = \int \mathrm{d}t \left(\frac{1}{2}\dot{x}^2 - V(x) \right)$$

$$S_E \sim \int d\tau \left(\frac{1}{2}\dot{x}^2 + V(x) \right)$$

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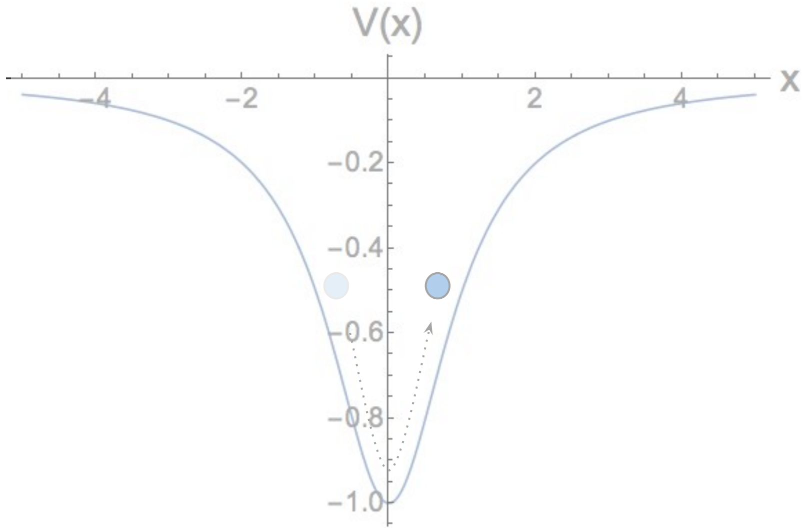

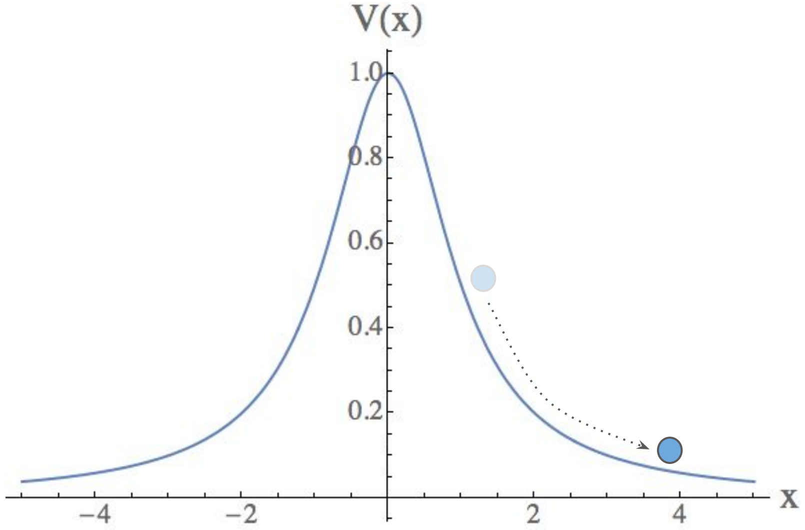

Insert Eucl. solution at an instant in time. Otherwise Lorentzian solution.

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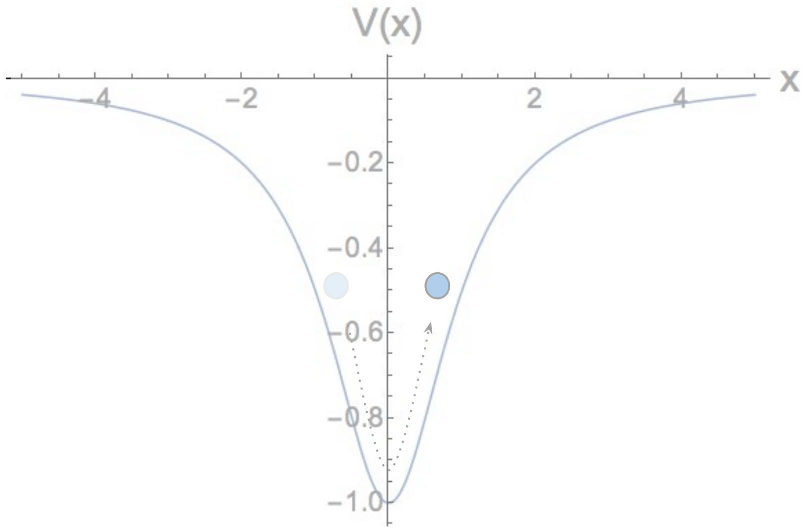

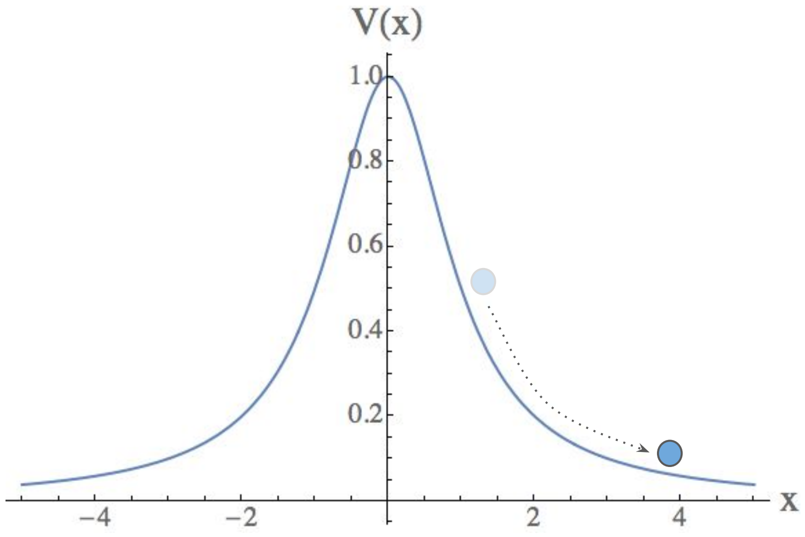

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Semi-classically - probability given by instanton action

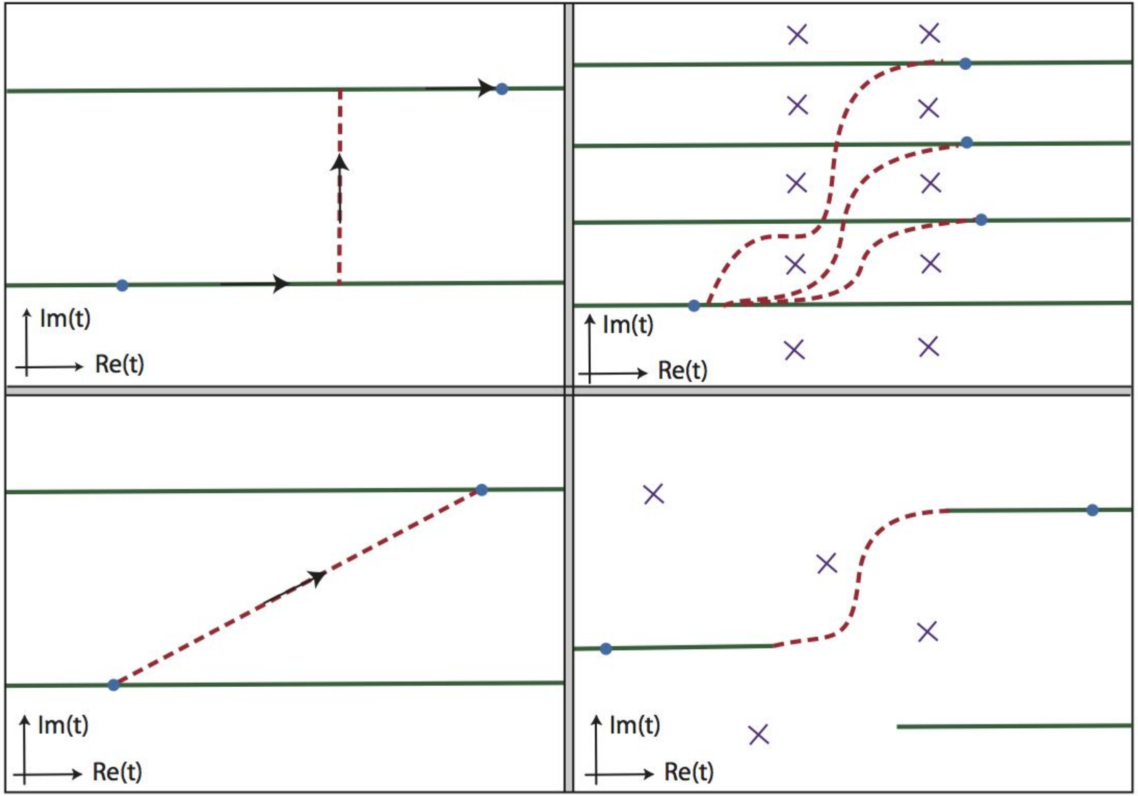
Generalizable?

