Multifield Dark Energy

(Part of the Master Thesis)

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Outline

1. General Introduction
2. Multiscalar Brans-Dicke Theory
3. Perturbations
4. Non Cancellation of Polynomials Order

Based on

V.V., Luca Amendola, "How can we tell whether dark energy is composed by multiple fields?", **gr-qc:1502.05922**

- Modifications of the standard theory of General Relativity to explain many fundamental phenomena in cosmology (the accelerated expansion of the Universe, Inflation, etc.)
- Brans-Dicke (strong motivations from higher dimensional theories, such as String Theory)

Standard Brans-Dicke Action

$$S_{BD} = \int d^4x\sqrt{-g}\left(\phi R - \frac{\omega_{BD}}{\phi} g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right) + \int d^4x\sqrt{-g}\mathcal{L}_{matter}$$

- One might think to generalize single-field theories to multi-field ones. (**From compactifications of extra dimensions.**)

2-Field Brans-Dicke Action

$S_{2-field} = \int d^4x\sqrt{-g}[\phi_1 R - \frac{\omega_1}{\phi_1}g^{\mu\nu}\partial_\mu\phi_1\partial_\nu\phi_1 + \phi_2 R - \frac{\omega_2}{\phi_2}g^{\mu\nu}\partial_\mu\phi_2\partial_\nu\phi_2$
$-V(\phi_1, \phi_2)] + \int d^4x\sqrt{-g}\mathcal{L}_{matter}.$

N-Field Brans-Dicke Action

$S_{N-field} = \int d^4x\sqrt{-g}[\sum_{i=1}^{N}(\phi_i R - \frac{\omega_i}{\phi_i}g^{\mu\nu}\partial_\mu\phi_i\partial_\nu\phi_i) - V(\phi_1, \phi_2, ..., \phi_N)]+$
$\int d^4x\sqrt{-g}\mathcal{L}_{matter}.$

- There exist a single-field potential $V(\phi)$ that reproduces any observed Hubble parameter.
  From Friedmann equations:

$$H(z)^2(1+z)^2\left(\frac{d\phi}{dz}\right)^2 = 2(1+z)H(z)\frac{dH}{dz} - \rho_m(z)(1+w_m(z))$$

  (Find $\rho_m(z)$ from continuity eq.)

$$V(z) = 3H(z)^2 - (1+z)H(z)\frac{dH(z)}{dz} + \frac{\rho_m(z)}{2}(w_m(z)-1)$$

- **Linear Cosmological Perturbations!!**

Perturbations

$g_{\mu\nu} = g^{(0)}_{\mu\nu} + \delta g_{\mu\nu}$ and $ds^2 = a^2(\eta)[-(1+2\Psi)d\eta^2 + (1+2\Phi)dx^i dx_i]$

also $\phi(t,\vec{x}) = \phi(t) + \varphi(t,\vec{x})$

Interested in:

- $\eta = -\frac{\Phi}{\Psi}$
- The effective gravitational constant Y which enters the modified Poisson equation: $\frac{k^2}{a^2}\Psi = -4\pi G Y \delta\rho$

η and Y for Standard Brans-Dicke Theory

$$\eta = h_2 \left(\frac{1+k^2 h_4}{1+k^2 h_5} \right) , \; Y = h_1 \left(\frac{1+k^2 h_5}{1+k^2 h_3} \right)$$

with h_i some known time-dependent functions.

η and Y for Multiscalar Brans-Dicke Theory

$$\eta = h_2 \frac{P_n^{(1)}(k)}{P_n^{(2)}(k)} , \; Y = h_1 \frac{P_n^{(2)}(k)}{P_n^{(3)}(k)}$$

The exact forms of the coefficients are important!!

