Phase diagram and fluctuations using PNJL model with eight-quark interactions

Paramita Deb

University of Calcutta

Plan of talk

- Restoration of chiral symmetry and deconfinement
- Nambu–Jona-Lasinio (*NJL*) Model
- Polyakov-Loop Extended NJL (*PNJL*) Model
- Phase diagram
- Fluctuations
- Conclusion

Restoration of Chiral Symmetry and Deconfin

- Restoration of chiral symmetry and deconfinement at high temperature and density.
- The phase transition associated with the chiral symmetry restoration in the vanishing quark mass limit is commonly referred to as the chiral phase transition.
- In pure gauge sector Polyakov loop serves as an order parameter for the low temperature Z_{N_c} symmetric confined phase to the high temperature deconfined phase, characterised by the spontaneous breaking of Z_{N_c} symmetry.

Polyakov loop

- The Polyakov Loop

$$L(\bar{x}) = \mathcal{P}exp[i\int_0^\beta d\tau A_4(\bar{x},\tau)]$$

$$\Phi = (Tr_c L)/N_c, \qquad \bar{\Phi} = (Tr_c L^\dagger)/N_c$$

- Φ can be rewritten in terms of free energy

$$\Phi = e^{-\beta \Delta F_q(x)}$$

- In the Z_{N_C} symmetric phase, $\Phi = 0$, an infinte amount of free energy is required to add an isolated heavy quark to the system.

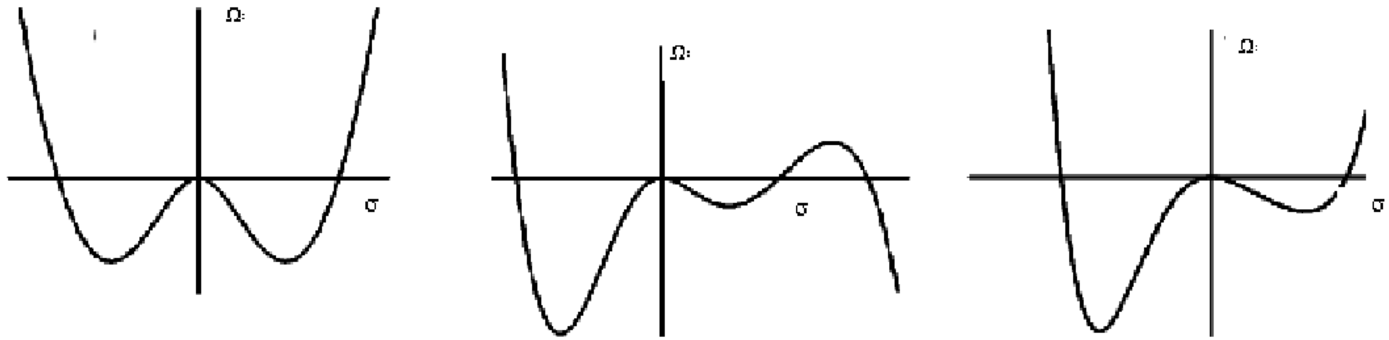
NJL model

NJL model has been widely used to study Chiral Phase Transition in QCD. $SU(3)_f$ version of NJL model described by the Lagrangian,

$$\mathcal{L} = \sum_{f=u,d,s} \bar{\psi}_f \gamma_\mu i\partial^\mu \psi_f - \sum_f m_f \bar{\psi}_f \psi_f$$
$$+ \frac{g_S}{2} \sum_{a=0,\ldots,8} [(\bar{\psi}\lambda^a\psi)^2 + (\bar{\psi}i\gamma_5\lambda^a\psi)^2]$$
$$- g_D[det_{f,f'}(\bar{\psi}_f P_L \psi_{f'}) + det_{f,f'}(\bar{\psi}_f P_R \psi_{f'})]$$
$$= \mathcal{L}_0 + \mathcal{L}_{\mathcal{SB}} + \mathcal{L}_s + \mathcal{L}_{\mathcal{KMT}}$$

- $\mathcal{L}_0$ is the Dirac term with gauge field interactions.
- $\mathcal{L}_{SB} \rightarrow$ mass term which breaks the symmetry explicitly.
- $\mathcal{L}_s$ is the four-fermi interaction term with coupling g_S.
- $\mathcal{L}_{\mathcal{KMT}}$ preserves $SU_L(3) \otimes SU_R(3)$ but breaks $U_A(1)$ symmetry. The determinant is taken over the flavor space. Mimics QCD Anomaly.

NJL model with Eight-quark interaction



- Six-quark interaction term introduces a serious problem of instability of the vacuum. To resolve the problem eight-quark interaction terms are added.

$$\mathcal{L}^{1}_{8q} = 8g_1[(\bar{\psi}_i P_R \psi_m)(\bar{\psi}_m P_L \psi_i]^2$$

$$\mathcal{L}^{2}_{8q} = 16g_2[(\bar{\psi}_i P_R \psi_m)(\bar{\psi}_m P_L \psi_j)(\bar{\psi}_j P_R \psi_k)(\bar{\psi}_k P_L \psi_i)]$$

Osipov *et.al* PLB 634: 48-54, (2006).

The mean field Lagrangian in presence of the eight-quark interaction is modified as

$$\mathcal{L}_{MFA} = \sum_{f=u,d,s} \bar{\psi}_f \gamma_\mu i\partial^\mu \psi_f - \sum_{f=u,d,s} M_f \bar{\psi}_f \psi_f + 2g_S \sum_{f=u,d,s} \sigma_f^2 - \frac{g_D}{2}\sigma_u\sigma_d\sigma_s + 3\frac{g_1}{2} \sum_{f=u,d,s} (\sigma_f^2)^2 + 3g_2 \sum_{f=u,d,s} \sigma_f^4$$

and the gap equation for the constituent masses can be written as

$$M_f = m_f - 2g_S\sigma_f + \frac{g_D}{2}\sigma_{f+1}\sigma_{f+2} - 2g_1\sigma_f(\sigma_u^2 + \sigma_d^2 + \sigma_s^2) - 4g_2\sigma_f^3$$