- Good:
  - Works also in QFT and with gravity

$\Longrightarrow$ Use general complexified time: $d\tau = N dt$

- Bad:
  - Seems unmotivated
  - Not continuous
  - What about singularities

Overview



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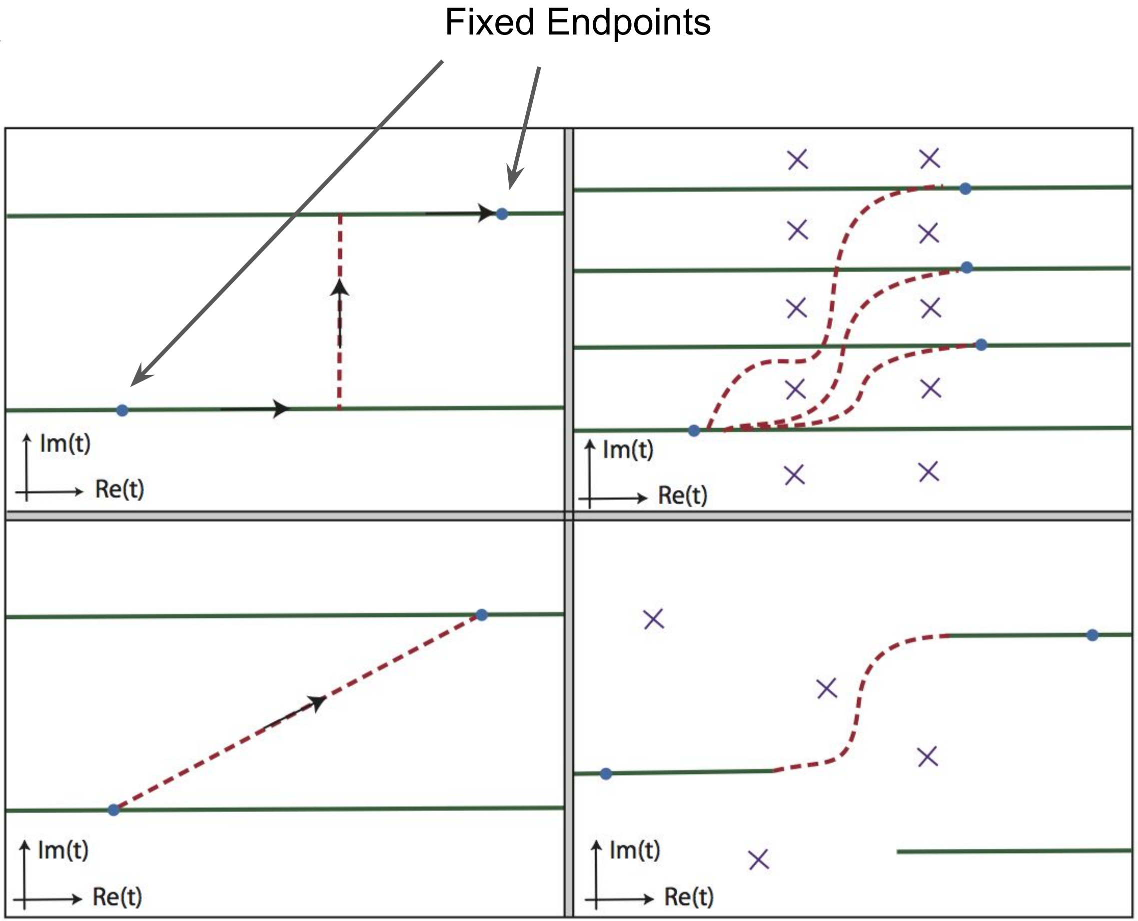

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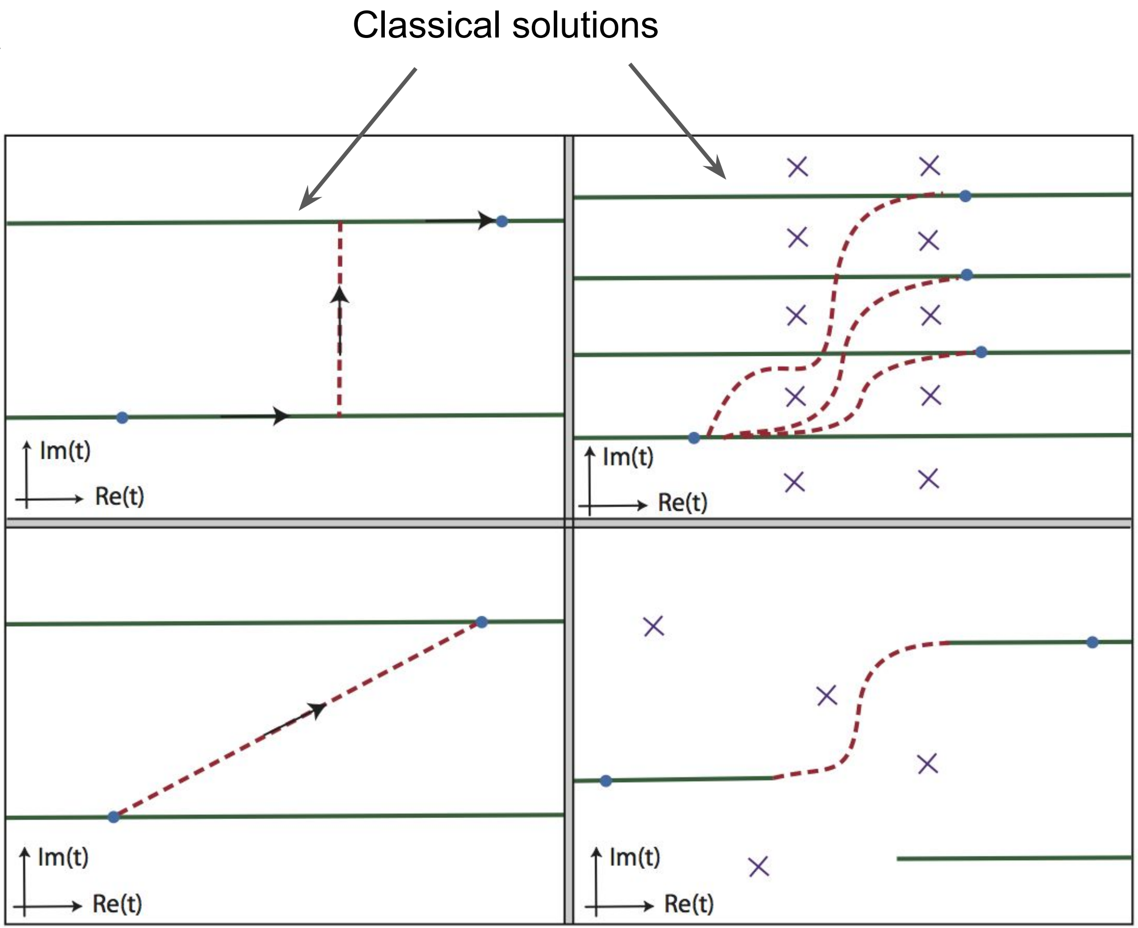

