The electroweak phase transition

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Outline

- introduction to the phase transition
  (strength of the transition & dynamics)

- baryogenesis
  (basic picture, examples of the MSSN and 2HDM)

- gravitational waves

- bubble wall velocity

- summary and outlook
The electroweak theory: SU(3) x SU(2) x U(1) model

Chiral (parity violating)

Matter: 3 generations of quarks and leptons

Symmetry breaking: Higgs boson

Fermion and gauge boson masses
Electroweak symmetry breaking

\[ V(H) = -m^2|H|^2 + \lambda|H|^4 \]

We have measured the vev, \( v = 246 \) GeV and the Higgs mass, \( m_h = 125 \) GeV.

The Higgs field generates masses for elementary particles (quarks, leptons, W, Z).
Here it is:
The Higgs boson

$m_h = 125.3 \pm 0.6$ GeV
looking closer....
• We are surrounded by a constant Higgs field
• This field was not present at high temperatures in the early universe
• How did it come about?
• Are there observable relics?

- Gravitational waves?
- Primordial magnetic fields?
- Baryon asymmetry?

First order electroweak phase transition
Electroweak Phase Transition
Bubbles

broken phase

symmetric phase

- temperature $T_c$
- vev in the broken phase $v_c$
- wall width $L_w$
- wall velocity $v_w$

$H(z)$
The strength of the PT

Thermal potential:

\[ V(H, T) = m^2(T)H^2 - E(T)H^3 + \lambda(T)H^4 \]

- Boson loops:
  - SM: gauge bosons
  - strong PT: \( m_h < 40 \text{ GeV} \) (no top)
    - never (with realistic top mass)
  - Lattice: crossover for \( m_h > 80 \text{ GeV} \) → no phase transition in the SM

Kajantie, Laine, Rummukainen, Shaposhnikov 1996
Csikor, Fodor, Heitger 1998
The strength of the PT

Thermal potential:

\[ V(H, T) = m^2(T)H^2 - E(T)H^3 + \lambda(T)H^4 \]

- Boson loops:
  - SM: gauge bosons
  - SUSY: light stops
  - 2HDM: extra Higgses

- tree-level: extra singlets: \( \lambda SH^2 \), NMSSM, etc.

- replace \( H^4 \) by \( H^6 \), etc.
2-loop contributions can be important:

\[
V_{SM}^{(2)} = \frac{g^2}{16\pi^2} T^2 \left[ M^2 \left( \frac{3}{4} \log \frac{M_L}{T} - \frac{51}{8} \log \frac{M}{T} \right) + \frac{3}{2} \left( M^2 - 4M_L^2 \right) \log \frac{M + 2M_L}{3T} + 3MM_L \right] + \frac{m^2(\phi)T^2}{64\pi^2} \left[ 16g_s^2 \left( \frac{8}{3} \log 2 - \frac{1}{2} - c_B \right) + 9h_t^2 \sin^2 \beta \left( \frac{4}{3} \log 2 - c_B \right) \right] \tag{22}
\]

Daisy resummation:
\[\rightarrow\text{thermal masses}\]

IR problems: loop expansion parameter is \(g^2T/m\) \(\rightarrow\text{lattice?}\)
Electroweak Baryogenesis
The baryon asymmetry

$$\eta_B = \frac{n_B}{n_\gamma} = (6.2 \pm 0.2) \times 10^{-10}$$

Two measurements:
1) CMBR+LSS
2) primordial nucleosynthesis
→ reasonable agreement
→ we understand the universe up to T~MeV

Can we repeat this success for the baryon asym.? problem: only 1 observable
→ need to be convinced by a specific model:
theory?, experiment? (insight ...??)

\[\text{T < TeV scale? } \rightarrow \text{ EWBG}\]
The basics

\[ \eta_B = \frac{n_B}{n_\gamma} = (6.2 \pm 0.2) \times 10^{-10} \]

- Baryon number
- C
- CP
- Equilibrium

SM

- Sphalerons +
- Gauge interactions +
- Yukawa interactions ?
- Electroweak phase transition ?

