

Debye screening mass of hot Yang-Mills theory to three loop order

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Outline

Motivation

Matching computations

Master sum-integrals

Results

Conclusion

Debye screening mass

- It parametrizes the dynamically generated screening of chromo-electric fields in hot QCD.
- Here defined as a matching coefficient of electrostatic QCD.
- The result contributes to the coefficient of $\mathcal{O}(g^7)$ of the pressure in hot QCD.
- During computation we developed new methods for computing (scalar as well as tensor) 3-loop sum-integrals.

Electrostatic QCD

- Thermodynamical equilibrium \Rightarrow static fields $\tau = 0 \Rightarrow d = 3$.
- Write down the most general Lagrangian that resembles all the symmetries of the original \mathcal{L}_{QCD} [Ginsparg 1980, Appelquist, Pisarski, 1981].

$$\mathcal{L}_{\text{EQCD}}^{3d} = \frac{1}{4} F_{ij}^a F_{ij}^a + \text{Tr}[D_i, A_0]^2 + m_E^2 \text{Tr}[A_0^2] + \lambda_E^{(1)} (\text{Tr}[A_0^2])^2 + \dots$$

$$D_i = \partial_i - i g_E A_i, \quad i, j = 1, 2, 3$$

- It is a super-renormalizable, universal theory.
- Describes dynamics at intermediate scale $p \leq 2\pi T$.
- Contains 2 scales: $\approx gT, \approx g^2 T$.

m_E to $\mathcal{O}(g^6)$

- Define the Debye mass as the pole of the static gluon propagator.
- on the QCD side:

$$\mathbf{p}^2 + \Pi_{00}(0, \mathbf{p}) \Big|_{\mathbf{p}^2 = -m_{\text{el}}^2} = 0$$

- on the EQCD side:

$$\mathbf{p}^2 + m_{E,\text{ren}}^2 + \delta m_E^2 + \Pi_{\text{EQCD}}(\mathbf{p}^2) \Big|_{\mathbf{p}^2 = -m_{\text{el}}^2} = 0 ,$$

Leading to:

$$m_{E,\text{ren}}^2 = m_{\text{el}}^2 - \delta m_E^2 - \Pi_{\text{EQCD}}(-m_{\text{el}}^2)$$

- m_E requires computation of the gluon self-energy.

Mass renormalization

- Mass parameter renormalization due to known UV divergences from $SU(N_c)$ +Adjoint Higgs Theory ($N_c \equiv C_A$):

$$\delta m_E^2 = \frac{2(N^2 + 1)}{(4\pi)^2} \frac{\mu^{-4\epsilon}}{4\epsilon} (-g_E^2 \lambda^{(1)} C_A + \lambda^{(1)2})$$

- Using g_E and $\lambda^{(1)}$ from matching EQCD to QCD $g_E = gT$ and $\lambda^{(1)} = \frac{20}{3} \frac{N^2}{N^2+1} \frac{g^4 T}{(4\pi)^2}$:

$$\delta m_E^2 = -\frac{10}{3} \frac{g^6 T^2}{(4\pi)^4} \frac{C_A^3}{\epsilon}$$

Gluon self-energy

- **4d QCD:** Perform an expansion both in g and around $\mathbf{p} = 0$ of gluon self-energy:

$$\Pi_{\mu\nu}^{QCD}(\mathbf{p}^2) = \sum_{n=1}^{\infty} \text{n-loop vacuum sum-integrals} \uparrow \Pi_{\mu\nu,n}(0) (g^2)^n + \mathbf{p}^2 \sum_{n=1}^{\infty} \text{n-loop vacuum sum-integrals} \uparrow \Pi'_{\mu\nu,n}(0) (g^2)^n + \dots$$

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- **3d EQCD:** $\Pi^{EQCD}(\mathbf{p}^2) = 0$ due to the absence of any scale in the vacuum integrals.
- This gives ($\Pi_{00} \equiv \Pi$):

$$\begin{aligned} m_{\text{el}}^2 &= g^2 \Pi_1(0) + g^4 \left[\Pi_2(0) - \Pi'_1(0) \Pi_1(0) \right] \\ &\quad + g^6 \left[\Pi_3(0) - \Pi'_1(0) \Pi_2(0) - \Pi'_2(0) \Pi_1(0) \right. \\ &\quad \left. + \Pi''_1(0) \Pi_1(0)^2 + \Pi_1(0) \Pi'_1(0)^2 \right] + \mathcal{O}(g^8). \end{aligned}$$

$\Pi_{\mu\nu}$ to 3-loop order

- Π_{00} enters m_E , Π_{ij} enters g_E .

