A cosmological exclusion plot:

Towards model-independent constraints on modified gravity from current and future growth rate data

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Contents

- Modified gravity theories $\longrightarrow$ Horndeski theory.

- How is possible to constrain modified gravity?

- Results for current data and forecasted data.

- Conclusion and future work.

EINSTEIN EQUATIONS

$$\boxed{R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G T_{\mu\nu}}$$

MODIFIED MATTER MODELS:

$$G_{\mu\nu} = 8\pi G T^{mod}_{\mu\nu}$$

Modify the matter sector:

- Quintessence
- K-Essence
- Coupled dark energy models...

MODIFIED GRAVITY MODELS:

$$G^{mod}_{\mu\nu} = 8\pi G T_{\mu\nu}$$

Modify the gravity sector:

- f(R) gravity
- Scalar-tensor theory
- .....

The Horndeski Theory

Dark energy Lagrangian: $\mathcal{L} = \sum_{i=2}^{5} \mathcal{L}_i$

Horndeski (1975)
Deffayet et al. (2012)

$$
\begin{aligned}
\mathcal{L}_2 &= K(\phi, X), \\
\mathcal{L}_3 &= -G_3(\phi, X)\Box\phi, \\
\mathcal{L}_4 &= G_4(\phi, X)R + G_{4,X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)(\nabla^\mu\nabla^\nu\phi)\right] \\
\mathcal{L}_5 &= G_5(\phi, X)G_{\mu\nu}(\nabla^\mu\nabla^\nu\phi) - \\
&- \frac{1}{6}G_{5,X}\left[(\Box\phi)^3 - 3(\Box\phi)(\nabla_\mu\nabla_\nu\phi)(\nabla^\mu\nabla^\nu\phi)\right] - \\
&- \frac{1}{3}G_{5,X}\left[(\nabla^\mu\nabla_\alpha\phi)(\nabla^\alpha\nabla_\beta\phi)(\nabla^\beta\nabla_\mu\phi)\right]
\end{aligned}
$$

- Most general Lagrangian for a single scalar field which gives second order equations of motion for both the scalar field and the metric.

$\Downarrow$

The theory is free from instabilities

- It includes: f(R) gravity, scalar-tensor theory, K-Essence, Galileon theory.

What we want to do…

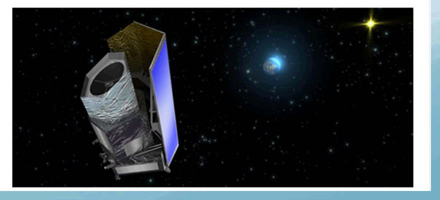
Test possible modifications of gravity at cosmological scales.

MODIFIED GRAVITY THEORY
(General and Horndeski case)

COSMOLOGICAL OBSERVATIONS FROM GALAXY SURVEYS
(Redshift Space Distortion)

CONSTRAINTS ON Y PARAMETER IN A MODEL INDEPENDENT WAY

FORECAST FOR FUTURE SURVEYS
(Euclid, 2020)



Modified gravity parameters

Friedman-Robertson-Walker perturbed metric:

$$ds^2 = -[(1+2\Psi(x,t))dt^2 + a^2(t)(1+2\Phi(x,t)]dx^2$$

MODIFIED POISSON EQUATION:

$$Y(a,k) \equiv -\frac{2k^2\Psi}{3(aH)^2\Omega_m\delta_m}$$

ANISOTROPIC STRESS:

$$\eta \equiv -\frac{\Phi}{\Psi}$$

Horndeski Theory in the QS limit:

$$\eta = h_2\left(\frac{1+k^2h_4}{1+k^2h_5}\right) \qquad Y = h_1\left(\frac{1+k^2h_5}{1+k^2h_3}\right)$$

$h_i(i=1...5)$ time-dependent functions

RSD observations

RSD only along the line-of-sight:

$$\delta_{gal}(k,z,\mu=1)=G(z)f(z)\sigma_8\delta_{m,0}(k)=f(z)\sigma_8(z)\delta_{m,0}(k)$$

RSD measurements

TWO DATASETS:

1. **Current dataset**
RSD measurements from 2dFGS, 6dFDG, LRG, BOSS, CMASS, WiggleZ, VIPERS Galaxy surveys.

