From Tensor Integral to IBP

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in collaboration with

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11. Sep. 2012

Contents

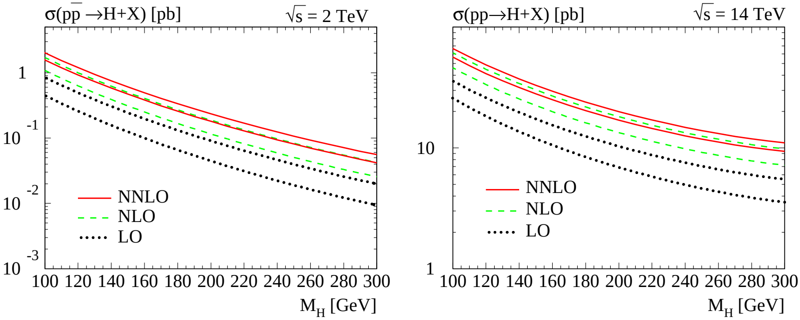
Multi loop calculation

Multi loop calculation

- *More precision in calculated results*

More precision

Ex.: Total cross section for Higgs production in gluon fusion



*[**R. Harlander, W. Kilgore Nov. '02**]*

- *Perturbative convergence* $LO \to NLO(\approx 70\%)$ *and* $NLO \to NNLO(\approx 30\%)$

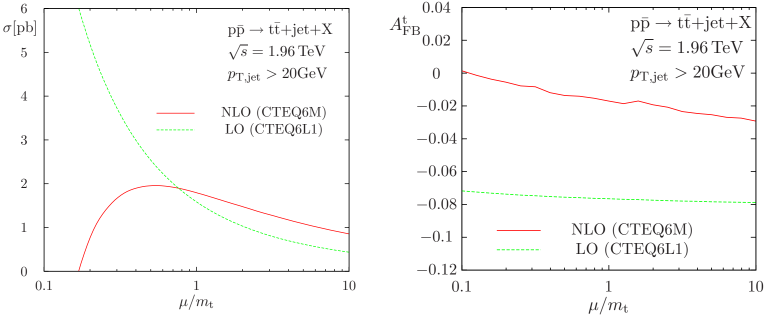
Multi loop calculation

Multi loop calculation

- *More precision in calculated results*
- *New effects*

New effects

Ex.: forward–backward charge asymmetry of the top quark



[S. Dittmaier, P. Uwer , S. Weinzierl Apr. '08]

Multi loop calculation

- *More precision in calculated results*
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Exact NLO or NNLO calculations of σ_{hard} *needed because of:*

Multi loop calculation

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Accurate and reliable predictions of parton–level observables.

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Backgrounds for New Physics Searches

NNLO

When do we need NNLO?

- *When NLO corrections are large*

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For instance ***single top quark*** *production :*

NNLO Single Top Quark Production

Single top quark production

Process	$\sqrt{S}$	$\sigma_{LO}(pb)$	$\sigma_{NLO}(pb)$
t–channel	2.0 *TeV* $p\bar{p}$	1.068	1.062
	14.0 *TeV* pp	152.7	155.9

[**B.Harris, E. Laenen, L.Phaf, Z. Sullivan, S. Weinzierl '02**]

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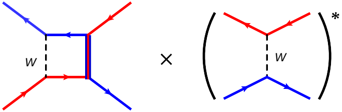
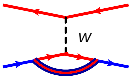


$\propto \; tr[T_a]\; tr[T_a] = 0$

Only vertex corrections contribute:



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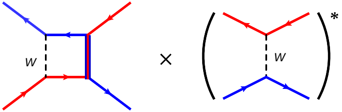

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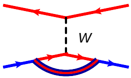

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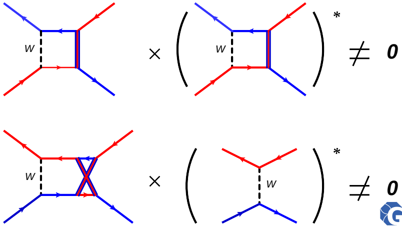