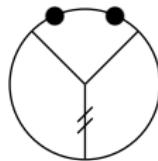
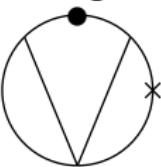
Automatized procedure:

- ~ 500 Feynman diagrams are generated.
- Taylor expansion, Lorentz contraction, color algebra yield $\approx 10^7$ sum-integrals.
- Systematic method for reducing the no. of sum-integrals:
Integration By Parts [Laporta 2000].
 - Result: $\mathcal{O}(10)$ master sum-integrals with rational functions in $d = 3 - 2\epsilon$ as coefficients.
 - **Problem:** All coefficients divergent in the limit $\epsilon \rightarrow 0$.
 - Non-trivial task: Trade divergent coefficients for more complicated sum-integrals via a suitable basis transformation

$$\text{Diagram} = \frac{94 - 48d + 6d^2}{3(d-3)^2(d-4)} \text{Diagram} + \frac{16}{3(d-3)^2} \text{Diagram}$$

Solving master sum-integrals

- Remaining master sum-integrals



- Laurent expansion in dim. reg. ($d = 3 - 2\epsilon$) parameter ϵ :
 $\frac{\#}{\epsilon^2} + \frac{\#}{\epsilon} + \# + \dots$
- No automatized procedure, everything beyond ϵ^0 very difficult to compute due to **discrete** Matsubara modes, \sum_{p_0} .
- Crucial: exploit **1-loop sub-structure** [Arnold, Zhai 1994]:

$$\text{Diagram} = \sum_{p_0=-\infty}^{\infty} \int \frac{d^d \mathbf{p}}{(2\pi)^d} \frac{[\Pi_{bce}(P)]^2}{(P^2)^a}$$

$$\Pi_{bce}(P^2) = \sum_{q_0=-\infty}^{\infty} \int \frac{d^d \mathbf{q}}{(2\pi)^d} \frac{q_0^e}{(Q^2)^b [(P+Q)^2]^c}, \quad P = (p_0, \mathbf{p}).$$

UV and IR divergences

- Subtract 0-temperature and leading UV parts from Π_{bce} to make sum-integrals finite.
- $\Pi^{0\text{-temp}}, \Pi_{UV} \propto \frac{1}{(P^2)^\alpha}$. Simple propagator-like structure: all UV divergences expressed in terms of Γ and ζ functions.
- IR divergences from $p_0 = 0$ mode due to

$$\int_p \frac{1}{(p^2)^a} \Pi_{bce}(p_0 = 0)^2$$

- “distribute” them on the other propagators through IBP’s.

$$\int_p \frac{\partial}{\partial \mathbf{p}_i} \mathbf{p}_i \frac{\Pi_{bc0}(p)^2}{(p^2)^a} = 0$$

$$\Rightarrow \int_p \frac{\Pi_{bc0}(p)^2}{(p^2)^a} = \#_1 \int_p \frac{\Pi_{bc0}(p)^2}{(p^2)^{a-1}} + \#_2 \int_p \frac{\Pi_{b-1c0}(p)^2}{(p^2)^a}$$

Result

$$\begin{aligned}
 \textcircled{\text{V}} = & \sum_P \frac{\Pi_{bce}^{0,UV} \Pi_{fgh}^{0;UV} + \text{other comb. thereof}}{(P^2)^a} \\
 & + \left. \int_p \frac{\Pi_{bce}(p_0 = 0) \Pi_{fgh}(p_0 = 0)}{(p^2)^a} \right|_{\text{IBP}} \\
 & + \left. \sum_{p_0 \neq 0} \int_p \frac{(\Pi_{bce} - \Pi_{bce}^0 - \Pi_{bce}^{UV})^2}{(P^2)^a} \right\} \mathcal{O}(\epsilon^0)
 \end{aligned}$$

- Divergent part done in **momentum** space.
- Finite part done in **configuration** space via Fourier transform.