2+1 PNJL model

- Major drawback of the *NJL* model is the lack of Confinement property as the gluons are integrated out with their only effect being the four-quark interaction terms.
- PNJL model attempts to cature the Confinement property of QCD transition by introducing a background temporal gluon field which is coupled to the quarks by the covarient derivative.
- $SU(3)_f$ version of PNJL model with eight-fermion interaction described by the Lagrangian,

$$
\begin{aligned}
\mathcal{L} = & \sum_{f=u,d,s} \bar{\psi}_f \gamma_\mu i D^\mu \psi_f - \sum_f m_f \bar{\psi}_f \psi_f + \sum_f \mu \gamma_0 \bar{\psi}_f \psi_f \\
& + \frac{g_S}{2} \sum_{a=0,\dots,8} [(\bar{\psi}\lambda^a\psi)^2 + (\bar{\psi} i\gamma_5 \lambda^a \psi)^2] \\
& - g_D [det\,(\bar{\psi}_f P_L \psi_{f'}) + det\,(\bar{\psi}_f P_R \psi_{f'})] \\
& + 8 g_1 [(\bar{\psi}_i P_R \psi_m)(\bar{\psi}_m P_L \psi_i)]^2 \\
& + 16 g_2 [(\bar{\psi}_i P_R \psi_m)(\bar{\psi}_m P_L \psi_j)(\bar{\psi}_j P_R \psi_k)(\bar{\psi}_k P_L \psi_i)] - \mathcal{U}'(\Phi[A], \bar{\Phi}[A], T)
\end{aligned}
$$

where $D_\mu \equiv \partial_\mu - iA_4$

Polyakov loop Potential

The potential $\mathcal{U}$ is written as

$$\frac{\mathcal{U}(\Phi,\bar{\Phi},T)}{T^4} = -\frac{b_2(T)}{2}\bar{\Phi}\Phi - \frac{b_3}{6}(\Phi^3+\bar{\Phi}^3) + \frac{b_4}{4}(\bar{\Phi}\Phi)^2$$

$$b_2(T) = a_0 + a_1(\frac{T_0}{T}) + a_2(\frac{T_0}{T})^2 + a_3(\frac{T_0}{T})^3,$$

b_3 and b_4 being constants.
We thus choose the following set of parameters,

$$a_0 = 6.75, a_1 = -1.95, a_2 = 2.625 a_3 = -7.44, b_3 = 0.75, b_4 = 7.5, T_0 = 190\,\mathrm{MeV}$$

———————C. Ratti, M. A. Thaler and W. Weise, Phys. Rev. D **73**, 014019 (2006)

We modify the potential by incorporating Vandermonde term:

$$\frac{\mathcal{U}'(\Phi[A],\bar{\Phi}[A],T)}{T^4} = \frac{\mathcal{U}(\Phi[A],\bar{\Phi}[A],T)}{T^4} - \kappa \ln[J(\Phi,\bar{\Phi})]$$

$J(\Phi,\bar{\Phi})$ is the Jacobian of transformation from Wilson line L to $(\Phi,\bar{\Phi})$ written as

$$J[\Phi,\bar{\Phi}] = (27/24\pi^2)(1 - 6\Phi\bar{\Phi} + 4(\Phi^3+\bar{\Phi}^3) - 3(\Phi\bar{\Phi})^2$$

$J(\Phi,\bar{\Phi})$ is known as Vandermonde determinant. Ghosh *et.al*, PRD 77, 094024, (2008)

Parameters

- We determine the input parameters of the NJL part of the Lagrangian by reproducing the physical variables. Two sets of input parameters with and without eight-quark interaction are given below.

set	m_u MeV	m_s MeV	Λ MeV	$g_S\Lambda^2$	$g_D\Lambda^5$	$g_1 \times 10^{-21}$ MeV^{-8}	$g_2 \times 10^{-22}$ MeV^{-8}
a	5.5	134.758	631.357	3.664	74.636	0.0	0.0
b	5.5	183.468	637.720	2.914	75.968	2.193	-5.890

- Bhattacharyya *et.al*; Phys. Rev D 82, 014021, (2010).

Set	a	b
$\kappa(\mathrm{MeV})$	0.13	0.06

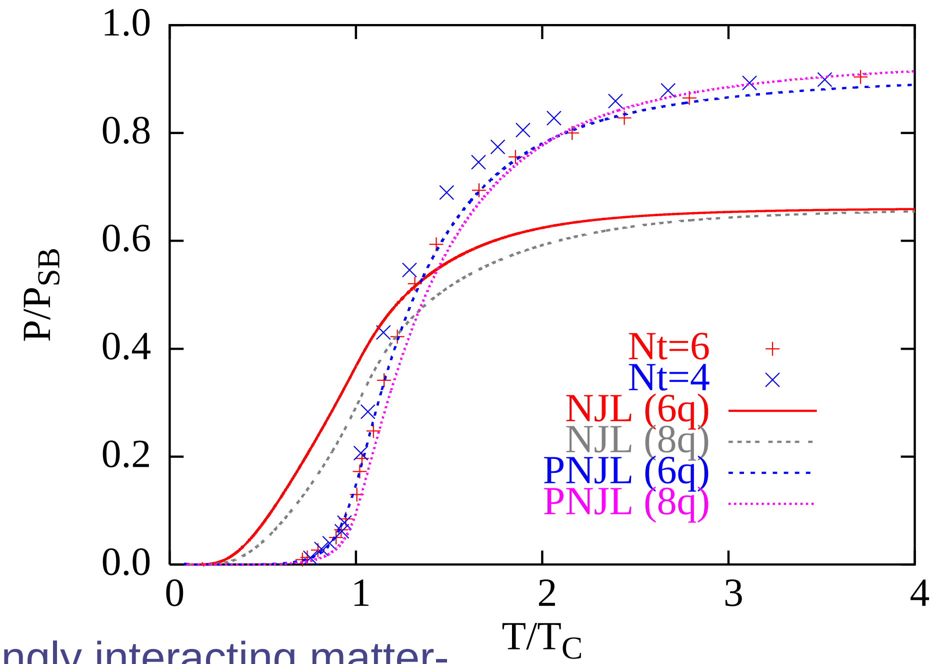
Thermodynamic potential of PNJL model

- *PNJL* model attempts to capture the Confinement property of QCD transition by introducing a background temporal gluon field which is coupled to the quarks by the covarient derivative.
- The thermodynamic potential –

