

Strong Coupling Thermodynamics

with
Ph. de Forcrand
arXiv:1111.1434

MDP / Hamiltonian QMC

1 Flavor

Many Flavors

Many-flavor Bulk Transition

with
Ph. de Forcrand and Seyon Kim
arXiv:1208.2148

- plaquette term **absent** in strong coupling limit
- link integrals can be integrated out analytically, since link integration factorizes:
- exact rewriting in terms of strong coupling graphs

$\frac{1}{N_c} \int d^4x \delta(x) \text{Tr}[U(x)] = \frac{1}{N_c} \int d^4x \delta(x) \text{Tr}[U(x) U(x+1) U(x+2) U(x+3)]$

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Strong coupling thermodynamics

with Ph. de Forcrand

arXiv:1111.1434

Chiral restoration

with Ph. de Forcrand

arXiv:1111.1434

Summary
The summary is in arXiv:1208.2148

Shown:
• Continuous Time
framework improves on
 μ -T phase diagram
• re-entrance vanishes

Prospects:
• $O(\beta)$ corrections may
help to understand the
connection to the (4 flavor)
continuum phase diagram

Argued:
• a strong first order bulk
transition exists
• finding in contrast to
meanfield theory
• chirally restored phase
extends to weaker
coupling

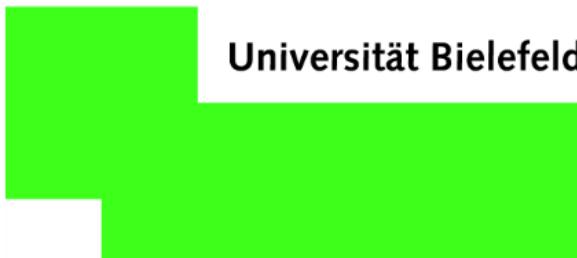
• strong coupling chirally
restored phase appears
to be IR-conformal
• this phase seems to have
a non-trivial IRFP

Final Colloquium Bielefeld GRK 881

New Developments in
Strong Coupling LQCD

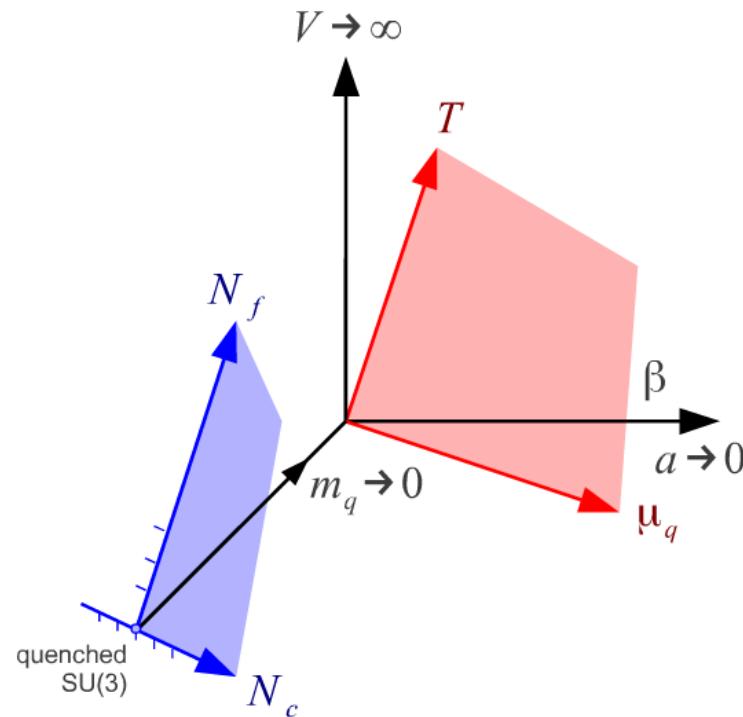
Wolfgang Unger, ETH Zürich

14.09.12



Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Motivation for Strong Coupling Studies



Systematic Errors

- thermodynamic limit
- continuum limit
- chiral limit

QCD Thermodynamics:

- temperature
- quark chemical potential
but: **sign problem**

QCD-like theories:

- Search for conformal window
- number of flavors
 - gauge group

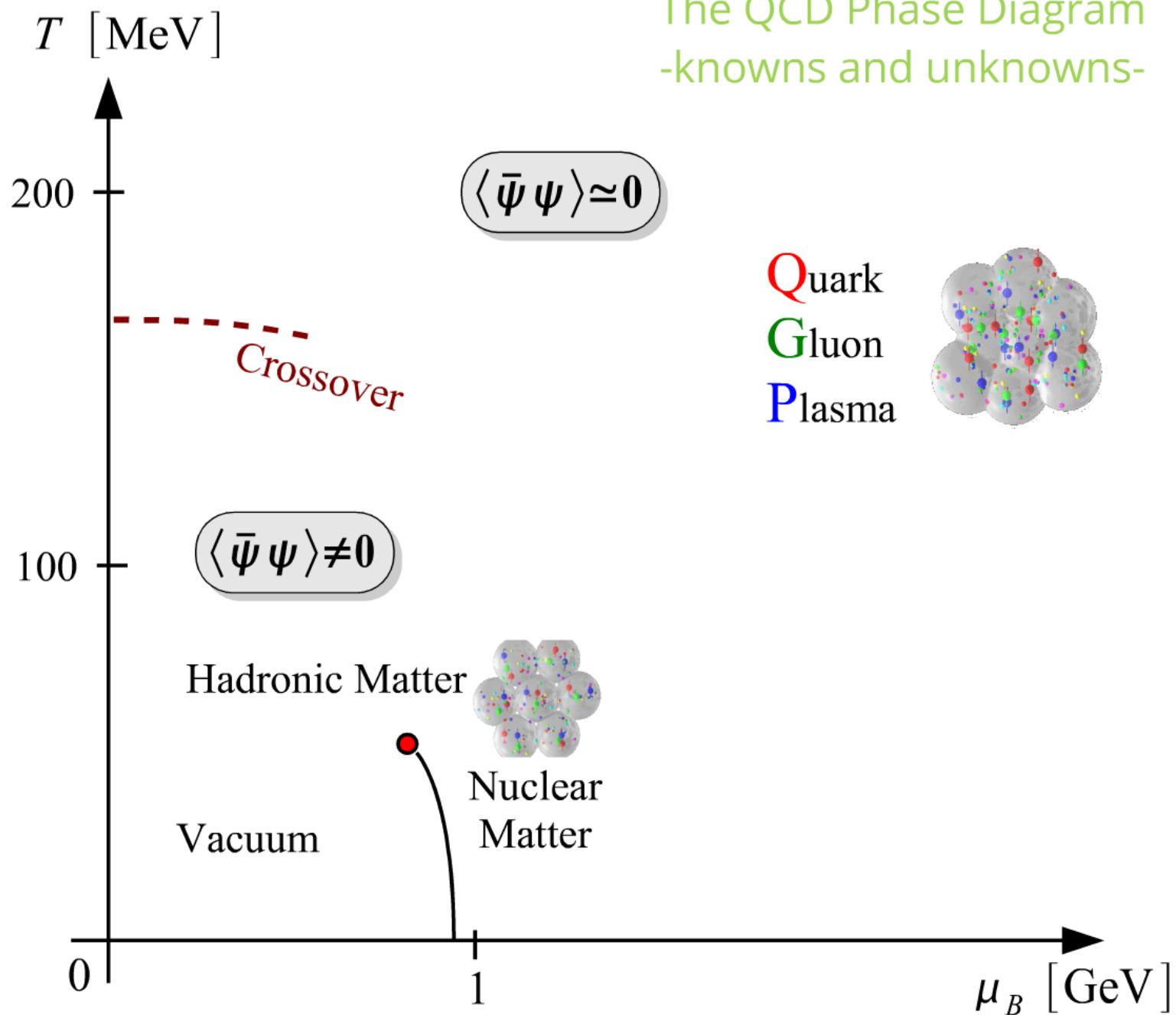
Strong Coupling Limit: lattice is maximally coarse, but:
long-distance properties can be studied more economically!

Look at Lattice QCD in a regime where the **sign problem** can be made mild:

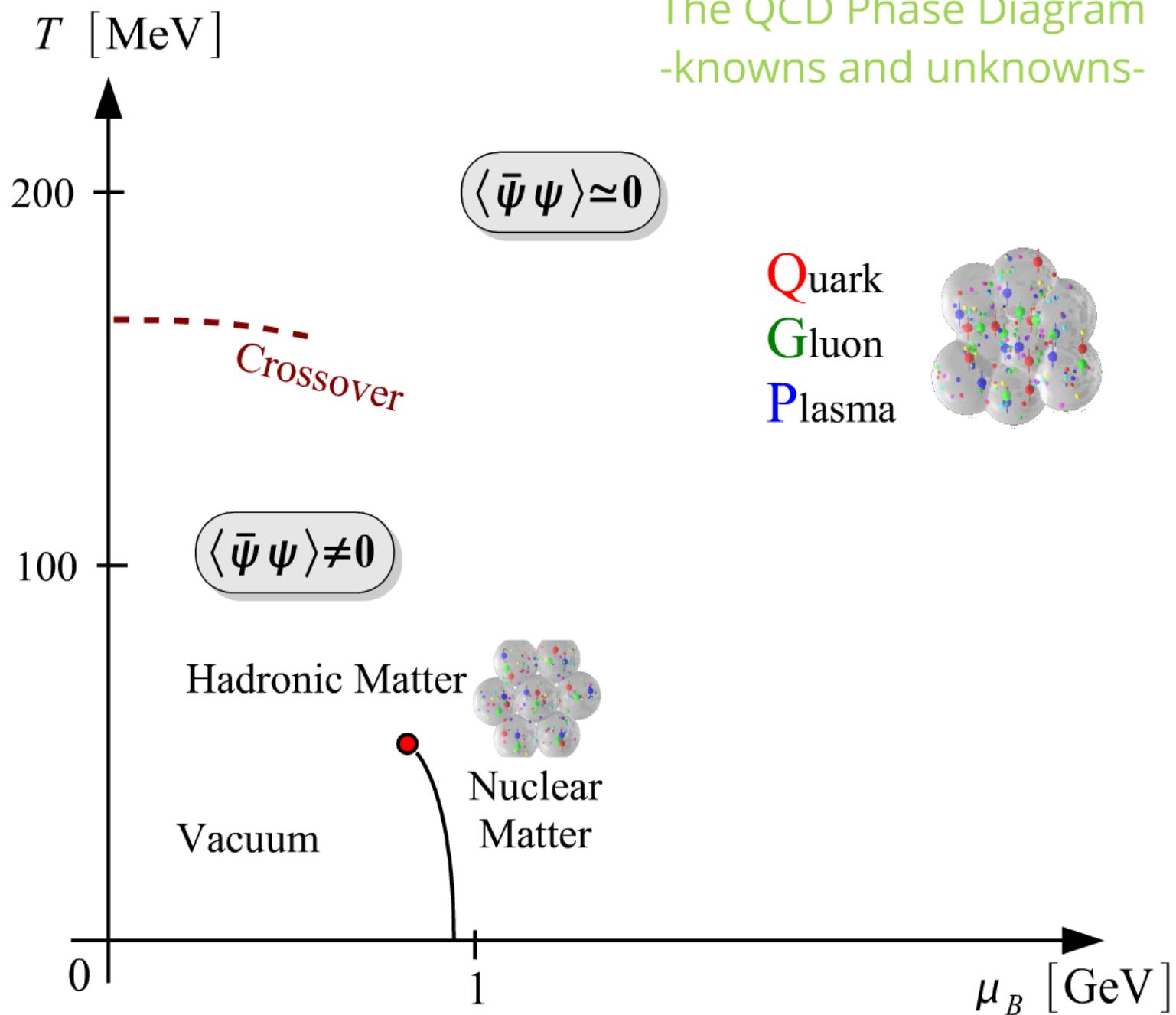
Strong Coupling Limit: $\beta = \frac{2N_c}{g^2} \rightarrow 0$

SC-LQCD is a 1-parameter deformation of QCD

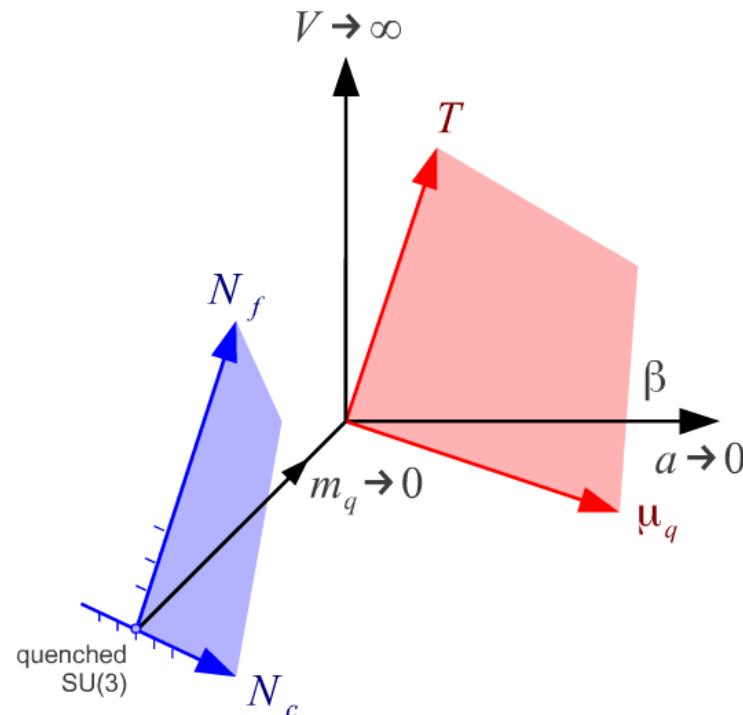
The QCD Phase Diagram -knowns and unknowns-



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$$g \rightarrow \infty, \beta = \frac{2N_c}{g^2} \rightarrow 0$$
$$-\frac{\beta}{2N_c} \sum_P (\text{tr } U_P + \text{tr } U_P^{\dagger}) + \mathcal{O}(a^2)$$

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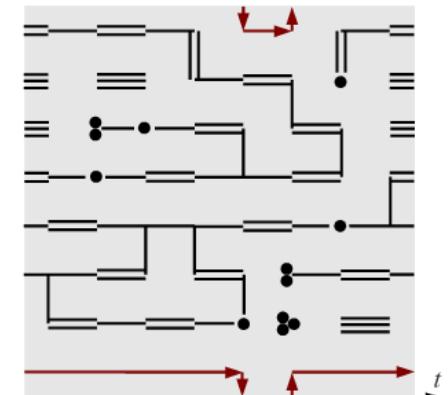
$$\int [dU] = \prod_{x,\mu} \int dU_{\mu}(x)$$

Thermodynamics at Strong Coupling?