Redshift interval from z = 0.07 to z = 0.8.

2. **Forecast dataset**
RSD forecasted measurements for the future Euclid mission.

Redshift interval from z = 0.5 to z = 2.1

A model independent approach

Evolution of density perturbations in the quasi-static limit:

$$\boxed{\delta_m'' + (2 + \frac{E'}{E})\delta_m' = \frac{3}{2}\frac{\delta_m}{a^3 E^2}\Omega_{m,0} Y(a,k)}$$

WHAT WE ASSUME:

1. **Background fixed to** $\Lambda\mathrm{CDM}$: $E^2 = \Omega_{m,0}^{(bg)} a^{-3} + 1 - \Omega_{m,0}^{(bg)}$

   ($\Omega_{m,0}^{(bg)} = 0.3$)

2. **Matter is a pressureless fluid.**

3. **Always consider linear scales in the sub-horizon regime.**

WHAT WE DON'T ASSUME:

1. Present value of the matter density fraction: $\Omega_{m,0}$.

2. Present value of the power spectrum amplitude: σ_8 .

3. Initial conditions for δ_m and δ'_m fixed to matter dominated Universe $\longrightarrow$ $\delta_{in} = e^{N_{in}}$ $\quad N_{in} = -1.5$

$$\alpha = \delta'_{in}/\delta_{in}$$

PARAMETERS: $\{ \hat{Y} \equiv \Omega_{m,0} Y \;,\; \sigma_8 \;,\; \alpha \}$

Likelihood Analysis

- Growth-rate data: $d_i = f(z_i)\sigma_8(z_i) = f(z_i)\sigma_8 G(z_i) = \sigma_8 \frac{\delta'}{\delta_0}$
- Theoretical estimates: $t_i = \delta_i'/\delta_0$

$$L' = \exp(-\bar{\chi}^2_{f\sigma_8}/2)$$

$$\bar{\chi}^2_{f\sigma_8} = (d_i - \sigma_8 t_i) C^{-1}_{ij} (d_j - \sigma_8 t_j)$$

Marginalizing on σ_8:

$$\boxed{\chi^2_{f\sigma_8} = S_{dd} - \frac{S^2_{dt}}{S_{tt}} + \log S_{tt} - 2\log(1 + \mathrm{Erf}(\frac{S_{dt}}{\sqrt{2S_{tt}}}))}$$

$$\boxed{\begin{aligned} S_{dt} &= d_i C^{-1}_{ij} t_j \\ S_{dd} &= d_i C^{-1}_{ij} d_j \\ S_{tt} &= t_i C^{-1}_{ij} t_j \end{aligned}}$$

Results

1. Current dataset

Survey	z	$f(z)\sigma_8(z)$	References
6dFGRS	0.067	0.423 ± 0.055	Beutler et al. (2012) [92]
LRG-200	0.25	0.3512 ± 0.0583	Samushia et al (2012) [93]
	0.37	0.4602 ± 0.0378	
LRG-60	0.25*	0.3665 ± 0.0601	Samushia et al (2012) [93]
	0.37*	0.4031 ± 0.0586	
BOSS	1) 0.30	0.408 ± 0.0552, $\rho_{12} = -0.19$	Tojeiro et al. (2012)[94]
	2) 0.60	0.433 ± 0.0662	
WiggleZ	1) 0.44	0.413 ± 0.080, $\rho_{12} = 0.51$	Blake (2011) [95]
	2) 0.60	0.390 ± 0.063, $\rho_{23} = 0.56$	
	3) 0.73	0.437 ± 0.072	
Vipers	0.8	0.47 ± 0.08	De la Torre et al (2013)[96]
2dFGRS	0.13	0.46 ± 0.06	Percival et al. (2004) [97]
LRG	0.35	0.445 ± 0.097	Chuang and Wang (2013) [98]
LOWZ	0.32	0.384 ± 0.095	Chuang at al (2013) [99]
CMASS	0.57*	0.348 ± 0.071	
	0.57*	0.423 ± 0.052	Beutler et al (2014)[100]
	0.57	0.441 ± 0.043	Samushia et al (2014) [77]
	0.57*	0.450 ± 0.011	Reid et al (2013) [101]

- $\hat{Y}$(k,a) constant in space and time.