$$
\begin{aligned}
\Omega = & \, \mathcal{U}'[\Phi, \bar{\Phi}, T] + 2g_S \sum_{f=u,d,s} \sigma_f^2 - \frac{g_D}{2} \sigma_u \sigma_d \sigma_s + 3\frac{g_1}{2} (\sum_{f=u,d,s} {\sigma_f}^2)^2 \\
& + 3g_2 \sum_{f=u,d,s} \sigma_f^4 - 6 \sum_{f=u,d,s} \int_0^\Lambda \frac{d^3p}{(2\pi)^3} E_f \Theta(\Lambda - |\vec{p}|) \\
& - 2T \sum_{f=u,d,s} \int_0^\infty \frac{d^3p}{(2\pi)^3} \ln \left[1 + 3(\Phi + \bar{\Phi} e^{-\frac{(E_f - \mu)}{T}}) e^{-\frac{(E_f-\mu)}{T}} + e^{-3\frac{(E_f-\mu)}{T}} \right] \\
& - 2T \sum_{f=u,d,s} \int_0^\infty \frac{d^3p}{(2\pi)^3} \ln \left[1 + 3(\bar{\Phi} + \Phi e^{-\frac{(E_f + \mu)}{T}}) e^{-\frac{(E_f+\mu)}{T}} + e^{-3\frac{(E_f+\mu)}{T}} \right]
\end{aligned}
$$

g_S and g_D are the four-quark and 6q coupling and g_1 and g_2 are the 8q coupling.

Pressure

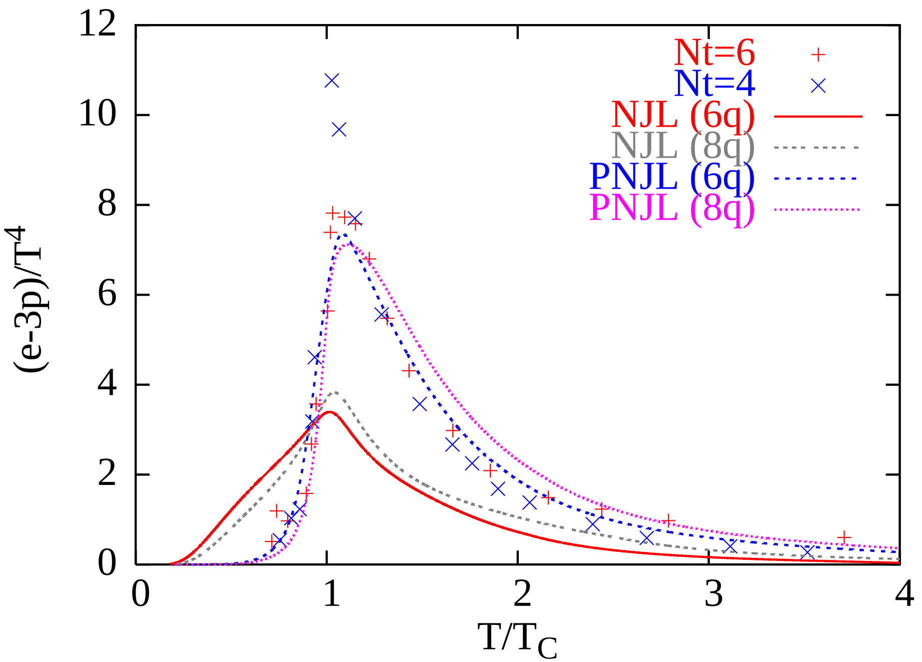


The pressure of the strongly interacting matter-

$$P(T, \mu_q, \mu_Q, \mu_S) = -\Omega(T, \mu_q, \mu_Q, \mu_S),$$

Pressure is very low at low temperature and starts to rise just before T_C. It rises till about $2T_C$ and then saturates. In NJL model pressure is found to be much lower compared to that in PNJL model or lattice. Near the transition region the value of pressure decreases due to introduction of eight-quark interaction.

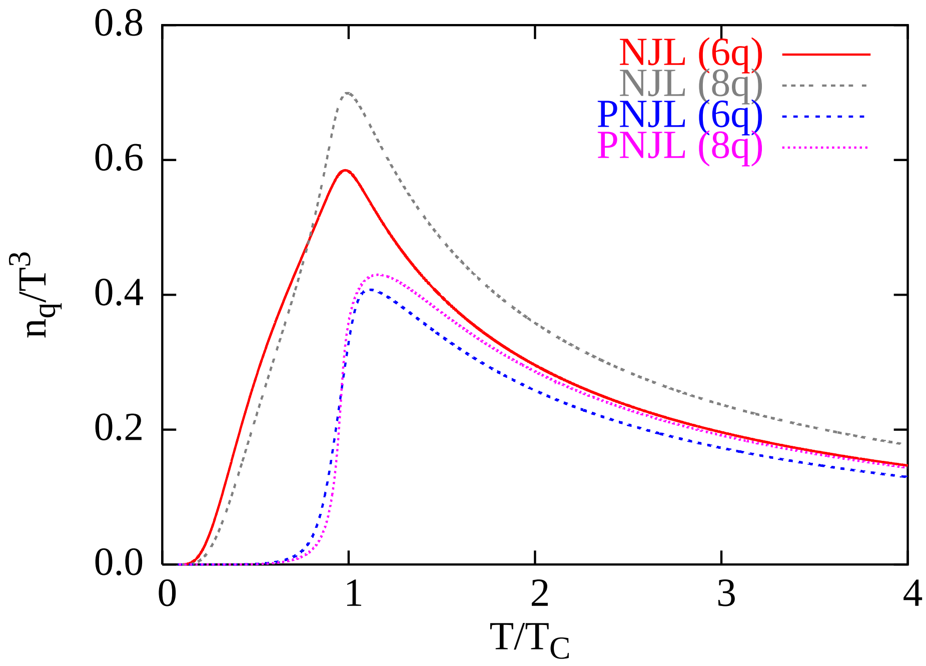
Trace-anomaly



Bhattacharyya *et.al*; M. Cheng *et.al* PRD 77, 014511, (2008).

- In conformally symmetric theory $\theta_{\mu\mu} = \epsilon - 3P = 0$
- $(\epsilon - 3P)$ is the measure of interaction, it signifies how quickly effective degrees of freedom grows with T.
- The plots show a peak due to largest deviation of $(\epsilon - 3P)$ from the conformal limit. Introduction of eight-quark interaction slightly lowers the peak position.
- The NJL model shows the value $(\epsilon - 3P)$ of much smaller magnitude. Thus $(\epsilon - 3p)$ above deconfinement temperature (T_d) is dominated by the Polyakov Loop.