Strong coupling LQCD shares important features with QCD:

- exhibits **confinement**, i.e. only color singlet degrees of freedom survive:
 - **mesons** (represented by monomers and dimers)
 - **baryons** (represented by oriented self-avoiding loops)
- and **spontaneous chiral symmetry breaking/restoration**: (restored at T_c)
⇒ SC-LQCD is a great laboratory to study the full (μ, T) **phase diagram**

SC-LQCD is a useful toymodel for **nuclear matter**



Strong Coupling LQCD has a long history:

Mean field ($1/d$ expansion):

1983: development of the technique [Kluberg-Stern, Morel, Petersson]

1985: first **finite density analysis** [Damgaard, Hochberg & Kawamoto]

1992: $T_c(\mu = 0) = 5/3$, $\mu_c(T = 0) = 0.66$ [Bilic *et al.*]

1995: entropy per baryon [Bilic & Cleymans]

2004: full phase diagram and location of (tri)critical point [Nishida *et al.*]

2009: include $\mathcal{O}(\beta)$ corrections [Ohnishi *et al.*]

Monte Carlo:

1984: formulation as a **dimer system** [Rossi & Wolff]

1989: first **finite density results** with MDP algorithm, $aT_c(\mu = 0) = 1.4$, $a\mu_c(T = 0) = 0.63$ [Karsch & Mütter]

2003: first **Worm algorithm** applied to U(3): **fast**, easy to do **chiral limit** [Adams & Chandrasekharan]

2008: spectroscopy for π , ρ , a_1 , a_0/f_0 , N : good agreement with HMC [de Forcrand & Kim]

2010: full phase diagram and nuclear potential for SU(3) [de Forcrand & Fromm]

Finite Temperature Partition Function

How to vary the temperature?

- $aT = 1/N_\tau$ is discrete with N_τ even
- $aT_c \simeq 1.5$, i.e. $N_\tau^c < 2 \Rightarrow$ we cannot address the phase transition!

Solution: introduce an **anisotropy** γ in the Dirac couplings:

$$\mathcal{Z}(m_q, \mu, \gamma, N_\tau) = \sum_{\{k, n, \ell\}} \prod_{b=(x, \mu)} \frac{(3 - k_b)!}{3! k_b!} \gamma^{2k_b \delta_{\mu 0}} \prod_x \frac{3!}{n_x!} (2am_q)^{n_x} \prod_\ell w(\ell, \mu)$$
$$k_b \in \{0, \dots N_c\}, n_x \in \{0, \dots N_c\}$$

Should we expect $a/a_\tau = \gamma$, as suggested at weak coupling?

- **No:** meanfield predicts $a/a_\tau = \gamma^2$, since $\gamma_c^2 = N_\tau \frac{(d-1)(N_c+1)(N_c+2)}{6(N_c+3)}$
 \Rightarrow sensible, N_τ -independent definition of the temperature: $aT \simeq \frac{\gamma^2}{N_\tau}$
- Moreover, SC-LQCD partition function is a function of γ^2

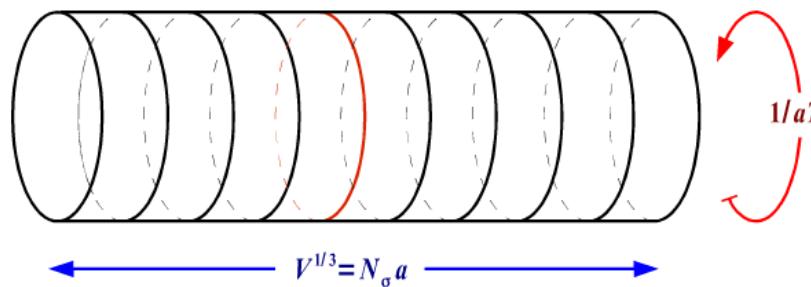
However: **precise correspondence between a/a_τ and γ^2 not known**

The Continuous Euclidean Time Limit

Strategy for **unambiguous** answer: the **continuous Euclidean time limit** (CT-limit):

$$N_\tau \rightarrow \infty, \quad \gamma \rightarrow \infty, \quad \gamma^2/N_\tau \equiv aT \quad \text{fixed}$$

- same as in analytic studies: $a_\tau = 0, aT = \beta^{-1} \in \mathbb{R}$



Several **advantages** of continuous Euclidean time approach:

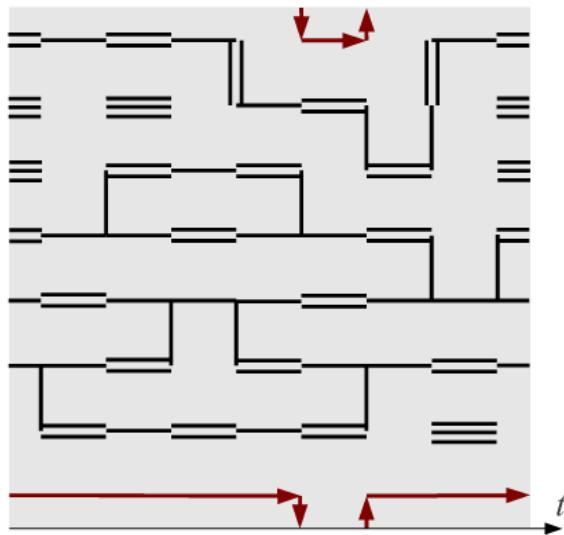
- ambiguities arising from the functional dependence of observables on the anisotropy parameter will be circumvented, **only one parameter** setting the temperature
- no need to perform the continuum extrapolation $N_\tau \rightarrow \infty$
- allows to estimate critical temperatures more precisely, with a faster algorithm (about 10 times faster than $N_t = 16$ at T_c)
- baryons become static in the CT-limit, the **sign problem is completely absent!**

Continuous Time Monte Carlo

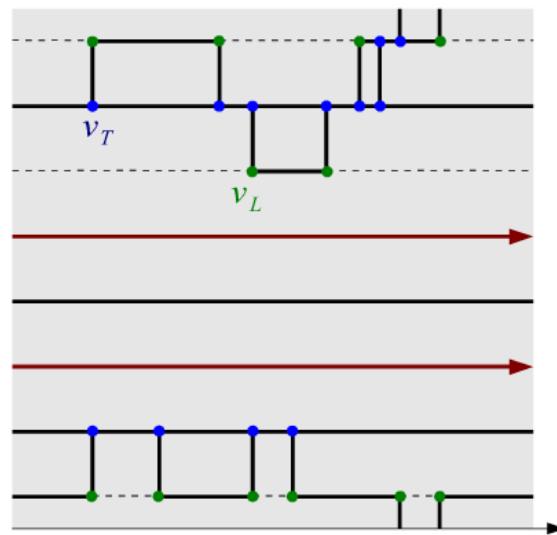
Partition function in inverse temperature $\beta = 1/aT$ and in the **chiral limit**:

$$\mathcal{Z}(\beta, \mu) = \sum_{\kappa \in 2\mathbb{N}} \frac{(\beta/2)^{\kappa}}{\kappa!} \sum_{\mathcal{C} \in \Gamma_\kappa} v_L^{n_L(\mathcal{C})} v_T^{n_T(\mathcal{C})} e^{3\beta \mu B(\mathcal{C})}, \quad v_L = 1, v_T = 2/\sqrt{3}$$

Discrete Time



Continuous Time



- multiple spatial dimers become resolved into **single dimers**
- baryons become static
- weight of a configuration solely given by the number of **vertices**
- also mesonic lines can be oriented:

New formulation allows to make use of

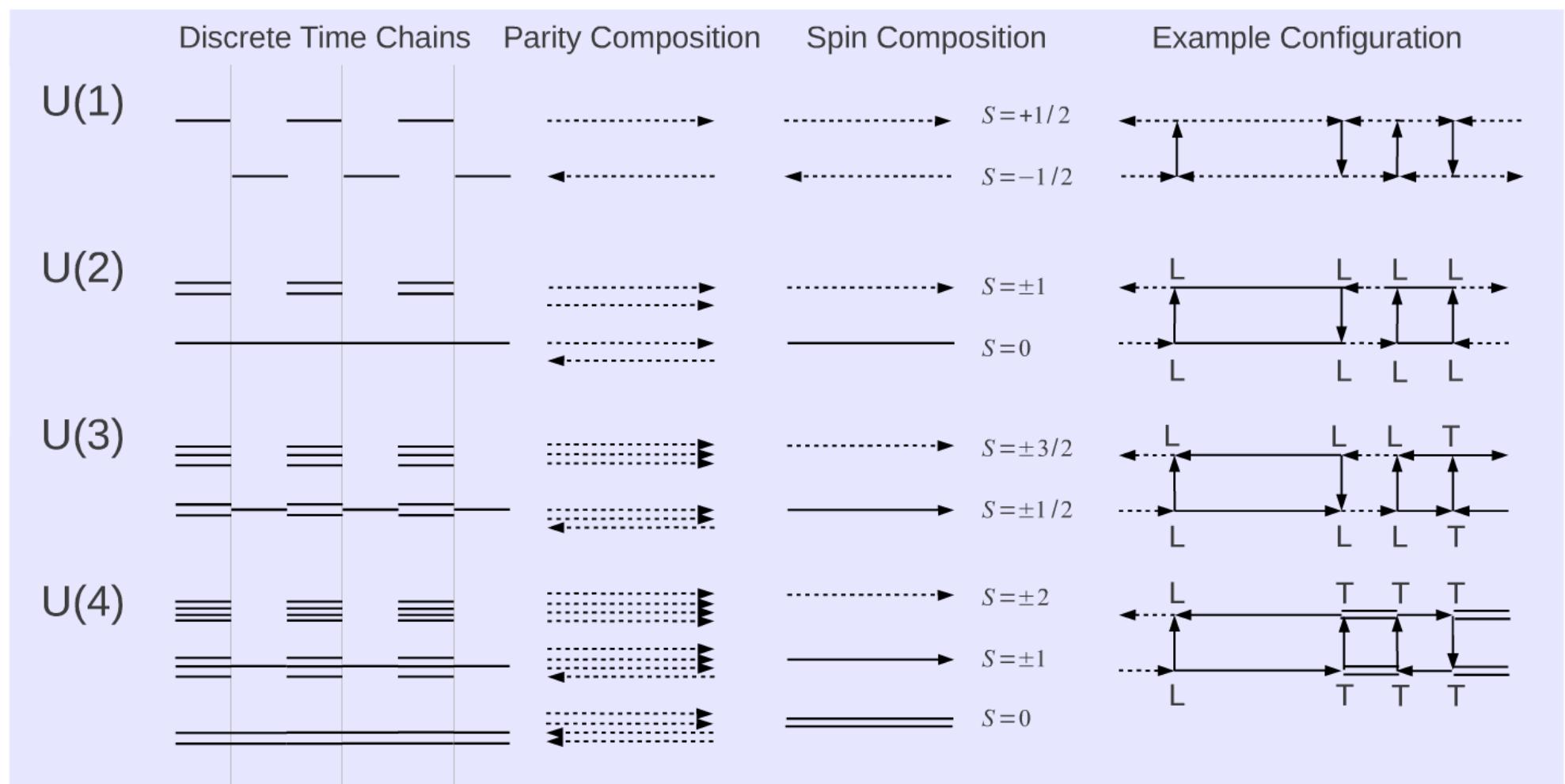
Quantum Monte Carlo techniques:

- Continuous Time Worm Algorithm [Beard & Wiese]
- Stochastic Series Expansion [Sandvik]

	Discrete Time Chains	Parity Composition	Spin Composition	Example Configuration
U(1)	—	$S=+1/2$ $S=-1/2$	
U(2)	= =	$S=\pm 1$	
U(3)	= = =	$S=0$ $S=\pm 1/2$	
U(4)	= = = =	$S=\pm 2$ $S=\pm 1$ $S=0$	

STRUCTURE OF VACUUM

- also mesonic lines can be oriented:

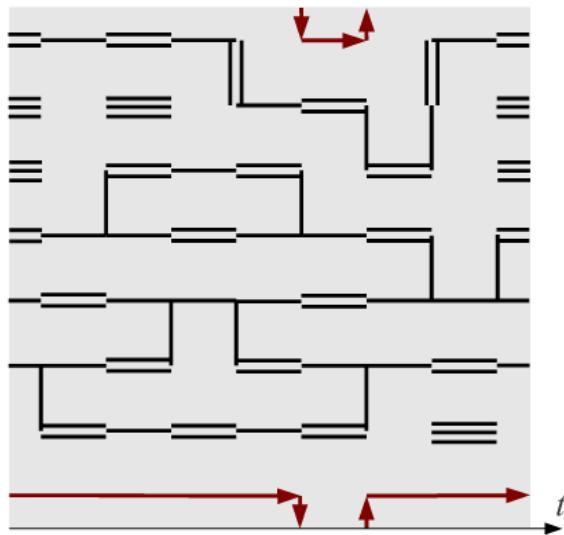


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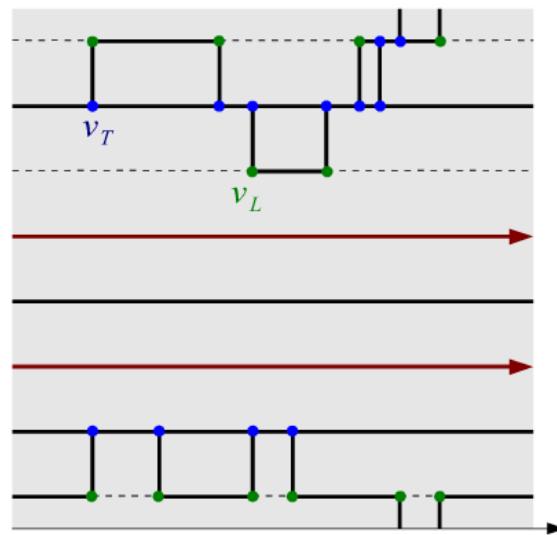
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		$S=\pm 2$	
		=====	$S=0$	

Hamiltonian Formulation

Strong Coupling $U(1)$ is identical to **XY Model** in zero field!