- *The true uncertainty due to missing higher order correction may be greater because new colour exchange diagrams first contribute at NNLO.*
- *A significant effect on kinematical distributions*
- *Important for studies of the V–A structure*
- *Source of polarised top quarks*
- *Access to the b quark pdfs*
- $\dots$

Tensor integral

In the NNLO–corrections occur tensor integrals:

$$\mathcal{I}(d, a_1, \cdots, a_n)[1, k_1^{\mu}, k_2^{\nu}, \cdots] = \int d^d k_1 \int d^d k_2 \frac{\prod_{ij} k_1^{\mu_i} k_2^{\nu_j}}{P_1^{a_1} \cdots P_n^{a_n}}$$

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- *By Schwinger parametrization*

  [O. V. Tarasov, Phys. Rev.'96, Nucl. Phys. '81]

- *By projection method*

  [T. Binoth, E.W.N. Glover, P. Marquard and J.J. van der Bij '02; E.W.N. Glover '04]

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Tensor reduction $\Rightarrow$ *various scalar integrals with the same structure of the integrand with different powers of propagators*

Possible approaches:

Integration by Parts

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[**Chetyrkin, Tkachov '81**]

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[**Chetyrkin, Tkachov '81**]

Reduction techniques:

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AIR	[**Anastasiou, Lazopoulos '04**]
FIRE	[**Smirnov '08**]
Crusher	[**Marquard, Seidel (to be published)**]
REDUZE 1&2	[**Studerus '09; Manteuffel, Studerus '12**]

Tarasov's method

Tensor reduction leads to a very large number of scalar integrals which are shifted in dimension and have other powers of propagators

$$\mathcal{I}(d, a_1, \cdots, a_n)[k_1^{\mu} k_2^{\nu}, \cdots] \rightarrow g^{\mu\nu} \sum_i \mathcal{I}(d + x_i, a_1^i, \cdots, a_n^i)[1]$$

Example for two loop corrections to Axial Vector Form Factors

$$\mathcal{I}(d, 1, 1, 1, 1, 1, 1)[1, k_1^{\mu_1} k_1^{\mu_2} k_2^{\nu_1} k_2^{\nu_2}] \rightarrow \mathcal{I}(2 + d, 2, 1, 1, 1, 1, 2) +$$

$$\cdots + \mathcal{I}(4 + d, 1, 1, 1, 2, 3, 1) + \cdots + \mathcal{I}(8 + d, 3, 3, 3, 2, 1, 2)$$

Tarasov's method

Shift in the dimension

Shift in the dimension

An arbitrary scalar Feynman integral:

$$\mathcal{J}^{(d)}(\{s_i\},\{m_s^2\}) \propto \prod_{j=1}^{N} c_j \int_0^\infty \cdots \int_0^\infty \frac{d\alpha_j \alpha_j^{a_j-1}}{[D(\alpha)]^{\frac{d}{2}}} e^{i\left[\frac{Q(\{s_i\},\alpha)}{D(\alpha)} - \sum_{l=1}^{N} \alpha_l (m_l^2 - i\epsilon)\right]}$$

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$D\left(\frac{\partial}{\partial m_j^2}\right)$ *(polynomial differential* operator*) obtained from* $D(\alpha)$ *by substituting* $\alpha_i \to \partial_j \equiv \partial/\partial m_j^2$*. The application of* $D(\partial_i)$ *to the scalar integral:*

$$\mathcal{J}^{(d-2)}(\{s_i\},\{m_s^2\}) \propto D(\partial_j)\ \mathcal{J}^{(d)}(\{s_i\},\{m_s^2\}),$$

