Perturbative structure of the Gribov-Zwanziger Lagrangian

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Outline

- Will briefly review the Gribov problem and the Gribov-Zwanziger localized Lagrangian for the Landau gauge
- Its infrared properties will be discussed and the inconsistency with lattice analyses of the gluon propagator
- An extension to the Gribov-Zwanziger Lagrangian can accommodate lattice data by introducing condensates
- Theoretical explanation of this decoupling solution from a Lagrangian point of view is not complete
- Examine the Gribov-Zwanziger Lagrangian from a more general point of view and suggest alternative tests for lattice analyses
- One test derives from the structure of 3-point vertices at the symmetric subtraction point examined at one loop and its power corrections

Background

- Yang-Mills action S is invariant under gauge transformations

$$A^a_\mu \;\rightarrow\; \tilde{A}^a_\mu \;=\; U^\dagger \partial_\mu U \;+\; U^\dagger A_\mu U$$

  where

$$S \;=\; -\,\frac{1}{4} \int d^4x \, G^a_{\mu\nu} G^{a\,\mu\nu}$$

  with $G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$

- So A^a_μ and $\tilde{A}^a_\mu$ define equivalent configurations but inequivalent configurations are required for a consistent quantum theory
- Apply Faddeev-Popov procedure and restrict gauge field to satisfy a gauge condition
- Will focus on Landau gauge $\partial^\mu A^a_\mu = 0$
- Thus path integral is

$$Z \;=\; \int \mathcal{D}A \, \delta(\partial^\mu A^a_\mu) \, \det\left(-\partial^\nu D_\nu\right) \; e^{-S}$$

  where $\partial^\mu D_\mu$ is the Faddeev-Popov operator

Gribov problem and QCD

- In non-abelian gauge theory there is a problem fixing the gauge globally [Gribov]

$$Z = \int \mathcal{D}A \, \delta(\partial^\mu A^a_\mu) \det\left(-\partial^\nu D_\nu\right) \, e^{-S}$$

- For a given A^a_μ satisfying the gauge condition $\partial^\mu A^a_\mu = 0$ there are (Gribov) copies $\tilde{A}^a_\mu$ obeying the same condition $\partial^\mu \tilde{A}^a_\mu = 0$
- Existence of Gribov copies which is equivalent to the Faddeev-Popov operator having zero eigenvalues

$$\partial^\mu D_\mu \Lambda^a \; = \; 0$$

- In the Landau gauge the Faddeev-Popov operator $\mathcal{M}^{ab}(A) = -\left(\partial_\mu D^\mu\right)^{ab}$ is hermitian and hence has *real* eigenvalues
- To avoid the copy problem the region of integration of the path integral must be restricted to a domain, Ω, called the Gribov horizon which contains the origin, $A^a_\mu = 0$
- Boundary, $\partial\Omega$, is defined by where the first zeroes of $\mathcal{M}^{ab}$ are

Consequences

- Gribov argued that this restriction to the first Gribov region or horizon had a marked effect on the infrared behaviour of QCD (Yang-Mills)
- A natural mass parameter, γ, emerges in the analysis reflecting the cutoff feature of the path integral
- It is central to the infrared or non-perturbative behaviour of the theory
- Gluon propagator form factor is

$$D_A(p^2) \; = \; \frac{(p^2)^2}{[(p^2)^2 + C_A \gamma^4]}$$

where

$$\langle A^a_\mu(p) A^b_\nu(-p) \rangle \; = \; - \, \frac{\delta^{ab} D_A(p^2)}{p^2} P_{\mu\nu}(p)$$

which has gluon suppression since $D_A(0) = 0$

- γ is not independent and satisfies the loop gap equation which at one loop is

$$1 \ = \ \frac{(d-1)}{d} \int \frac{d^d k}{(2\pi)^d} \frac{C_A g^2}{[(k^2)^2 + C_A \gamma^4]}$$

implying the $\overline{\text{MS}}$ result, with $a = g^2/(16\pi^2)$,

$$1 \ = \ C_A \left[\frac{5}{8} - \frac{3}{8} \ln \left(\frac{C_A \gamma^4}{\mu^4} \right) \right] a \ + \ O(a^2)$$

or

$$\frac{C_A \gamma^4}{\mu^4} \ = \ \exp \left[\frac{5}{3} - \frac{8}{3 C_A a} \right]$$