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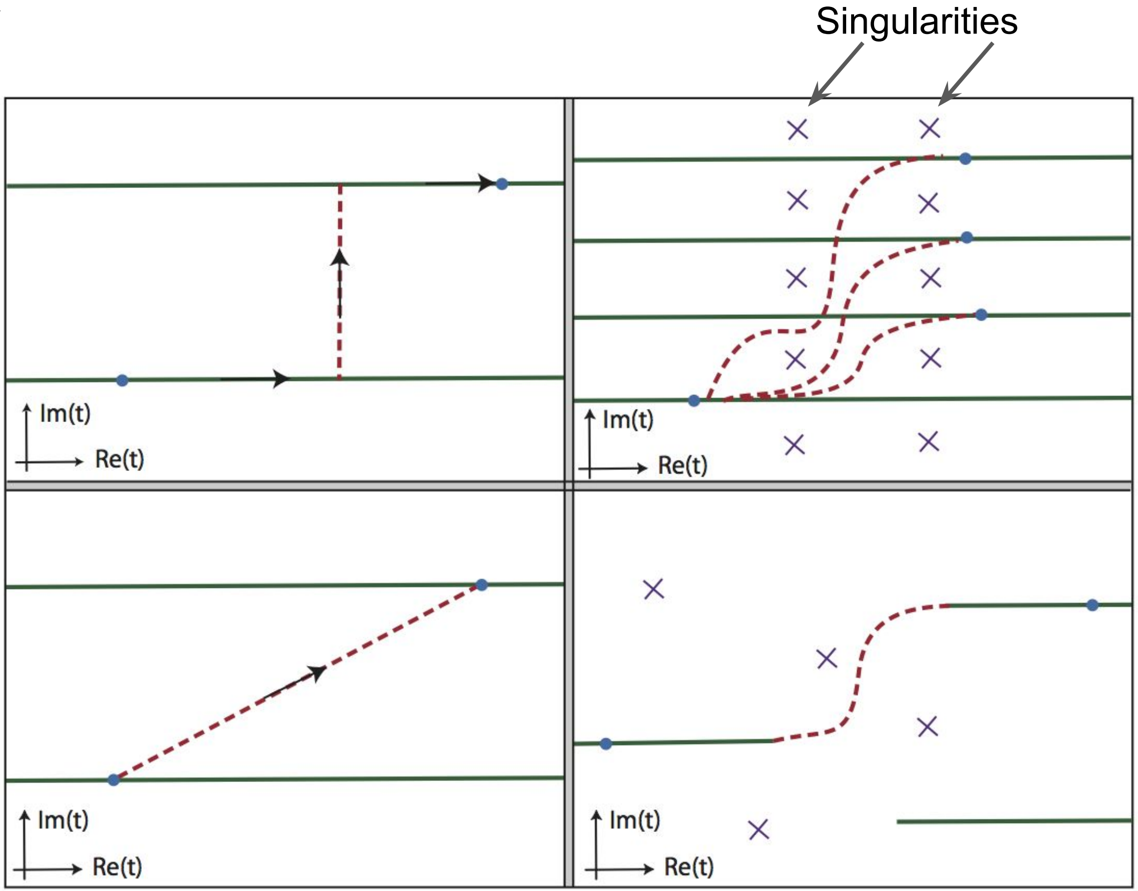

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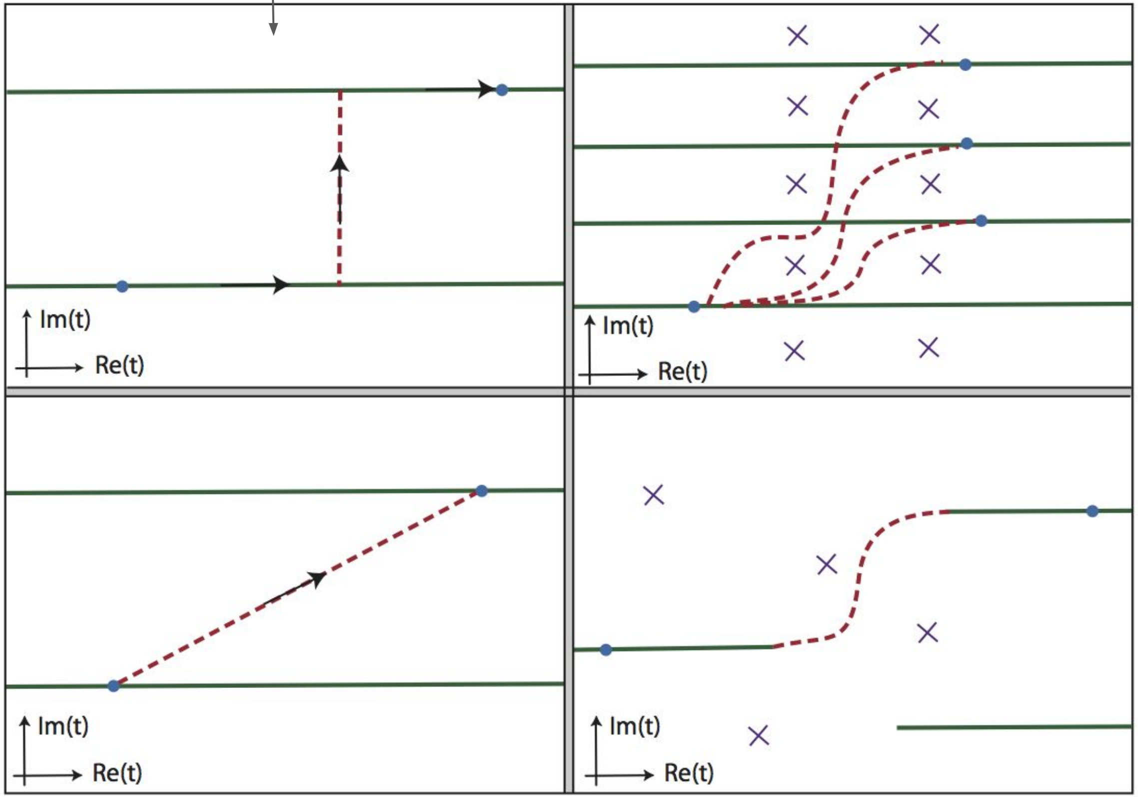

