pres_zif2016

August 10, 2016

0.1 Transport in small correlated disordered networks

0.1.1 Adrian Ortega, Manan Vyas, Thomas Stegmann, Luis Benet

ICF -UNAM

ZIF - Randomness in Physics and Mathematics

To run the presentation just type: jupyter nbconvert pres-CIC_Ene2016.ipynb –to slides –post serve

Next copy paste the http direction in your favorite browser.

```
In [2]: %matplotlib inline
        from matplotlib import pyplot as plt
        import numpy as np
        import random
        import scipy.integrate
        import scipy.stats as spy
        from pylab import rcParams
        rcParams['figure.figsize'] = 10, 5
        from lib_read import *
        from bw_funcs import *
        from plot_transmissions import *
        import scipy.stats as stats
        import scipy.optimize as opt
        from IPython.display import Image
```

1 Outline

1.0.1 1. Motivation

1.0.2 2. Historical account for Embedded Ensembles

1.0.3 2. The model

1.0.4 3. Closed system, Efficiency

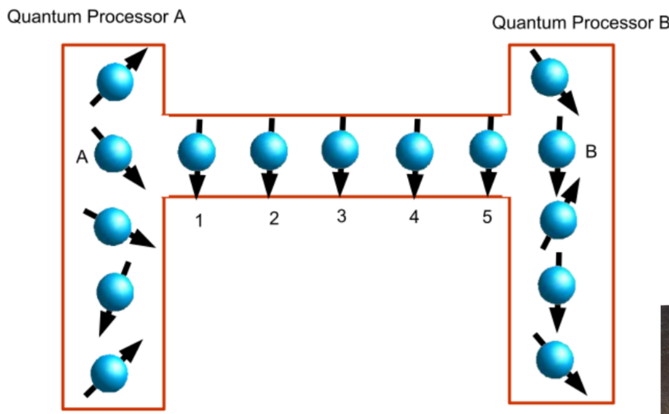
1.0.5 4. Open system (Scattering)

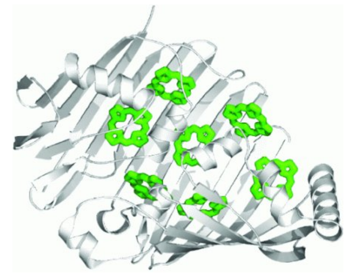
1.0.6 5. Summary

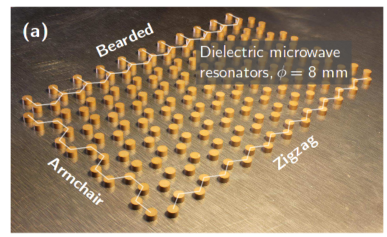
1.1 Motivation

```
In [2]: Image(filename='physical_examples.png')
```

Out[2]:







1. Quantum buses: S. Bose 2008 arXiv 0802.1221v1. Nikolopoulos, Jex. *Quantum State Transfer and Networking Engineering*. Springer 2014.
2. FMO: S. Hoyer et al 2010 New J. Phys. 12 065041.
3. Artificial graphene: Nice-France group. arxiv.org/pdf/1406.6409v2.pdf

1.2 Characteristics:

1. Finite interacting systems.
2. Can be modeled as networks.
3. The quantum bus is a many-body interacting quantum system. The FMO and the artificial graphene are systems for one particle in a discrete network.
4. Quantum bus/artificial graphene $\sim$ ordered system. FMO system at room temperature (Disordered system).

1.3 Historical account about Embedded Ensembles

1. Two-Body Random Ensemble (TBRE) 1970 – 71 [1]. The mathematical treatment of TBRE fails [2].
2. With the Embedded Gaussian Ensembles (EGE) it is possible to derive analytical results [3].
3. Some analytical results were obtained by the Heidelberg group in [4]. A nice integration of many results are available in Kota's book [5].

---

[1] B. French y S.S.M. Wong 1970 Phys. Lett. B 449(7). O. Bohigas y J. Flores 1971 Phys. Lett. B 34(4), 261.
[2] T.A. Brody et al 1981 Rev. Mod. Phys. 53,3.
[3] K.K. Mon y J.B. French 1975 Ann. Phys. 95:90-11.
[4] L. Benet et al 2001 Ann. Phys. 292, (67-94).
[5] V.K.B. Kota. *Embedded Random Matrix Ensembles in Quantum Physics*. Springer 2014.