Kuzmin, Rubakov, Shaposhnikov ‘85

Sakharov ‘67
Electroweak baryogenesis?

There are testable consequences:

• New particles (scalars?!?) at the LHC
  (Higgs sector is crucial!)

• New sources of CP violation which should show up soon in electric dipole experiments

• Could the electroweak phase transition produce observable gravitational waves?

If confirmed, it would constrain the early universe up to $T \sim 100\,\text{GeV}$ (nano sec.), like nucleosynthesis does for the MeV-scale (min.)
The mechanism

broken phase

symmetric phase

$H(z)$

$V_w$
The mechanism

\[ \Gamma_{\text{sph}} \sim T_c^4 \exp(-\kappa v_c/T_c) \ll 1 \]

\[ \frac{v_c}{T_c} > 1.0 \]

“strong PT”

CP violation \( \rightarrow \) left-h. quark number

\[ \Gamma_{\text{sph}} \sim T_c^4 \rightarrow \beta \]

diffusion

H(z)
Transport equations

EWBG relies on diffusion of charges: use Boltzmann equations.

The interaction with the bubble wall induces a force on the particles, which is different for particles and antiparticles if CP is broken.

\[
(\partial_t + \dot{z} \partial_z + \dot{p}_z \partial_{p_z}) f = C[f]
\]

Compute the force from dispersion relations

\[
\dot{p}_z = -\partial_z E(z, p_z)
\]

Z is the coordinate along the wall profile with wall width \( L_w \).
Elektroweak bubbles have typically thick walls: \( L_w >> (T_c)^{-1} \)
\( (L_w)^{-1} << p \) for a typical particle in the plasma

Compute the dispersion relation via an expansion in \( 1/(L_w T_c) \)

Consider a free fermion with a complex mass

\[
M(z) = m(z)e^{i\theta(z)}
\]

\[
(i\partial - P_{\perp} M(z) - P_{\parallel} M^*(z))\psi = 0
\]

\[
\psi \sim \exp(-iEt - i\int z' p_z(z')dz')
\]

\[
E_{\pm} = E_0 \pm \Delta E_0
\]

\[
= \sqrt{p^2 + m^2} \pm \theta' \frac{m^2}{2(p^2 + m^2)}
\]

only a varying \( \theta \) contributes!

- Joyce, Prokopec, Turok ’95
- Cline, Joyce, Kainulainen ’00
- more rigorous, using the Schwinger-Keldysh formalism:
  - Kainulainen, Prokopec, Schmidt, Weinstock ’01-’04
  - Konstandin, Prokopec, Schmidt, Seco ’05
- (Carena, Moreno, Quiros, Seco, Wagner ’00)
Diffusion equations

Fluid ansatz for the phase space densities:

to arrive at diffusion equations for the $\mu$'s

\[-(D_i \mu_i'' + v_w \mu_i') + \Gamma_{ij} \mu_j = S_i\]

- diffusion constant
- wall velocity ($v_w < v_s = 0.58$)
- interaction rates
- CP violating source terms

relevant particles: top, Higgs, super partners, ...

interactions: top Yukawa interaction
- strong sphalerons
- top helicity flips (broken phase)
- super gauge interactions (equ.)

Step 1: compute

Step 2: switch on the weak sphalerons

\[\eta_B \sim \Gamma_{WS} \int_{-\infty}^{\infty} dz \ n_{BL}(z)\]
And so it looks in practice…

\[
\begin{align*}
(3\kappa_t + 3\kappa_b)v_w\mu_{q_3,2}' - (3K_{1,t} + 3K_{1,b})v_{q_3,2}' + 6\Gamma_y(\mu_{q_3,2} + \mu_{t,2}) \\
- 6\Gamma_m(\mu_{q_3,2} + \mu_{t,2}) - 6\Gamma_{ss} [(2 + 9\kappa_t + 9\kappa_b)\mu_{q_3,2} + (1 - 9\kappa_t)\mu_{t,2}] \\
= -3K_{6,t}m_t^2\theta_t'\mu_{q_3,1}' \\
\end{align*}
\]

