Beamforming
(and LOFAR)

Andreas Horneffer
(Using Material from J. Anderson and M. Kuniyoshi)
Effective collecting area $A(\nu, \theta, \phi)$ m$^2$

On-axis response $A_0 = \eta A$
$\eta = $ aperture efficiency

Normalized pattern (primary beam)
$A(\nu, \theta, \phi) = A(\nu, \theta, \phi)/A_0$

Beam solid angle
$\Omega_A = \int \int_A A(\nu, \theta, \phi) \, d\Omega$

$A_0 \Omega_A = \lambda^2$
$\lambda = $ wavelength, $\nu = $ frequency
Aperture-Beam Fourier Transform Relationship

\[ f(u,v) = \text{aperture field distribution} \]
\[ u,v = \text{aperture coordinates} \]
\[ (\text{in wavelengths}) \]

\[ F(l,m) = \text{far-field voltage pattern} \]
\[ l = \sin\theta \cos\phi, \ m = \sin\theta \sin\phi \]

\[ F(l,m) = \int\int f(u,v) \exp(2\pi i (ul + vm)) \, du \, dv \]

\[ f(u,v) = \int\int F(l,m) \exp(-2\pi i (ul + vm)) \, dl \, dm \]
Pointing a Dish Antenna

θ = Beamwidth

Main Lobe

Null

Side Lobe

Antenna Site

Normalized Relative

-20 dB

-10 dB

-3 dB

0 dB
Beamforming means “pointing” an antenna array
And shaping the beam
In short: by giving each antenna the right delay and weight, and then adding the signals.
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The Principle of Beamforming

**Note:** Antenna patterns are the same when transmitting or receiving. Thus receiving works the same as the transmitting case shown here.
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The coaxial cables have the same length.

Beam pattern

The coaxial cables have the same length.
The Principle of Beamforming

The coaxial cables have the same length.

same phase

Beam pattern

The coaxial cables have the same length.

generator

a

b
The Principle of Beamforming

The coaxial cables have the same length.

The coaxial cables have the same length.

Beam pattern

generator

θ
The Principle of Beamforming

The coaxial cables have the same length.

Beam pattern

The coaxial cables have the same length.
The Principle of Beamforming

same phase

different phase

Resulting beam pattern

The coaxial cables have the same length.

generator

\[ a \quad b \]
The Principle of Beamforming

The coaxial cables have the same length.

\[ \text{generator} \]

\[ (a+b)\sin\theta \]

\[ a \sin\theta \]

added cables
The Principle of Beamforming

Wave front

Same phase

Same phase

θ

a

asinθ

b

(a+b)sinθ

generator

coaxial cable
The Principle of Beamforming

different phase

coaxial cable

generator

\[ (a+b)\sin\theta \]
The Principle of Beamforming

Resulting beam pattern

same phase

different phase

coaxial cable

generator

\( a \sin \theta \)

\( (a+b) \sin \theta \)
The Principle of Beamforming

\[(a+b)\sin\theta\]
The Principle of Beamforming

Resulting beam pattern

same phase

different phase

(a+b)sinθ

b

asinθ

a

θ

generator
Digital Beamforming

- A shift in time is a multiplication with a phase gradient in frequency
  (Fourier shift theorem)
- If $\Delta v$ is small then the phase gradient is a phase factor

![Diagram of digital beamforming process](attachment:image.png)
LOFAR High Band Antennas

HBA Tiles
Dipole Beam
HBA Tiles
Tile Beam

generator
HBA Tiles
Station Beam 1

Added cable

generator
HBA Tiles
Station Beam 2

- Station beam outside the tile Beam

Added cable

generator
Single- and Double-Slit Experiment

- You see the main pattern from the single slit
- In the double slit experiment the brightness of the double-slit maxima is modulated with the single-lit pattern
- See same effect when forming a station beam within a tile-beam

Images from Wikipedia
LOFAR Tile Beamformer

1) Delay Lines on Frontend Boards
   5 bit (32 steps); 0.5 ns resolution

2) Analog signal Addition
LOFAR Station Beamformer

1) Polyphase Filterbank

2) Multiply with phase weight

\[ S_{f,\text{out}}[t] = S_{f,\text{in}}[t] \times A_f e^{i\theta_f} \]

3) Add to data on ring

\[ S_i[t] \]

\[ \sum_{j<i} S_j[t] \]

\[ \sum_{j<i+1} S_j[t] \]
HBA Multi-Beaming

- Only one tile beam!
- Can point several stations beams within the tile beam.
- Can point station beam outside the tile beam, but with reduced sensitivity.
Grating Lobes

Intended Direction

Grating Lobe Direction

Same phase

$\theta$

a

b

generator
Grating Interferometer with Isotropic Sources

Array of $n$ isotropic sources of equal amplitude $E_0$ and spacing $d$
Grating Interferometer with isotropic sources
Grating Interferometer with Dipoles

- Array of the same spacing
Pseudorandom Spacing

Array of pseudorandom spacing
Grating vs Pseudorandom

Station beam at 60MHz

Station beam at 20MHz

Station Beam

Grating lobe

(-60,0)

θ[^\circ\ ]  0  20  40  60  -60

φ[^\circ\ ]  0  20  40  -40

0  0.2  0.4  0.6  0.8  1

(-60,0)

θ[^\circ\ ]  0  20  40  60  -60

φ[^\circ\ ]  0  20  40  -40

0  0.2  0.4  0.6  0.8  1
Crab Nebula

- regular HBA spacing creates grating lobes
- Position of grating lobes changes with frequency

by S. ter Veen
Grating Lobes in LOFAR Visibilities

- Two Baselines
- One of the stations of the second baseline had a grating-lobe on a strong source.
Station Primary Beam

That's actually a LWA simulation.
Asymmetric Station Beam

The ratio of the HPBW of down side to up side

That's actually a LWA simulation.
Pointing error

H = -100d (-6.7h)

Dec = 40.7d

El = +14.1d

Pointing error (0,0)
Pointing error

$H = -80\,^d\, (-5.3\,^h)$

$\text{Dec} = 40.7\,^d$

$\text{El} = +27.7\,^d$
H = -60d (-4h)
Dec = 40.7d
El = +42.3d

Pointing error
Pointing error

Pointing error as a function of elevation angle (degree).

That's actually a LWA simulation.
Shaping the Beam

- Shape the beam by giving different weights to the elements
- In this case: get a circular beam at all elevations

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- Shape the beam by giving different weights to the elements
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Disadvantage: loose a lot of sensitivity!
The End!
Beam Rotation on the Sky

Parallactic angle
Primary Beam Shape

LOFAR

Antenna Performance
Parameters

DIFFRACTION PATTERN

ERROR SCATTER PATTERN

dB

πD1

Elevation (arc min)

Azimuth (arc min)
Pointing Accuracy
\[ \Delta \theta = \text{rms pointing error} \]