Lowest I=0 scalar meson state in the scalar meson nonet - A case study

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Motivation

Formalism

Result & Discussion

Chiral Phase transition and sigma-mesonic mode

Quantum fluctuation of the order parameter of chiral phase transition

Higgs Particle in QCD → Existence in real world is still unclear

$f_0(600)$ (or σ) → Mass (400 -1200) MeV
Full Width → (600 -1000) MeV

Composition: $q\bar{q}$ state or tetra-quark state?

Scalar meson puzzle

18 scalar meson resonances below 2 GeV → two many resonances

Too many scalar to fit into a multiplet

I = 1	$a_0(980)$	$\bar{u}d, \bar{d}u, \sqrt{1/2}(\bar{u}u - \bar{d}d)$
I = 0	$f_0(600)$	$\sqrt{1/2}(\bar{u}u + \bar{d}d)$
	$f_0(980)$	$\bar{s}s$
I = 1/2	k(800)	$\bar{u}s, \bar{s}u, \bar{d}s, \bar{s}d$

$$I = 1 : m[\rho(776)] \approx 776\ \mathrm{MeV} \quad n\bar{n}$$
$$I = 0 : m[\omega(783)] \approx 783\ \mathrm{MeV} \quad n\bar{n}$$
$$I = 1/2 : m[K^*(892)] \approx 892\ \mathrm{MeV} \quad n\bar{s}$$
$$I = 0 : m[\phi(1020)] \approx 1020\ \mathrm{MeV} \quad s\bar{s}$$

Light scalar meson → Two quark stare or four quark state

Model

Three types of fields:

Two chiral effective nonet fields (M, M') describing the two quark and four quark states.

Spurion field (Y) representing pure ground gluon-bound state.

Note: The four quark field may, most generally, be imagined as some linear combination of a diquark-antidiquark and a “molecule” made of two quark-antiquark “atoms”.

At the symmetry level we are working: we are not interested in the underlying quark structure.

Model (contd ...)

Transformation properties:

M, M' transformed in the same way under $SU(3)_L \times SU(3)_R$

$$M \rightarrow U_L M U_R^{\dagger},$$

However under $U(1)_A$ $\quad M \rightarrow e^{2i\nu} M.$

$$M' \rightarrow e^{-4i\nu} M'$$

M and M' fields can be distinguished from their $U(1)_A$ transformation properties.

Model (contd ...)

Assumptions in our model:

We interpret the spurion field as effective glueball field.
To accommodate realistic glueball field it is widely used practice to introduce a flavor singlet complex field to the linear/non-linear sigma model.

Interaction between the glueball field and the quark fields:
in principle should be determined from the symmetry properties of the QCD. At the pehnomenological level many choices are possible.
Best ---->????
We couple Instanton induced effective Determinant term with the glueball field. This choice is motivated from the observation that instantons are important to determine the properties of the glueball states.

Model (contd ...)

$$\mathcal{L} = [\mathrm{Tr}(\partial_\mu M \, \partial^\mu M^\dagger) + \mathrm{Tr}(\partial_\mu M' \, \partial^\mu M'^\dagger) + \partial_\mu Y \, \partial^\mu Y^*] - m^2_Y$$
$$Y^* Y - m^2_M \mathrm{Tr}(M^\dagger M) - - m^2_{M'} \mathrm{Tr}(M'^\dagger M') - \lambda_1 \mathrm{Tr}(M^\dagger M M^\dagger M) - \lambda_{11} \mathrm{Tr}(M'^\dagger M' M'^\dagger M')$$
$$-\lambda_2 \mathrm{Tr}(M^\dagger M M'^\dagger M') - \lambda_3(\epsilon_{abc}\epsilon^{def} M_d{}^a M_e{}^b M_f'{}^c + h.c.) - \lambda_Y (Y^* Y)^2 + K[Y$$
$$\mathrm{Det}(M) + h.c.] + [\mathrm{Tr}(B.M) + h.c.] + [\mathrm{Tr}(B'.M') + h.c.] + (D.Y + h.c.)$$

9 External Symmetry breaking parameters: $\mathbf{T_a B_a}$

We consider only two dominant one '0' and '8' components.

$$M = T_a(\sigma_a + i\,\pi_a); \qquad M' = T_a(\sigma_a' + i\,\pi_a'); \qquad Y = \sqrt{\frac{1}{2}}\,(y_1 + i\,y_2);$$

Model (contd ...)