- The theory has no meaning if γ is treated as independent and does not satisfy this condition
- Using gap equation ghost propagator form factor is enhanced

$$D_c(p^2) \ \sim \ \frac{1}{p^2} \quad \text{as} \ \ p^2 \ \to \ 0$$

Zwanziger construction

- Suggests gluon propagator can be reproduced from an effective non-local Lagrangian

$$L^\gamma = -\frac{1}{4} G^a_{\mu\nu} G^{a\,\mu\nu} + \frac{C_A \gamma^4}{2} A^a_\mu \frac{1}{\partial^\nu \partial_\nu} A^{a\,\mu} + \ldots$$

- In 1989 Zwanziger managed to localize the (cut-off) Gribov path integral and the horizon operator was determined to be

$$f^{acp} f^{bdp} A^a_\mu \left(\frac{1}{\partial^\nu D_\nu} \right)^{cd} A^{b\,\mu}$$

and not, for example,

$$A^a_\mu \left(\frac{1}{\partial^\nu D_\nu} \right)^{ab} A^{b\,\mu}$$

- A naive localization of the latter leads to a non-renormalizable Lagrangian
- Horizon conditions defines gap equation satisfied by Gribov mass γ
- While L^γ incorporates the horizon condition it is clearly not useful for practical calculations due to non-locality

Local Gribov-Zwanziger Lagrangian

- Zwanziger constructed a completely local Lagrangian
- In essence the non-local projection of the gauge field, $(\partial D)^{-1} A_\mu$, was defined to be a localizing ghost field

$$
\begin{aligned}
L^{GZ} \;\; = \;\; & L^{QCD} + \frac{1}{2} \rho^{ab\,\mu} \partial^\nu \left(D_\nu \rho_\mu \right)^{ab} + \frac{i}{2} \rho^{ab\,\mu} \partial^\nu \left(D_\nu \xi_\mu \right)^{ab} \\
& - \frac{i}{2} \xi^{ab\,\mu} \partial^\nu \left(D_\nu \rho_\mu \right)^{ab} + \frac{1}{2} \xi^{ab\,\mu} \partial^\nu \left(D_\nu \xi_\mu \right)^{ab} \\
& - \bar{\omega}^{ab\,\mu} \partial^\nu \left(D_\nu \omega_\mu \right)^{ab} - \frac{1}{\sqrt{2}} g f^{abc} \partial^\nu \bar{\omega}^{ae}_\mu \left(D_\nu c \right)^b \rho^{ec\,\mu} \\
& - \frac{i}{\sqrt{2}} g f^{abc} \partial^\nu \bar{\omega}^{ae}_\mu \left(D_\nu c \right)^b \xi^{ec\,\mu} - i \gamma^2 f^{abc} A^{a\,\mu} \xi^{bc}_\mu - \frac{d N_A \gamma^4}{2g^2}
\end{aligned}
$$

- Fields ω^{ab}_μ and $\bar{\omega}^{ab}_\mu$ are anti-commuting
- 2-point mixing term leads to complicated propagator structure
- Lagrangian is *local* and *renormalizable*, [Schaden, Maggiore; Sorella et al], so that it can be used to perform calculations

Propagators

- Propagators include Gribov's suppressed gluon

$$
\begin{aligned}
\langle A^a_\mu(p) A^b_\nu(-p) \rangle &= -\frac{\delta^{ab} p^2}{[(p^2)^2 + C_A \gamma^4]} P_{\mu\nu}(p) \\
\langle A^a_\mu(p) \xi^{bc}_\nu(-p) \rangle &= \frac{i f^{abc} \gamma^2}{[(p^2)^2 + C_A \gamma^4]} P_{\mu\nu}(p) \\
\langle A^a_\mu(p) \rho^{bc}_\nu(-p) \rangle &= 0 \\
\langle \xi^{ab}_\mu(p) \xi^{cd}_\nu(-p) \rangle &= -\frac{\delta^{ac}\delta^{bd}}{p^2} \eta_{\mu\nu} + \frac{f^{abe} f^{cde} \gamma^4}{p^2 [(p^2)^2 + C_A \gamma^4]} P_{\mu\nu}(p) \\
\langle \xi^{ab}_\mu(p) \rho^{cd}_\nu(-p) \rangle &= 0 \\
\langle \rho^{ab}_\mu(p) \rho^{cd}_\nu(-p) \rangle &= \langle \omega^{ab}_\mu(p) \bar{\omega}^{cd}_\nu(-p) \rangle = -\frac{\delta^{ac}\delta^{bd}}{p^2} \eta_{\mu\nu}
\end{aligned}
$$

where $P_{\mu\nu}(p) = \eta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}$

- In quantum computations implementing the gap equation produces suppressed gluon and an enhanced Faddeev-Popov ghost