1.4 The model - Embedded gaussian ensemble (EGE)

Quantum many-body fermionic interacting system. Parameters: n-number of particles, k-rank of interaction, l-sp states. The interaction hamiltonian is:

$$V_k = \sum_{\alpha,\gamma} v_{k;\alpha,\gamma} \Psi^{\dagger}_{k;\alpha} \Psi_{k;\alpha}, \quad (1)$$

The basis in which we represent the hamiltonian, here the ocupation number basis, also generates the network for the system.

1.5 Centrosymmetry (CS)

One dimensional quantum buses and GOE-disordered networks benefits from the CS [1].

The CS for these cases is just

$$[H, J] = 0,$$

J is the exchange matrix.

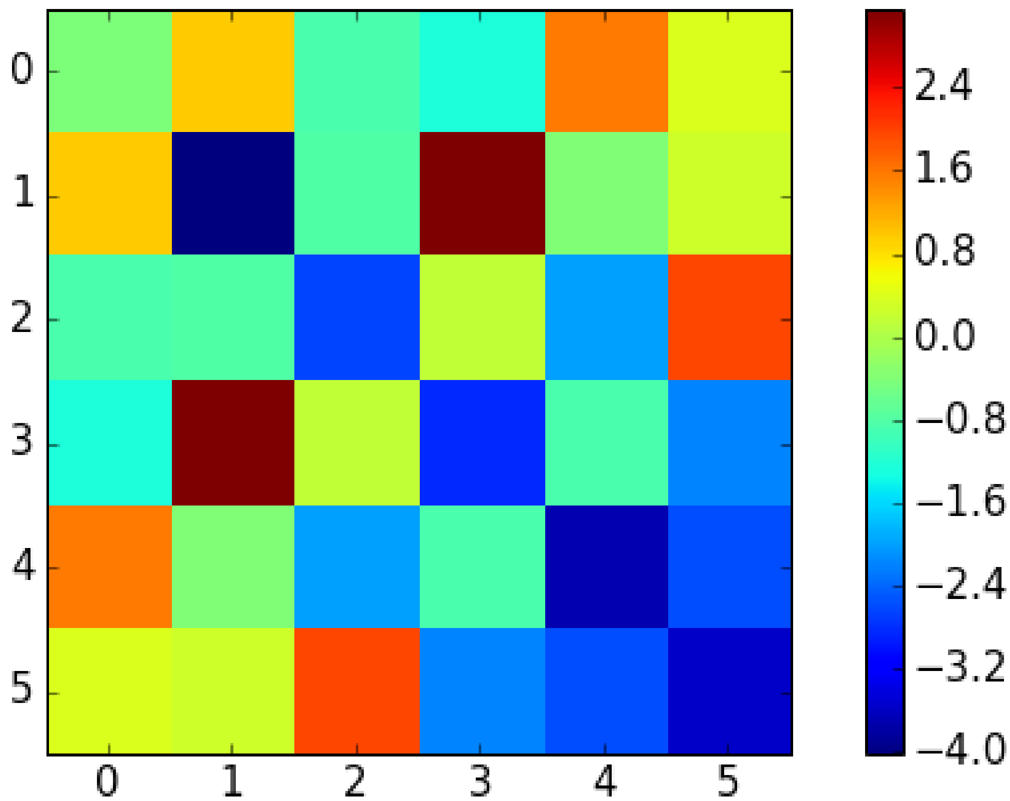
We have two types of ensembles, **EGE and csEGE**.