1. Since we are treating the M and M' fields on the same footing and not consideringtheir underlying quark structure, we have retained the quartic coupling terms for the M' fields. If we constrained our terms on the basis of the underlying quark lines then this term would correspond to the higher order terms. That is why A.H. Fariborz et. al. have not considered these quartic term. But we are considering this term for the reason stated above.

2. we have not changed the basis for the I=0 fields to the strange and non−strange basis. The reason that, we are interested in the relative 2−quark, 4−quark and glueball components of these mesons. So the above mentioned basis can also provide the same information.

Mixing Scheme

For I = 1/2, 1 states: Two and four quarks states mixed with each other ---> doublet sector

For I = 0 scalar and pseudo-scalar states two, four quarks as well as glueball states mixed with each other ---> 5 states each for scalar and pseudo-scalar.

Particle content in each Sector:

Pseudo-Scalars: { π $\underline{\pi'}$ }, { K K' }, { η_1 η_2 $\underline{\eta_3}$ η_4 }

Scalars: { a a' }, { κ κ' }, { f_1 f_2 f_3 $\underline{f_4}$ }

Solving Algorithm

19 Parameters.

Vacuum stability conditions + Physical Input Mass

Highly non linear in vacuum condensates of each fields ---> difficult to proceed.

Input parameters:

. a) We randomly generate the mixing angles for the scalar 'a' and pesudoscalar pion doublet fields Range $(-\pi/2, \pi/2)$.

b) Randomly generate the vacuum condensates for the 2−quark and 4−quark fields. Range (0.0001, 1.0)

Solving Algorithm

From the mass matrix of pions and 'a' fields we solve all the parameters relevant to doublet sectors.

Mixing in the doublet sector are only between 2 and
4 quark states ---> parameters related to glueball remain undetermined.

However the doublet sector put a constraint on the
product of vacuum scalar condensates of the glueball field and the coupling constant 'k' of the instanton term.

Solving Algorithm

From I = 0 Pseudo-scalar

$$: Tr[M^2_\eta]=Tr[M^2_\eta]_{exp} \text{ and } Det[M^2_\eta]=Det[M^2_\eta]_{exp}$$

Vacuum scalar condensate of the glueball field

instanton coupling constant 'k'

a constraint in the form of sum between m^2_Y and λ_Y'

Result

DIFFERENT POSSIBLE SCENARIOS:

1. The ratio of B8/B0 is the ratio of the strange quark mass to u,d quark mass. In our present calculation we have taken the value of this ratio to be 30. We can change this ratio, for e.g., 25 or 20 to see how the different quantites change.

2. I have taken the mass values of the I=0 Pseudoscalar mesons as : 1.76, 1.475, 1.295, 0.958, 0.547. Keeping the last two lightest mesons same we can use differenet meson mass valuea also.

3. There is a huge uncertainity in the π' mass values from 1.2−1.4 Gev. We change the mass value for π' meson and see how different quantites respond to this scenario.

Result

PREDICTION:

From our model we can predict:

1. In the doublet sector the Kaons and the kapp mass spectrum and mixzing angles

2. I=0 scalar meson mass spectrum and mixing angles

3. The mass spectrum and mixing angles for the I=0 pesudoscalars are also can be considered as a prediction (As we have put only two broad constraints the trace and determinant of the physical particles which is far less than the five mass values and 10 mixing angles involved).

Result

π' mass = 1.35 GeV

Pseudo-scalar Mass [GeV] : 1.68 (1.76), 1.55 (1.475), 1.23 (1.295), 1.07 (0.958), 0.509 (0.547)

Scalar Mass [GeV] : 1.708 (1.71), 1.503 (1.5), 1.29 (1.37), 1.10 (0.98), 0.708 (0.4 - 1.0)

% Composition for the lowest scalar:
Two-quark ~ 80%
Tetra quark: 19.19%
Glueball: 0.01%