Extension to $U(N_c)$ for SC-LQCD straightforward:

$$\mathcal{H} = \frac{1}{2} \sum_{\langle x,y \rangle} J_x^+ J_y^-$$

with $J^+ = \begin{pmatrix} 0 & & & & \\ v_1 & 0 & & & \\ & v_2 & 0 & & \\ & & \ddots & \ddots & \\ & & & v_{N_c} & 0 \end{pmatrix}$, $v_k = \sqrt{\frac{k(1+N_c-k)}{N_c}}$

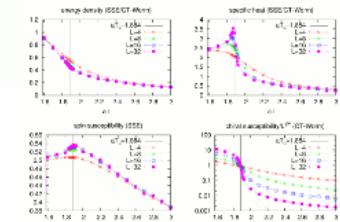
and $J^- = (J^+)^T$ for absorption/emission

- $N_c = 3$: $v_L \equiv v_1 = v_3 = 1$, $v_T \equiv v_2 = 2/\sqrt{3}$

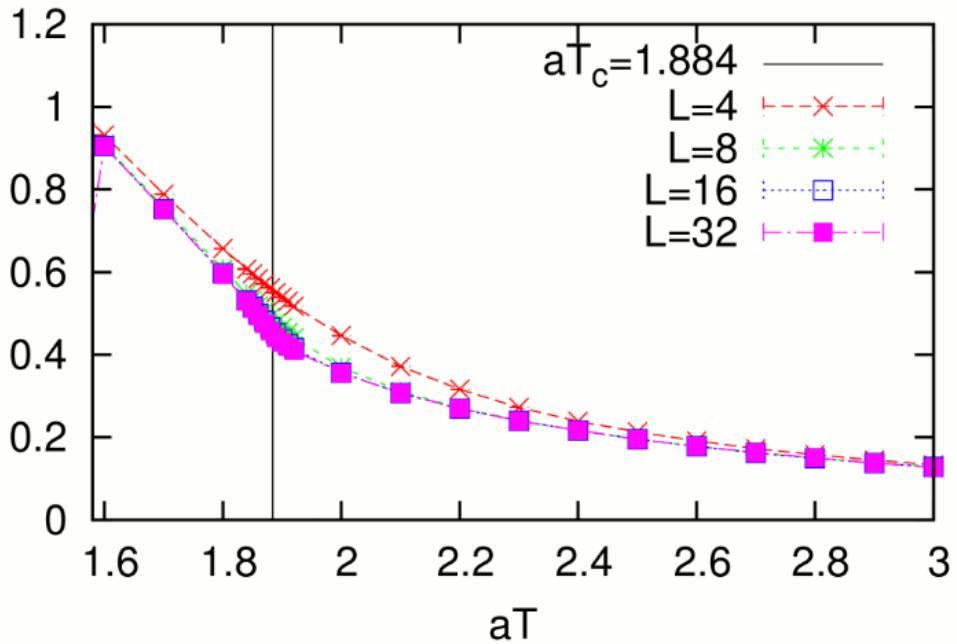
- state vector characterizing time slice:

$$|S^z\rangle(t) \in \left\{ \bigotimes_{\vec{x} \in V} S_{\vec{x}}^z \mid S_{\vec{x}}^z \in \{-N_c/2, \dots, N_c/2\} \right\}$$

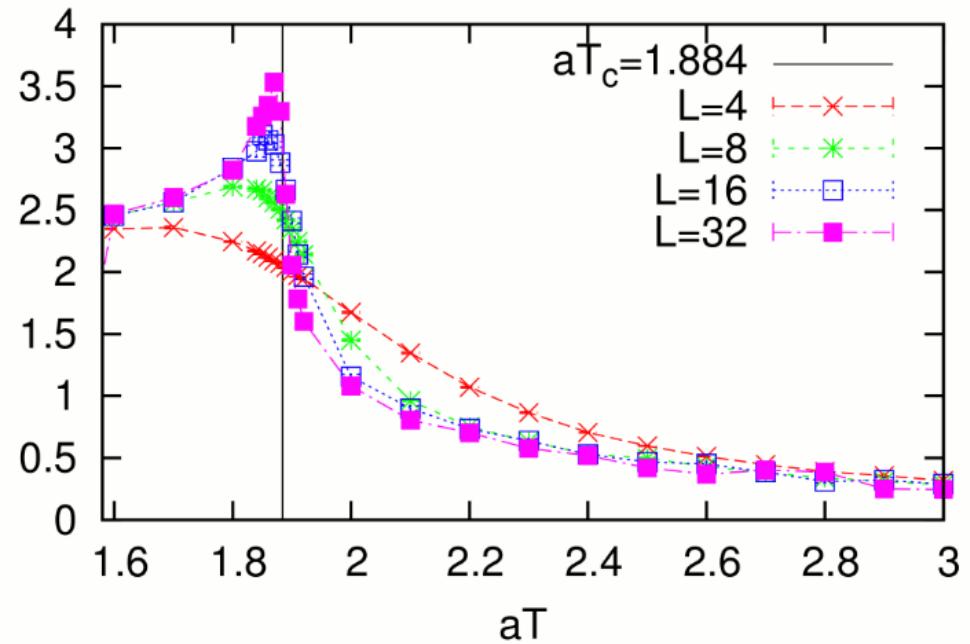
- **oriented spatial dimers** act at time t_i on $|S_x\rangle$ by raising/lowering spin at absorption/emission site
- lowest/highest weight: $J^+|N_c/2\rangle = 0$, $J^-|-N_c/2\rangle = 0$
- S^z counts net number of (odd-even) time like meson sites at each site
- $\frac{N_c}{2}[J^+, J^-] = J^z = \text{diag}(-N_c/2, \dots, N_c/2)$ fulfilled, $J^z|S^z\rangle = S^z|S^z\rangle$
- new observable: **spin susceptibility** $\chi_s = \beta \langle (\sum_i S_i^z)^2 \rangle / N$



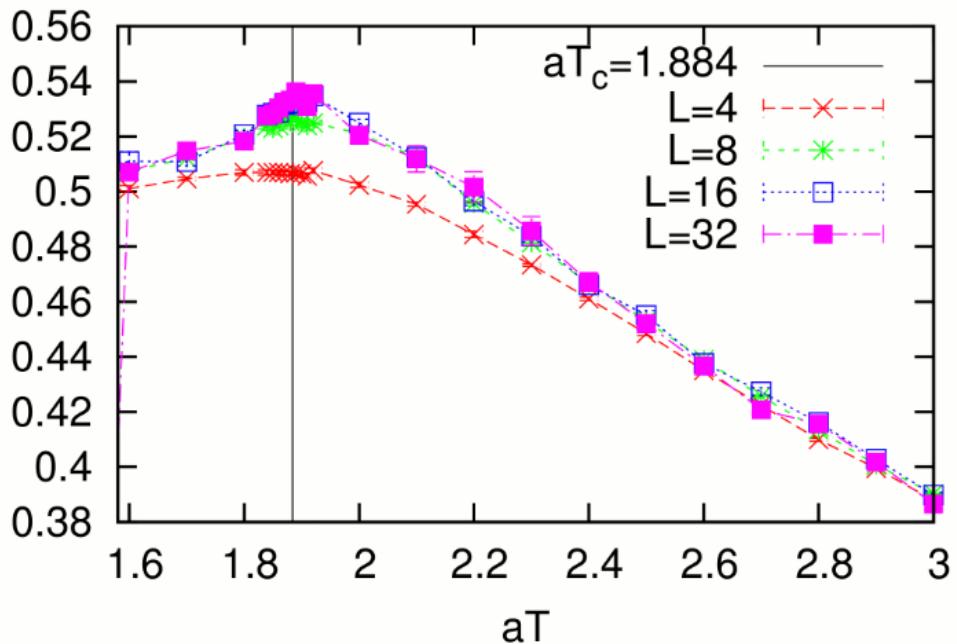
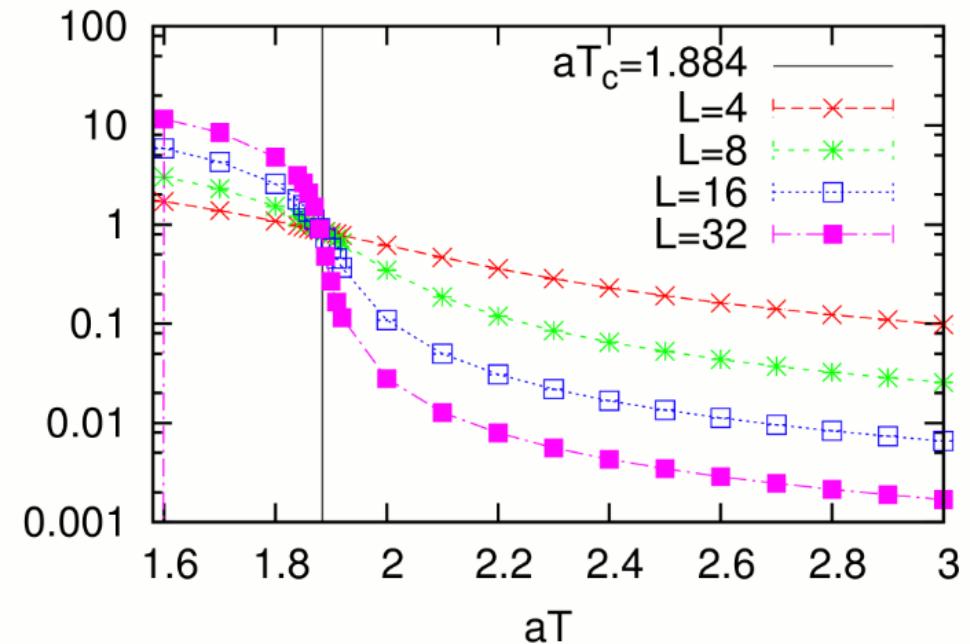
energy density (SSE/CT-Worm)



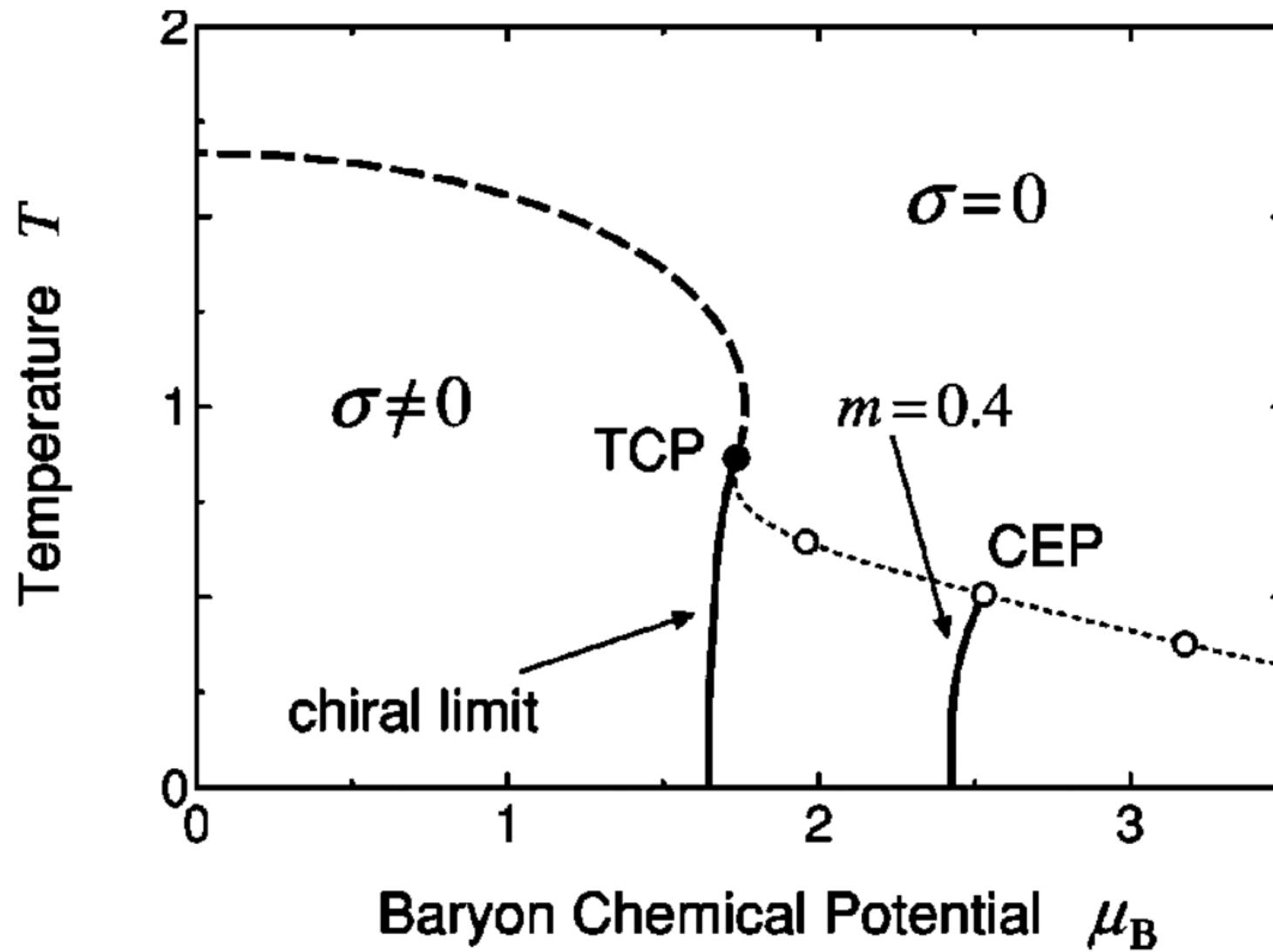
specific heat (SSE/CT-Worm)



spin susceptibility (SSE)

chiral susceptibility/ $L^{\gamma/\nu}$ (CT-Worm)