Handling tensor structures

- Of the form:

$$\oint_{PQR} \frac{\{Q_\mu R_\mu, (Q_\mu R_\mu)^2\}}{(P^2)^a Q^2 R^2 (P+Q)^2 (P+R)^2} \rightarrow \oint_Q \frac{\{Q_\mu, Q_\mu Q_\nu\}}{Q^2 (P+Q)^2}$$

- Usual projection techniques introduce $1/\mathbf{p}^2$ -like structures in sum-integrals:

$$\oint_Q \frac{q_0 \mathbf{q}}{Q^2 (P+Q)^2} = \frac{p_0 \mathbf{p}}{\mathbf{p}^2} \oint_Q \left(\frac{1}{Q^2} + \frac{P^2/2 - p_0^2}{Q^2 (P+Q)^2} \right)$$

- Idea from 0-temperature technique [Tarasov, 1996].
- Remove **tensor**-structure of master sum-integral:
tensor sum-integral in d dim. = **scalar** sum-integral in $d + 2(4)$ -dim.

From tensor to scalar

Define:

$$I_{\nu_1 \dots \nu_5}^d = \int_{pqr} \frac{e^{-2\alpha \vec{q}\vec{r}}}{(p^2 + m_1^2)^{\nu_1} (q^2 + m_2^2)^{\nu_2} (r^2 + m_3^2)^{\nu_3} ((p+q)^2 + m_4^2)^{\nu_4} ((p+r)^2 + m_5^2)^{\nu_5}}$$

Via $\Gamma(\nu) A^{-1} = \int_0^\infty d\alpha \alpha^{\nu-1} e^{-\alpha A}$ we get:

$$I_{\nu_1 \dots \nu_5}^d \propto \int_0^\infty \frac{\prod_1^5 d\alpha_i \alpha_i^{\nu_i-1} e^{-\alpha_i m_i^2}}{[D(\alpha) + 2\alpha \alpha_4 \alpha_5 + \alpha^2 \dots]^{d/2}}$$

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Master formula:

$$\begin{aligned} \partial_{-2\alpha} I_{\nu_1, \nu_2, \nu_3, \nu_4, \nu_5}^d |_{\alpha=0} &= \int_{pqr} \frac{\vec{q}\vec{r}}{(p^2 + m_1^2)^{\nu_1} \dots ((p+r)^2 + m_5^2)^{\nu_5}} \\ &= \int \left(\prod_1^5 d\alpha_i \alpha_i^{\nu_i-1} e^{-\alpha_i m_i^2} \right) \frac{\alpha_4 \alpha_5}{[D(\alpha)]^{(d+2)/2}} \\ &= \partial_{m_4^2} \partial_{m_5^2} I_{\nu_1, \nu_2, \nu_3, \nu_4, \nu_5}^d \Big|_{\alpha=0} = I_{\nu_1, \nu_2, \nu_3, \nu_4+1, \nu_5+1}^{d+2} \Big|_{\alpha=0} \end{aligned}$$

Applying to sum-integrals

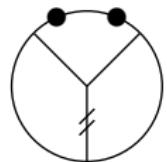
Via:

$$\oint_P \frac{\vec{qr}}{(P^2)^a} \Pi_{bce} \Pi_{fgh} = T^3 \sum_{p_0, q_0, r_0} (q_0)^e (r_0)^h \partial_{-2\alpha} I_{a,b,c,f,g}^{3-2\epsilon} \Big|_{m_1=p_0, \dots, \alpha=0}$$