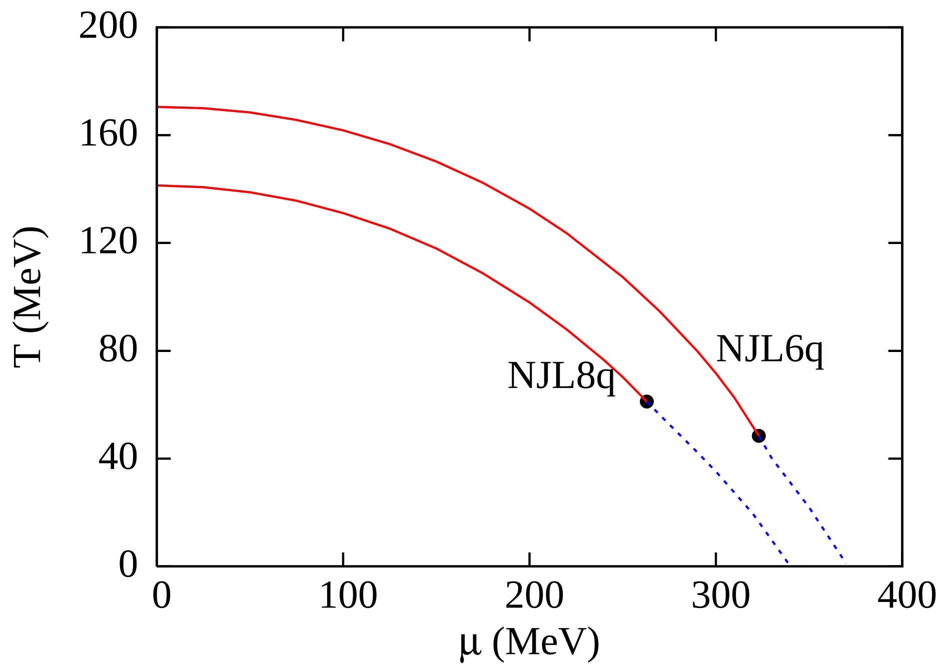
Number Density

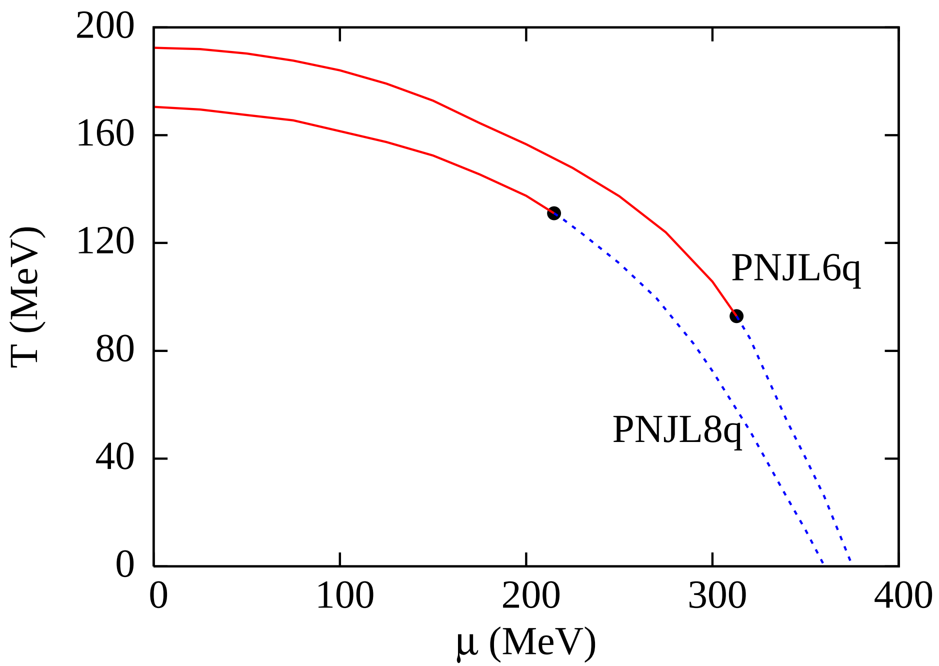


$$n = \partial P/\partial\mu$$

The coupling of quark quasiparticle to the Φ reduces the weight of n_q as thermodynamically active degrees of freedom below T_c. The effect of Confinement is evident.

Phase diagram

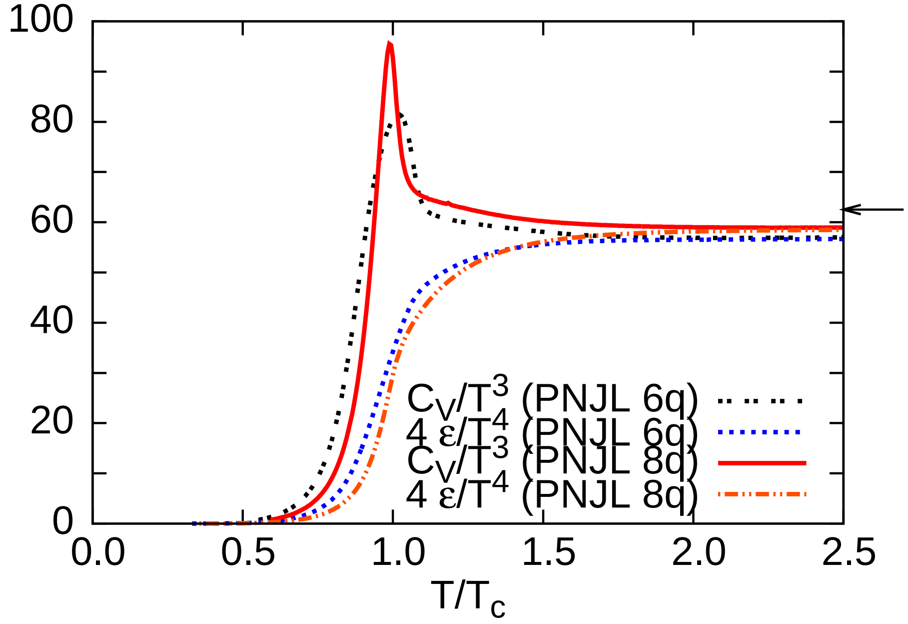




The addition of eight-quark interaction shifts the CEP towards lower μ and higher T value, which is closer to the Lattice result.