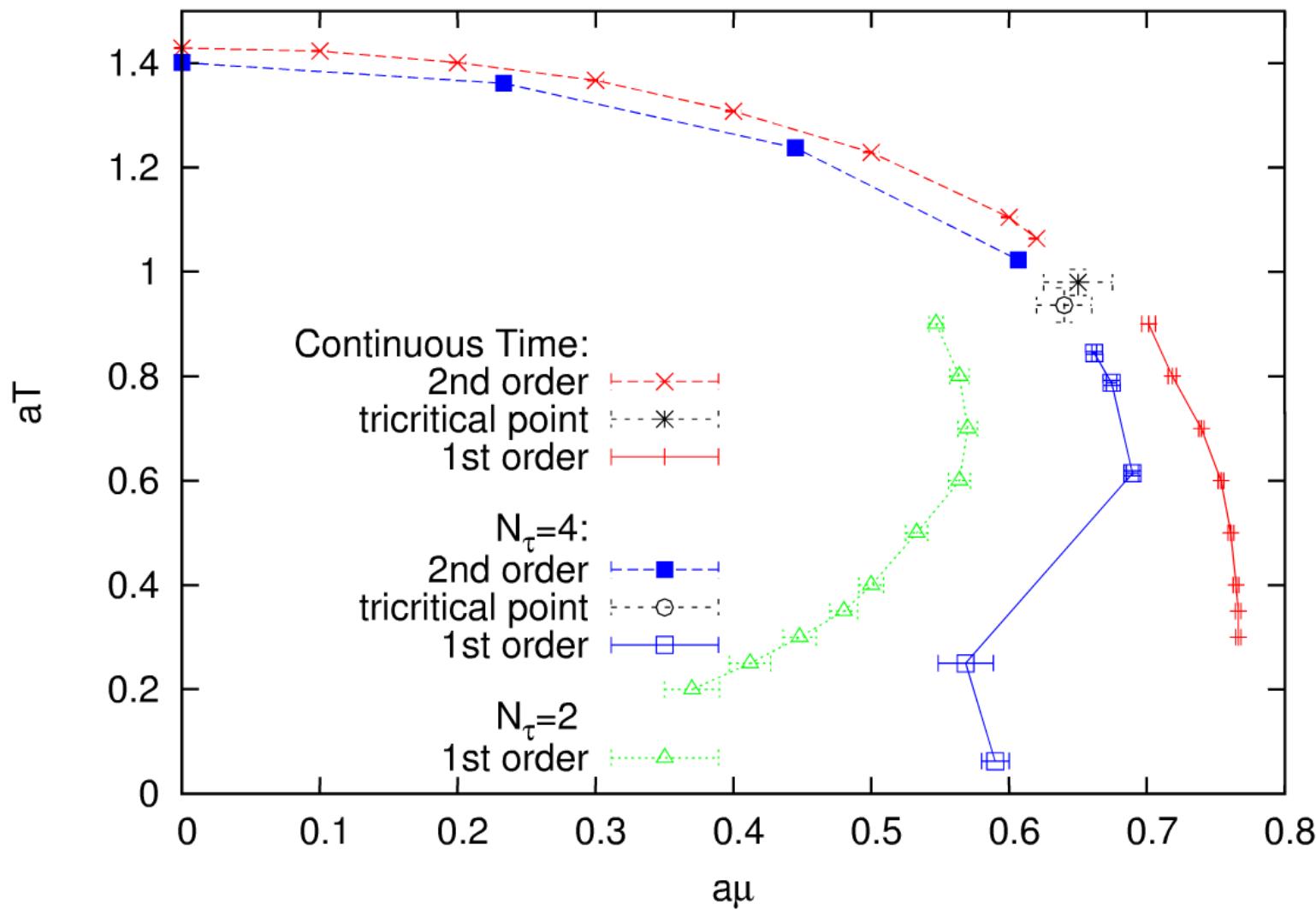
Strong Coupling Phase Diagram (MF)



chiral phase transition:

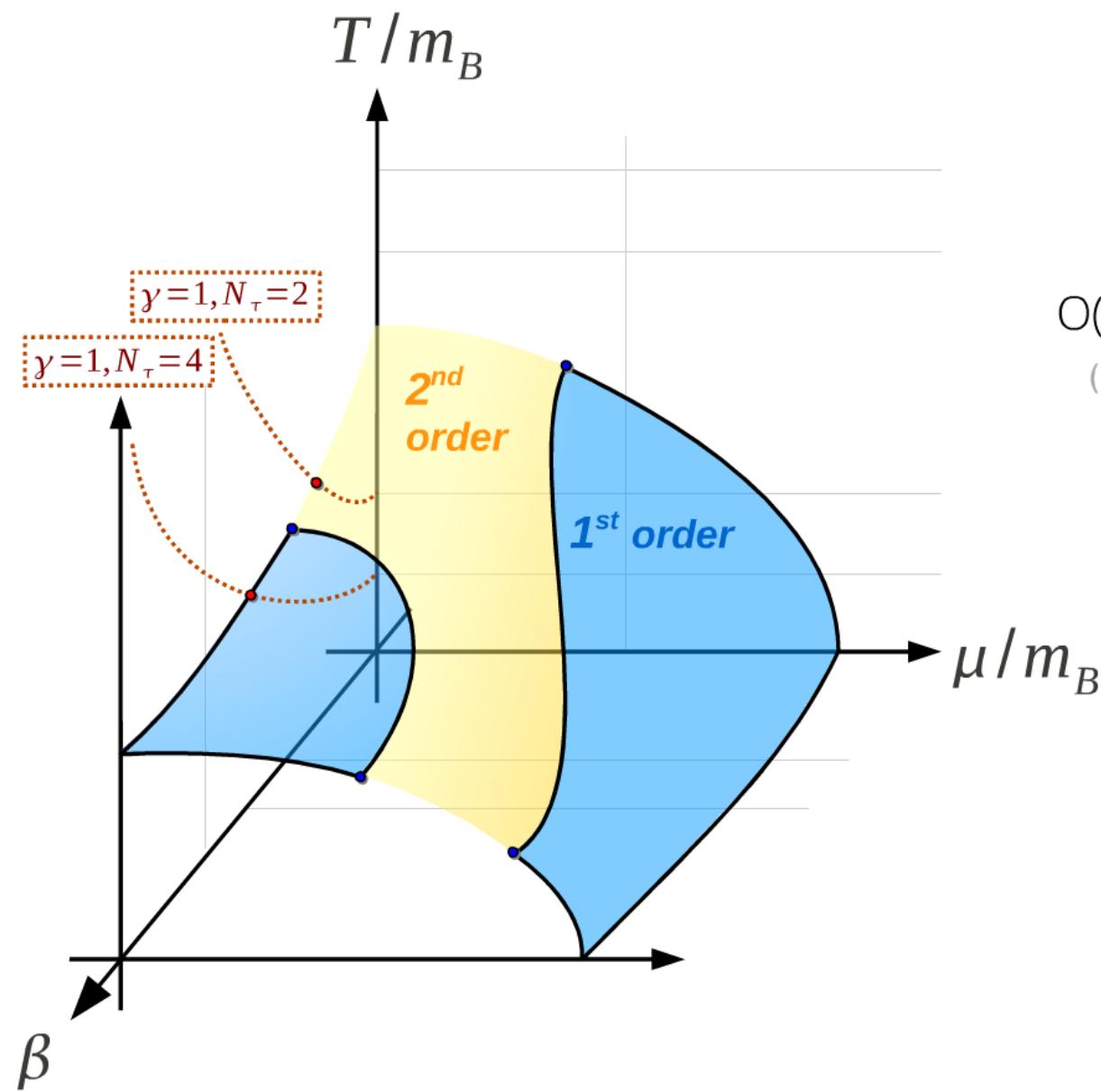
- 2nd order at low densities, 1st order at low temperatures
- finite quark mass: TCP trans to CEP

Strong Coupling Phase Diagram (MC)



- finite N_τ effects severe for low temperatures
- no re-entrance as you consider the continuous time limit

The Phase Diagram in the Chiral Limit



1 flavor at strong coupling
connected to
4 continuum flavors

O(beta) corrections might tell
(with J. Langelage, M. Fromm,
K. Miura, O. Philipsen)

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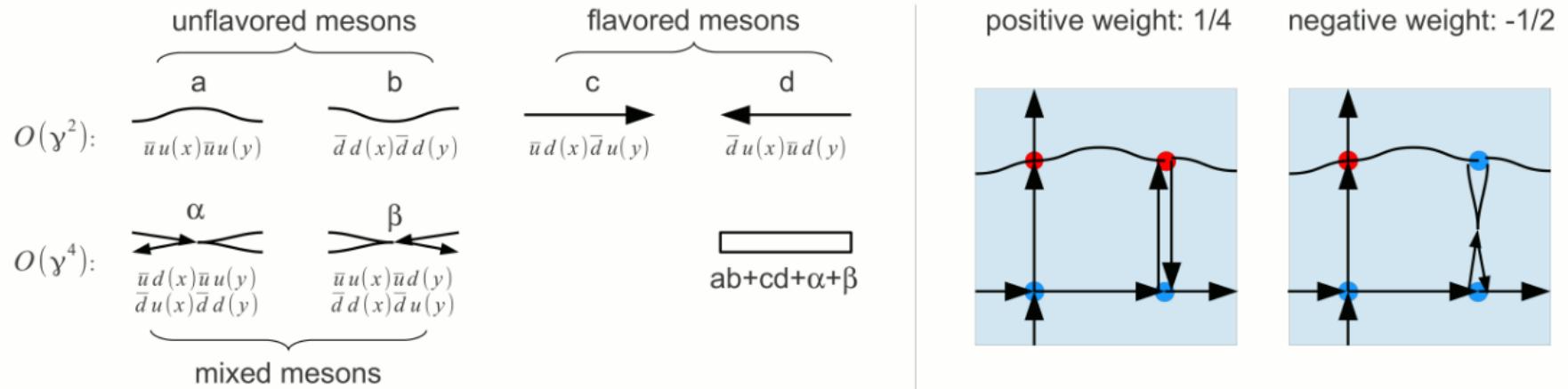
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Getting more realistic: 2 Flavors

Aim: obtain phase diagram for 2-flavor SC-LQCD, where **pion exchange** may play a crucial role for nuclear transition, but:

- at present, no 2-flavor formulation for staggered SC-LQCD suitable for MC
- already the mesonic sector has a severe **sign problem** (worse than for finite μ HMC)
- 2 new types of mixed dimers give negative sign in mesonic loops already for U(2):



Observation in Continuous Time:

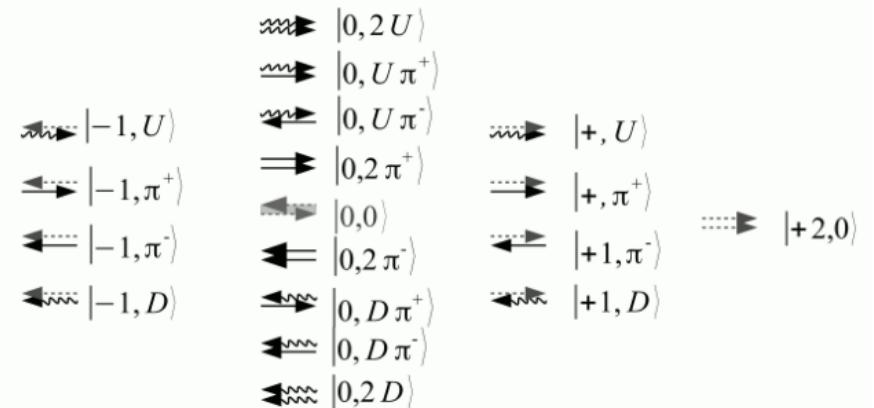
- static lines for 2 staggered flavors have all **positive weight**
-> **no sign problem**
- again: only single spatial dimers
- Hamiltonian formulation now also involves flavor charges

Flavored static lines:

- new classification in terms of **quantum numbers**

$$|S_U^z, S_D^z, Q_{\pi^+}\rangle, \quad S^z = S_U^z + S_D^z$$

- $N_c = 2$: in total 19 types of lines in the mesonic sector, 10 with $|B|=1$ and 2 with $B=|2|$



Transition Rules:

- “spin” S^z counts number of emission/absorption events (remnant of even/odd decomposition) with $S^z = -\frac{1}{2}N_c N_f, \dots, +\frac{1}{2}N_c N_f$
- “charge” $Q_\pi^+ = -N_c, \dots, +N_c$ denote the flavor content (flavor neutral, $\bar{u}d$, $\bar{d}u$)
- **spin/charge conservation:** transitions at spatial dimers, raising charges at one site, lowering at a neighboring site:

$$|\Delta S_U^z| + |\Delta S_D^z| = 1, \quad |\Delta Q_{\pi^+}| = 1$$

Hamiltonian Formulation

1) is identical to **XY Model** in zero field!
for SC-LQCD straightforward:

with $J^+ = \begin{pmatrix} 0 & & & & \\ v_1 & 0 & & & \\ & v_2 & 0 & & \\ & & \ddots & \ddots & \\ & & & v_{N_c} & 0 \end{pmatrix}$,

and $J^- = (J^+)^T$ for absorption/emission

$$v_1 = v_3 = 1, v_T \equiv v_2 = 2/\sqrt{3}$$

characterizing time slice:

2 Flavor Hamiltonian

$$\mathcal{H} = \frac{1}{2} \sum_{\langle x,y \rangle} \left(J_{U(x)}^+ J_{U(y)}^- + J_{D(x)}^+ J_{D(y)}^- + J_{\pi^+(x)}^+ J_{\pi^+(y)}^- + J_{\pi^-(x)}^+ J_{\pi^-(y)}^- \right)$$

- Absorption (J^+ , lower left triangle) and Emission (J^- , upper triangle), state vector:

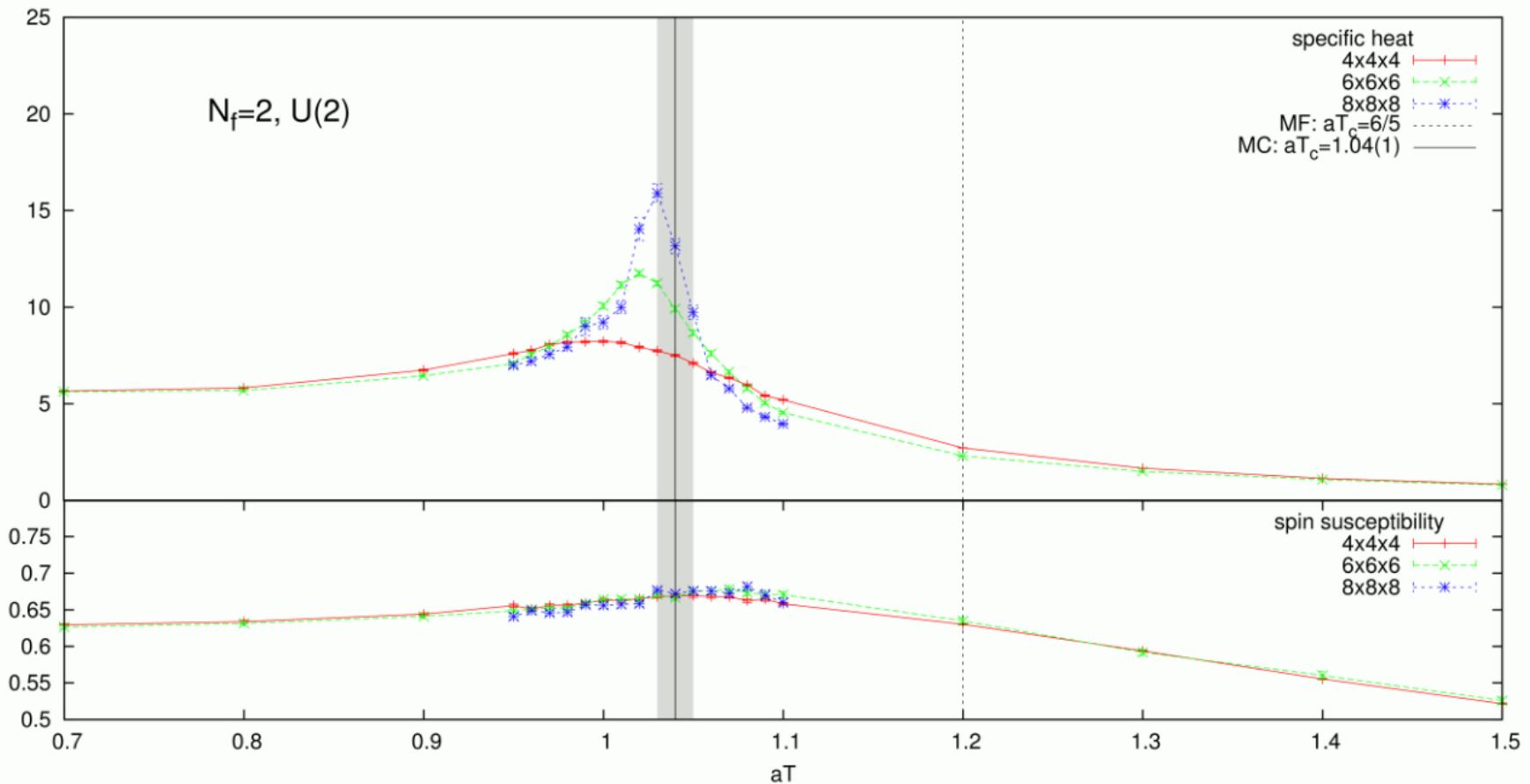
$$J_{\pi_i}^{+/-} = \begin{pmatrix} & U & D & \pi^+ & \pi^- & & & & \\ & U & & & & \hat{D} & \hat{\pi}^+ & \hat{\pi}^- & \\ & D & & D & & \hat{U} & & & \\ & \pi^+ & & \pi^+ & & \hat{\pi}^- & \hat{U} & & \\ & \pi^- & & \pi^- & \pi^+ & & \hat{U} & \hat{D} & \\ \hline U & & & & & & & & \\ D & & & & & & & & \\ \pi^+ & & & & & & & & \\ \pi^- & & & & & & & & \\ \hline U & & & & & & & & \\ D & & & & & & & & \\ \pi^+ & & & & & & & & \\ \pi^- & & & & & & & & \\ \hline D & & & & & & & & \\ U & & & & & & & & \\ \pi^+ & & & & & & & & \\ \pi^- & & & & & & & & \\ \hline D & & & & & & & & \\ U & & & & & & & & \\ \pi^+ & & & & & & & & \\ \pi^- & & & & & & & & \\ \hline \end{pmatrix}, \quad \chi = \begin{pmatrix} -2, 0 \\ -1, U \\ -1, D \\ -1, \pi^+ \\ -1, \pi^- \\ 0, 2U \\ 0, 2D \\ 0, 2\pi^+ \\ 0, 2\pi^- \\ 0, 0 \\ 0, U\pi^+ \\ 0, U\pi^- \\ 0, D\pi^+ \\ 0, D\pi^- \\ +1, U \\ +1, D \\ +1, \pi^+ \\ +1, \pi^- \\ +2, 0 \end{pmatrix}$$

- vertex weights are $v_{\hat{\pi}_i} = \frac{1}{\sqrt{2}}$ if states with $S = 0$, $|Q| = 1$ are involved, $v_{\pi_i} = 1$ else

Preliminary Results on 2-Flavors

Comparison of aT_c from MC data with mean field:

N_f	$N_c = 1$	$N_c = 2$	$N_c = 3$
1	$3/2 [1.102(1)]$	$4/2 [1.467(1)]$	$5/2 [1.884(1)]$
2	$5/5 [0.77(1)]$	$6/5 [1.04(1)]$	$7/5$



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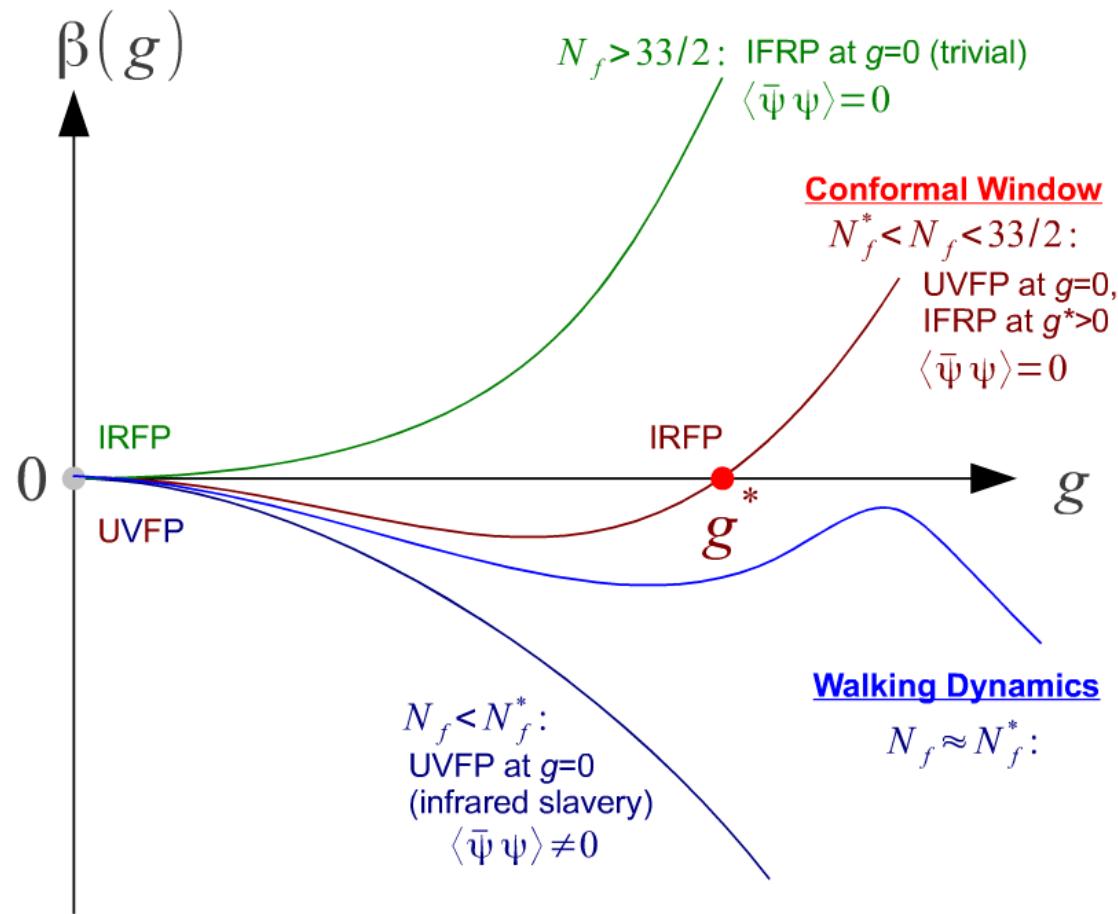
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QCD Beta-Function and Conformality



Literature on the Determination of N_f^*

- Red: conformal Blue: χ SB Black: unclear
- $SU(3) + N_f = 8-16$ fundamental rep:
 - ▶ $N_f = 8$: Appelquist et al; Deuzeman et al; Fodor et al; Jin et al
 - ▶ $N_f = 9$: Fodor et al
 - ▶ $N_f = 10$: Hayakawa et al; Appelquist et al
 - ▶ $N_f = 12$: Hasenfratz; Appelquist et al; Deuzeman et al; Xin and Mawhinney; Fodor et al
 - ▶ $N_f = 16$: Damgaard et al; Heller; Hasenfratz; Fodor et al
- $SU(2) +$ fundamental rep fermions:
 - ▶ $N_f = 4$: Karavirta et al
 - ▶ $N_f = 6$: Del Debbio et al; Karavirta et al; Appelquist et al (unclear)
 - ▶ $N_f = 8$: Iwasaki et al
 - ▶ $N_f = 10$: Karavirta et al
- $SU(2) + N_f = 2$ adjoint rep: Catterall et al; Bursa et al; Hietanen et al; De Grand et al
- $SU(3) + N_f = 2$ 2-index symmetric rep: DeGrand et al; Sinclair and Kogut; Fodor et al
- $SU(4) + N_f = 2$ 2-index symmetric rep: DeGrand et al

Literature on the Determination of N_f^*

Red: conformal Blue: χ SB Black: unclear

- $SU(3) + N_f = 8\text{--}16$ fundamental rep:
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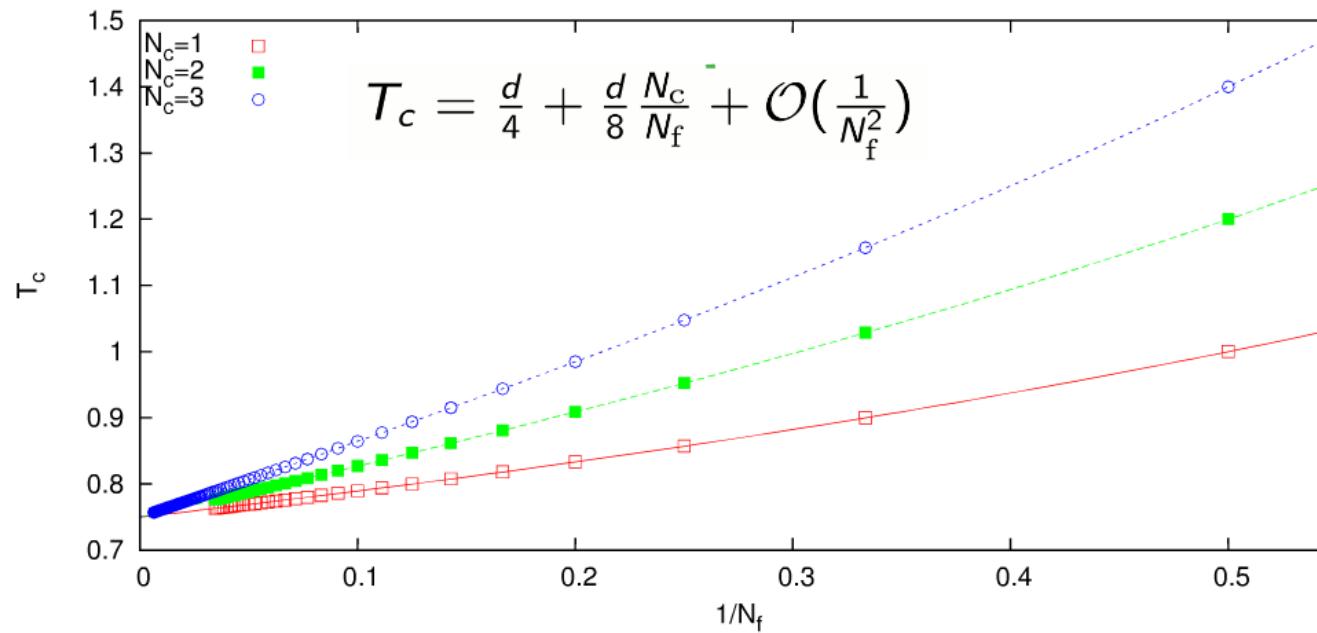
Flavor Dependence at Strong Coupling

Mean Field: chiral symmetry is **always broken** in the strong-coupling limit of staggered fermions at $T = 0$ **for all values of N_f and N_c**

Zero Temperature:

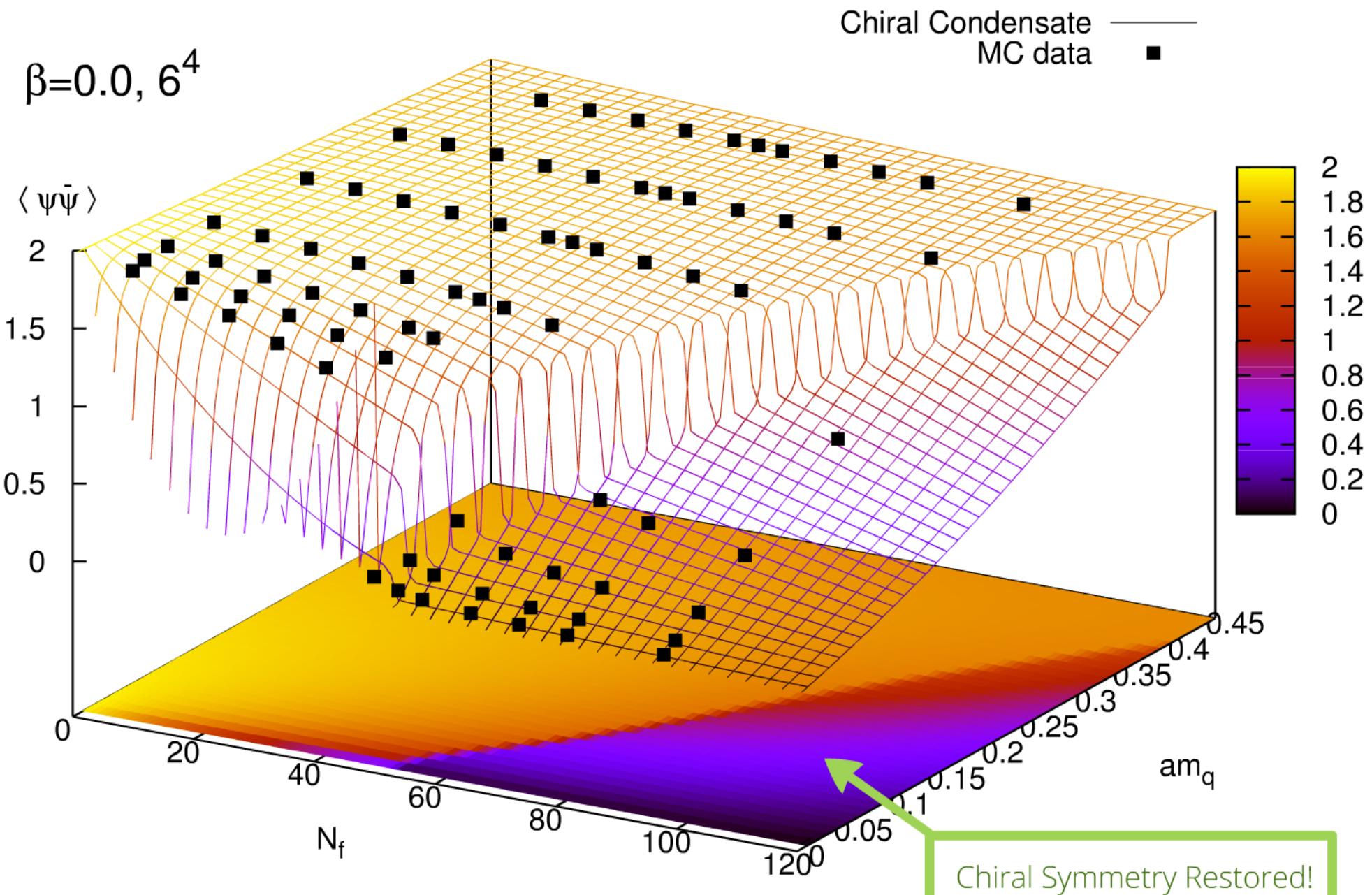
$$\langle \bar{\psi} \psi \rangle(T=0) = \frac{((1+d^2)^{1/2}-1)/2}{d}^{1/2}$$