2-Field Brans-Dicke Action

- $S_{2-BD} = \int d^4x\sqrt{-g}[\phi_1 R - \frac{\omega_1}{\phi_1}g^{\mu\nu}\partial_\mu\phi_1\partial_\nu\phi_1 + \phi_2 R - \frac{\omega_2}{\phi_2}g^{\mu\nu}\partial_\mu\phi_2\partial_\nu\phi_2 - V(\phi_1,\phi_2)] + \int d^4x\sqrt{-g}\mathcal{L}_{matter}$

Equations of Motion

- $(\phi_1 + \phi_2)G_{\mu\nu} + [\Box(\phi_1 + \phi_2) + \frac{1}{2}\frac{\omega_1}{\phi_1}(\nabla\phi_1)^2 + \frac{1}{2}\frac{\omega_2}{\phi_2}(\nabla\phi_2)^2]g_{\mu\nu} - \nabla_\mu\nabla_\nu(\phi_1 + \phi_2) - \frac{\omega_1}{\phi_1}\nabla_\mu\phi_1\nabla_\nu\phi_1 - \frac{\omega_2}{\phi_2}\nabla_\mu\phi_2\nabla_\nu\phi_2 + \frac{V}{2}g_{\mu\nu} = 8\pi G T_{\mu\nu}$
- $\phi_i R + 2\omega_i\Box\phi_i - \frac{\omega_i}{\phi_i}\partial^\alpha\phi_i\partial_\alpha\phi_i - \phi_i V_{,\phi_i} = 0,\ i = 1,2$
- $(3 + 2\omega_1)\Box\phi_1 + (3 + 2\omega_2)\Box\phi_2 + 2V(\phi_1,\phi_2) - V_{,\phi_1} - V_{,\phi_2} = 8\pi G T.$

Perturbations

Continuation

2-Field Perturbation Equations

- $2(\phi_1 + \phi_2)\frac{k^2}{a^2}\Phi + 2\frac{k^2}{a^2}(\varphi_1 + \varphi_2) + 2(\phi_1 + \phi_2)\frac{k^2}{a^2}\Psi = -24\pi G c_s^2 \delta\rho$
- $4\frac{k^2}{a^2}\Phi + 2\frac{k^2}{a^2}\Psi - [2\frac{\omega_i}{\phi_i}\frac{k^2}{a^2} + M_i^2]\varphi_i = 0, \ i = 1, 2$
- $2(\phi_1 + \phi_2)\frac{k^2}{a^2}\Phi + \frac{k^2}{a^2}(\varphi_1 + \varphi_2) = 8\pi G \delta\rho$

N-Field Perturbation Equations

- $2\sum\phi_i\frac{k^2}{a^2}\Phi + 2\frac{k^2}{a^2}\sum\varphi_i + 2\sum\phi_i\frac{k^2}{a^2}\Psi = -24\pi G c_s^2 \delta\rho$
- $4\frac{k^2}{a^2}\Phi + 2\frac{k^2}{a^2}\Psi - [2\frac{\omega_i}{\phi_i}\frac{k^2}{a^2} + M_i^2]\varphi_i = 0, \ i = 1, 2, ..., N$
- $2\sum\phi_i\frac{k^2}{a^2}\Phi + \frac{k^2}{a^2}\sum\varphi_i = 8\pi G \delta\rho$

η and Y

$$\eta = C_0 \frac{1+\sum_{i=1}^{N} C_i k^{2i}}{1+\sum_{i=1}^{N} D_i k^{2i}}, \quad Y = \bar{C}_0 \frac{1+\sum_{i=1}^{N} \bar{C}_i k^{2i}}{1+\sum_{i=1}^{N} \bar{D}_i k^{2i}}$$

Coefficients

where

$C_0 = \frac{1}{1+3c_s^2}$, $\bar{C}_0 = \frac{1+3c_s^2}{\sum_{i=1}^{N} \phi_i}$, aaaand...

Coefficients

$$C_d = \frac{1}{a^{2d} C_\star}[(\sum_{i=1}^{N} \phi_i) 2^d \sum_{i_1>i_2>...>i_d} \frac{\omega_{i_1}}{\phi_{i_1}} \frac{\omega_{i_2}}{\phi_{i_2}} \cdots \frac{\omega_{i_d}}{\phi_{i_d}} \prod_{j \neq i_1, i_2, ..., i_d} M_j^2 +$$