---

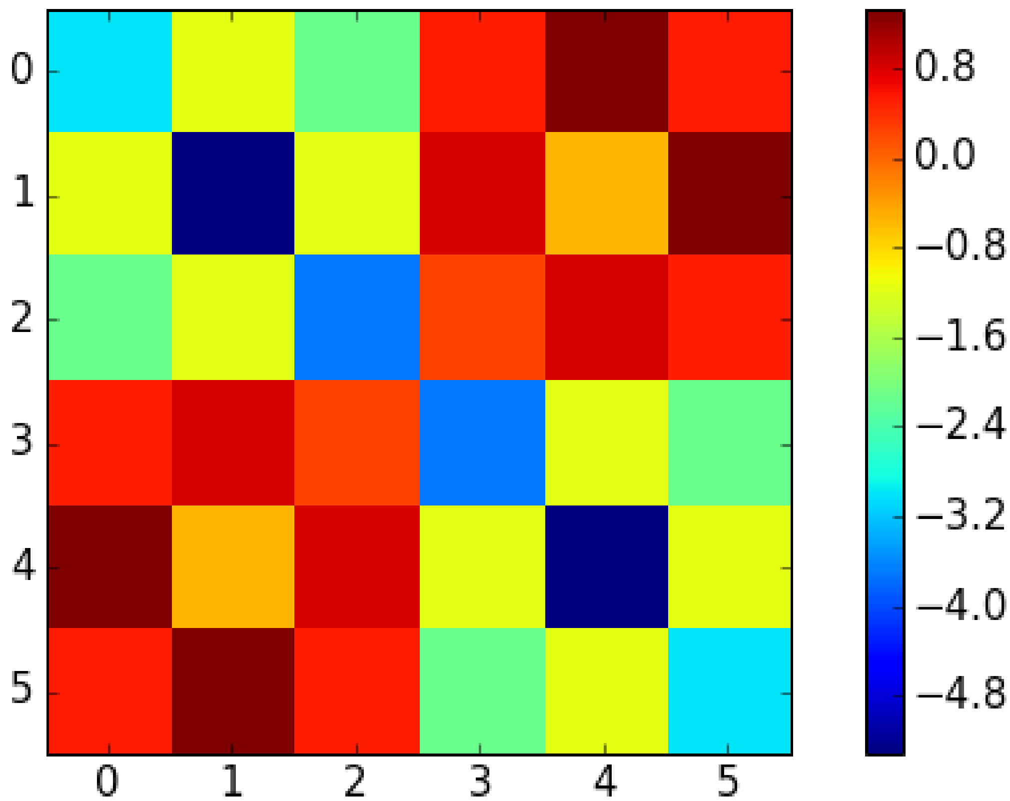
[1] S. Bose 2008 arXiv 0802.1221v1. M. Walschaers et. al. 2013 PRL 111, 180601

In [10]:
```
lniv = 6; nPart = 5; NHn = binomial(lniv,nPart)
eta=1.0; s=1; d=NHn; iRealiz=1
kPart=2
filename = '../fermionictranspor_greensfuncs/random_interac_fixedrandband/best_worst/Hmat_l'+st
print filename
Hmat = matstruct(filename, NHn)
plot_mat(Hmat, NHn)
filename = '../fermionictranspor_greensfuncs/csrandom_interac_fixedrandband/best_worst/Hmat_l'+
print filename
Hmat = matstruct(filename, NHn)
plot_mat(Hmat, NHn)
```

```
../fermionictranspor_greensfuncs/random_interac_fixedrandband/best_worst/Hmat_l6n5k2dim6_eta1.0s1d6_ireal
```