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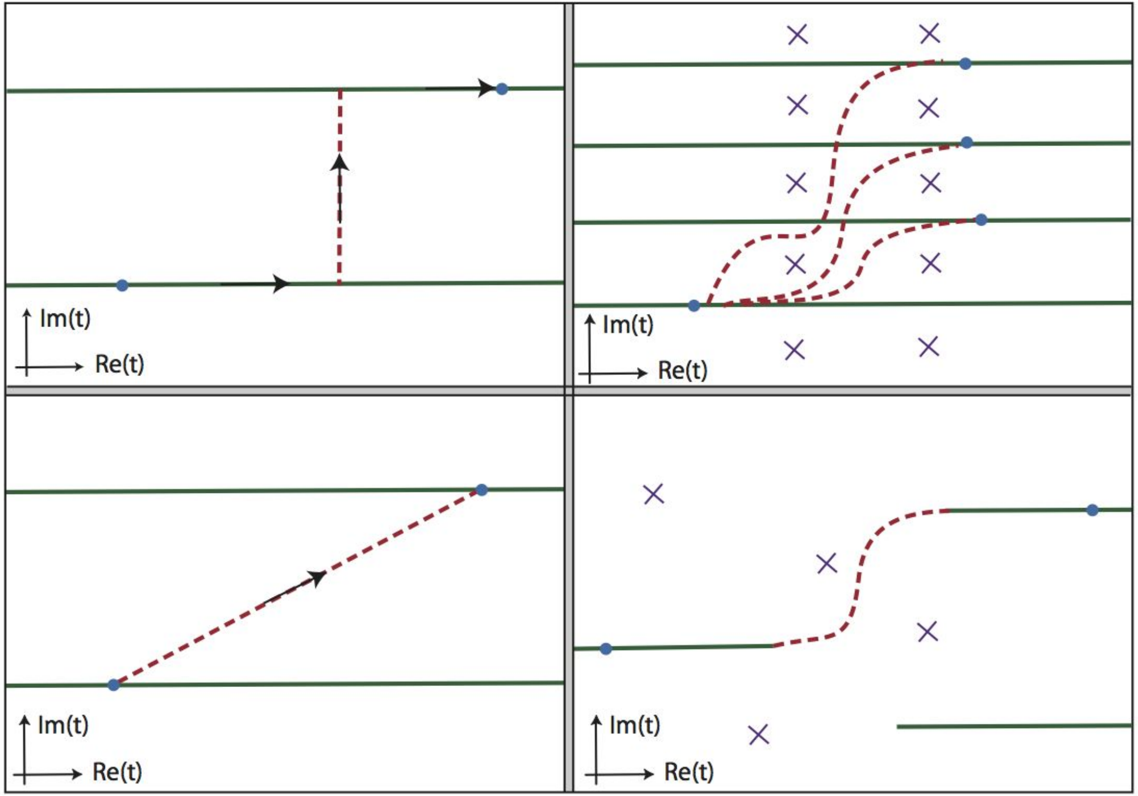
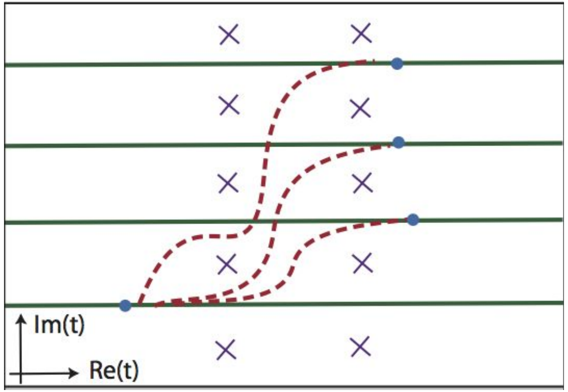


Do all paths matter?

Cauchy’s Theorem - deformed paths are equivalent when there are no singularities present.

Should we sum over all inequivalent paths?



A: No but have a prescription for which paths contribute to tunneling

Which path matters?

Evaluate perturbations!

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Expand action to second order around the saddle point

$$\langle x_f, t_f \mid x_i, t_i \rangle = \mathcal{N} \int_{x_i,t_i}^{x_f,t_f} D[x(t)]\, e^{iS} \sim e^{-S_E(x_{cc})} \frac{1}{\sqrt{\Pi_n \omega_n}}$$

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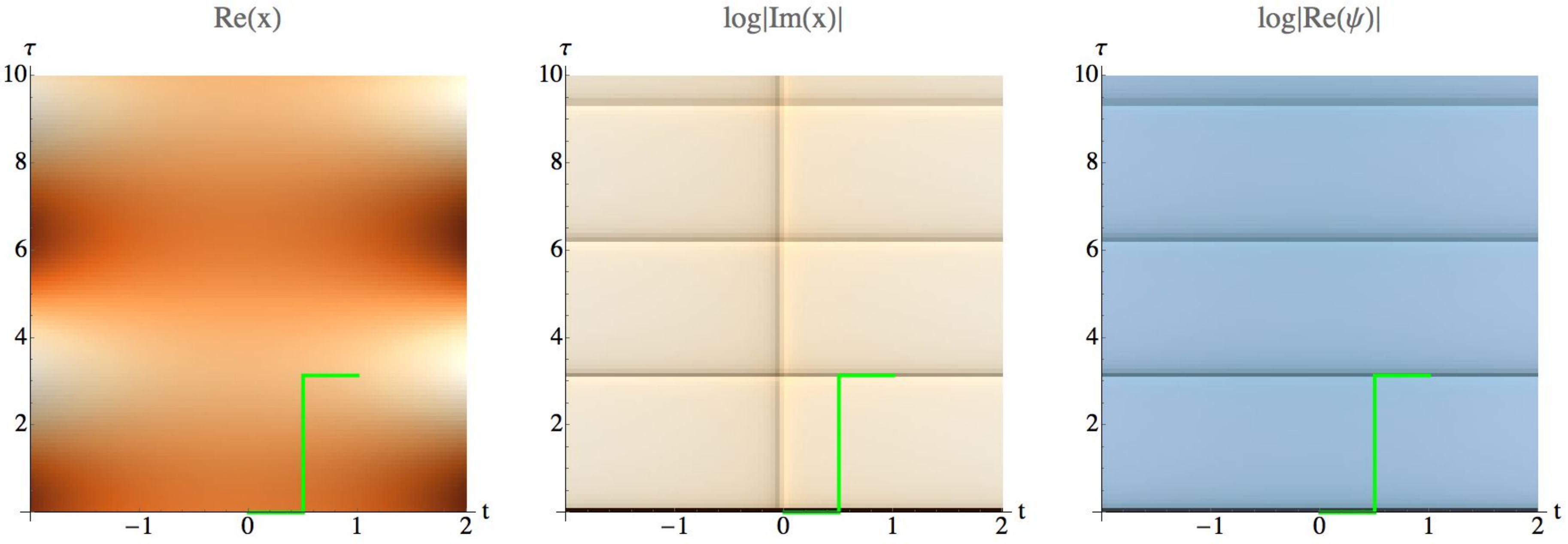
Numerical Studies



Quantum Cosmology

Canonical quantization of gravity.

$$S = \int d^4x \sqrt{-g} \left(\frac{R}{2\kappa^2} - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \, \partial_\nu \phi - V(\phi) \right)$$

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Work within semi-classical “minisuperspace” setting.

$$ds^2 = -\tilde{N}^2(\lambda) d\lambda^2 + a^2(\lambda) d\Omega_3^2$$

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$$ds^2 = -\tilde{N}^2(\lambda)d\lambda^2 + a^2(\lambda)d\Omega_3^2$$

$$S = \frac{6\pi^2}{\kappa^2}\int d\lambda\tilde{N}\left(-a\frac{\dot{a}^2}{\tilde{N}^2} + a + \frac{\kappa^2 a^3}{3}\left(\frac{1}{2}\frac{\dot{\phi}^2}{\tilde{N}^2} - V\right)\right)$$