Zero momentum limit

- More recently Zwanziger has proved that the bosonic localizing enhances too
- This has been verified explicitly at one loop in three and four dimensions
- Compute one loop corrections to 2-point functions, expand in powers of p^2, set gap equation and invert to get propagator behaviour for low momenta, $a = g^2/(16\pi^2)$,
- As $p^2 \to 0$ one loop four dimensional bosonic localizing ghost propagators behave as

$$
\begin{aligned}
\langle \xi^{ab}_\mu(p) \xi^{cd}_\nu(-p) \rangle &\sim \frac{4\gamma^2}{\pi\sqrt{C_A}(p^2)^2 a} \left[\delta^{ad}\delta^{bc} - \delta^{ac}\delta^{bd} \right] \eta_{\mu\nu} \\
&\quad + \frac{8\gamma^2}{\pi C_A^{3/2}(p^2)^2 a} f^{abe} f^{cde} P_{\mu\nu}(p) \\
\langle \rho^{ab}_\mu(p) \rho^{cd}_\nu(-p) \rangle &\sim - \frac{8\gamma^2}{\pi\sqrt{C_A}(p^2)^2 a} \delta^{ac}\delta^{bd} \eta_{\mu\nu}
\end{aligned}
$$

- Suppression is absent for the adjoint projection of transverse part consistent with Zwanziger

Adjoint projection

- More specifically taking the adjoint projection of the ξ^{ab}_μ field produces the propagator

$$f^{apq} f^{brs} \langle \xi^{pq}_\mu(p) \xi^{rs}_\nu(-p) \rangle \;=\; - \, \frac{C_A p^2}{[(p^2)^2 + C_A \gamma^4]} \delta^{ab} P_{\mu\nu}(p) \;-\; \frac{C_A}{p^2} \delta^{ab} L_{\mu\nu}(p)$$

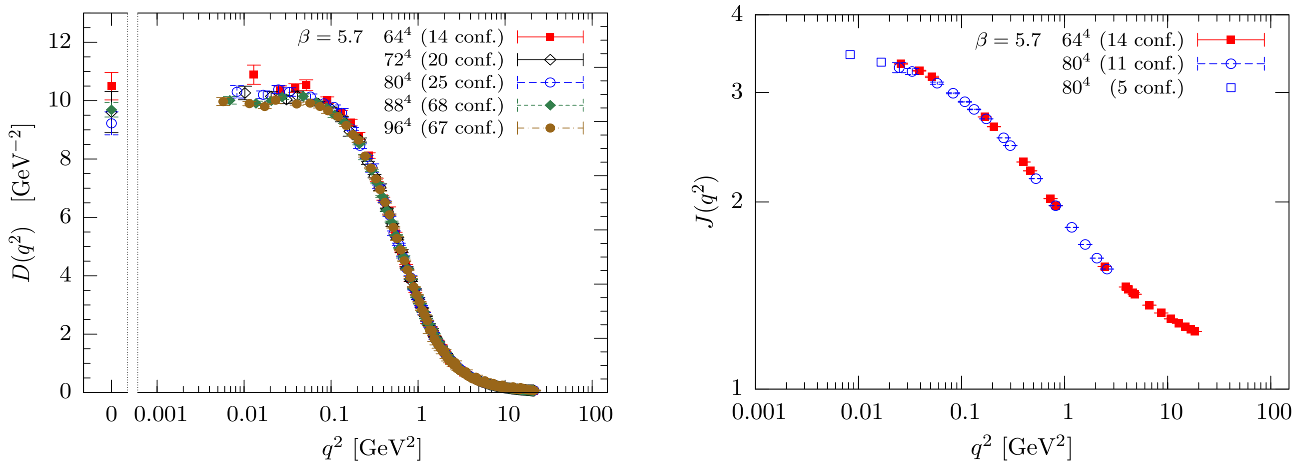
- Transverse part is in effect the same as the gluon
- In the $p^2 \to 0$ limit, after implementing the gap equation,

$$\begin{aligned} f^{apq} f^{brs} \langle \xi^{pq}_\mu(p) \xi^{rs}_\nu(-p) \rangle \quad &\sim \quad - \, \delta^{ab} \left[\frac{69 \pi C_A^2 a}{128 \sqrt{C_A} \gamma^2} \;+\; \frac{p^2}{\gamma^4} \right] P_{\mu\nu}(p) \\ &\quad\;\; - \, \delta^{ab} \left[\frac{8 C_A \gamma^2}{\pi \sqrt{C_A} (p^2)^2 a} \;+\; \frac{4}{\pi^2 p^2 a} \right] L_{\mu\nu}(p) \end{aligned}$$