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$$\mathcal{J}^{(d-2)}(\{s_i\},\{m_s^2\}) \propto D(\partial_j)\ \mathcal{J}^{(d)}(\{s_i\},\{m_s^2\}),$$

apply this to master integrals

$$\mathcal{J}_{master}(d-2, a_1, \cdots, a_n) = \sum_i c_i \mathcal{J}(d, a_1^i, \cdots, a_n^i),$$

Tarasov's method

all scalar integrals in rhs. of that equation have to be replaced by master integrals.
i.e.

$$\mathfrak{I}_{master}(d-2, a_1, \cdots, a_n) = \sum_j D_j \mathfrak{I}_{master}(d, a_1^j, \cdots, a_n^j),$$

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$$\mathcal{J}_{master}(d-2, a_1, \cdots, a_n) = \sum_j D_j \mathcal{J}_{master}(d, a_1^j, \cdots, a_n^j),$$

we have for all master integrals:

$$\begin{pmatrix} \mathcal{J}_1^{d-2} \\ \vdots \\ \mathcal{J}_l^{d-2} \end{pmatrix}_{master} = D_{l\,l} \cdot \begin{pmatrix} \mathcal{J}_1^{d} \\ \vdots \\ \mathcal{J}_l^{d} \end{pmatrix}_{master}$$

where l *is the number of master integrals.*

Projection method

By this method we get scalar products between loop momenta and external momenta and no shift in the dimension of integrals

$$\mathcal{I}(d, a_1, \cdots, a_n)_{[1, k_1^{\mu}, k_2^{\mu}, \cdots]} \to g^{\mu\nu} \sum_{ij} \mathcal{I}(d, a_1, \cdots, a_n)_{[1]} k_i p_j$$

Example for two loop corrections to Axial Vector Form Factors

$$\mathcal{I}(d, 1, 1, 1, 1, 1, 1)_{[1, k_1^{\mu_1} k_1^{\mu_2} k_2^{\nu_1} k_2^{\nu_2}]} \to \mathcal{I}(d, -2, 1, 1, 1, 1, 1) + \cdots$$

$$+ \mathcal{I}(d, -1, 0, 1, 1, 1, 1) + \cdots + \mathcal{I}(d, 0, 1, 1, 1, -2, -2)$$

Projection method

The general tensor structure for the amplitude $\mathcal{A}$:

$$\mathcal{A} = \sum_{i=1}^{n} B_i(t, u, s)\mathcal{S}_i,$$

where t, u *and* s *are the Mandelstam variables and* $\mathcal{S}_i$ *are the Dirac structures.*
Projectors for the tensor coefficients:

$$\mathcal{S}_j^\dagger \mathcal{A} = \sum_{i=1}^{n} B_i(t, u, s)\underbrace{\left(\mathcal{S}_j^\dagger \mathcal{S}_i\right)}_{\mathcal{M}_{ji}} \Rightarrow B_i(t, u, s) = \sum_i \mathcal{M}_{ij}^{-1}\left(\mathcal{S}_j^\dagger \mathcal{A}\right)$$

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essential for this method : *to be able to calculate the inverse matrix* $\mathcal{M}_{ij}^{-1}$

Tarasov's method vs. Projection method

Tarasov's method

- *Positive powers for propagators (the sum of the powers of all propagators is large)*
- *Calculate the inverse matrix in order to shift back the dimension*

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- *Negative powers for propagators*
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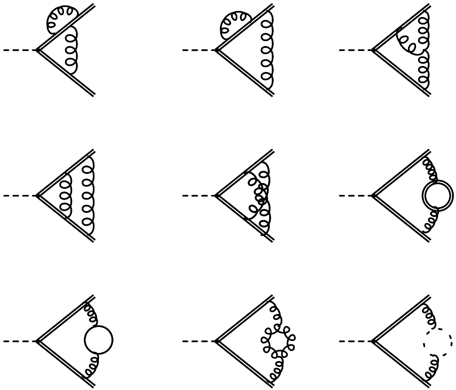
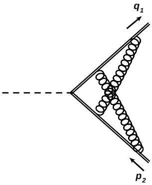
We implemented both methods to calculate the two loop corrections to Heavy Quark Vector and axial Vector Form Factors:

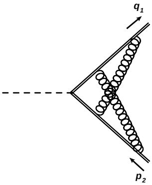


[W. Bernreuther, R. Bonciani, T. Gehrmann, R. Heinesch, T. Leineweber, P. Mastrolia, E. Remiddi '04]

[J. Gluza, A. Mitov, S. Moch, T. Riemann '09]

Two loop corrections to Heavy Quark Form Factors





There are 6 Dirac structures

(Heavy Quark Vector and axial Vector

Form Factors):

$$\mathcal{S}_1 = \bar{u}(q_1)(1+\gamma_5)u(p_2)p_2^{\mu}$$
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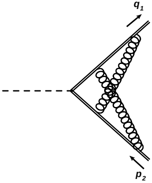

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	Projection	*Tarasov*
number of integrals	*564*	*671*
max sum of powers of propagators	*6*	*14*
max sum of negative powers of propagators	*4*	*0*
reduction time	*7500 s*	*433260 s*

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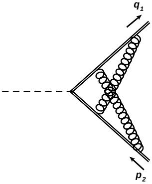

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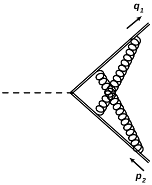

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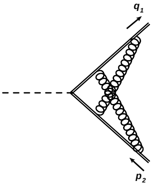
$\Rightarrow$ *projection method is an alternative method for the multi loop calculation!*



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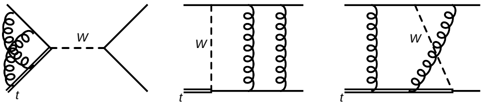
but $\cdots$

There are three topological families:



Vertex corrections **Planar double boxes** **Non Planar double boxes**

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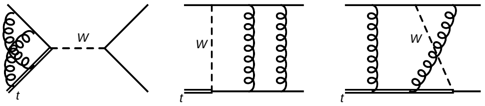

and 11 Dirac structures:

$$
\begin{aligned}
S_1 &= \overline{u}(q_1)\,\gamma_7\; u(p_2)\overline{u}(q_2)\,\gamma_6\gamma_{q_1}\; u(p_1)\\
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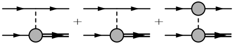
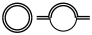
S_4 &= \overline{u}(q_1)\,\gamma_7\gamma_{\mu_1}\gamma_{p_1}\; u(p_2)\overline{u}(q_2)\,\gamma_6\gamma_{\mu_1}\; u(p_1)\\
S_5 &= \overline{u}(q_1)\,\gamma_7\gamma_{\mu_1}\gamma_{\mu_2}\; u(p_2)\overline{u}(q_2)\,\gamma_6\gamma_{\mu_1}\gamma_{\mu_2}\gamma_{q_1}\; u(p_1)\\
S_6 &= \overline{u}(q_1)\,\gamma_6\gamma_{\mu_1}\gamma_{\mu_2}\gamma_{p_1}\; u(p_2)\overline{u}(q_2)\,\gamma_6\gamma_{\mu_1}\gamma_{\mu_2}\gamma_{q_1}\; u(p_1)\\
S_7 &= \overline{u}(q_1)\,\gamma_6\gamma_{\mu_1}\gamma_{\mu_2}\gamma_{\mu_3}\; u(p_2)\overline{u}(q_2)\,\gamma_6\gamma_{\mu_1}\gamma_{\mu_2}\gamma_{\mu_3}\; u(p_1)\\
S_8 &= \overline{u}(q_1)\,\gamma_7\gamma_{\mu_1}\gamma_{\mu_2}\gamma_{\mu_3}\gamma_{p_1}\; u(p_2)\overline{u}(q_2)\,\gamma_6\gamma_{\mu_1}\gamma_{\mu_2}\gamma_{\mu_3}\; u(p_1)\\
S_9 &= \overline{u}(q_1)\,\gamma_7\gamma_{\mu_1}\gamma_{\mu_2}\gamma_{\mu_3}\gamma_{\mu_4}\; u(p_2)\overline{u}(q_2)\,\gamma_6\gamma_{\mu_1}\gamma_{\mu_2}\gamma_{\mu_3}\gamma_{\mu_4}\gamma_{q_1}\; u(p_1)\\
S_{10} &= \overline{u}(q_1)\,\gamma_6\gamma_{\mu_1}\gamma_{\mu_2}\gamma_{\mu_3}\gamma_{\mu_4}\gamma_{p_1}\; u(p_2)\overline{u}(q_2)\,\gamma_6\gamma_{\mu_1}\gamma_{\mu_2}\gamma_{\mu_3}\gamma_{\mu_4}\gamma_{q_1}\; u(p_1)\\
S_{11} &= \overline{u}(q_1)\,\gamma_6\gamma_{\mu_1}\gamma_{\mu_2}\gamma_{\mu_3}\gamma_{\mu_4}\gamma_{\mu_5}\; u(p_2)\overline{u}(q_2)\,\gamma_6\gamma_{\mu_1}\gamma_{\mu_2}\gamma_{\mu_3}\gamma_{\mu_4}\gamma_{\mu_5}\; u(p_1)
\end{aligned}
$$