Inflation and Ekpyrosis

$$\epsilon = \frac{V_{,\phi}^2}{2V^2}$$

$\epsilon < 1$ — Inflation

$\epsilon > 3$ — Ekpyrosis

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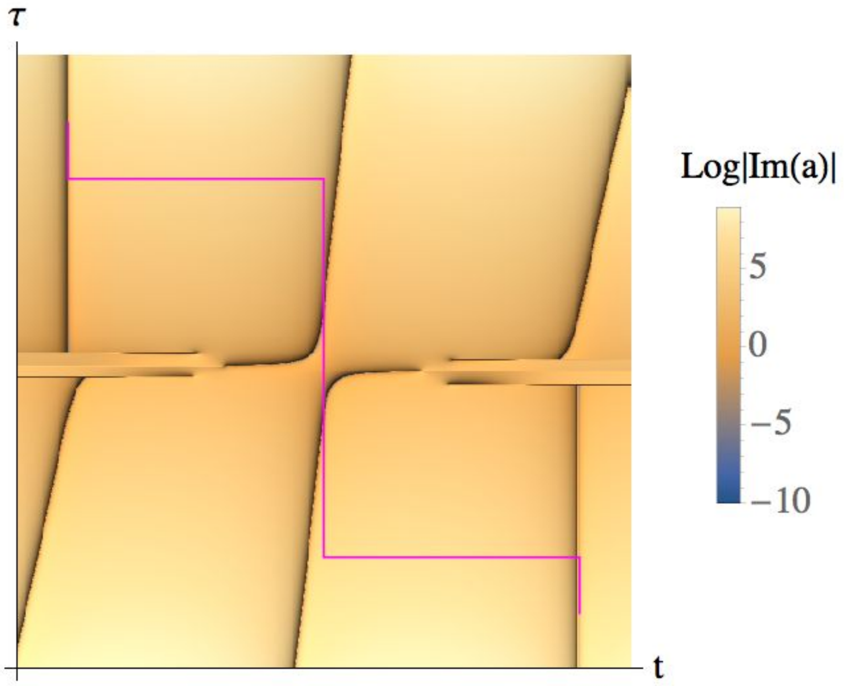
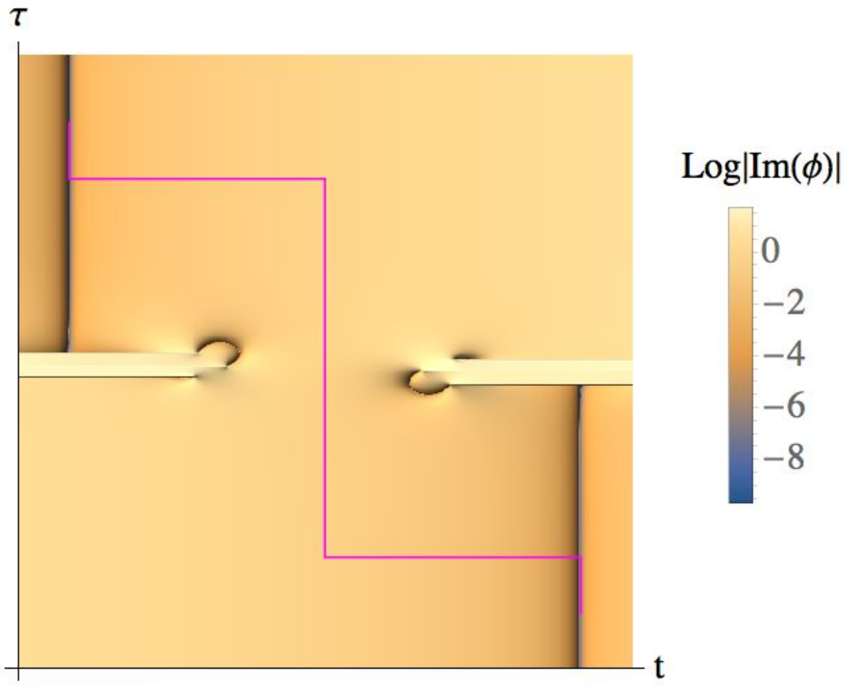
Can we transition between two classical regions of the universe via a quantum transition?

Inflation to Inflation





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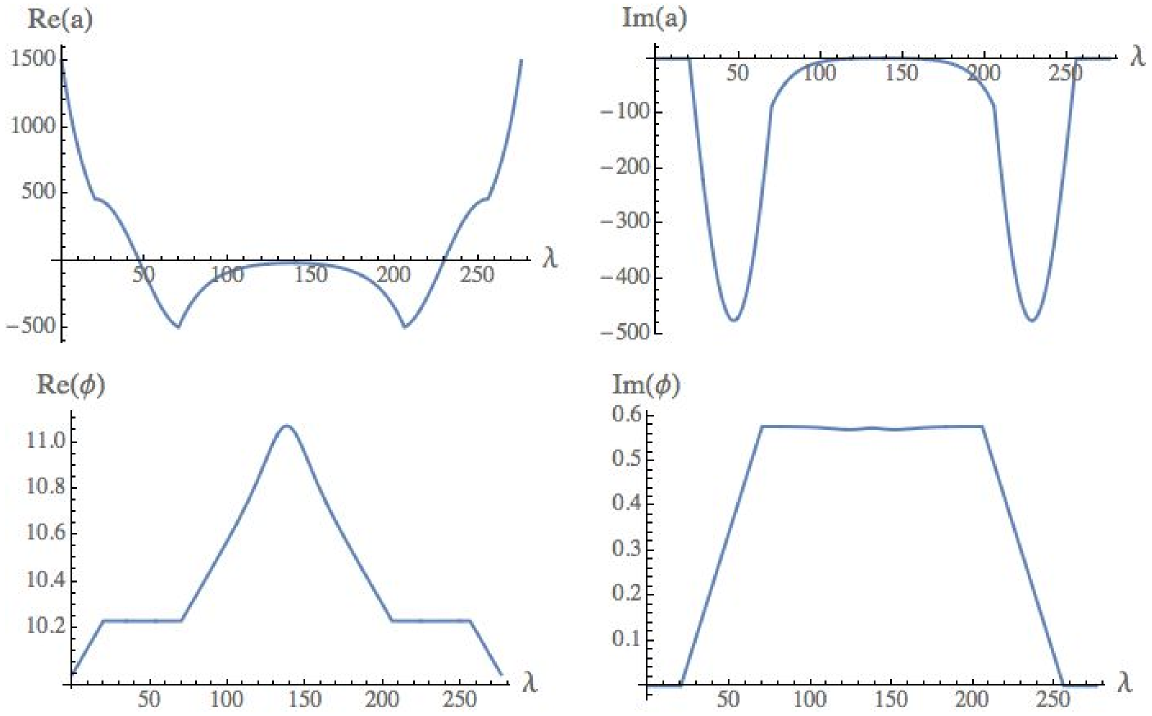