- Transverse part freezes to a finite part but longitudinal part enhances
- Three dimensions is similar

Lattice results

- Original lattice study of the infrared propagators was by Cucchieri, Mendes and Maas
- Lattice does not find a suppressed gluon or enhanced Faddeev-Popov ghost
- Gluon propagator freezes to a finite non-zero value and ghost propagator has slight deviation from free behaviour
- From Bogolubsky et al, arXiv:0901.0736



- Effective coupling vanishes in the infrared limit
- This is referred to as the decoupling solution which is also observed in DSE studies [Alkofer et al]
- Suggests gluon is massive and probably contradicts global BRST invariance [Fischer et al]

BRST invariant operator

- Decoupling solution can be modelled with massive localizing ghosts
- Include BRST invariant dimension two operator built from localizing ghosts in Lagrangian giving the general mass term

$$
\begin{aligned}
\mathcal{O} \;&=\; \left[\mu_{\mathcal{Q}}^2 \delta^{ac} \delta^{bd} \;+\; \mu_{\mathcal{W}}^2 f^{ace} f^{bde} \;+\; \frac{\mu_{\mathcal{R}}^2}{C_A} f^{abe} f^{cde} \right. \\
&\quad \left. +\; \mu_{\mathcal{S}}^2 d_A^{abcd} \;+\; \frac{\mu_{\mathcal{P}}^2}{N_A} \delta^{ab} \delta^{cd} \;+\; \mu_{\mathcal{T}}^2 \delta^{ad} \delta^{bc} \right] \mathcal{O}^{abcd}
\end{aligned}
$$

where

$$
\mathcal{O}^{abcd} \;=\; \frac{1}{2} \left[\rho^{ab} \rho^{cd} \;+\; i \xi^{ab} \rho^{cd} \;-\; i \rho^{ab} \xi^{cd} \;+\; \xi^{ab} \xi^{cd} \right] \;-\; \bar{\omega}^{ab} \omega^{cd}
$$

and d_A^{abcd} is totally symmetric rank four adjoint tensor

- Mass parameters μ_i^2 tag the colour tensors
- Structure of the propagators has been investigated

$\mathcal{R}$ channel

- Consider $\mu_i^2 = 0$ for $i \neq \mathcal{R}$ then propagators are

$$
\begin{aligned}
\langle A^a_\mu(p) A^b_\nu(-p) \rangle_{\mathcal{R}} &= - \frac{\delta^{ab}[p^2 + \mu^2_{\mathcal{R}}]}{[(p^2)^2 + \mu^2_{\mathcal{R}} p^2 + C_A \gamma^4]} P_{\mu\nu}(p) \\
\langle A^a_\mu(p) \xi^{bc}_\nu(-p) \rangle_{\mathcal{R}} &= \frac{i f^{abc} \gamma^2}{[(p^2)^2 + \mu^2_{\mathcal{R}} p^2 + C_A \gamma^4]} P_{\mu\nu}(p) \\
\langle A^a_\mu(p) \rho^{bc}_\nu(-p) \rangle_{\mathcal{R}} &= \langle \xi^{ab}_\mu(p) \rho^{cd}_\nu(-p) \rangle_{\mathcal{R}} \; = \; 0 \\
\langle \xi^{ab}_\mu(p) \xi^{cd}_\nu(-p) \rangle_{\mathcal{R}} &= - \frac{\delta^{ac}\delta^{bd}}{p^2} \eta_{\mu\nu} + \frac{f^{abe} f^{cde} [\mu^2_{\mathcal{R}} p^2 + C_A \gamma^4]}{C_A p^2 [(p^2)^2 + \mu^2_{\mathcal{R}} p^2 + C_A \gamma^4]} P_{\mu\nu}(p) \\
&\quad + \frac{f^{abe} f^{cde} \mu^2_{\mathcal{R}}}{C_A p^2 [p^2 + \mu^2_{\mathcal{R}}]} L_{\mu\nu}(p) \\
\langle \rho^{ab}_\mu(p) \rho^{cd}_\nu(-p) \rangle_{\mathcal{R}} &= \langle \omega^{ab}_\mu(p) \bar{\omega}^{cd}_\nu(-p) \rangle_{\mathcal{R}} \\
&= - \frac{\delta^{ac}\delta^{bd}}{p^2} \eta_{\mu\nu} \; + \; \frac{f^{abe} f^{cde} \mu^2_{\mathcal{R}}}{C_A p^2 [p^2 + \mu^2_{\mathcal{R}}]} \eta_{\mu\nu}
\end{aligned}
$$