$$(2+3c_s^2)\, 2^{d-1} \sum_{i=1}^{N} \sum_{\substack{i_j \neq i, \\ j=1,...,d-1 \\ i_1>i_2>...>i_{d-1}}} \frac{\omega_{i_1}}{\phi_{i_1}} \frac{\omega_{i_2}}{\phi_{i_2}} \cdots \frac{\omega_{i_{d-1}}}{\phi_{i_{d-1}}} \prod_{j \neq i, i_1, i_2, ..., i_{d-1}} M_j^2]$$

$$C_\star = (\sum_{i=1}^{N} \phi_i) \prod_{j=1}^{N} M_j^2;$$

$$(d = 1, ..., N)$$

Perturbations

Continuation

Coefficients

$$\bar{C}_d = \frac{1}{a^{2d}\bar{C}_\star}[\left(1+3c_s^2\right)\left(\sum_{i=1}^N \phi_i\right)2^d \sum_{i_1>i_2>\ldots>i_d} \frac{\omega_{i_1}}{\phi_{i_1}}\frac{\omega_{i_2}}{\phi_{i_2}}\cdots\frac{\omega_{i_d}}{\phi_{i_d}} \prod_{j\neq i_1,i_2,\ldots,i_d} M_j^2 + 2\left(2+3c_s^2\right)2^{d-1}\sum_{i=1}^N \sum_{\substack{i_j\neq i,\\ j=1,\ldots,d-1\\ i_1>i_2>\ldots>i_{d-1}}} \frac{\omega_{i_1}}{\phi_{i_1}}\frac{\omega_{i_2}}{\phi_{i_2}}\cdots\frac{\omega_{i_{d-1}}}{\phi_{i_{d-1}}} \prod_{j\neq i,i_1,i_2,\ldots,i_{d-1}} M_j^2]$$

Coefficients

$$\bar{D}_d = \frac{1}{a^{2d}\bar{D}_\star}[\left(\sum_{i=1}^N \phi_i\right)^2 2^d \sum_{i_1>i_2>\ldots>i_d} \frac{\omega_{i_1}}{\phi_{i_1}}\frac{\omega_{i_2}}{\phi_{i_2}}\cdots\frac{\omega_{i_d}}{\phi_{i_d}} \prod_{j\neq i_1,i_2,\ldots,i_d} M_j^2 + 3\left(\sum_{i=1}^N \phi_i\right)2^{d-1}\sum_{i=1}^N \sum_{\substack{i_j\neq i,\\ j=1,\ldots,d-1\\ i_1>i_2>\ldots>i_{d-1}}} \frac{\omega_{i_1}}{\phi_{i_1}}\frac{\omega_{i_2}}{\phi_{i_2}}\cdots\frac{\omega_{i_{d-1}}}{\phi_{i_{d-1}}} \prod_{j\neq i,i_1,i_2,\ldots,i_{d-1}} M_j^2]$$

$$\bar{C}_\star = \left(1+3c_s^2\right)\left(\sum_{i=1}^N \phi_i\right)\prod_{j=1}^N M_j^2;\ \bar{D}_\star = \left(\sum_{i=1}^N \phi_i\right)^2\prod_{j=1}^N M_j^2;$$

(Note: $D_d = \bar{C}_d$)

Non-Triviality

Effectively lower order polynomials only in one of the following cases:

- *a*) $c_s^2 = 1/3$,
- *b*) at least one of the coupling constants is infinitely large,
- *c*) one or more $M_i^2 \to \infty$,
- *d*) one or more $\phi_i \to 0$,
- *e*) there are i and j such that $\omega_{i,j} \to 0$,
- *f*) there are i and j such that $\frac{\omega_i}{\phi_i} = \alpha \frac{\omega_j}{\phi_j}$ and $M_i^2 = \alpha M_j^2$. (Note the difference between $\alpha = -1$ and all the other α's.)

Summary

- Multiscalar Brans-Dicke Theory.
- Generic fluid matter source with equation of state w and sound speed c_s. This might turn out useful to compare real data to models in which dark matter includes hot or warm components.
- There are no non-trivial cases in which the higher-order k terms in the polynomials cancel out $\Rightarrow$ the k-structure of Y, η is a unique signature of the number of dark energy fields coupled to gravity.


THANK
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