ModelIndex	$(\mu_C, T_C)_{(set1)}$MeV
NJL(6q)	$(323, 49)$
NJL(8q)	$(263, 61)$
PNJL(6q)	$(313, 93)$
PNJL(8q)	$(260, 119)$

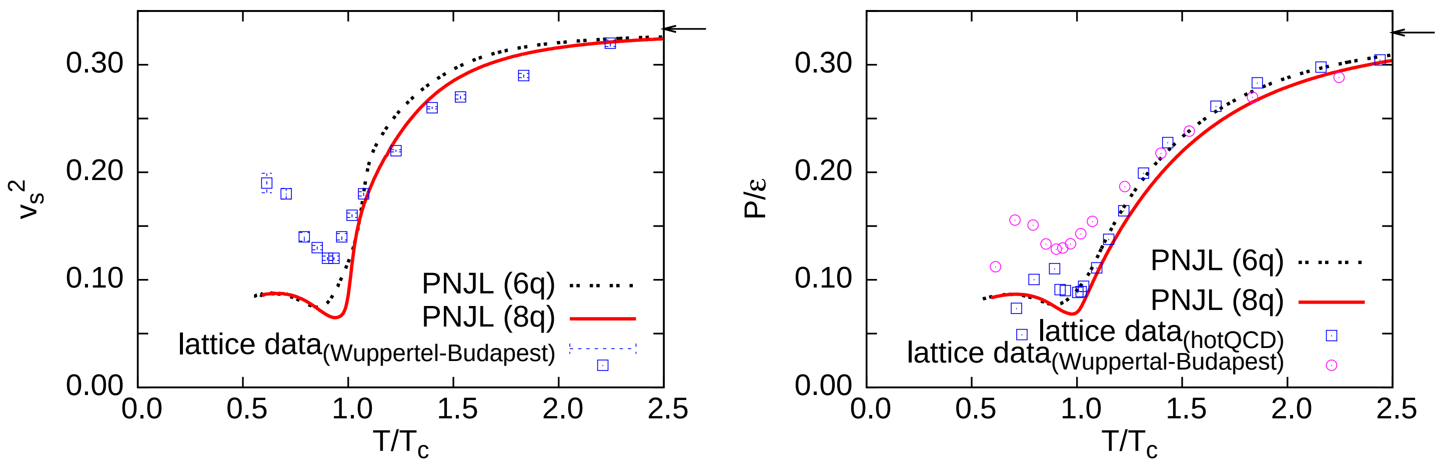
Specific heat



The specific heat is the rate of change of energy density with temperature at constant volume–

$$C_V = \frac{\partial \epsilon}{\partial T}\Big|_V = -T\frac{\partial^2 \Omega}{\partial T^2}\Big|_V .$$

Speed of Sound



- The square of speed of sound at constant entropy S –

$$v_s^2 = \frac{\partial P}{\partial \epsilon}\bigg|_S = \frac{\partial P}{\partial T}\bigg|_V \bigg/ \frac{\partial \epsilon}{\partial T}\bigg|_V = \frac{\partial \Omega}{\partial T}\bigg|_V \bigg/ T\frac{\partial^2 \Omega}{\partial T^2}\bigg|_V \quad . \tag{1}$$

- v_s^2 is related to elliptic flow and the rapidity distribution.
- $\Delta/\epsilon(\Delta = \epsilon - 3P)$ measures the similarity between QCD and the conformal theories.
- The softest point of P/ϵ is 0.07 for 6q, 0.06 for 8q and 0.08 for the Hot-QCD group.
  For Lattice data— M. Cheng *et.al*, PRD 79, 074505, (2009). Bhattacharyya *et.al*; PRD. 82, 114028, (2010).

Fluctuations

- Fluctuations are important characteristics of any physical system. They provide essential information about the effective degrees of freedom and their possible quasi-particle nature.
- Fluctuations are important signatures for investigating the phase transition of QCD.
- Fluctuations can be of different origins like, quantum fluctuations, dynamical fluctuations, measurement induced fluctuations etc.
- If we look into the entire system none of the conserved charges will fluctuate. If we consider the small subsystem, that may exchange the conserved quanta with the rest of the system.
- Flavour chemical potentials μ_u, μ_d, μ_s are related to μ_q, μ_Q, μ_S by,

$$\mu_u = \mu_q + \frac{2}{3}\mu_Q, \quad \mu_d = \mu_q - \frac{1}{3}\mu_Q, \quad \mu_s = \mu_q - \frac{1}{3}\mu_Q - \mu_S$$