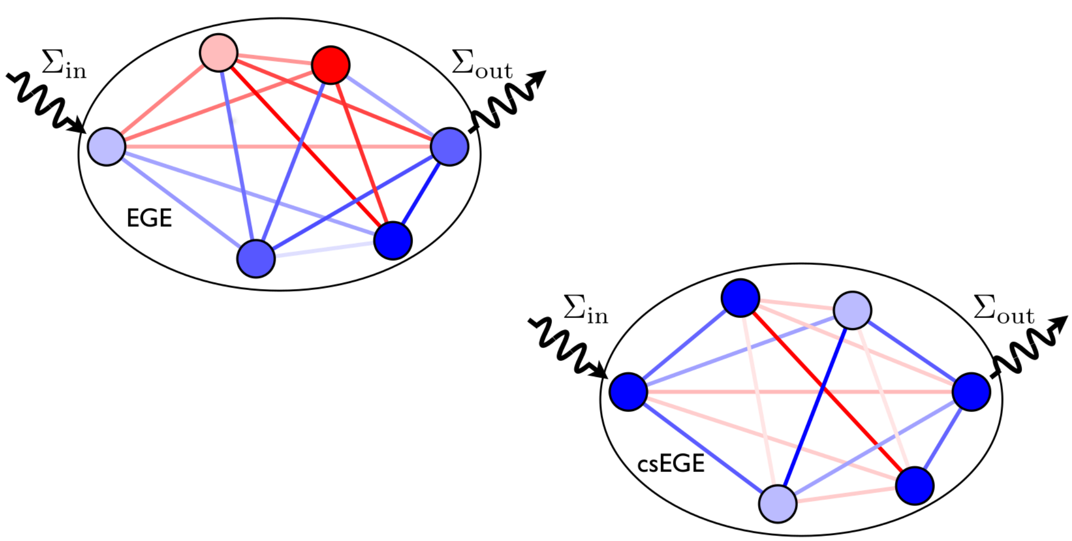
../fermionictranspor_greensfuncs/csrandom_interac_fixedrandband/best_worst/Hmat_l6n5k2dim6_eta1.0s1d6_irea




```
<matplotlib.figure.Figure at 0x7f389c5b2510>
```

In [3]: `Image(filename='nets.png')`

Out[3]:



1.6 Efficiency

In order to quantify the degree of optimization in our system to develop the task $|in\rangle \to |out\rangle$ in a certain time we define the *Efficiency* as

$$\mathcal{P}_{\mu,\nu} = \max_{t\in[0,T_{max})} |\langle \mu, e^{-iV_k t}\nu\rangle|^2, \hbar = 1.$$

That is, the maximum probability of finding the initial state $|\nu\rangle$ in the state $|\mu\rangle$ in an interval of time $[0, T_{max})$ $(\mu > \nu)$.

The ensemble will be efficient if most of its best efficiencies are $\sim 95\%$ [2]

---

[2] M. Walschaers et. al. 2013 PRL 111, 180601.

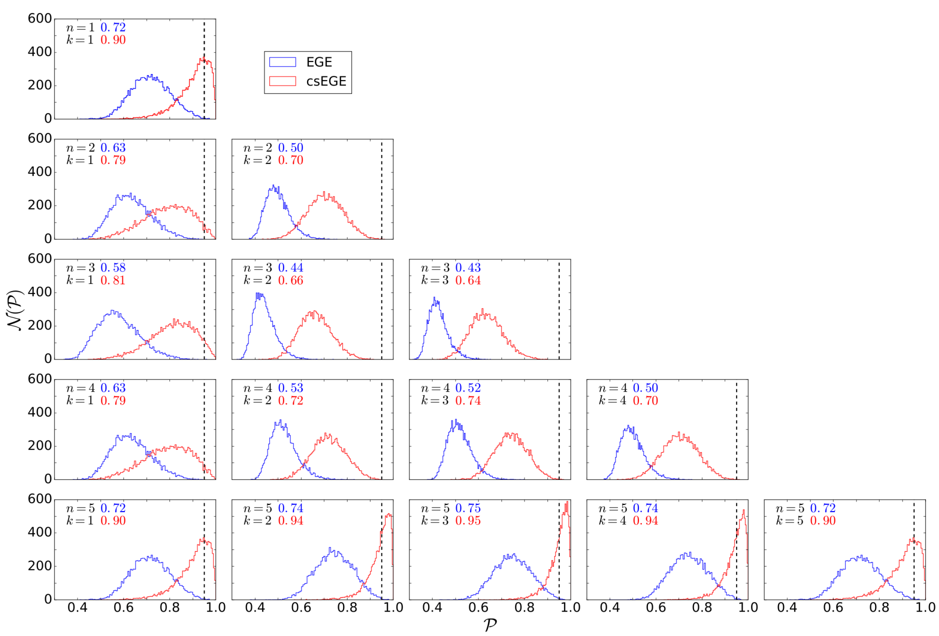
1.7 Numerical simulations

Parameters $l = 6$, n, k, and 10000-realizations.