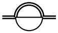
- *Vertex corrections: both methods*

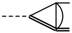
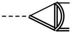
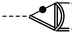
Vertex corrections



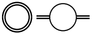
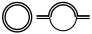
$$= \Big[N_C^2\, T C_F g_s^4 \Big\{ S_1 \Big[\frac{1}{m_t^5} \left(\frac{-8-11t+t^2}{2\epsilon(t-1)^3} - \frac{-71-275t+44t^2}{12(t-1)^3} + \mathcal{O}(\epsilon) \right)$$

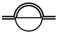
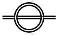
$$- \left(\frac{44t}{3m_t^3(t-1)^3} + \mathcal{O}(\epsilon) \right)$$


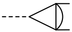
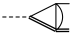
$$- \left(\frac{2n_f}{3m_t^3(t-1)} + \mathcal{O}(\epsilon) \right)$$

$$+ \frac{1}{m_t^3} \left(-\frac{-8-11t+t^2}{3\epsilon(t-1)^3} + \frac{-103-226t+41t^2}{18(t-1)^3} + \mathcal{O}(\epsilon) \right)$$

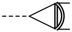
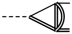

$$+ \left(\frac{2n_f}{3m_t(t-1)} + \mathcal{O}(\epsilon) \right)$$

$$+ \left(\frac{12t}{m_t(t-1)^3} + \mathcal{O}(\epsilon) \right)$$

$$- \left(\frac{2m_t(t+1)}{3(t-1)} + \mathcal{O}(\epsilon) \right)$$

$$\Big]$$

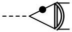
$$+ S_3 \Big[\frac{1}{m_t^4} \left(-\frac{11+12t+13t^2}{24\epsilon(t-1)^2} + \frac{-72+169t-324t^2+303t^3+76t^4-24t^5}{72(t-1)^3 t} + \mathcal{O}(\epsilon) \right)$$


$$+ \frac{1}{m_t^2} \left(\frac{2}{3\epsilon} - \frac{46+28t}{9t} + \mathcal{O}(\epsilon) \right)$$

$$+ \frac{1}{m_t^2} \left(\frac{2t}{3\epsilon(t-1)} + \frac{2t(1+5t-14t^2)}{9(t-1)^3} + \mathcal{O}(\epsilon) \right)$$


$$+ \frac{1}{m_t^2} \left(\frac{n_f(5-19t)}{36(t-1)} - \frac{n_f(t+1)}{6\epsilon(t-1)} + \mathcal{O}(\epsilon) \right)$$