\[
\begin{align*}
-(K_{1,t} + K_{1,b})\mu_{q_3,2}' + (K_{2,t} + K_{2,b})v_wv_{q_3,2}' - \left(\frac{K_{1,t}^2}{\kappa_t D_Q} + \frac{K_{1,b}^2}{\kappa_b D_Q}\right)v_{q_3,2}' \\
= K_{4,t}v_wm_t^2\theta_t'' + K_{5,t}v_w(m_t^2)'\theta_t' - K_{7,t}m_t^2\theta_t'v_{q_3,1}' \\
\end{align*}
\]

\[
\begin{align*}
3\kappa_tv_w\mu_{t,2}' - 3K_{1,t}v_{t,2}' - 6\Gamma_y(\mu_{q_3,2} + \mu_{t,2}) - 6\Gamma_m(\mu_{q_3,2} + \mu_{t,2}) \\
- 3\Gamma_{ss} [(2 + 9\kappa_t + 9\kappa_b)\mu_{q_3,2} + (1 - 9\kappa_t)\mu_{t,2}] \\
= -3K_{6,t}m_t^2\theta_t'\mu_{t,1}' \\
\end{align*}
\]

\[
\begin{align*}
-K_{1,t}\mu_{t,2}' + K_{2,t}v_wv_{t,2}' - \frac{K_{1,t}^2}{\kappa_t D_Q}v_{t,2}' \\
= K_{4,t}v_wm_t^2\theta_t'' + K_{5,t}v_w(m_t^2)'\theta_t' - K_{7,t}m_t^2\theta_t'v_{t,1}' \\
\end{align*}
\]

[Bodeker, Fromme, Huber, Seniuch]
Models of new physics
The 2HDM

\[ V(H_1, H_2) = \mu_1^2 |H_1|^2 + \mu_2^2 |H_2|^2 + \mu_3^2 e^{i\phi} H_1^\dagger H_2 + \lambda_1 |H_1|^4 + \ldots \]

→ 4 extra physical Higgs degrees of freedom: 2 neutral, 2 charged
→ CP violation, phase \( \Phi \) (\( \mu_3 \) breaks \( Z_2 \) symmetry softly)
→ there is a phase induced between the 2 Higgs vevs

\[ v_1 = \langle H_1 \rangle, \quad v_2 e^{i\theta} = \langle H_2 \rangle \]

simplified parameter choice: only 2 scales

1 light Higgs \( m_h \) → SM-like
3 degenerate heavy Higgses \( m_H \) → keeps EW corrections small

early work:
Turok, Zadrozny ’91
Davies, Froggatt, Jenkins, Moorhouse ’94
Cline, Kainulainen, Vischer ’95
Cline, Lemieux ‘96
The phase transition

Evaluate 1-loop thermal potential:
loops of heavy Higgses generate a cubic term
→ strong PT for
   \( m_H > 300 \text{ GeV} \)
   \( m_h \) up to 200 GeV
→ PT ~ independent of \( \Phi \)
→ thin walls only for very strong PT (agrees with Cline, Lemieux ’96)

missing: 2-loop analysis of the thermal potential; lattice; wall velocity

[Fromme, S.H., Senuich ’06]
The bubble wall

Solve the field equations with the thermal potential $\rightarrow$ wall profile $\Phi_i(z)$

kink-shaped with wall thickness $L_w$  \hspace{2cm} $\theta$ becomes dynamical

( numerical algorithm for multi-field profiles, T. Konstandin, S.H. ´06)
The baryon asymmetry

The relative phase between the Higgs vevs, $\theta$, changes along the bubble wall.