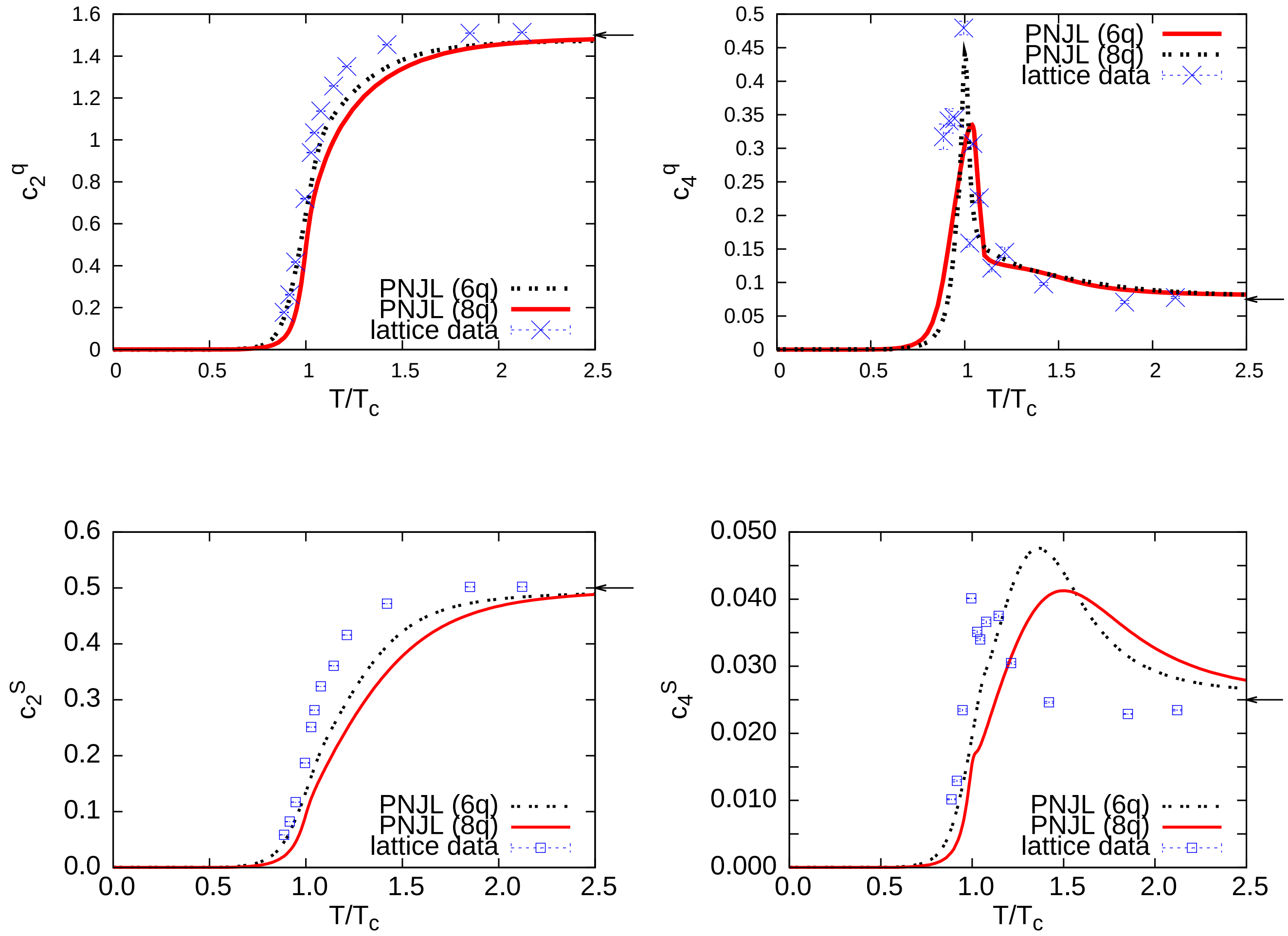
- Expanding the scaled pressure in a Taylor series around the zero chemical potentials μ_q, μ_Q, μ_S as

$$\frac{p(T,\mu_q,\mu_Q,\mu_S)}{T^4} = \sum_{i,j,k} c_{i,j,k}^{q,Q,S} (\frac{\mu_q}{T})^i (\frac{\mu_Q}{T})^j (\frac{\mu_S}{T})^k$$

where,

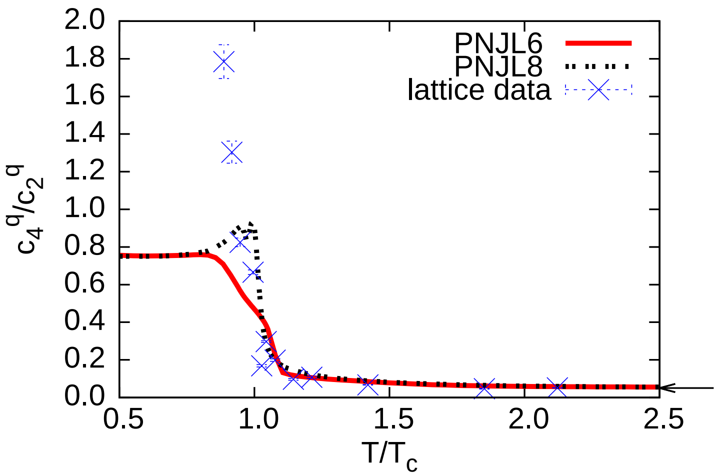
$$c_{i,j,k}^{q,Q,S}(T) = \frac{1}{i!j!k!} \frac{\partial^i}{\partial(\frac{\mu_q}{T})^i} \frac{\partial^j}{\partial(\frac{\mu_Q}{T})^j} \frac{\partial^k (P/T^4)}{\partial(\frac{\mu_S}{T})^k}\Big|_{\mu_{q,Q,S}=0}$$

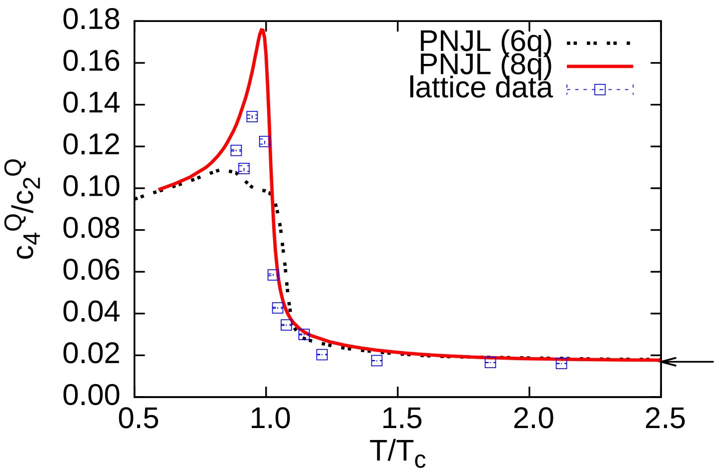
Diagonal Susceptibility (2+1 flavor)



—Bhattacharyya *et.al*; PRD 82, 114028, (2010).