Example of sum-integral containing a tensor-structure:

$$\begin{aligned}
 & \text{Diagram } \{d\} = 4 \oint_{PQR} \frac{(Q \cdot R)^2}{P^6 Q^2 R^2 (P+Q)^2 (P+R)^2} - 2 \left(\text{Diagram } \{d\} \right) \\
 & + 2 \left(\text{Diagram } \{d\} \right) \\
 & = 4 \text{Diagram } \{d\} + 4d \text{Diagram } \{d+2\} + 3d \text{Diagram } \{d+2\} + 2d \text{Diagram } \{d+2\} \\
 & + 4d(d+2) \text{Diagram } \{d+4\} + (\text{1-loop in } d)^3 = \dots
 \end{aligned}$$

...Result

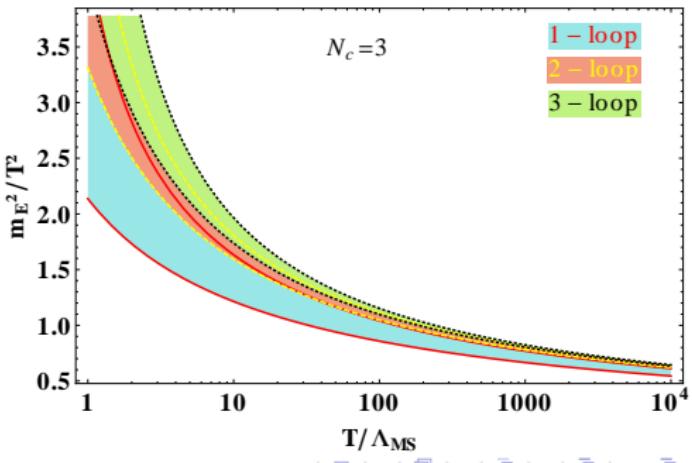


$$\begin{aligned} &= -\frac{5}{36} \frac{T^2}{(4\pi)^4} \left(\frac{\mu^2}{4\pi T^2} \right)^{3\epsilon} \frac{1}{\epsilon^2} \\ &\quad \times \left[1 + \epsilon \left(\frac{71}{30} + \gamma_E + 2 \frac{\zeta'(-1)}{\zeta(-1)} \right) + 44.6299(1)\epsilon^2 + \mathcal{O}(\epsilon^3) \right] \end{aligned}$$

Renormalized mass parameter

$$\begin{aligned}
 m_{E,\text{ren}}^2 = & T^2 g^2(\bar{\mu}) \frac{C_A}{3} \left\{ 1 + \frac{g^2(\bar{\mu})}{(4\pi)^2} \frac{C_A}{3} \left(22 \ln \frac{\bar{\mu} e^{\gamma_E}}{4\pi T} + 5 \right) \right. \\
 & + \frac{g^4(\bar{\mu})}{(4\pi)^4} \left(\frac{C_A}{3} \right)^2 \left(484 \ln^2 \frac{\bar{\mu} e^{\gamma_E}}{T} - 116 \ln \frac{\bar{\mu} e^{\gamma_E}}{T} + \frac{1091}{2} \right. \\
 & \left. \left. - 144\gamma_E + 324 \ln 4\pi - 180 \frac{\zeta'(-1)}{\zeta(-1)} - \frac{56}{5} \zeta(3) \right) + \mathcal{O}(g^6) \right\}
 \end{aligned}$$

- Gauge invariant.
- Band due to variation of arbitrary scale $\bar{\mu}$.
- Shows better convergence properties in comparison to 1 and 2 loop result.



Conclusion and outlook

- Debye mass calculated via a matching computation between QCD and EQCD. It enters also the $\mathcal{O}(g^7)$ coefficient of the QCD pressure.
- Side benefit: We generalized the computation of a class of three loop sum-integrals.
- In addition: New method for computing **tensor** sum-integrals.
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[hep-ph/1208.0284](https://arxiv.org/abs/hep-ph/1208.0284)

Next steps:

- Find a suitable basis transformation for Π_{ij} which enters the computation of g_E .
- Compute g_E with the methods developed here and use it in determining the spatial string tension of QCD.