A. Ortega, M. Vyas, L. Benet. Ann. der Phys. 527 9-10 2015.

```
In [5]: Image(filename='efficiencies_beta1.png')
```

```
Out[5]:
```



1.8 Open system (Scattering)

We use NEGF (Non-equilibrium Green's function) formalism and calculate transmission and current.

To open the system, we attach *broad-band contacts* to the Fock states $|1\rangle$ and $|N\rangle$, and calculate the transmission:

$$T(E) = 4\,\mathrm{Tr}(\ \mathrm{Im}(\Sigma_S)\ G(E)\ \mathrm{Im}(\Sigma_D)\ G^{\dagger}(E)\ )$$

where Σ_{SD} are the self-energies of the contacts and $G(E)$ is the Green's function of the total system defined as

$$G(E) = (E - V_k - \Sigma_S - \Sigma_D)^{-1}$$

We also calculate the total current:

$$I = \int_{E_{min}}^{E_{max}} T(E)dE.$$

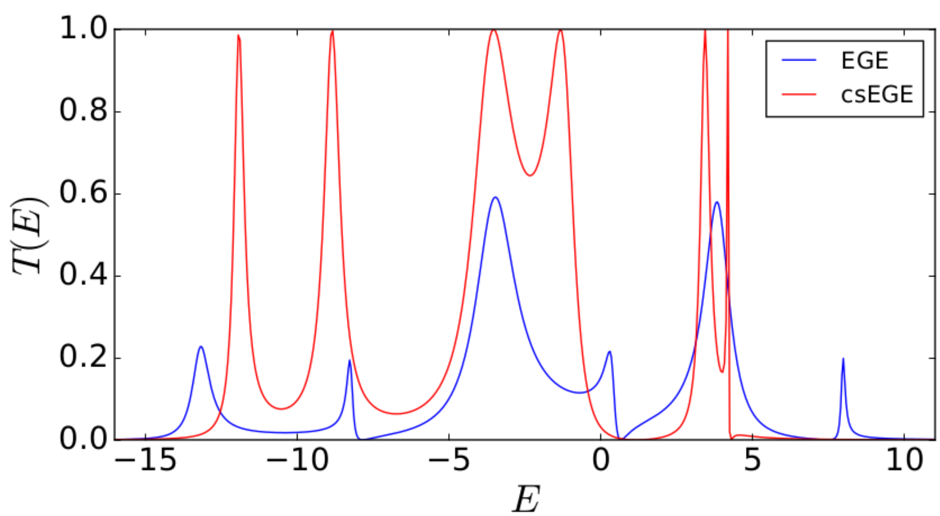
Σ_{SD} depend on the coupling parameter η, which is fixed to 1 in all the numerical simulations.