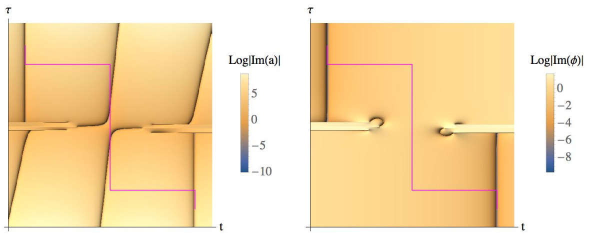

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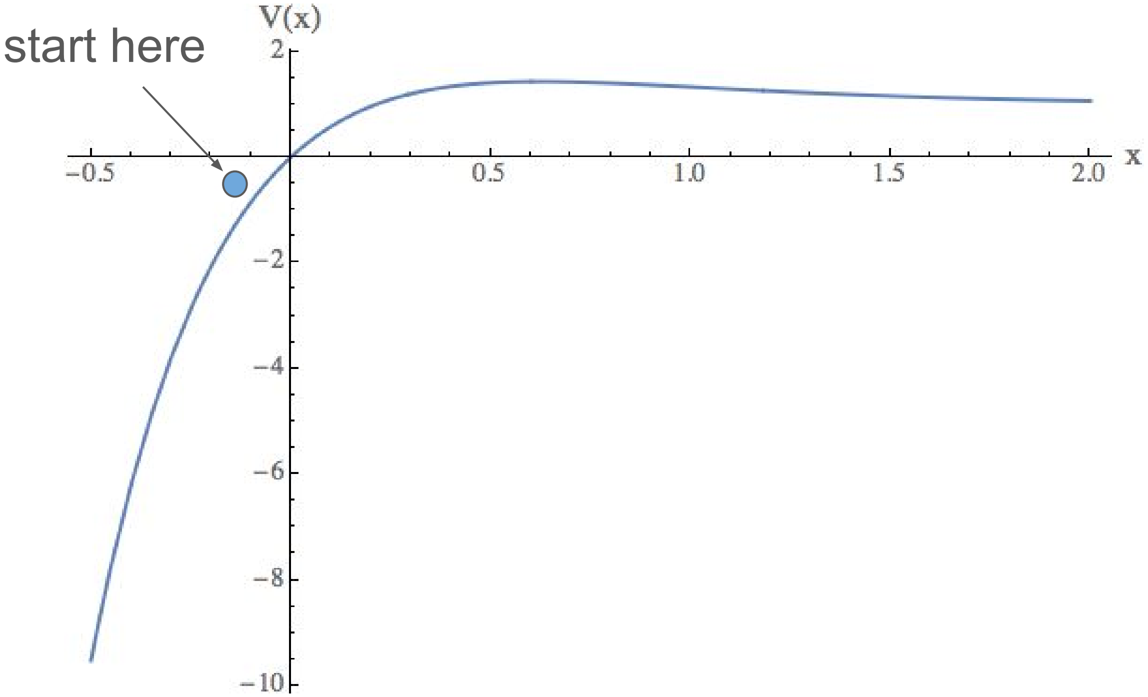

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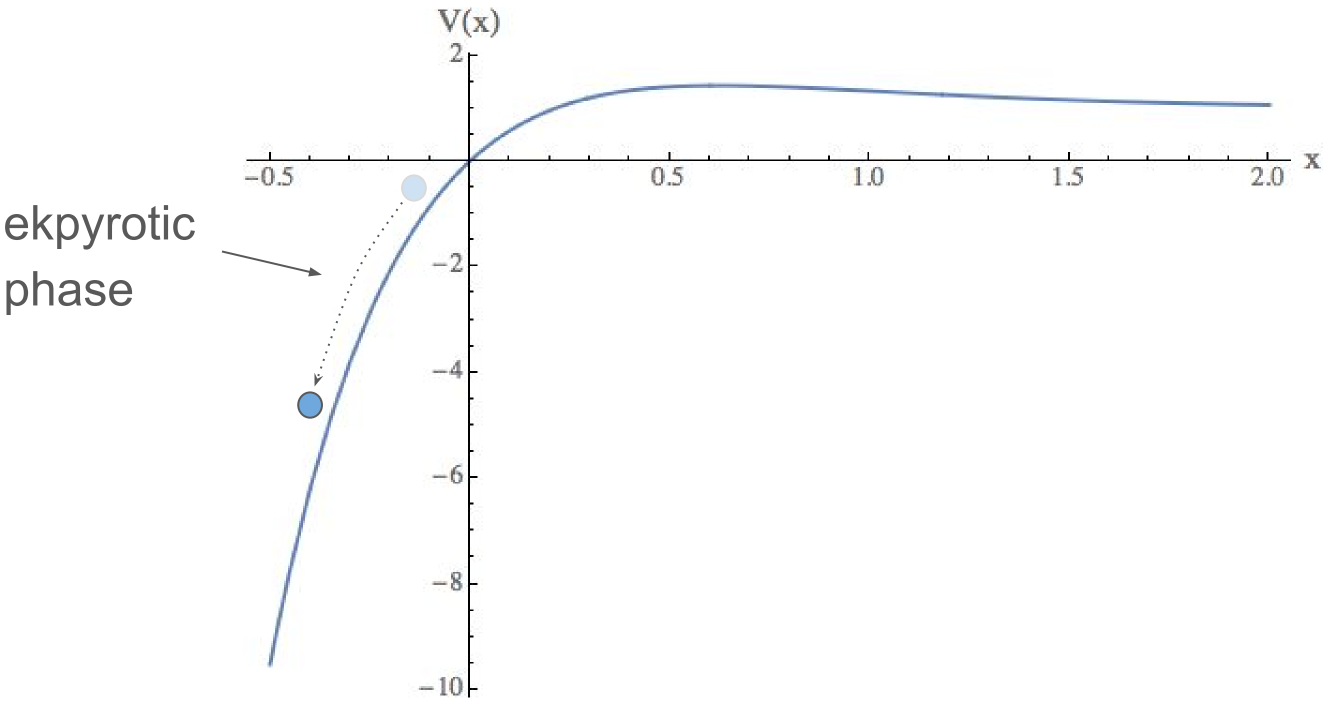

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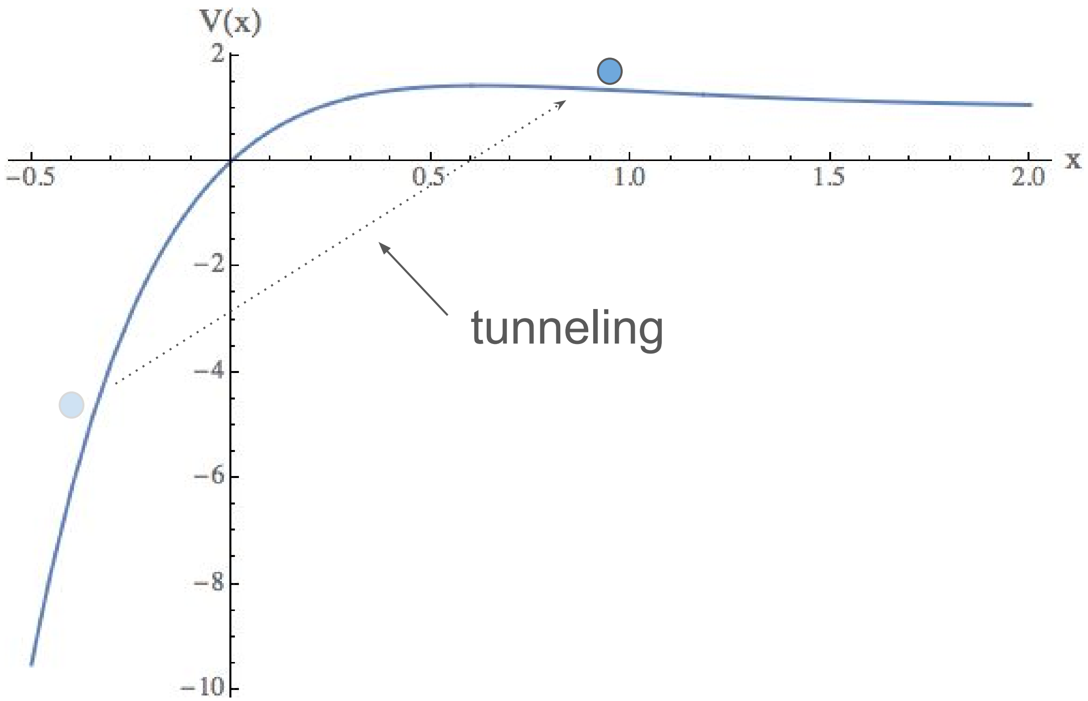