FOUR DIFFERENT CASES:
(in increasing order of "model independence")

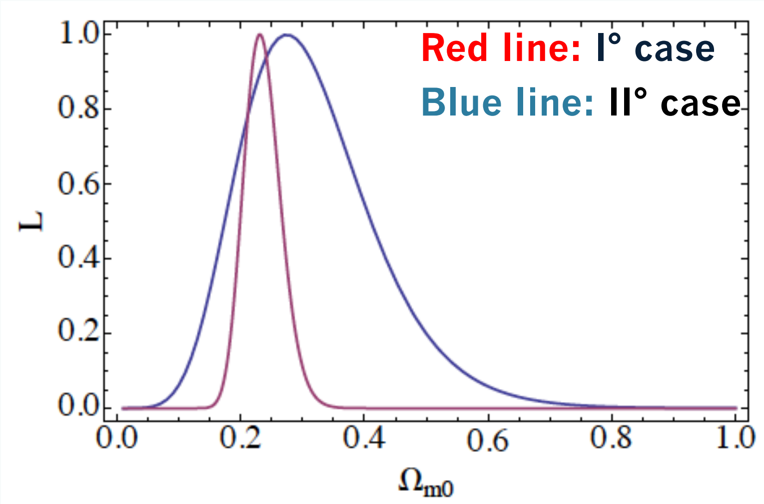
I° case: $\Lambda\mathrm{CDM}$ model, $Y = 1$, $\sigma_8 = 0.83$

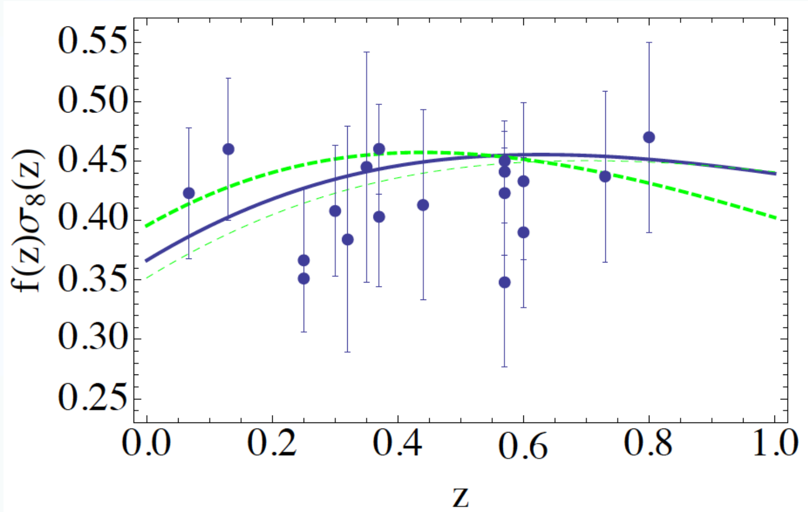
II° case: $\Lambda\mathrm{CDM}$ model, $Y = 1$, marginalizing on σ_8.

III° case: Uniform prior on $\hat{Y}$ fixing the IC.