- By contrast $\mathcal{Q}$ channel propagators have no massless poles in p^2

Faddeev-Popov ghost propagator corrections

- If $D_c(p^2)$ is ghost propagator form factor then at one loop

$$D_c(p^2) = -\left[1 - C_A\left[\frac{5}{8} - \frac{3}{8}\ln\left[\frac{C_A\gamma^4}{\mu^4}\right] + \frac{3\mu_{\mathcal{R}}^2}{8\sqrt{\mu_{\mathcal{R}}^4 - 4C_A\gamma^4}}\ln\left[\frac{\mu_+^2}{\mu_-^2}\right] + O\left(p^2\right)\right] a + O(a^2)\right]^{-1}$$

- Gap equation, with $\mu_\pm^2 = \frac{1}{2}\left[\mu_{\mathcal{R}}^2 \pm \sqrt{\mu_{\mathcal{R}}^4 - 4C_A\gamma^4}\right]$, is

$$1 = C_A\left[\frac{5}{8} - \frac{3}{8}\ln\left[\frac{C_A\gamma^4}{\mu^4}\right] - \frac{3\mu_{\mathcal{R}}^2}{8\sqrt{\mu_{\mathcal{R}}^4 - 4C_A\gamma^4}}\ln\left[\frac{\mu_+^2}{\mu_-^2}\right]\right] a + O(a^2)$$

- No enhancement, unless $\mu_{\mathcal{R}}^2 = 0$
- So ghost propagator mimics lattice measurements, similar to $\mathcal{Q}$ channel
- Lack of ghost enhancement can be traced to frozen gluon propagator

Localizing ghost propagator corrections

- In $\mathcal{Q}$ channel there are no massless poles in p^2 in propagators
- So form factors will all freeze to finite non-zero values
- For $\mathcal{R}$ channel have to compute one loop corrections and implement the gap equation similar to pure Gribov-Zwanziger case
- Zero momentum limit of localizing propagators is similar to Faddeev-Popov ghosts

$$\begin{aligned}
\langle \xi^{ab}_\mu(p) \xi^{cd}_\nu(-p) \rangle_{\mathcal{R}} &\sim \frac{1}{2\mathcal{Q}_0 p^2 a} \left[\delta^{ac}\delta^{bd} ~-~ \delta^{ad}\delta^{bc} ~-~ \frac{2}{C_A} f^{abe} f^{cde} \right] \eta_{\mu\nu} \\
\langle \rho^{ab}_\mu(p) \rho^{cd}_\nu(-p) \rangle_{\mathcal{R}} &\sim \frac{1}{2\mathcal{Q}_0 p^2 a} \left[\delta^{ac}\delta^{bd} ~-~ \delta^{ad}\delta^{bc} ~-~ \frac{2}{C_A} f^{abe} f^{cde} \right] \eta_{\mu\nu}
\end{aligned}$$

where

$$\begin{aligned}
\mathcal{Q}_0 &= \left[\frac{1}{8} \sqrt{\mu^4_{\mathcal{R}} - 4C_A\gamma^4} \ln \left[\frac{\mu^2_+}{\mu^2_-} \right] ~-~ \frac{1}{8} \ln \left[\frac{C_A\gamma^4}{(p^2)^2} \right] ~-~ \frac{11}{24} \right] \frac{\mu^2_{\mathcal{R}}}{C_A\gamma^4} \\
&+ \frac{1}{4\sqrt{\mu^4_{\mathcal{R}} - 4C_A\gamma^4}} \ln \left[\frac{\mu^2_+}{\mu^2_-} \right]
\end{aligned}$$

Contrast

- This produces the same behaviour as the Faddeev-Popov ghost propagator in the same $p^2 \to 0$ limit
- Same colour tensor emerges as for pure Gribov-Zwanziger case
- Adjoint projection of ξ^{ab}_μ propagator freezes to zero or a finite non-zero value
- $\mathcal{Q}$ channel produces frozen gluon propagator with frozen Bose localizing ghost propagator
- By contrast $\mathcal{R}$ channel produces frozen gluon propagator but with Bose localizing ghost propagator similar to lattice behaviour of Faddeev-Popov ghost propagator
- Refer to former as gluon-like and the latter as ghost-like
- Need lattice data on these propagators in the zero momentum limit to resolve
- Problem in trying to resolve propagator properties is that gauge fixing and trying to go to zero momentum on the lattice is technically very difficult
- Need to develop alternatives in order to give insight into which scenario is correct