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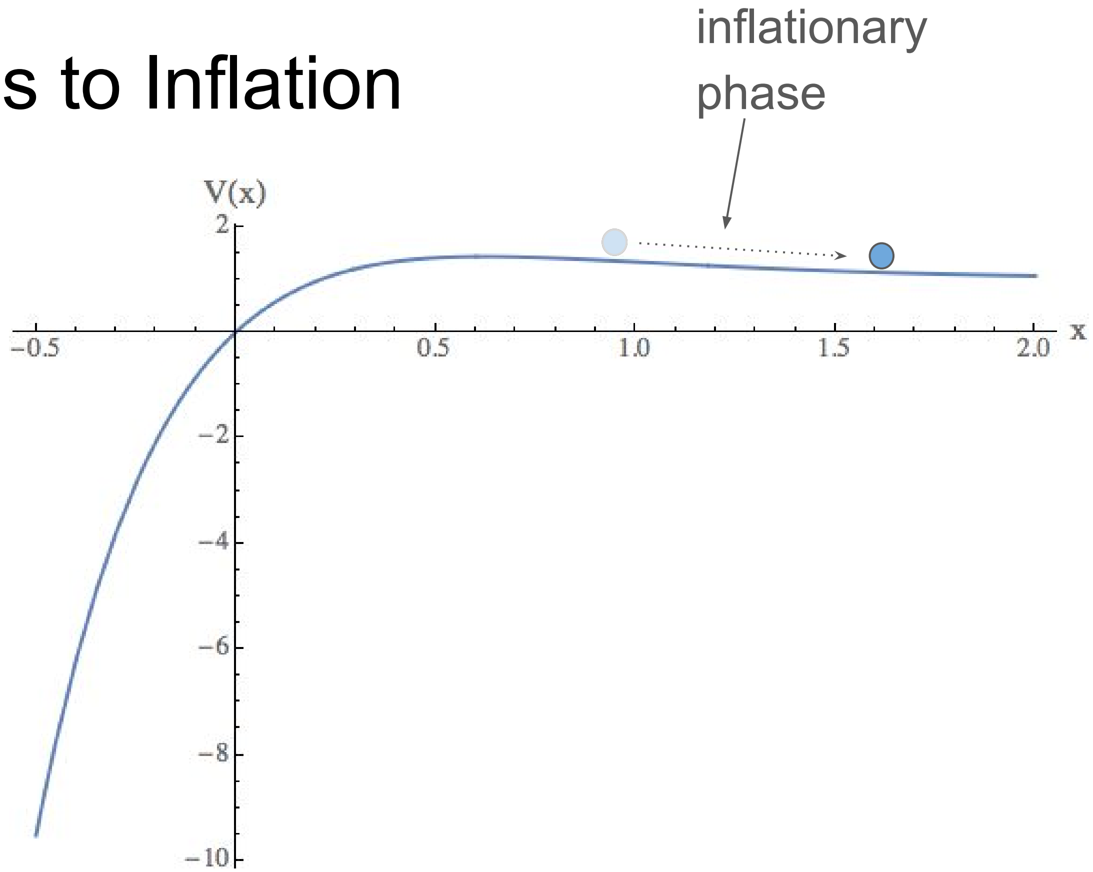

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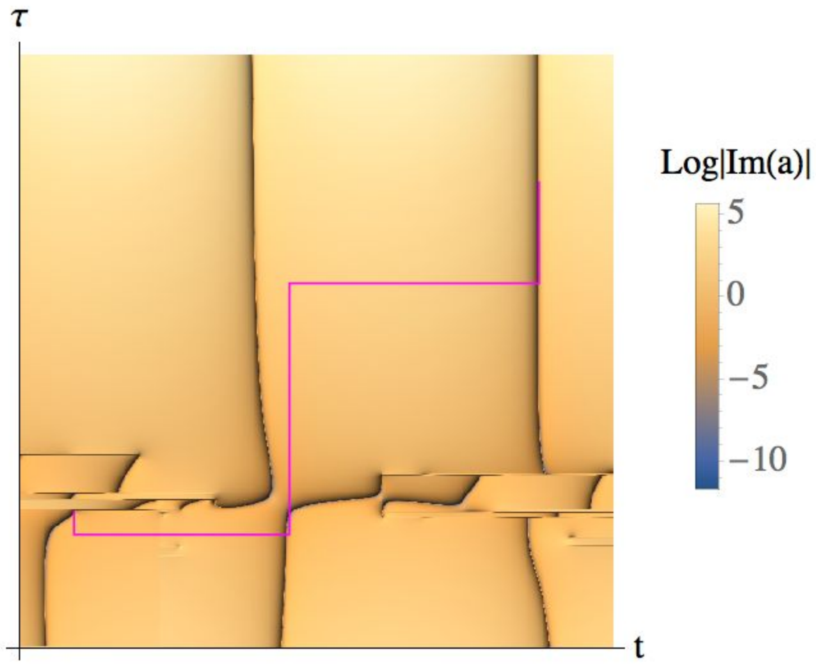

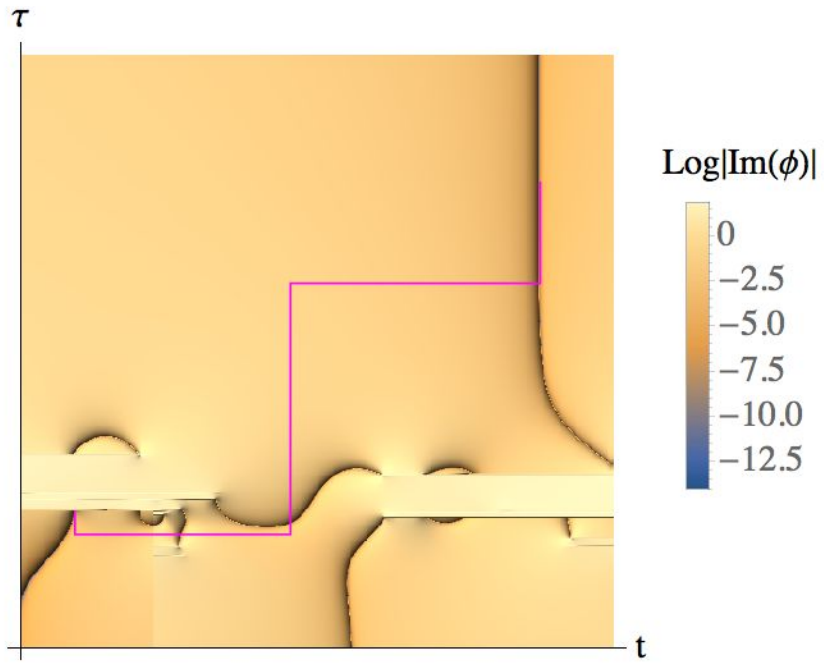

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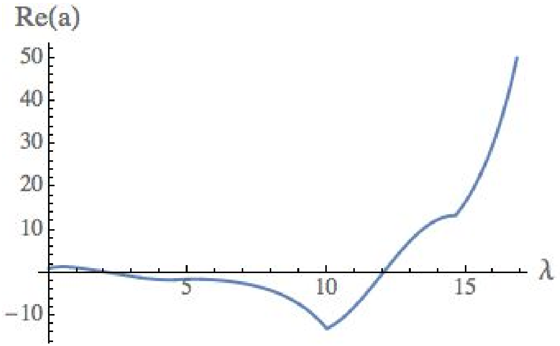