Finite Temperature:

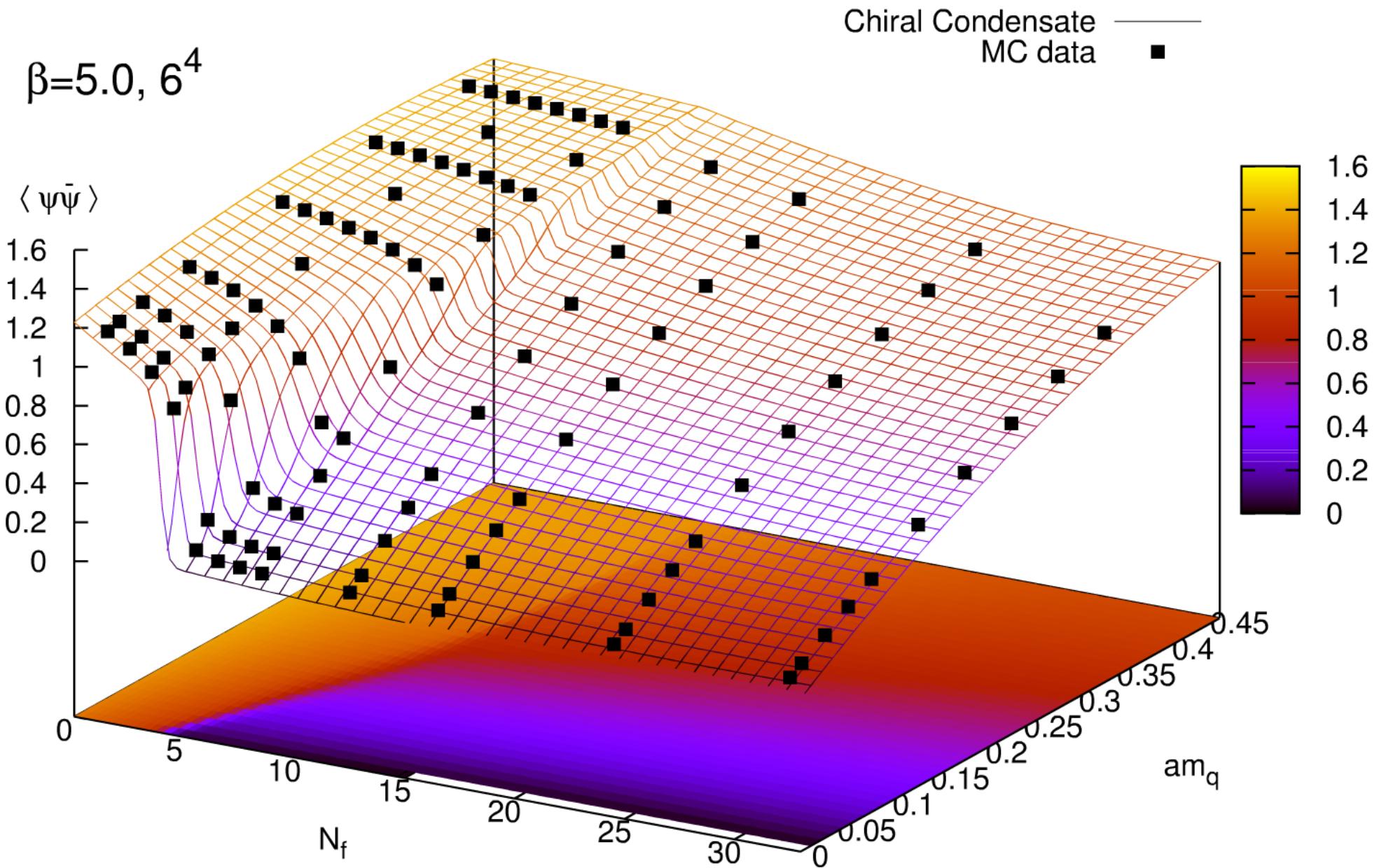


On the other hand: loop expansion of the determinant shows that dynamical fermions induce a plaquette coupling $\propto N_f/m_q^4$, as studied numerically by [A. Hasenfratz, T. DeGrand PRD49 (1994)]: fermions have ordering effect
⇒ suggests **chiral symmetry restoration** for sufficiently large N_f ?

Strong First Order Bulk Transition!



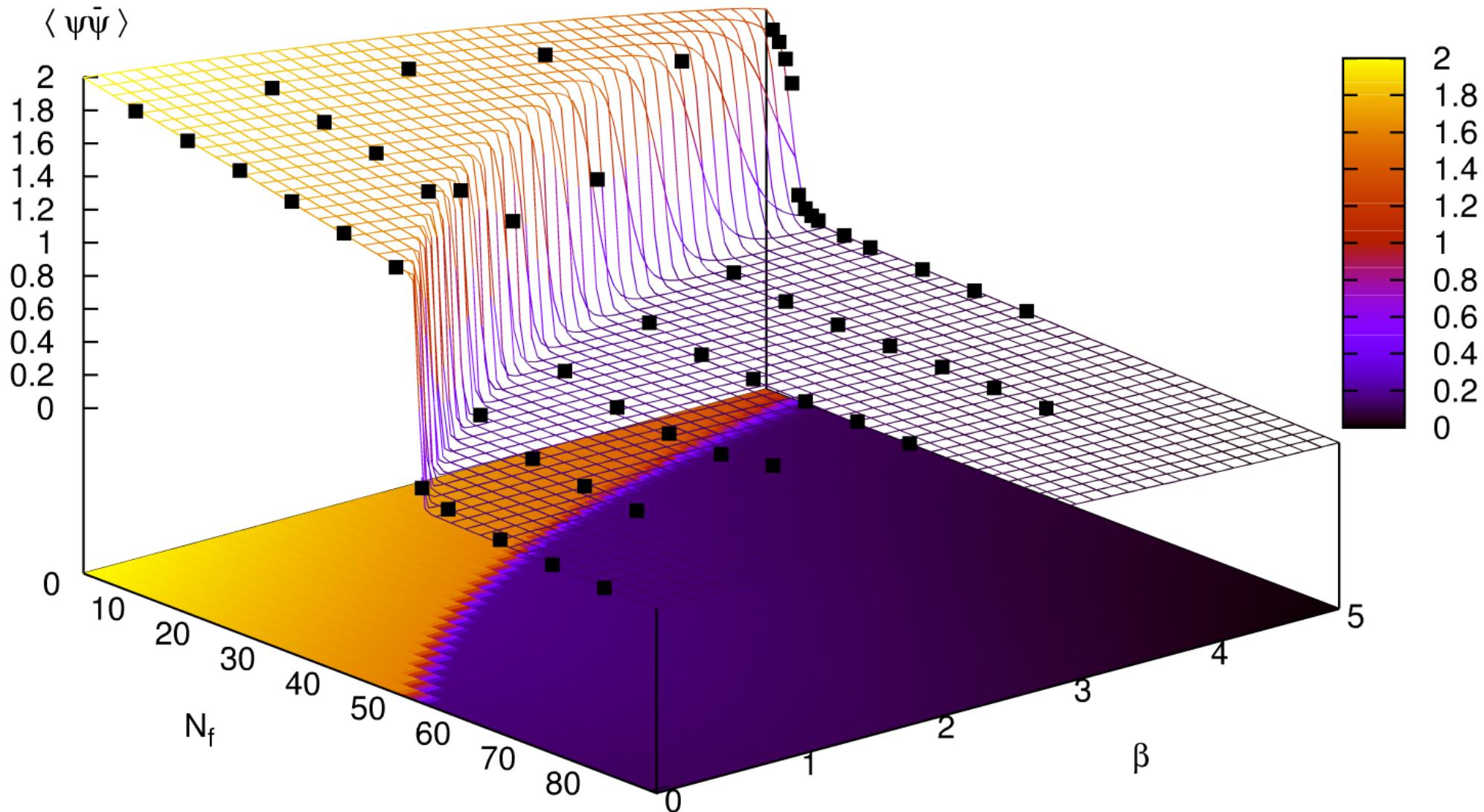
Bulk Transition also at Weaker Coupling



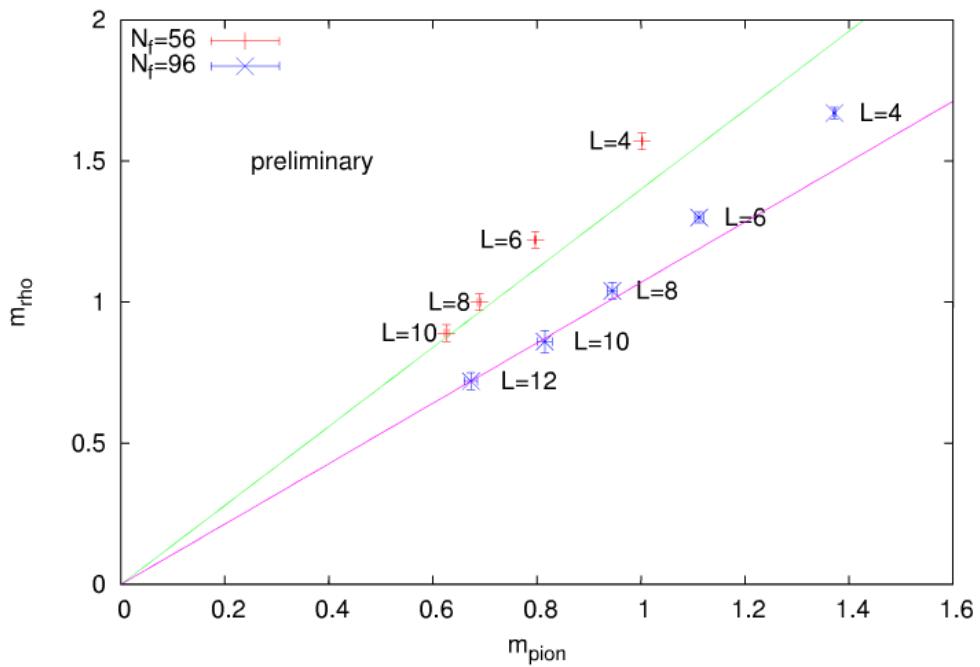
Transition Extends to Weak Coupling

$am_q=0.025, 6^4$

Chiral Condensate
MC data



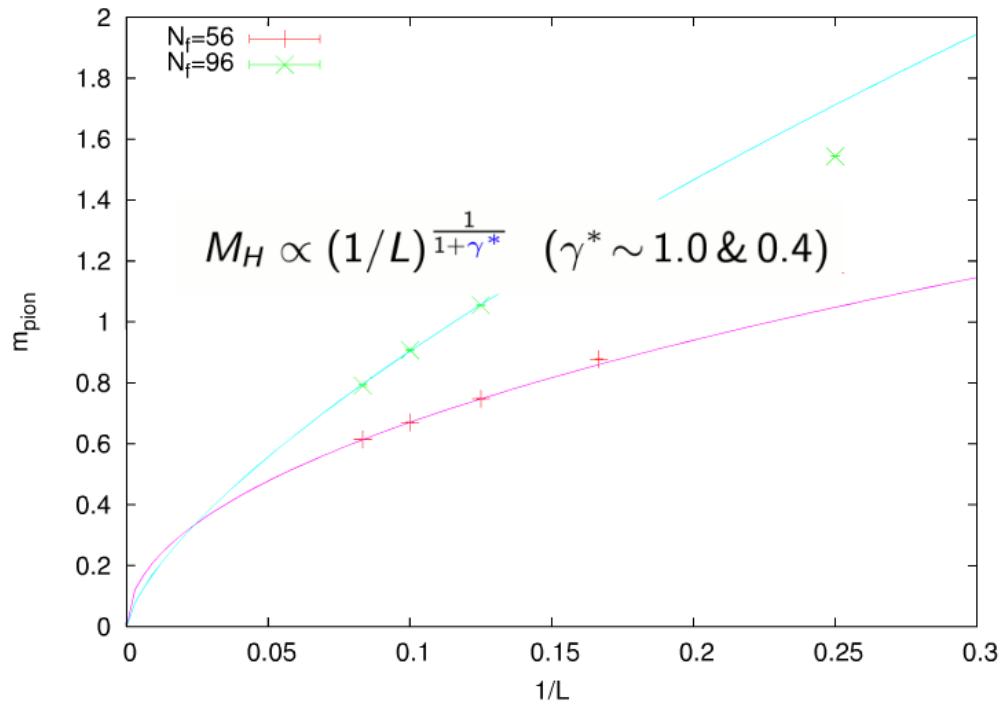
Hadron Masses



hadron spectrum obtained for $N_f=56$ and $N_f=96$ at zero quark mass:

- hadron masses in chiral limit are nonzero
- but masses decrease a lot as L is increased

- parity partners degenerate
- mass ratios should become independent of L
- L -dependence governed by mass anomalous dimension