Power corrections

- Early lattice work by Boucaud et al measured the running of the coupling constant
- Computed an effective coupling constant defined from the propagator form factors and vertex functions
- For intermediate range of momenta they observed deviation from expected perturbative behaviour
- Discrepancy could be explained by power corrections derived from a dimension *two* operator rather than dimension four
- It was assumed that the operator was $\frac{1}{2} A^a_\mu A^{a\,\mu}$
- Vertices measured at symmetric point which is a non-exceptional momentum configuration
- Can mimic such effects with Gribov-Zwanziger Lagrangian
- Examine power corrections to the vertex functions at both the asymmetric and symmetric subtraction points
- Effectively a next-to-high-energy expansion

Triple gluon vertex

- Define vertex functions at symmetric subtraction point by

$$\left\langle A^a_\mu(p) A^b_\nu(q) A^c_\sigma(-p-q) \right\rangle \Big|_{p^2=q^2=-\mu^2} = f^{abc} \, \Sigma^{ggg}_{\mu\nu\sigma}(p,q) \Big|_{p^2=q^2=-\mu^2}$$

with

$$\Sigma^{ggg}_{\mu\nu\sigma}(p,q) \Big|_{p^2=q^2=-\mu^2} = \sum_{k=1}^{3} \mathcal{P}^{ggg}_{(k)\,\mu\nu\sigma}(p,q) \, \Sigma^{ggg}_{(k)}(p,q)$$

- For non-Gribov Lagrangian there are 8 one loop diagrams
- Used Laporta algorithm encoded in GiNaC based Reduze algorithm to handle reduction to master integrals
- Numerical evaluation of arbitrary gauge exact result to two loops in Landau gauge in $\overline{\mathrm{MS}}$ is

$$\begin{aligned} \Sigma^{ggg}_{(1)}(p,q) &= -1 - \left[1.1212444 - 0.0417366 N_f \right] a \\ &\quad + \left[29.7530676 - 11.5677203] N_f \right] a^2 + O(a^3) \end{aligned}$$

- Other amplitudes similar and finite parts used to define MOM schemes and extend Celmaster and Gonsalves to two loops

MOMggg scheme mapping

- Landau gauge coupling constant mapping is

$$
\begin{aligned}
&a_{\mathrm{MOMggg}} \\
&= a_{\overline{\mathrm{MS}}} + \left[\left[69\psi'(\frac{1}{3}) - 46\pi^2 + 1188 \right] C_A \right. \\
&+ \left. \left[128\pi^2 - 192\psi'(\frac{1}{3}) - 432 \right] T_F N_f \right] \frac{a^2_{\overline{\mathrm{MS}}}}{162} \\
&+ \left[\left[19044(\psi'(\frac{1}{3}))^2 - 25392\pi^2\psi'(\frac{1}{3}) - 6938784\psi'(\frac{1}{3}) \right.\right. \\
&\quad - 100602\psi'''(\frac{1}{3}) - 72643392 s_2(\frac{\pi}{6}) + 145286784 s_2(\frac{\pi}{2}) \\
&\quad + 121072320 s_3(\frac{\pi}{6}) - 96857856 s_3(\frac{\pi}{2}) + 276736\pi^4 \\
&\quad + 4625856\pi^2 - 113724\Sigma + 8301852\zeta(3) \\
&\quad + 40126833 - 504468\frac{\ln^2(3)\pi}{\sqrt{3}} + 6053616\frac{\ln(3)\pi}{\sqrt{3}} \\
&\quad \left. +541836\frac{\pi^3}{\sqrt{3}} \right] C_A^2
\end{aligned}
$$

$$
\begin{aligned}
&+\Big[141312\pi^2\psi'(\frac{1}{3}) - 105984(\psi'(\frac{1}{3}))^2 - 2960064\psi'(\frac{1}{3}) + 33592320 s_2(\frac{\pi}{6}) \\
&\quad - 67184640 s_2(\frac{\pi}{2}) - 55987200 s_3(\frac{\pi}{6}) + 44789760 s_3(\frac{\pi}{2}) - 47104\pi^4 \\
&\quad + 1973376\pi^2 + 2239488\Sigma - 8957952\zeta(3) - 26695008 \\
&\quad + 233280\frac{\ln^2(3)\pi}{\sqrt{3}} - 2799360\frac{\ln(3)\pi}{\sqrt{3}} - 250560\frac{\pi^3}{\sqrt{3}}\Big] C_A T_F N_f \\
&+\Big[124416\psi'''(\frac{1}{3}) - 1492992\psi'(\frac{1}{3}) - 331776\pi^4 + 995328\pi^2 \\
&\quad - 4478976\Sigma + 6718464\zeta(3) - 7138368\Big] C_F T_F N_f \\
&+\Big[147456(\psi'(\frac{1}{3}))^2 - 196608\pi^2\psi'(\frac{1}{3}) + 2322432\psi'(\frac{1}{3}) + 65536\pi^4 \\
&\quad - 1548288\pi^2 + 2923776\Big] T_F^2 N_f^2\Big] \frac{a^3_{\overline{\mathrm{MS}}}}{419904} + O\left(a^4_{\overline{\mathrm{MS}}}\right)
\end{aligned}
$$