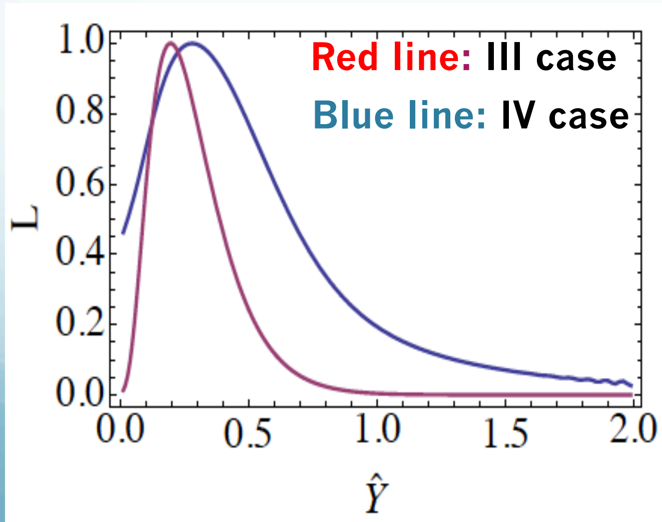
IV° case: Uniform prior on $\hat{Y}$ and α.

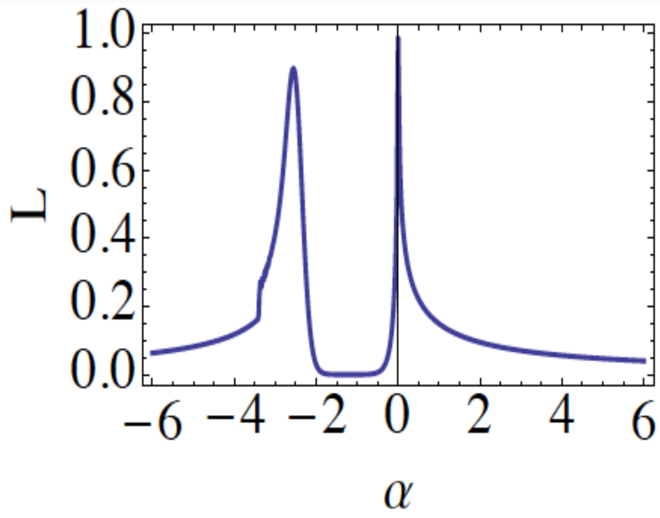
I, II case:

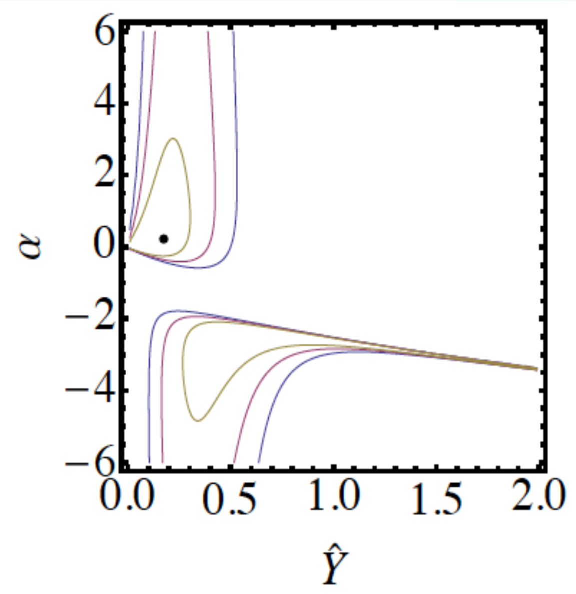




III, IV case:







SUMMARY OF RESULTS FOR CURRENT DATA

	case	α (best fit)	$\Delta\alpha$ (95%)	$\Delta\alpha$ (68%)	$\Omega_{m,0}^{(bg)}$ (best fit)	$\Delta\Omega_{m,0}^{(bg)}$ (95%)	$\Delta\Omega_{m,0}^{(bg)}$ (68%)
ΛCDM, $Y = 1$, $\sigma_8 = 0.83$	I	1	-	-	0.23	[0.18, 0.29]	[0.20, 0.26]
ΛCDM, $Y = 1$, marg. on σ_8	II	1	-	-	0.27	[0.12, 0.54]	[0.18, 0.39]
	case				$\hat{Y}$	$\Delta\hat{Y}$ (95%)	$\Delta\hat{Y}$ (68%)
Uniform prior on $\hat{Y}$	III	1	-	-	0.20	[0.040, 0.60]	[0.095, 0.36]
Uniform prior on $\hat{Y}, \alpha$	IV	-0.015	$\leq$-2.08 and $\geq$ -0.67	[-0.40, 1.32] and [-4.05, -2.20]	0.28	[0, 1.35]	[0.048, 0.63]