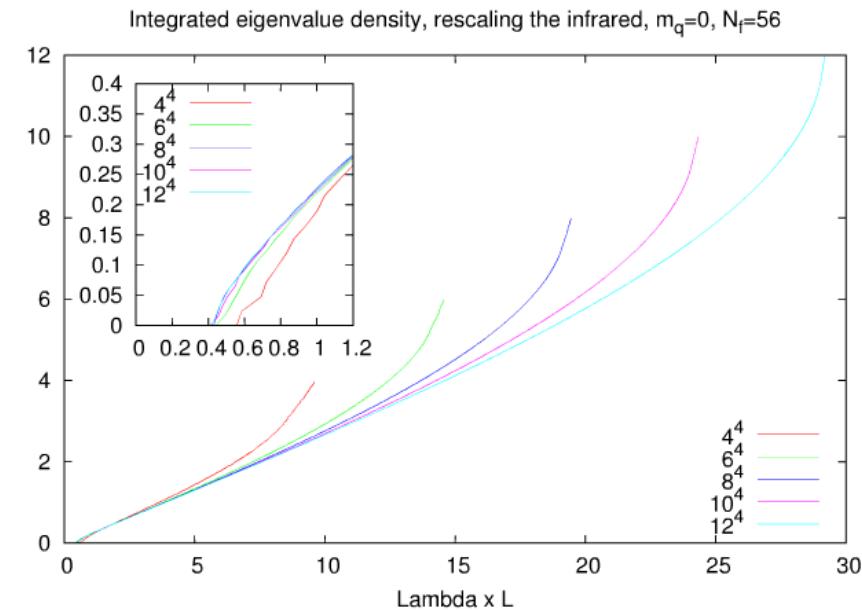
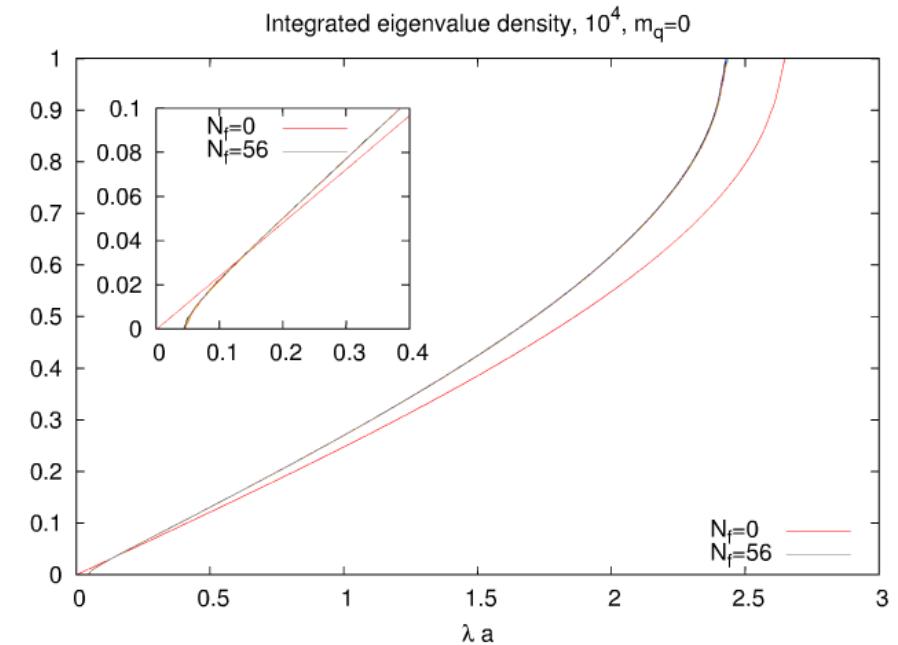


Dirac Eigenvalue Spectrum

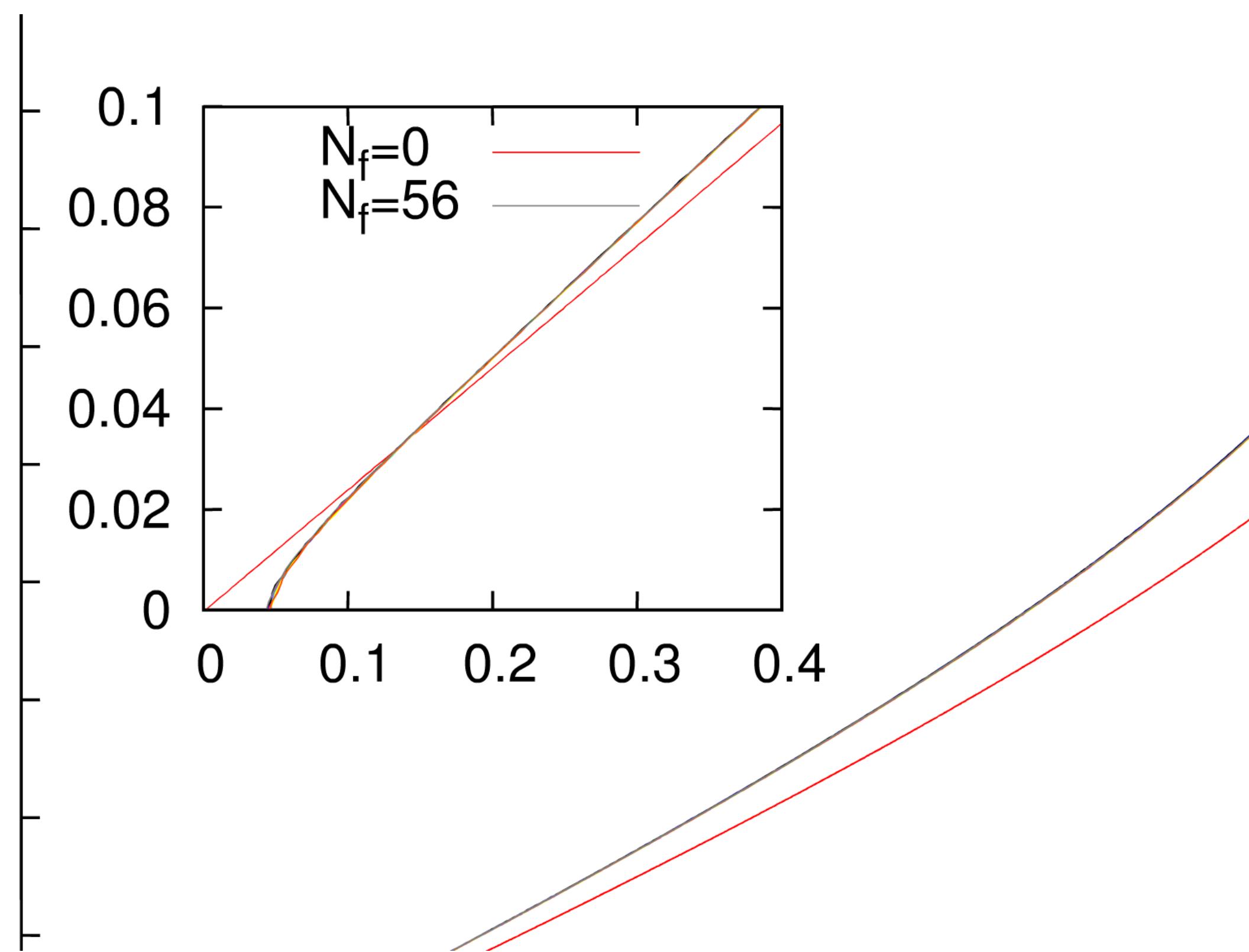
Integrated eigenvalue density:

$$\int_0^\lambda \rho(\bar{\lambda}) d\bar{\lambda} = \frac{\text{rank}(\lambda)}{\text{rank(Dirac matrix)}} \in [0, 1]$$

- measures fraction of EVs smaller than λ
- derivative gives $\rho(\lambda)$ measured for $N_f=0$ (quenched) and $N_f=56$ (chirally symmetric phase) at zero quark mass
- $N_f=56$ shows spectral gap, consistent with chiral symmetry restoration, $\rho(0)=0$
- IR-spectrum invariant after rescaling: spectral gap $\sim 1/L$
- IR physics only depends on L ,
- UV physics on a



→ Dirac Spectrum consistent with IR-conformal theory

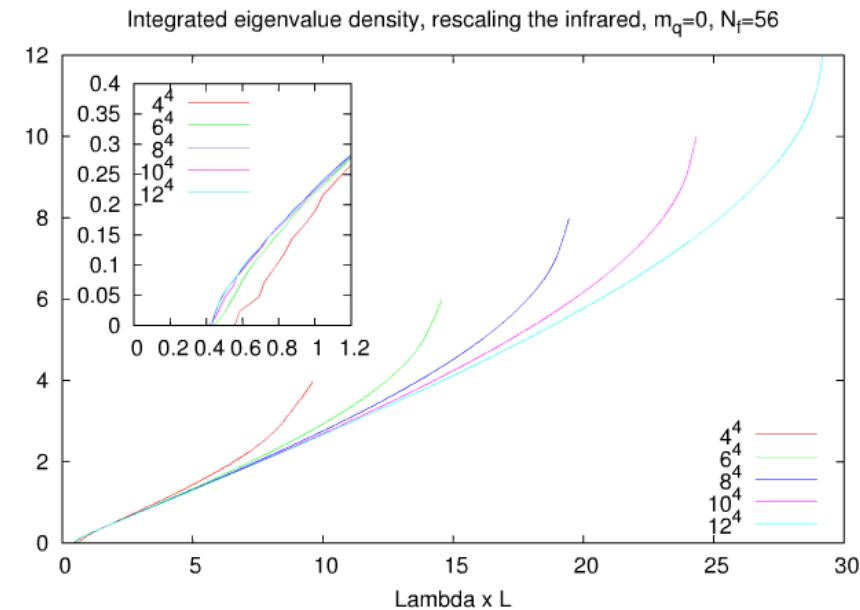
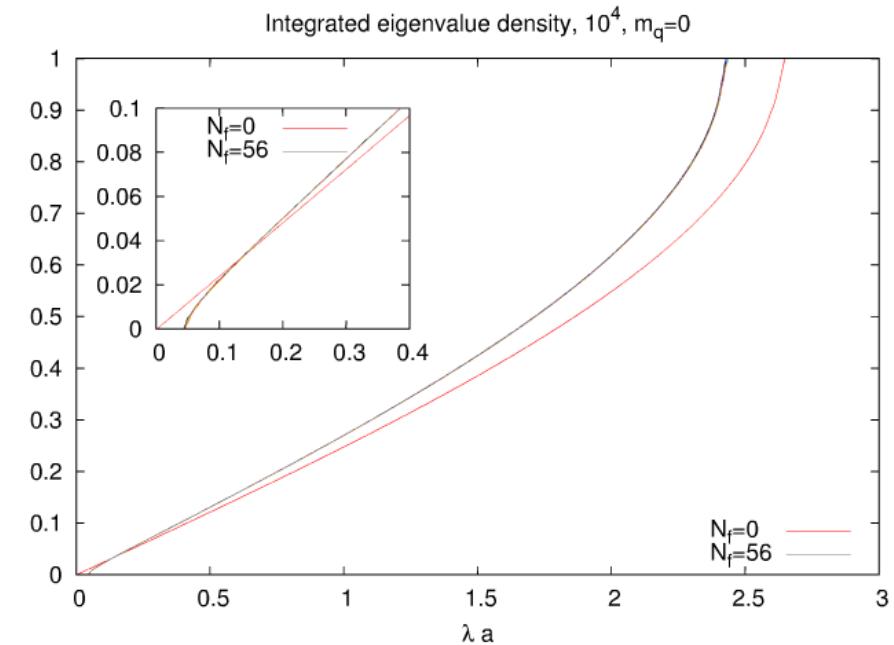


Dirac Eigenvalue Spectrum

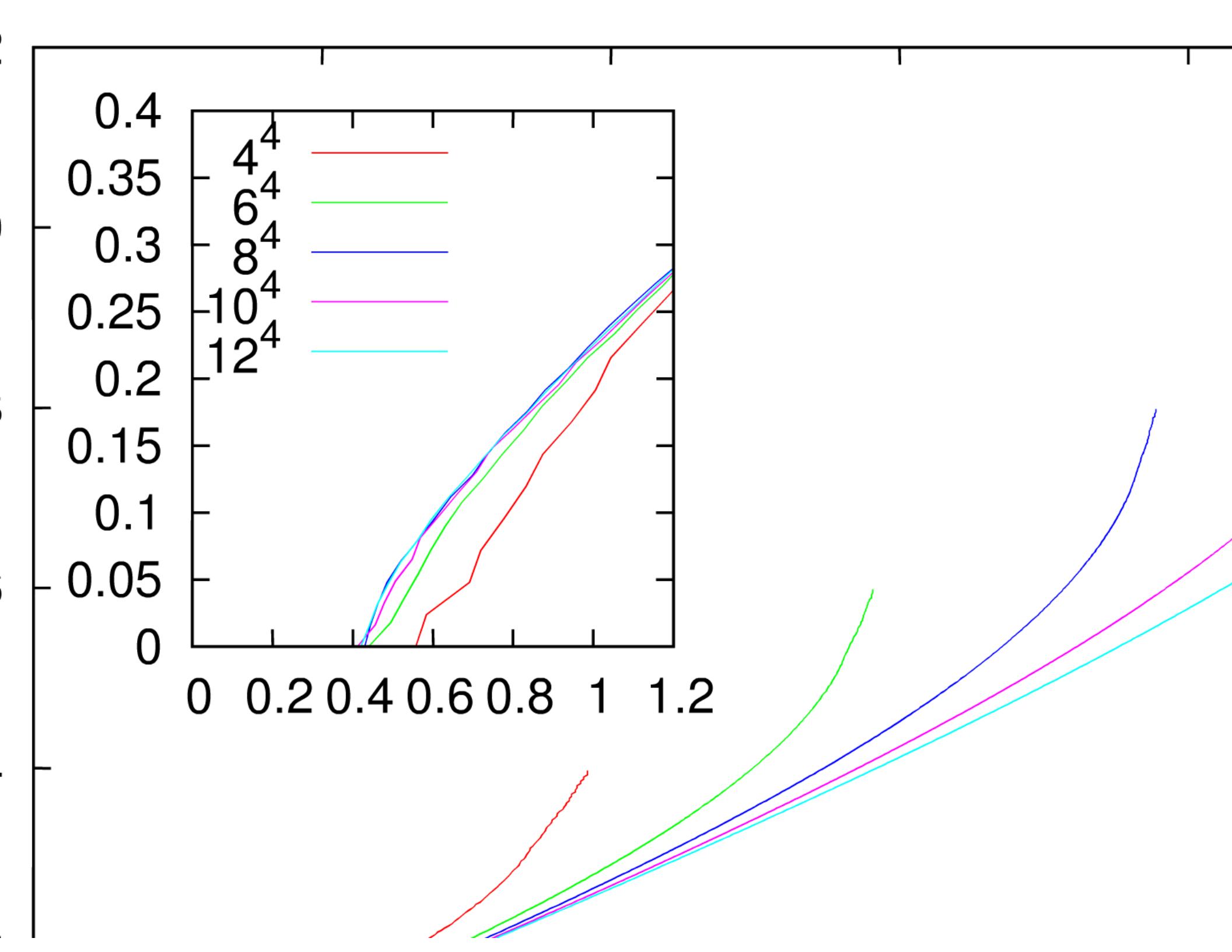
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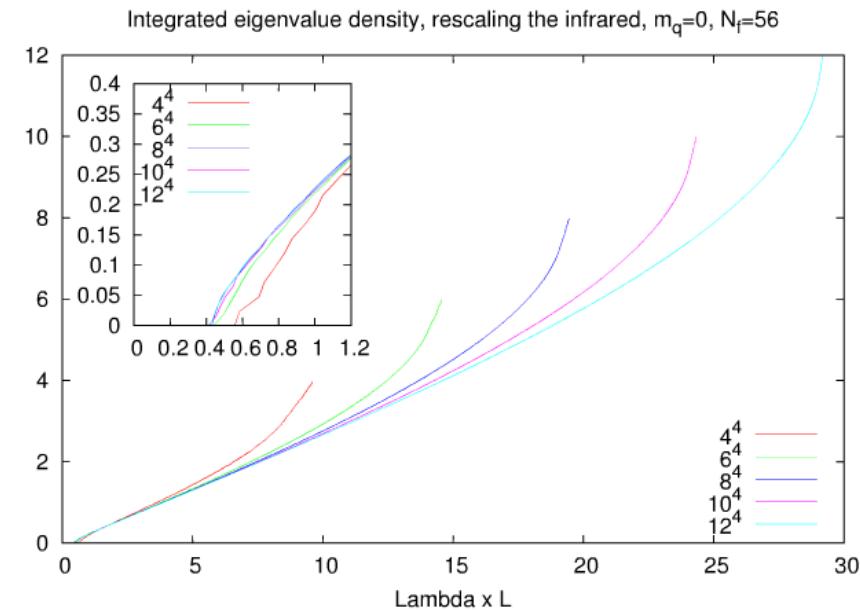
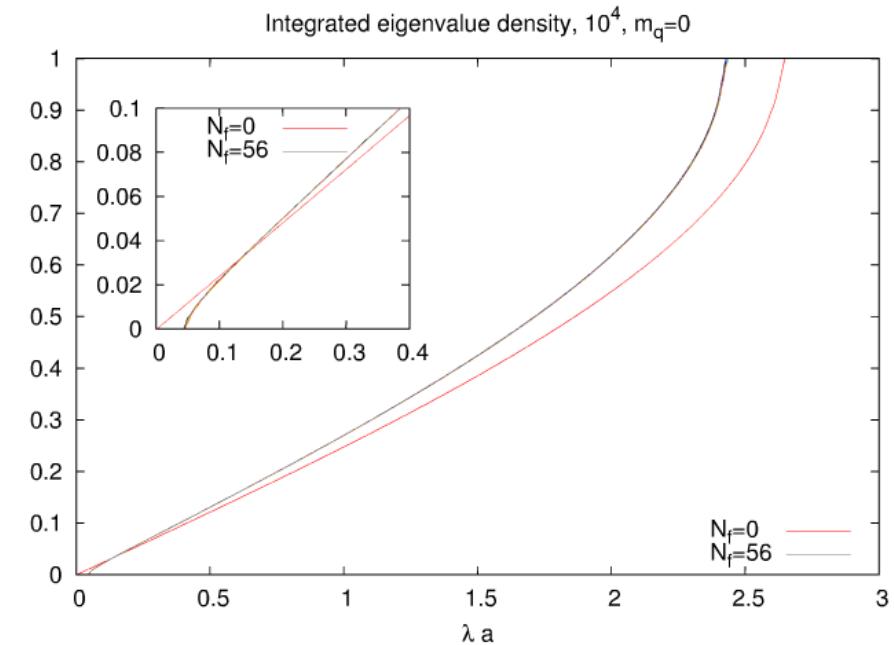