$$+ \frac{1}{m_t^2} \left(\frac{5t^2+12t+19}{36\epsilon(t-1)^2} + \frac{29t^2-86t-87}{72(t-1)^2} + \mathcal{O}(\epsilon) \right)$$


$$+ \left(-n_f/9 + n_f/(3\epsilon) + \mathcal{O}(\epsilon) \right)$$

$$+ \left(\frac{n_f t}{3\epsilon(t-1)} - \frac{n_f(3+t)}{9(t-1)} + \mathcal{O}(\epsilon) \right)$$


$$+ \left(2/t - t/3 + \mathcal{O}(\epsilon) \right)$$

$$+ \left(\frac{(1+3t+3t^2-t^3)}{3(t-1)^2} + \mathcal{O}(\epsilon) \right)$$


$$- \left(\frac{m_t^2(3t^2+19t+28)}{9} + \mathcal{O}(\epsilon) \right)$$

$$- \left(\frac{m_t^2 t(3t^2+4t+1)}{9(t-1)} + \mathcal{O}(\epsilon) \right)$$

$$\Big] \Big\}$$

$$+ N_C^2 C_F^2 \Big\{ \cdots \Big\} + N_C^2 C_F C_A \Big\{ \cdots \Big\} \Big] \frac{1}{m_t^2 \left(t - m_W^2 \right)}$$

Two loop corrections to single Top Quark Production

- *Vertex corrections: both methods* ✓

- *Vertex corrections: both methods* ✓
- *Planar double boxes : projection method*

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problem: build the inverse matrix $\mathcal{M}_{ji} = \mathcal{S}_j^\dagger \mathcal{S}_i$

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- *Planar double boxes : projection method*

problem: build the inverse matrix $\mathcal{M}_{ji} = \mathcal{S}_j^\dagger \mathcal{S}_i$

try with common computer algebra system, e.g. Mathematica or Maple

runtime $\approx$ *1 month with 64GB RAM*

- *Vertex corrections: both methods* ✓
- *Planar double boxes : projection method*

problem: build the inverse matrix $\mathcal{M}_{ji} = \mathcal{S}_j^\dagger \mathcal{S}_i$

try with common computer algebra system, e.g.

Mathematica or Maple

runtime $\approx$ *1 month with 64GB RAM*

- *We calculated* $\mathcal{M}_{ji}^{-1}$ *and all Planar double boxes diagrams* ✓

- *Vertex corrections: both methods* ✓
- *Planar double boxes : projection method*

problem: build the inverse matrix $\mathcal{M}_{ji} = \mathcal{S}_j^\dagger \mathcal{S}_i$

try with common computer algebra system, e.g.

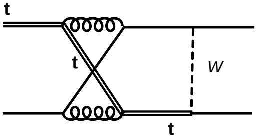
Mathematica or Maple

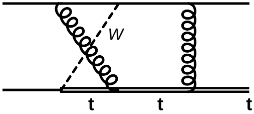
runtime $\approx$ *1 month with 64GB RAM*

- *We calculated* $\mathcal{M}_{ji}^{-1}$ *and all Planar double boxes diagrams* ✓
- *Non Planar double boxes: a challenge !*

Topology	*# Diagrams*	*reduction*	*performed checks*
Vertex corrections	*29*	✓	✓
Planar double boxes	*6*	✓	*work in progress*
Non Planar double boxes	*12*	*work in progress*	–

There are two most complicated topologies, which could not be reduced completely until now :





Conclusions

- *We have seen two possibilities to reduce tensor integrals to scalar integrals*
- *The choice of reductions method determines how difficult the next step (IBP) is*
- *As a test of our setup, we have calculated the* $\mathcal{O}(\alpha_s^2)$ *contributions to the Heavy Quark Vector and Axial Vector Form Factors, confirming the results of* `Bernreuther et al.` *and* `Gluza et al.`
- *We have also calculated the two loop vertex corrections to single Top Quark Production*