$\theta_t = \theta/(1 + \tan^2 \beta)$

top transport generates a baryon asymmetry

$\eta_B$ in units of $10^{-11}$, $\varphi = 0.2$

[Fromme, S.H., Senuich ’06]

$d_n = 0.1 \times 10^{-26} - 7 \times 10^{-26}$ e cm

exp. bound: $d_n < 3.0 \times 10^{-26}$ e cm

Could LHC see these extra Higgses?
Inert 2HDM:

\[ V(\Phi_1, \Phi_2) = m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 + \frac{\lambda_1}{2} (\Phi_1^\dagger \Phi_1)^2 + \frac{\lambda_2}{2} (\Phi_2^\dagger \Phi_2)^2 \\
+ \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) + \frac{\lambda_5}{2} \left[(\Phi_1^\dagger \Phi_2)^2 + (\Phi_2^\dagger \Phi_1)^2\right] \]

doublet 2 does **not** get a vev

→ Dark matter

CP violation from higher-dim. operators

[Gil, Chankowski, Krawczyk 2012]
SM + higher-dim. operators

\[ V(H) = -\mu^2 |H|^2 + \lambda |H|^4 + \frac{1}{M^2} |H|^6 \]

maybe related to strong dynamics at the TeV scale, such as technicolor or gravity?
(or simply comes from integrating out extra scalars)

two parameters, \((\lambda, M) \leftrightarrow (m_h, M)\)

\(\lambda\) can be negative \(\rightarrow\) bump because of \(|H|^4\) and \(|H|^6\)

\[
V_{\text{eff}}(\phi, T) = \frac{1}{2} \left( -\mu^2 + \left( \frac{1}{2} \lambda + \frac{3}{16} g_2^2 + \frac{1}{16} g_1^2 + \frac{1}{4} y_t^2 \right) T^2 \right) \phi^2 \\
- \frac{g_2^3}{16\pi} T \phi^3 + \frac{\lambda}{4} \phi^4 + \frac{3}{64\pi^2} y_t^4 \phi^4 \ln \left( \frac{Q^2}{c_F T^2} \right) \\
+ \frac{1}{8M^2} (\phi^6 + 2\phi^4 T^2 + \phi^2 T^4).
\]

Zhang ‘93
Grojean et al. ‘04
Results for the PT

Evaluating the 1-loop thermal potential:

strong phase transition for $M<850 \text{ GeV}$
up to $m_h \sim 170 \text{ GeV}$

wall thickness $2 < L_w T_c < 16$

$\xi \equiv \frac{v_c}{T_c}$

Bödeker, Fromme, S.H., Seniuch ’04

Similar results, including Higgs cubic terms
Delaunay, Grojean, Wells ‘07
SM + higher-dim. operators

\[ V(H) = -\mu^2 |H|^2 + \lambda |H|^4 + \frac{1}{M^2} |H|^6 \]

Zhang ‘93

maybe related to strong dynamics at the TeV scale, such as technicolor or gravity?

two parameters, \((\lambda, M) \leftrightarrow (m_h, M)\)

\(\lambda\) can be negative \(\rightarrow\) bump because of \(|H|^4\) and \(|H|^6\): \(M < \sim 800\) GeV

**CP violation:**

\[
\frac{x}{M^2} (H^\dagger H) H t^c q
\]

Zhang, Lee, Whisnant, Young ‘94

contributes to the top mass:

\[ m_t = y H + \frac{x}{M^2} (H^\dagger H) H \]

induces a varying phase in \(m_t\) if \(xy^*\) is complex, with

\[
\theta \sim \frac{|II|^2 \text{Im}(x)}{M^2 y}
\]
The baryon asymmetry

for \( \text{Im}(x)=1 \) and 
\( v_w=0.01, 0.3 \)

\( \eta_B \) increases rapidly with smaller \( M \) because of the stronger PT prediction for EDMs with M. Pospelov, A. Ritz → testable with next generation experiments!

→ LHC: deviations in the triple Higgs coupling?