- Σ is a combination of harmonic polylogarithms

Basis tensors

- Overall there are 14 different possible basis tensors for decomposition of Green's function built from p_μ, q_μ and $\eta_{\mu\nu}$
- Explicit computation produces three combinations

$$
\begin{aligned}
\mathcal{P}^{ggg}_{(1)\mu\nu\sigma}(p,q) &= \eta_{\mu\nu}p_\sigma - \eta_{\mu\nu}q_\sigma - 2\eta_{\mu\sigma}p_\nu - \eta_{\sigma\mu}q_\nu + \eta_{\nu\sigma}p_\mu + 2\eta_{\nu\sigma}q_\mu \\
\mathcal{P}^{ggg}_{(2)\mu\nu\sigma}(p,q) &= [2p_\mu p_\nu p_\sigma + p_\mu q_\nu p_\sigma - p_\mu q_\nu q_\sigma \\
&\quad +2q_\mu p_\nu p_\sigma - 2q_\mu p_\nu q_\sigma - 2q_\mu q_\nu q_\sigma] \frac{1}{2\mu^2} \\
\mathcal{P}^{ggg}_{(3)\mu\nu\sigma}(p,q) &= [p_\mu p_\nu q_\sigma - q_\mu p_\nu p_\sigma + q_\mu p_\nu q_\sigma - q_\mu q_\nu p_\sigma] \frac{1}{\mu^2}
\end{aligned}
$$

- $\mathcal{P}^{ggg}_{(1)\mu\nu\sigma}(p,q)$ corresponds to the original Feynman rule for the vertex when $r = -p - q$
- Channel 1 amplitude can be used to define an effective running coupling constant for the vertex

Power corrections in Gribov-Zwanziger

- Repeat one loop symmetric point vertex calculation using the Gribov-Zwanziger Lagrangian
- There are 30 one loop graphs for triple gluon vertex
- At symmetric point the external momenta do not need to be small which avoids zero momenta issues
- Can expand Feynman integrals in powers of γ^2/μ^2 using method of Smirnov et al
- This is a two stage process
- First stage is to reduce all one loop Feynman graphs to a set of masters which are either 2-point or 3-point
- Use the Laporta algorithm for seven different possibilities of masses in diagrams at the symmetric point when there are two mass scales
- Schematically these are $(0,0,0)$, $(m_1,0,0)$, $(m_1,m_1,0)$, $(m_1,m_2,0)$, (m_1,m_1,m_1), (m_1,m_1,m_2) and (m_1,m_2,m_3) where none of the m_i are equal
- Have used the REDUZE implementation based on GINAC
- Write integration routines in FORM with diagrams generated by QGRAF

Power corrections in Gribov-Zwanziger

- Second stage is to substitute the expansion for the master diagrams as explicit expressions for one loop 3-point not known explicitly
- Extend method of Davydychev, Smirnov and Tausk developed explicitly for 2-point diagrams to 3-point case
- If the Feynman integral is J then

$$J_\Gamma \sim \Sigma_\gamma J_{\Gamma/\gamma} \circ \mathcal{T}_{\{m_i\};\{k_i\}} J_\gamma$$

  where Γ is the original graph, γ are subgraphs and $\mathcal{T}_{\{m_i\};\{k_i\}}$ is the Taylor expansion in masses and momenta q_i which are external to the subgraph γ
- The operation $\circ$ takes the Taylor expansion and inserts it in the numerator of the integrand of the reduced graph $J_{\Gamma/\gamma}$
- The construction of the set of graphs γ is the same as for the 2-point case but here there are *two* external momenta
- There are four terms in J_Γ
- First is the expansion of the original diagram in masses
- The other three are the three possible permutations of the two external momenta around the diagram