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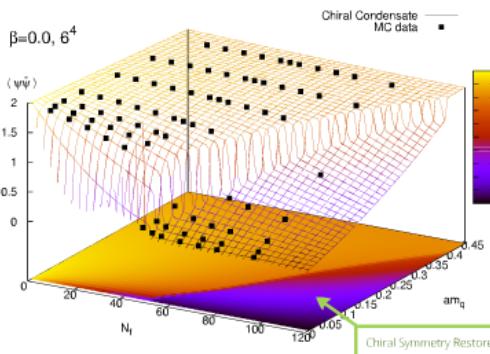
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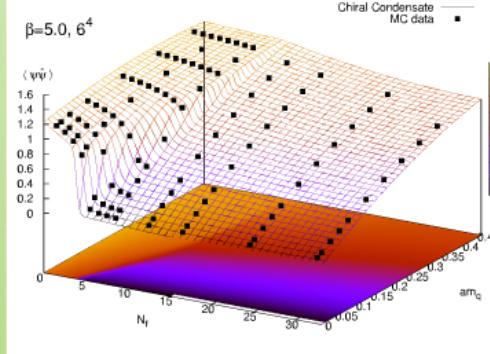
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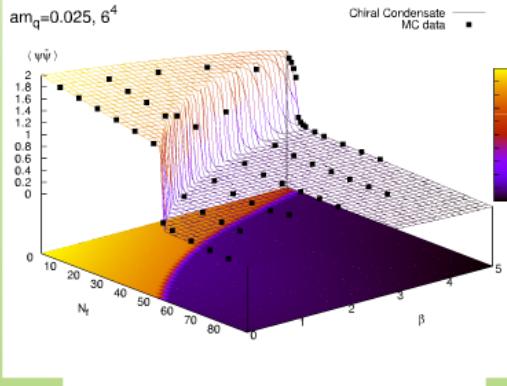
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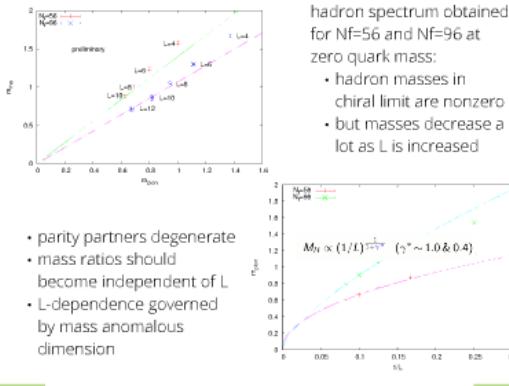
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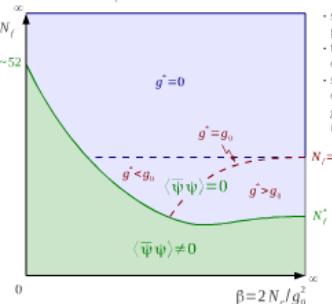
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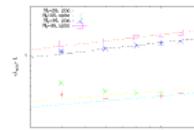
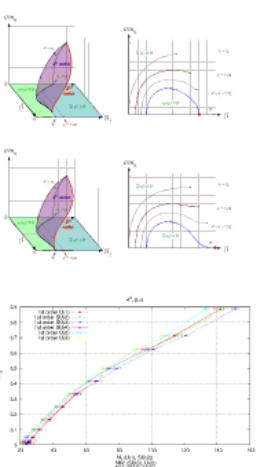
The Conjectured Phase Diagram

The strong coupling IR-conformal phase is analytically connected with the weak coupling, continuum IR-conformal phase

$am_q=0, T=0, SU(3)$



- strong coupling chirally restored phase seems to be conformal
- triviality ($g^*=0$) cannot be ruled out, but is not favored by our data
- strong coupling limit is laboratory of choice to study IR-conformal gauge theory: range of conformal invariance is maximized



Dirac Eigenvalue Spectrum

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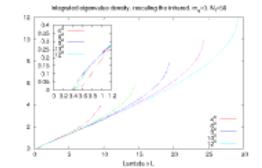
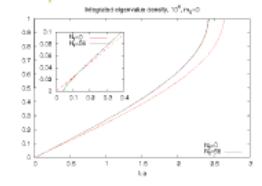
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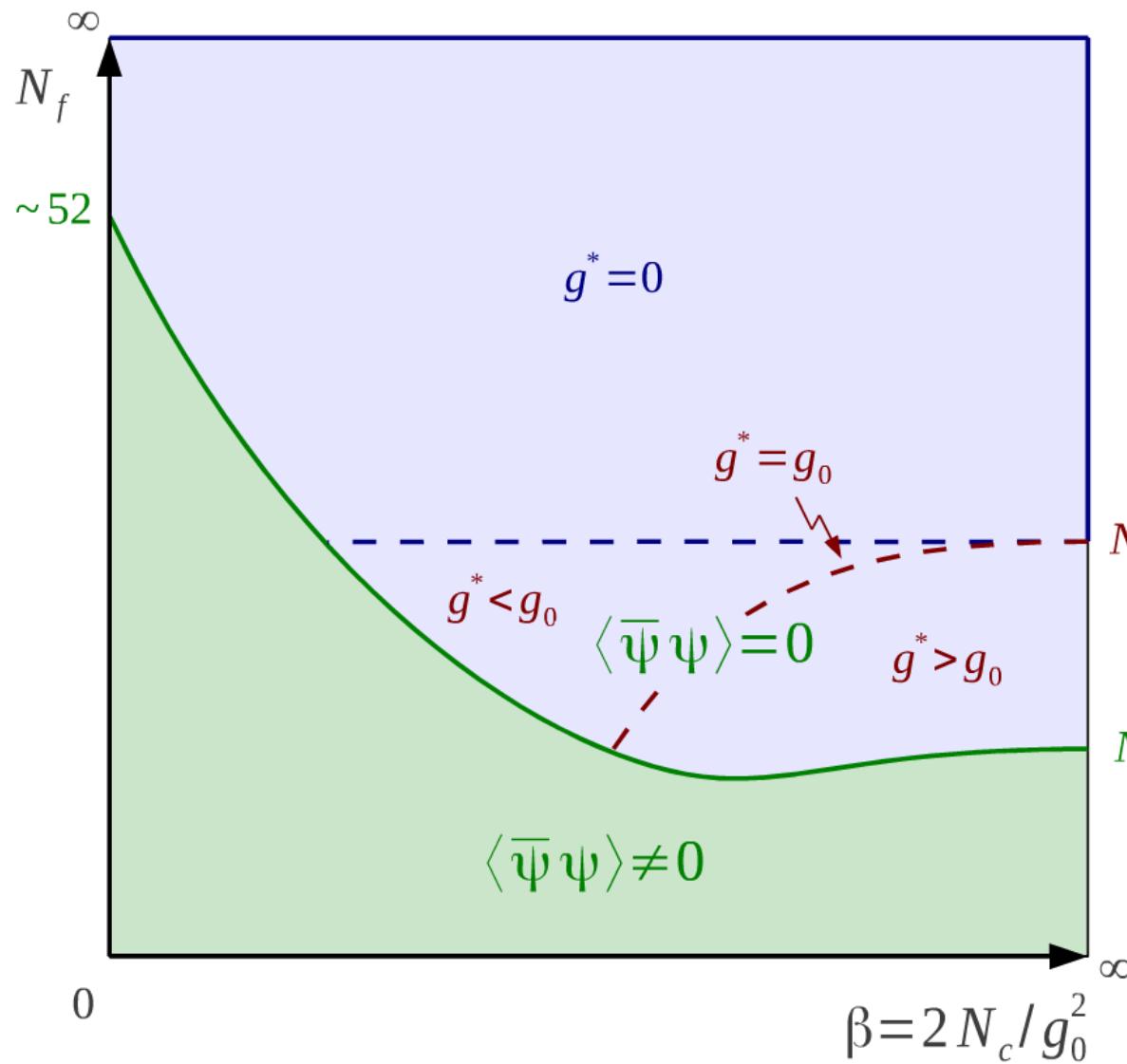
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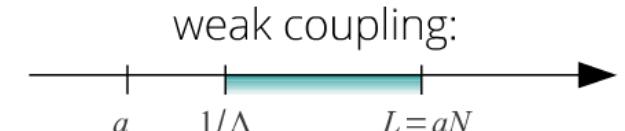
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$$N_f = 11N_c/2$$



of choice to study IR-conformal gauge theory: range of conformal invariance is maximized:

$$N_f = 11 N_c / 2$$

$$g^* > g_0$$

$$N_f^*$$

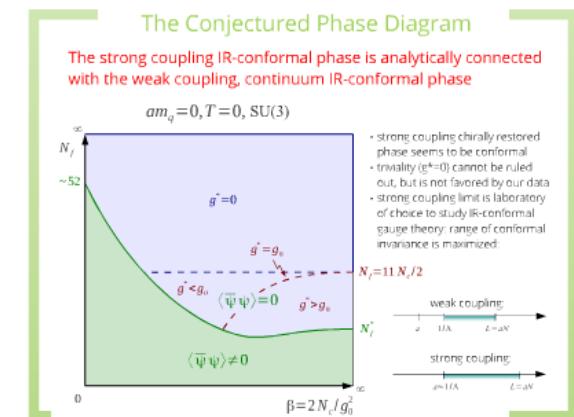
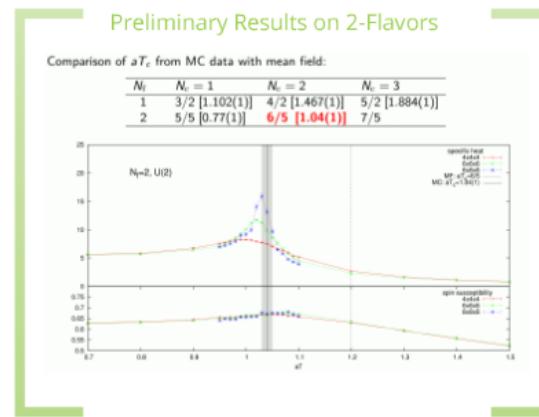
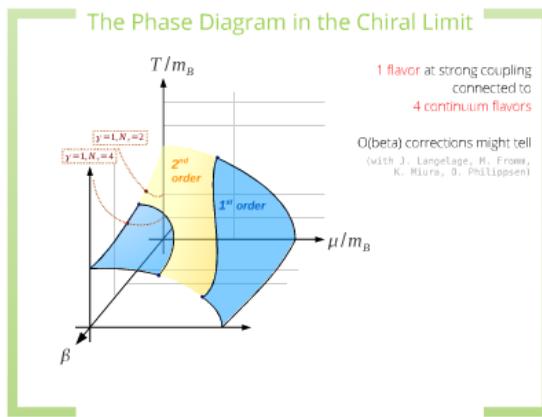


strong coupling:



$$-2 M / \sigma^2$$

Summary



Shown:

- Continuous Time framework improves on mu-T phase diagram
- re-entrance vanishes

Prospects:

- O(beta) corrections may help to understand the connection to the (4 flavor) continuum phase diagram

Shown:

- Continuous Time partition function can be mapped on a **quantum spin system**, Hamiltonian allows for **Quantum Monte Carlo**

Prospects:

- 2-Flavor Phase Diagram feasible
- maybe: nuclear potential incorporating pion exchange

Shown:

- a **strong first order bulk transition** exists
- finding in contrast to meanfield theory
- chirally restored phase extends to weaker coupling

Argued:

- strong coupling chirally restored phase appears to be **IR-conformal**
- this phase seems to have a **non-trivial IRFP**

Strong Coupling QCD remains exciting both from algorithmic and physics point of view