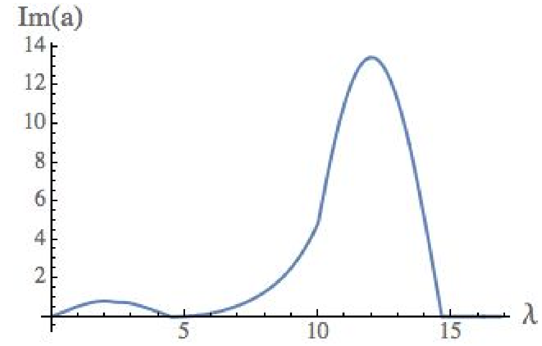

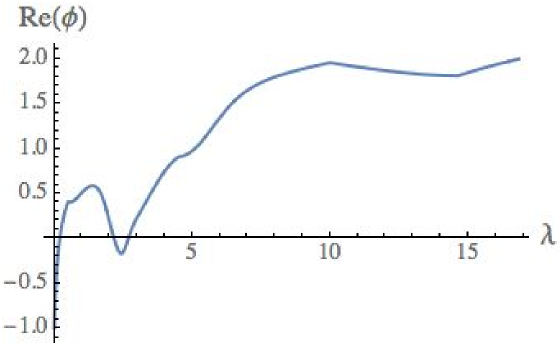

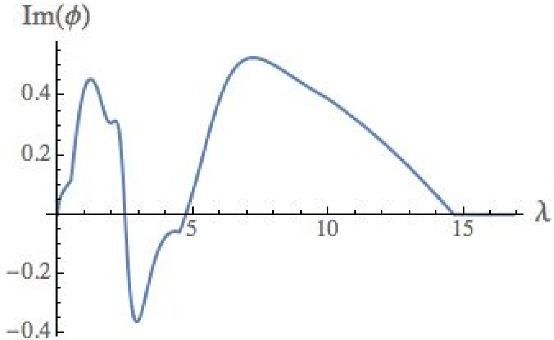

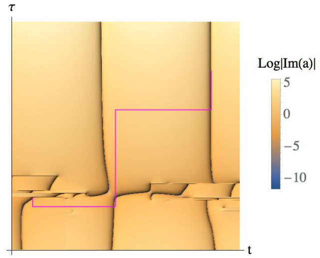

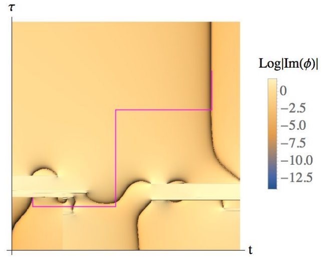

The Road So Far

Path integral description of 1D quantum tunneling using complex time paths

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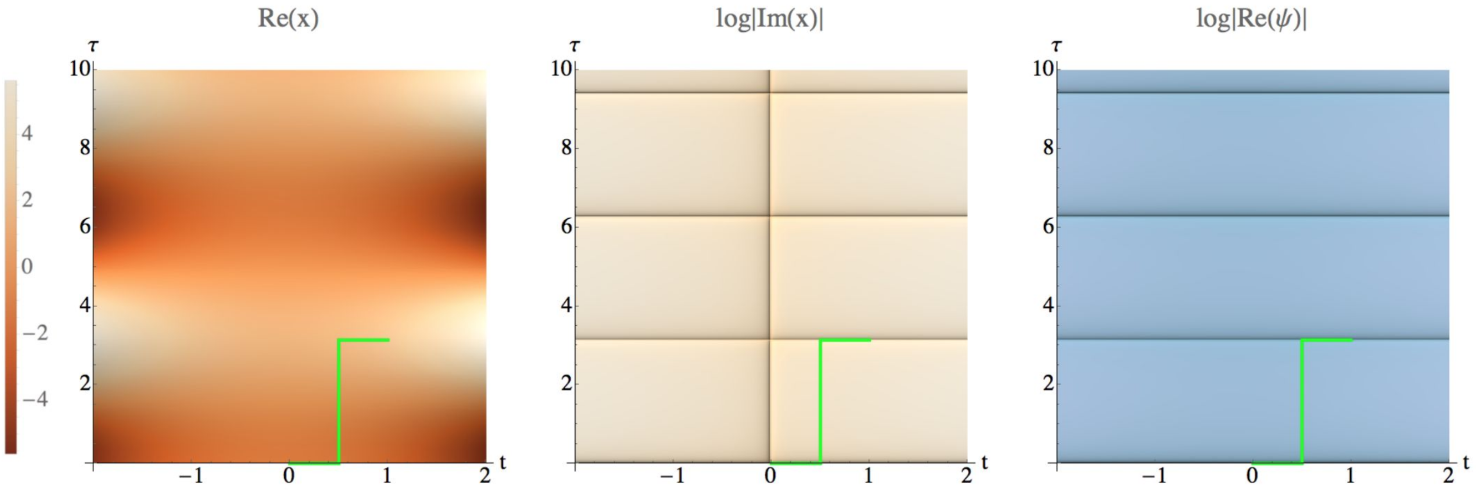

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Path integral description of 1D quantum tunneling using complex time paths

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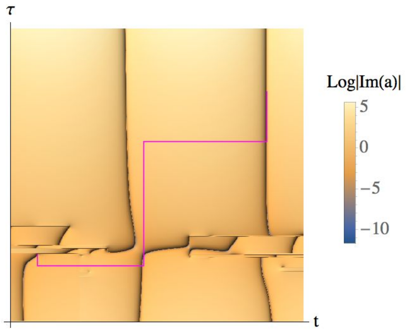
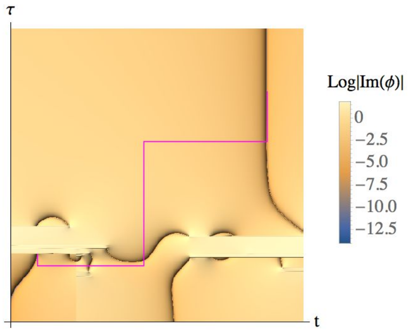
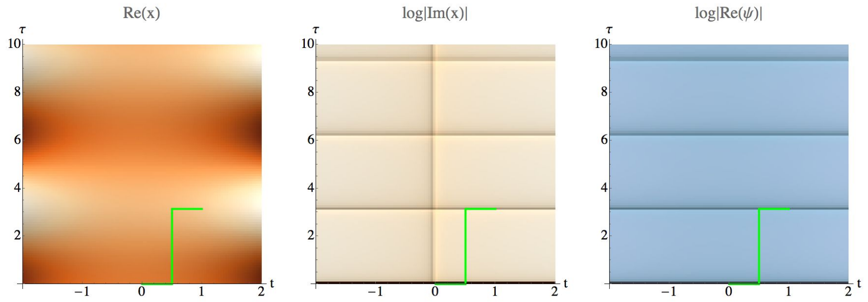
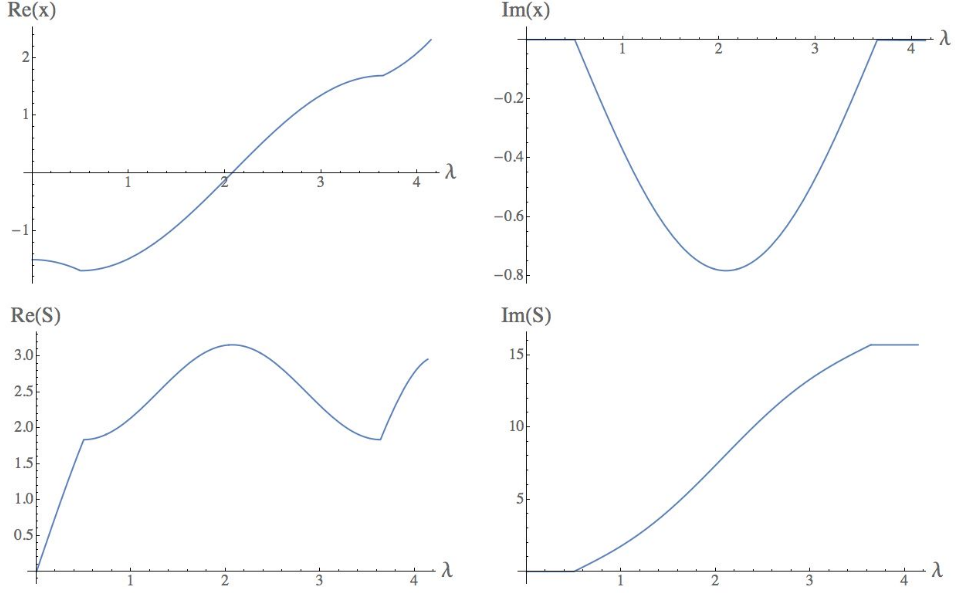




Numerical Studies





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