Kurtosis

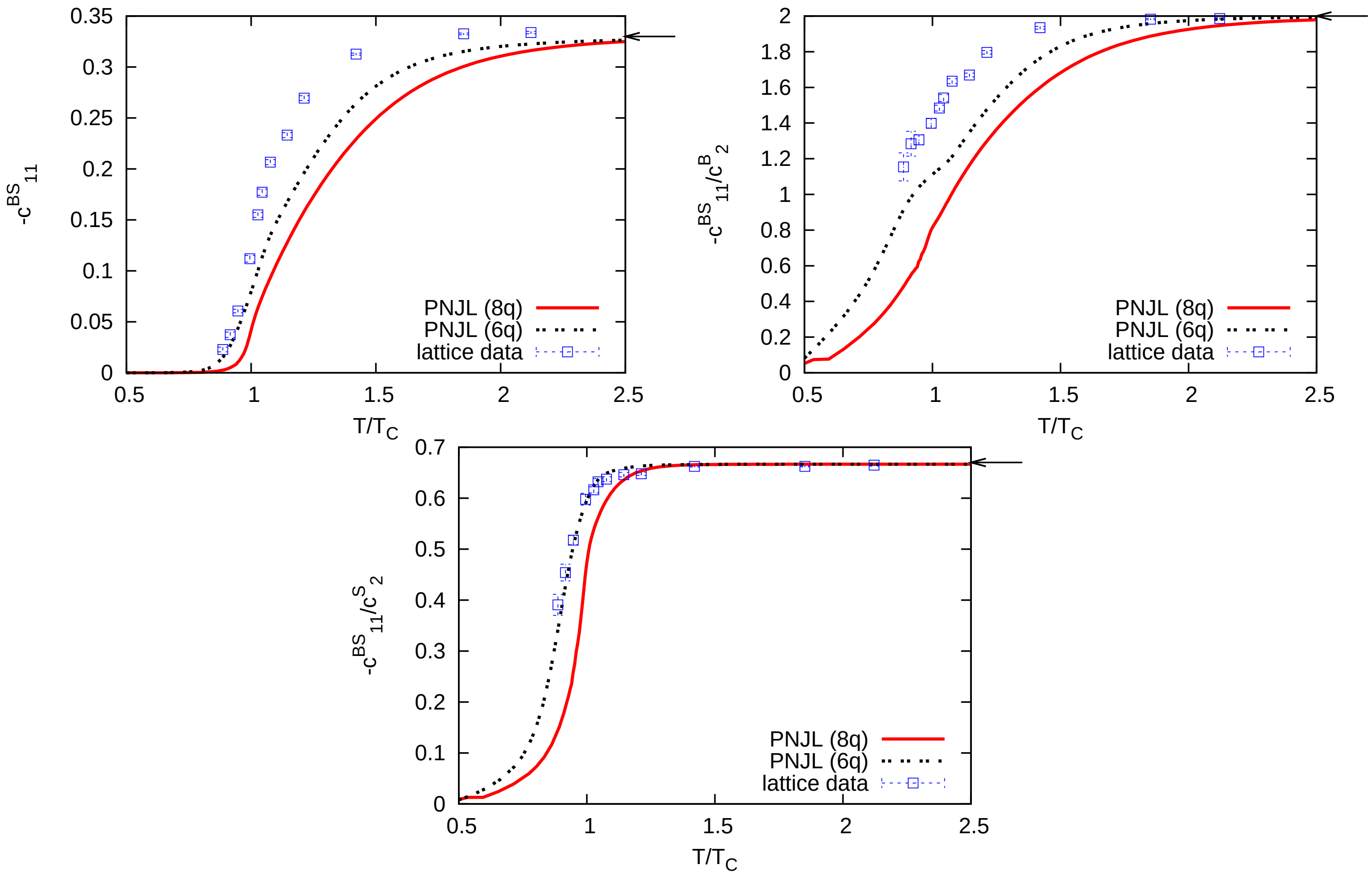




$$\sigma = \sqrt{\langle (N - \langle N \rangle)^2 \rangle} \; ; \kappa = \frac{\langle (N - \langle N \rangle)^4 \rangle}{\sigma^4}$$

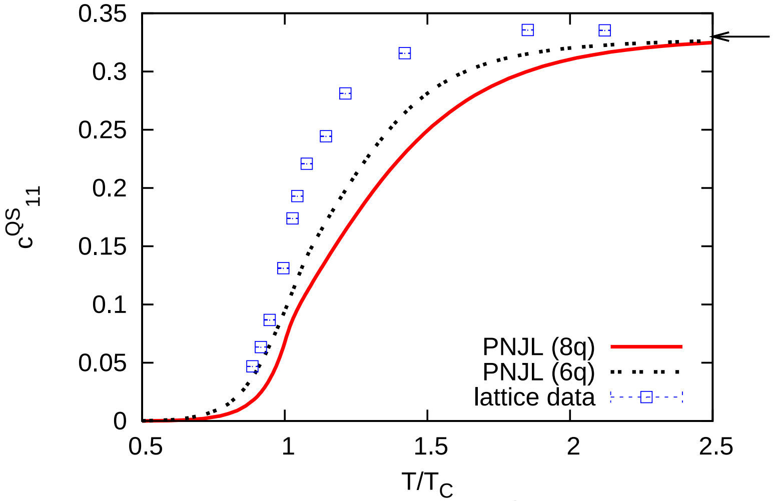
- Kurtosis tell us about the quark content of the system and sensitive probe of deconfinement.
- Kurtosis for baryon number chemical potential in hadron phase to be 1 and in QGP phase to be $\frac{1}{9}$.
- $R_q = (N_C B)^2$ is 9 at low temperature and 1 at high temperature.