1.9 Numerical results

1.9.1 Current distributions for EGE and csEGE (10^4 - realizations)

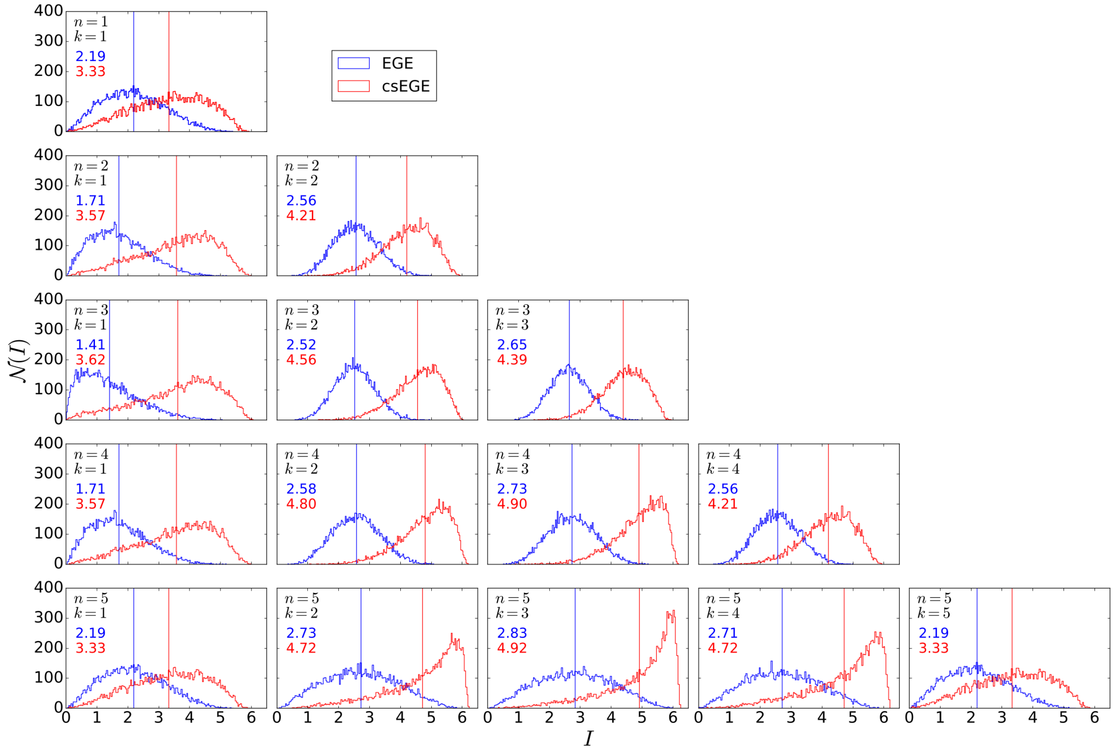
```
In [17]: Image(filename='1realizT.png')
```

Out[17]:




```
In [13]: Image(filename='currents.png')
```

Out[13]:



1.9.2 Important observations

1. Centrosymmetry is an important property to have in a disordered interacting quantum system, if one has to preserve good transport rates.

2. Similar results are found in A. Ortega, M. Vyas, L. Benet. Ann. der Phys. 527 9-10 2015 (1) [3] for the efficiency and A. Ortega, T. Stegmann, L. Benet. sent-to-PRE (2) for the current distributions over the ensemble.

---

(1) http://onlinelibrary.wiley.com/doi/10.1002/andp.201500140/abstract

(2) https://arxiv.org/abs/1605.01445

1.10 Why csEGE is better than EGE?

To answer this question, we rewrite $T(E)$ as

$$T(E) = 4|G_{1N}|^2,$$

and notice that all the information is contained in the Green's function for the central system G. In particular

$$G_{1N}(E) = \sum_{k=1}^{N} \frac{\Psi_{1,k}\Psi_{N,k}}{E-\epsilon_k} = \sum_{k=1}^{N} \frac{\Upsilon_{1,N,k}}{E-\epsilon_k},$$

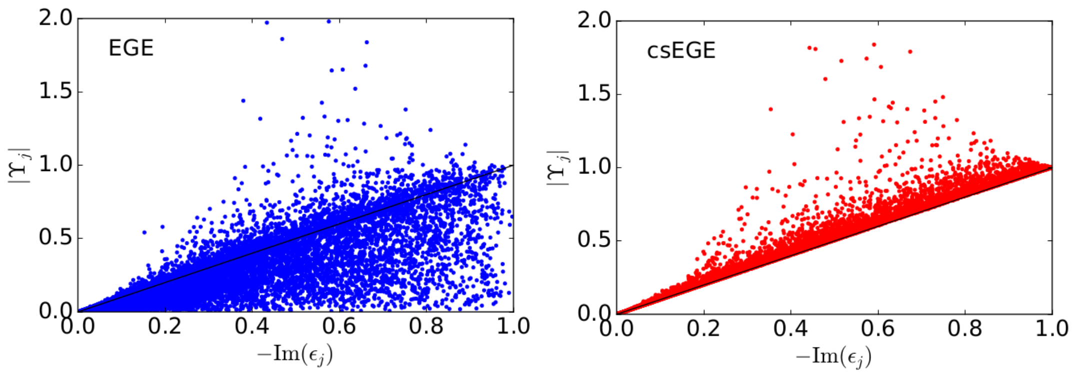
and we analyse what happens with the eigenvalues and eigenvectors of $H = V_k + \Sigma_S + \Sigma_D$.