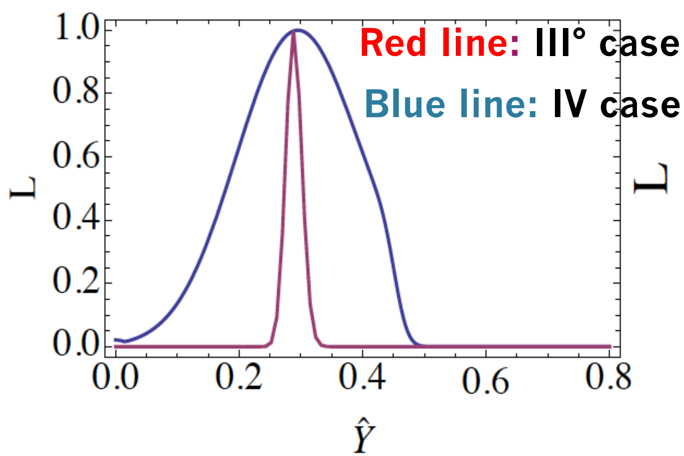
Relative percent error of order of 100%

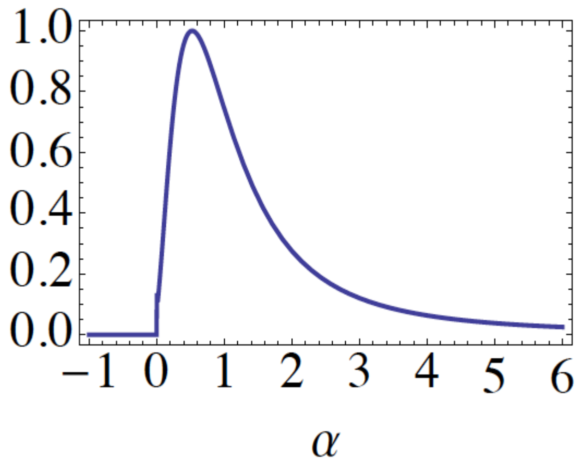
2. Forecast dataset

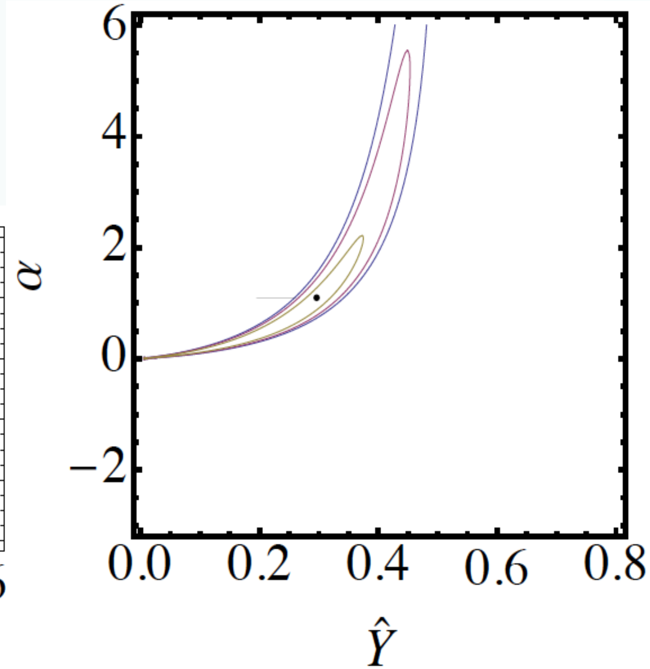
A. Z BINNING case:

$\bar{z}$	$f\sigma_8$	$\Delta f\sigma_8$ (68% c.l.)
0.6	0.469	0.0092
0.8	0.457	0.0068
1.0	0.438	0.0056
1.2	0.417	0.0049
1.4	0.396	0.0047
1.8	0.354	0.0039

III, IV case:







SUMMARY OF RESULTS FOR FORECASTED EUCLID DATA

	case	Best fit α	$\Delta\alpha$ (95%)	$\Delta\alpha$ (68%)	Best fit $\hat{Y}$	ΔY (95%)	ΔY (68%)
Uniform prior on $\hat{Y}$	III	1	-	-	0.29	[0.26, 0.32]	[0.28, 0.30]
Uniform prior on $\hat{Y}, \alpha$	IV	0.53	[0, 4.0]	[0.12, 1.6]	0.30	[0.12, 0.43]	$\boxed{[0.21, 0.38]}$

Relative percent error of order of 30%.

- Varying the IC, the uncertainty on $\hat{Y}$ increases from 0.03 to 0.15 at 95% c.l.

B. K BINNING case: The quasi-static Horndeski results

$\bar{z}$	$k_{min} - k_1$	$k_1 - k_2$	$k_2 - k_{max}$
0.6	0.007-0.022	0.022-0.063	0.063-0.180
0.8	0.007-0.023	0.023-0.071	0.071-0.215
1.0	0.007-0.024	0.024-0.078	0.078-0.249
1.2	0.007-0.026	0.026-0.086	0.086-0.287
1.4	0.007-0.027	0.027-0.094	0.094-0.329
1.8	0.007-0.029	0.029-0.112	0.112-0.426

k-bins for every redshif bin

$\bar{z}$	i	$f\sigma_8(z)$	$\Delta f\sigma_8(z)$	$\Delta f\sigma_8(z)\%$
	1		0.07	15
0.6	2	0.469	0.017	3.6
	3		0.0097	2.1
	1		0.05	11
0.8	2	0.457	0.012	2.6
	3		0.0074	1.6
	1		0.039	8.9
1.0	2	0.438	0.0089	2
	3		0.0062	1.4
	1		0.032	7.7
1.2	2	0.417	0.0072	1.7
	3		0.0055	1.3
	1		0.028	7
1.4	2	0.396	0.0065	1.6
	3		0.0057	1.4
	1		0.015	4.3
1.8	2	0.354	0.0047	1.3
	3		0.0061	1.7

- For Horndeski Theory: $\boxed{Y = h_1 \frac{1 + (k/k_p)^2 h_5}{1 + (k/k_p)^2 h_3}}$ $k_p = 1h/\mathrm{Mpc}$

- $\hat{h}_1, h_3$ and h_5 are constant (we neglect the time variation in the observed range).

- $h_5 = 0$ due to the degeneracy between h_5 and h_3.