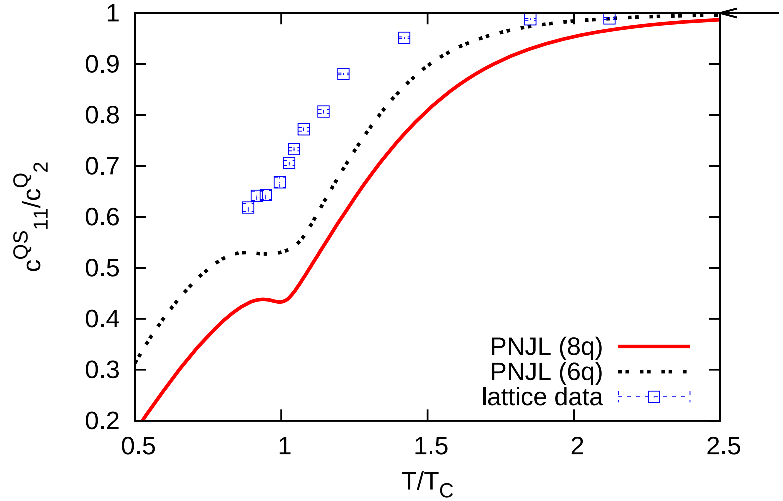
BS correlation

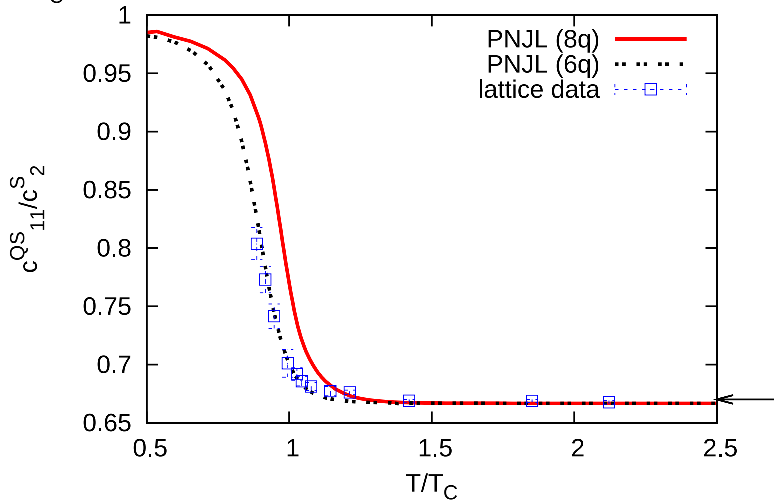


Lattice data taken from Phys. Rev. D79, 074505 (2009). Lightest baryons carry no strangeness ; so $\frac{c_{11}^{BS}}{c_2^B}$ tends to zero at low T. In quark phase baryon number and strangeness are strongly correlated through the strange quark indicating C_{BS} should approach to SB limit.

QS correlation

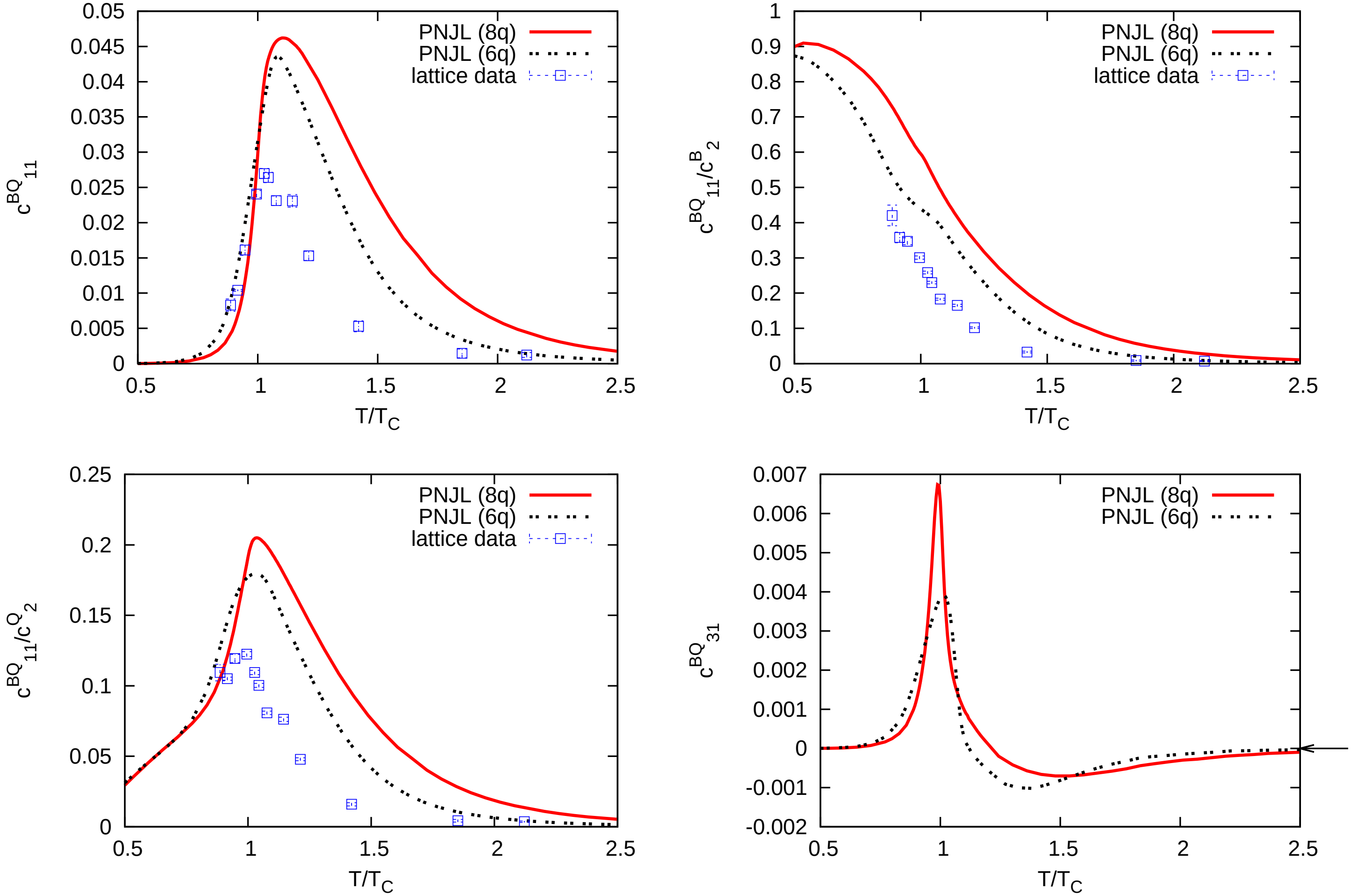






c_{11}^{QS} also shows crossover feature. Charge is strongly correlated to strangeness through strange quark, so its value approaches to SB limit. $\frac{c_{11}^{QS}}{c_2^Q}$ tends to zero at low T because lightest charged particles do not carry strangeness. Plateau indicates hadronic degrees of freedom is dominated

BQ correlation



At low T the contributions from heavy baryons decrease and at high T in the weakly interacting phase, baryon and charge quantum numbers are completely independent of each other. c_{31}^{BQ} is best indicator of crossover.

Concluding Remarks...

- Introduction of eight-quark interaction term stabilizes the vacuum.
- Eight-quark interaction affects the bulk thermodynamic properties at zero density.
- Eight-quark interaction shifts the CEP to the low μ and high T value.
- Fluctuations of different conserved charges show significant behavior near the phase transition temparature.

List of Collaborators

- Abhijit Bhattacharyya
  University of Calcutta
- Saumen Datta
  TIFR
- Sanjoy K. Ghosh
  Rajarshi Ray
  Anirban Lahiri
  Bose Institute

THANK YOU !!