Role of the eigevalues and eigenvectors of H to the transmission

1. The real part of the eigenvalues fixes aproximately the position of the resonances in the system.
2. The imaginary part of the eigenvalues fixes the width of the resonances.
3. The eigenfunctions fix the height of such resonances.

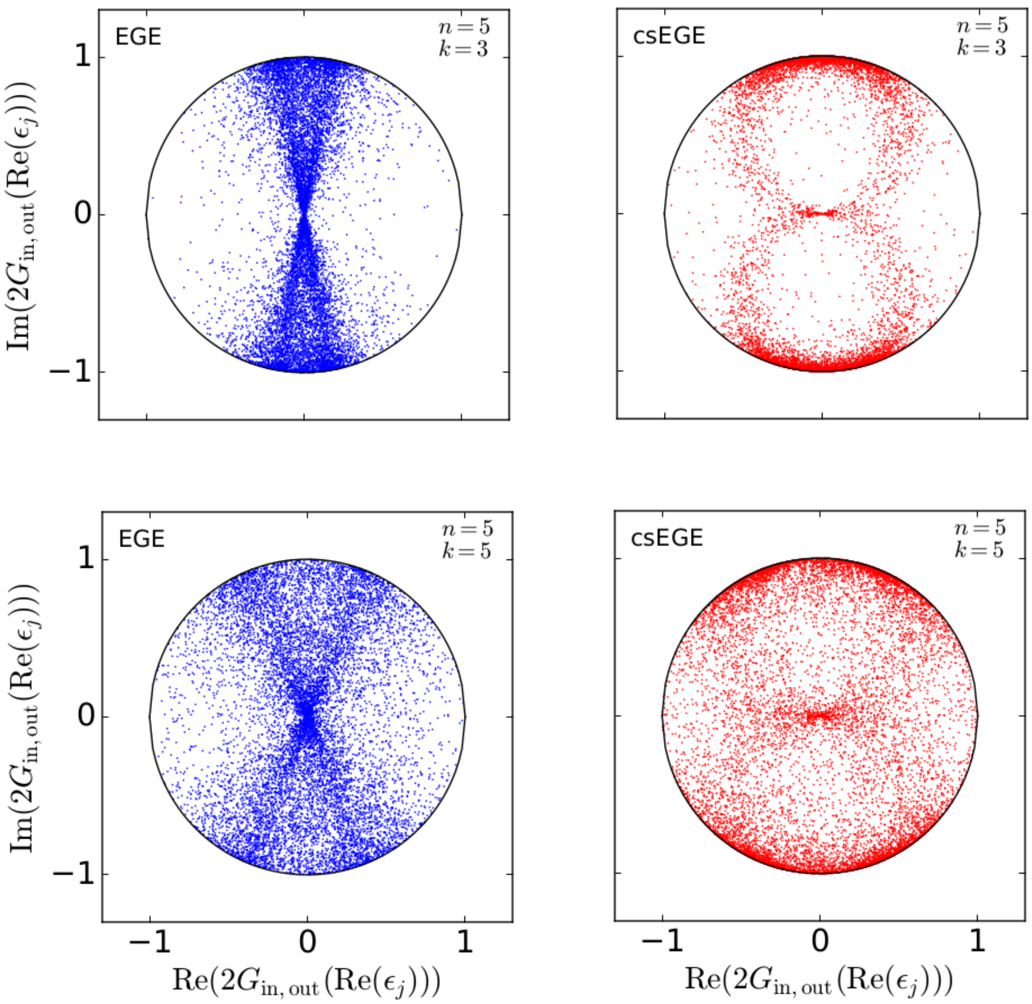
```
In [15]: Image(filename='correlations.png')
```

Out[15]:




```
In [16]: Image(filename='greenfuncs.png')
```

Out[16]:



1.11 Summary

- We analized the transport properties for an open system for the Embedded Ensemble model using Efficiency and NEGF. We found that the CS yields better transport properties.
- In using NEGF, we know why the curent is better in the CS case by using the spectral representation of the Green's function.
- We want to study the magnetic-field case. Also the system with spin. We want to study also the role of decoherence using NEGF.
- We would like to find a connection with the application of quantum buses, or custom physical systems such as the artifical graphene.