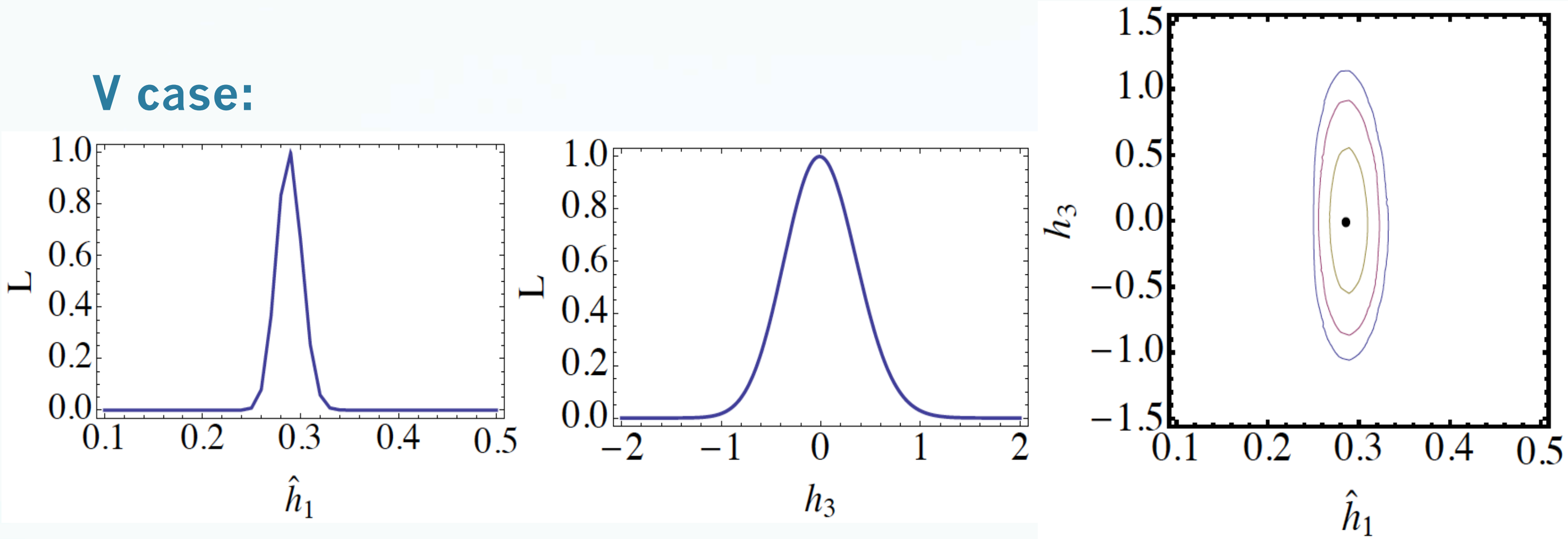
- Parameters: { $\hat{h}_1 = \Omega_{m,0} h_1, h_3$, α }

TWO DIFFERENT CASES:

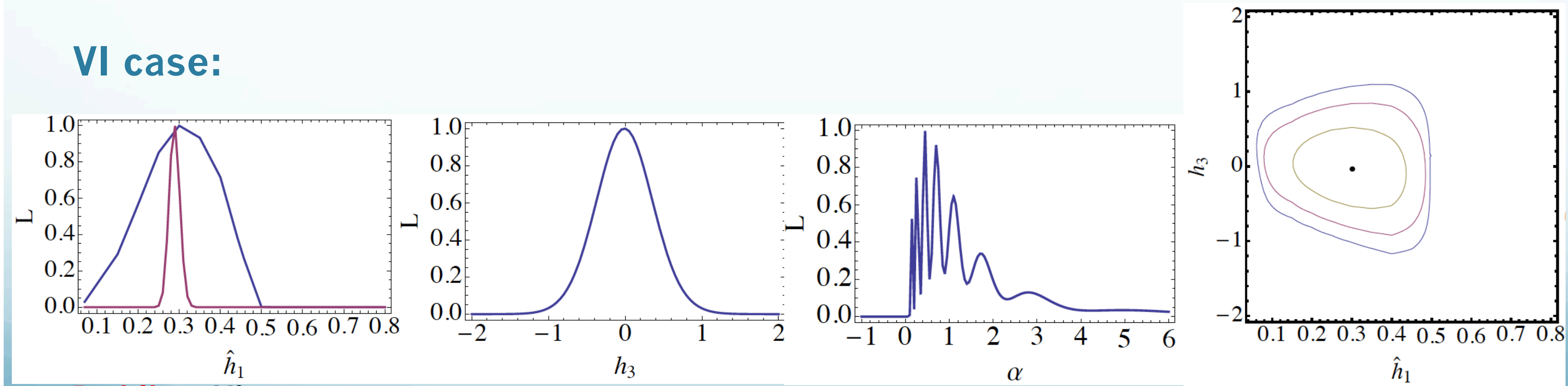
V case: Horndeski case fixing the IC.

VI case: Horndeski case varying the IC.