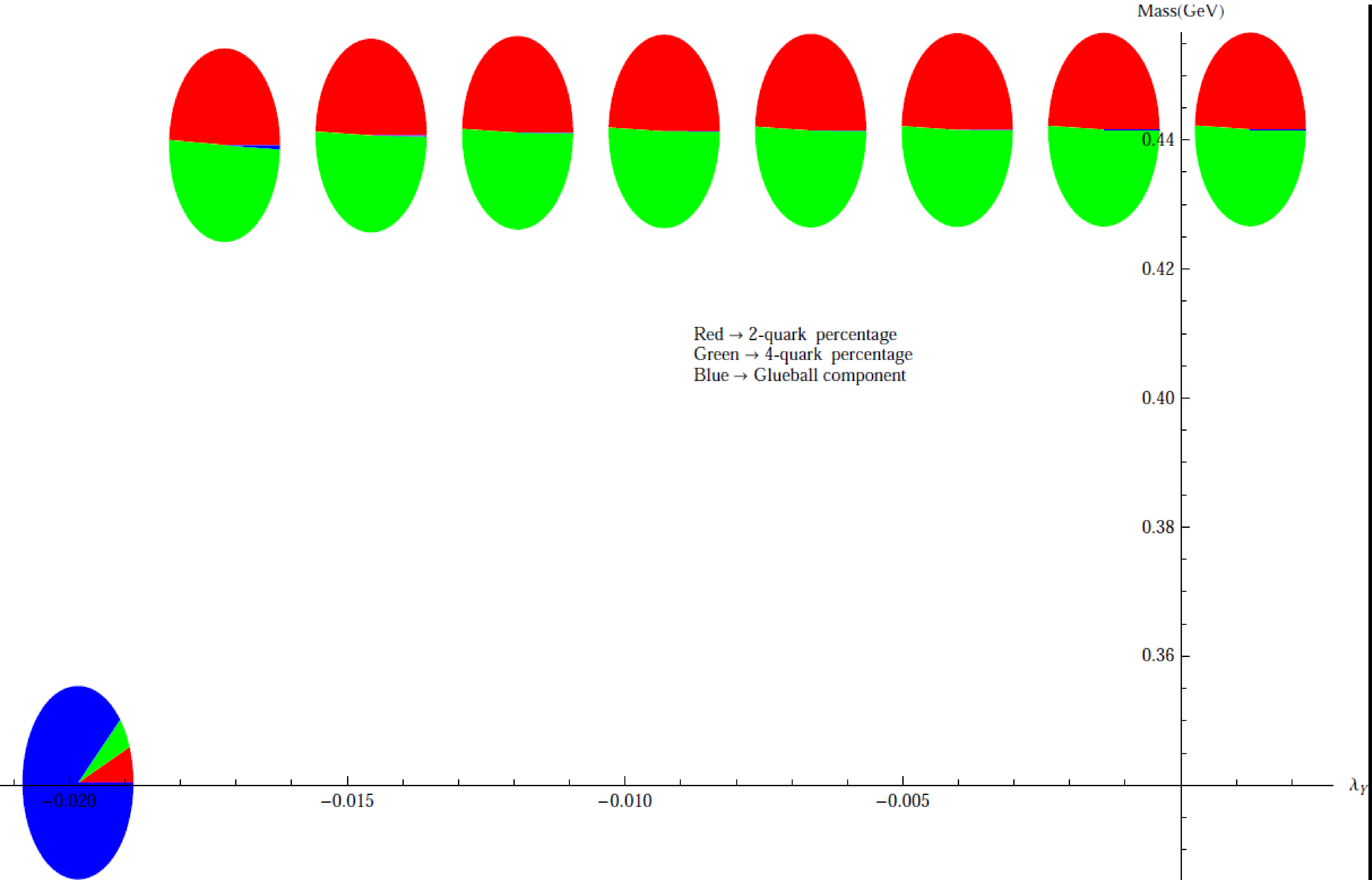
Result

π' mass = 1.3 GeV

Pseudo-scalar Mass [GeV] : 1.79 (1.76), 1.56 (1.475), 1.195 (1.295), 0.829 (0.958), 0.636 (0.547)

Scalar Mass [GeV] : 1.75 (1.71) , 1.68 (1.5), 1.35 (1.37), 1.128 (0.98), 0.393 (0.4-1.0)

Result


Mass(GeV)
0.44
0.42
0.40
0.38
0.36
Red → 2-quark percentage
Green → 4-quark percentage
Blue → Glueball component
−0.020
−0.015
−0.010
−0.005
λ_y

Discussion

After the global fit analysis we will be able to make prediction about
The nature of the lowest I = 0 scalar meson.

Nature of other members in I =0 scalar mesons

Understanding of the vacuum phenomenology → medium behaviors

Implication for chiral symmetry restoration