Results

- Results for triple gluon vertex

$$
\begin{aligned}
\Sigma^{ggg}_{(1)}(p,q,\gamma^2) &= \Sigma^{ggg}_{(1)}(p,q,0) \\
&\quad + \left[\frac{13}{6} + \frac{7\pi^2}{36} - \frac{7}{24}\psi'\left(\frac{1}{3}\right) - \frac{1}{2}\ln\left[\frac{C_A\gamma^4}{\mu^4}\right]\right] \frac{C_A^2\gamma^4}{\mu^4} a \\
\Sigma^{ggg}_{(2)}(p,q,\gamma^2) &= \Sigma^{ggg}_{(2)}(p,q,0) + \frac{3\pi}{32}\frac{C_A^{3/2}\gamma^2}{\mu^2} a \\
\Sigma^{ggg}_{(3)}(p,q,\gamma^2) &= \Sigma^{ggg}_{(3)}(p,q,0) + \frac{3\pi}{32}\frac{C_A^{3/2}\gamma^2}{\mu^2} a
\end{aligned}
$$

- Power corrections for channel 1 are dimension four; others are dimension two
- At asymmetric subtraction point all channels are dimension two
- For ghost-gluon and quark-gluon corrections in all channels at symmetric and asymmetric points are dimension two
- Need to consider pure gluon mass case for comparison

Gluon mass

- Repeat analysis with either Yang-Mills with a massive gluon propagator

$$\langle A^a_\mu(p) A^b_\nu(-p) \rangle ~=~ - \frac{\delta^{ab}}{[p^2 + \mu^2_{\mathcal{X}}]} P_{\mu\nu}(p)$$

or use Gribov-Zwanziger with a mass term $\frac{1}{2} \mu^2_{\mathcal{X}} A^a_\mu A^{a\,\mu}$

$$\begin{aligned}
\Sigma^{ggg}_{(1)}(p,q,\mu^2_{\mathcal{X}}) &= \Sigma^{ggg}_{(1)}(p,q,0) \\
&\quad + \left[\frac{7}{4} + \frac{14\pi^2}{27} - \frac{1}{3} \psi^\prime \left(\frac{1}{3} \right) \right] \frac{C_A \mu^2_{\mathcal{X}}}{\mu^4} a \\
\Sigma^{ggg}_{(2)}(p,q,\mu^2_{\mathcal{X}}) &= \Sigma^{ggg}_{(2)}(p,q,0) \\
&\quad + \left[\frac{673}{96} - \frac{4\pi^2}{27} + \frac{2}{9} \psi^\prime \left(\frac{1}{3} \right) - \frac{3}{16} \ln \left[\frac{\mu^2_{\mathcal{X}}}{\mu^2} \right] \right] \frac{C_A \mu^2_{\mathcal{X}}}{\mu^2} a \\
\Sigma^{ggg}_{(3)}(p,q,\mu^2_{\mathcal{X}}) &= \Sigma^{ggg}_{(3)}(p,q,0) \\
&\quad + \left[\frac{596}{96} - \frac{4\pi^2}{9} + \frac{2}{3} \psi^\prime \left(\frac{1}{3} \right) - \frac{211}{24} \ln \left[\frac{\mu^2_{\mathcal{X}}}{\mu^2} \right] \right] \frac{C_A \mu^2_{\mathcal{X}}}{\mu^2} a
\end{aligned}$$

- Corrections are all dimension two

Test

- For separate $\mathcal{Q}$ and $\mathcal{R}$ solutions all Lorentz channels in each vertex have dimension two corrections except for channel 1 of the triple gluon vertex
- Can formulate a test of whether the power corrections derive from the Gribov mass or a gluon mass term
- Compute the behaviour of all channels of the triple gluon vertex at the symmetric point
- If deviation from expected behaviour is commensurate in all three channels then it suggests a gluon mass
- If deviation from expected behaviour is weaker in channel 1 compared with 2 and 3 then suggests Gribov mass is present
- To determine whether the behaviour is pure Gribov-Zwanziger, $\mathcal{Q}$ or $\mathcal{R}$ would require comparison with numerical values in expansion as well as other vertices to establish consistency
- For the latter two there are various combinations of the mass parameters such as $\sqrt{\mu_{\mathcal{R}}^4 - 4C_A\gamma^4}$

Conclusions

- Explanation of lattice data using the Gribov-Zwanziger approach has not been fully resolved
- Presented tests which bypass the focus on the gluon and ghost propagator form factors in the infrared limit
- Triple gluon vertex power corrections at one loop suggest a test of the effective running coupling constant
- Absence of dimension two corrections at one loop needs to be tested at two loops at the symmetric subtraction point
- Ultimately to resolve these issues then more lattice data is needed