V case:



VI case:



Red line: V° case

Blue line: VI° case

SUMMARY OF RESULTS FOR HORNDESKI THEORY

	case	α (best fit)	$\Delta\alpha$ (95%)	$\Delta\alpha$ (68%)	$\hat{h}_1$ (best fit)	$\Delta\hat{h}_1$ (95%)	$\Delta\hat{h}_1$ (68%)	h_3 (best fit)	Δh_3 (95%)	Δh_3 (68%)
Horndeski	V	1	-	-	0.3	[0.26, 0.32]	[0.27, 0.32]	0	[-0.70, 0.72]	[-0.37, 0.35]
Horndeski	VI	0.85	[0.10, 2.2]	[0.22, 1.9]	0.3	[0.097, 0.44]	[0.17, 0.40]	0	[-0.72, 0.73]	[-0.36, 0.36]

- Varying the IC, the error on $\hat{h}_1$ increases from 0.02 to 0.1 at 95% c.l.
- Error on h_3 remains unchanged, since we assume k-independent I.C.

3. A cosmological exclusion plot

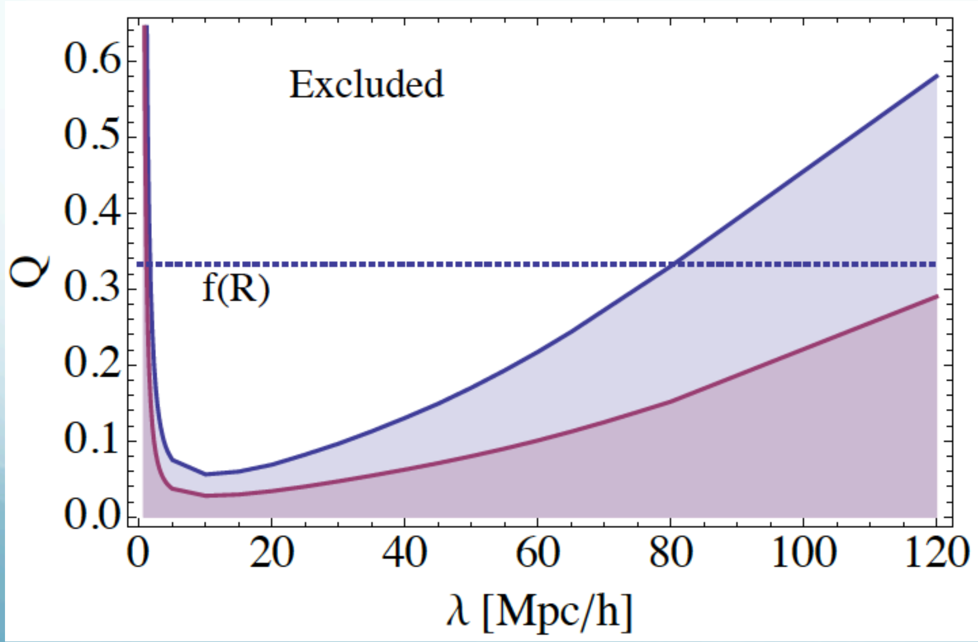
Fourier-space

$$Y(a,k) \equiv -\frac{2k^2\Psi}{3(aH)^2\Omega_m\delta_m}$$

Real-space

$$\Psi(r) = -\frac{G_0 M}{r} h_1 \left(1 + Qe^{-r/\lambda}\right)$$

YUKAWA-LIKE GRAVITATIONAL POTENTIAL



PARAMETERS:

$$h_3 = \lambda^2$$

$$h_5 = (1+Q)\lambda^2$$

$$\hat{h}_1 = \Omega_{m,0} h_1$$

$$\alpha$$

Conclusion

- The current growth-rate data can't constrain $\hat{Y}$ to better than an order of 100% error.

- For the forecasted–Euclid data, we find that the relative error on $\hat{Y}$ reduce to 30% at 68% c.l.

- For a Euclid-like experiment, the strength Q can be confined within 3% of the Newtonian gravity at 68% c.l. if the range is around 10 Mpc.

- <u>Extension of the work</u>: constrain $\hat{Y}$ parameter as much as possible model independent by using SNIa.

Thank you!