Fromme, S.H. ‘06
**Classic: The MSSM**

strong PT from stop loops
→ right-handed stop mass \( \sim 100 \) GeV
left-handed stop mass \( \sim 1000 \) TeV

[Carena, Nardini, Quiros, Wagner]

CP violation from varying chargino mixing
\[
\theta(z) = \arg(M_{2\mu}^*) f(H_1(z), H_2(z))
\]

resonant enhancement of \( \eta \) for \( M_2 \sim \mu \)
chargino mass \( < \sim 300 \) GeV
large phases \( > 0.2 \) required
→ 1st and 2nd generation squarks
heavy to keep 1-loop EDMs small

→ “Split SUSY + light stop”

Konstandin, Prokopec, Schmidt, Seco ‘05
\( \nu_w = 0.05, M_2 = 200 \) GeV, maximal phase

\[
\text{obs: } \eta = 0.9 \times 10^{-10}
\]

similar but somewhat more optimistic
results in Carena, Quiros, Seco, Wagner ‘02
Cirigliano, Profumo, Ramsey-Musolf ‘06

→ scenario is tightly constrained!
Possible test: modified Higgs branching ratios, e.g. into two photons:

\[
\frac{(\sigma \times BR)/\sigma_{BR}^{\text{SM}}}{MSSM-\text{like}, m_h = 125 \text{ GeV}}
\]

[Cohen, Morrissey, Pierce 2012]
[see also e.g. Curtin, Jaiswal, Meade 2012]
singlets models contain cubic terms: $\sim$$\text{SHH}$
at tree-level $\rightarrow$ stronger PT
also new sources of CP violation

model building problems: domain walls vs.
destabilization of the weak scale

which model to take?
$Z_3$ symmetry (NMSSM)
$Z_{5,7}$ R-symmetries (nMSSM)
extra U(1)'s (ESSM, ...)
fat Higgs…

Pietroni '92
Davies, Froggatt, Moorhouse '96
S.H., Schmidt '98
Bastero-Gil, Hugonie, King, Roy, Vespati '00
Kang, Langacker, Li, Liu '04
Menon, Morrissey, Wagner '04
S.H., Konstandin, Prokopec, Schmidt '06
Balazs, Carena, Freitas, Wagner '07
(Profumo, Ramsey-Musolf, Shaughnessy '07)

problem with 1-loop EDM's remains!

Konstandin, S.H. '06

computation of bubble profiles?
in the general singlet model the **broken** minimum can be CP conserving, but the **symmetric** minimum violates CP

→ CP violating wall profile

CP conservation at $T=0$

\[ L_w = 20, 10, 5, 3 \]

\[ L_w \sim 3 \]

S.H., John, Laine, Schmidt ‘99

S.H., Schmidt ‘00
A composite Higgs model

Higgs arising as a pseudo Nambu Goldstone boson, eg. in
SO(6) → SO(5): Higgs + singlet  [Espinosa, Gripaios, Konstadin, Riva ‘11]

\[ V = V^{\text{even}} + V^{\text{odd}}, \]
\[ V^{\text{even}} \equiv -\mu_h^2 |H|^2 + \lambda_h |H|^4 - \frac{1}{2} \mu_s^2 s^2 + \frac{1}{4} \lambda_s s^4 + \frac{1}{2} \lambda_m s^2 |H|^2, \]
\[ V^{\text{odd}} \equiv \frac{1}{2} \mu_m s |H|^2 + \mu_1^3 s + \frac{1}{3} \mu_3 s^3, \]

CP violation:

\[ \frac{s}{f} H \bar{Q}_3(a + ib\gamma_5)t \]
Gravitational waves
Gravitational waves

sources of GW’s: direct bubble collisions
turbulence
magnetic fields

key parameters: available energy

\[ \alpha = \frac{\text{latent heat}}{\text{radiation energy}} \]

typical bubble radius

\[ \langle R \rangle \propto v_b \tau \approx \frac{v_b}{\beta} \]

\( v_b \) wall velocity: supersonic!?

Grojean, Servant ‘06
\[ f_{\text{turb}} = 3.4 \times 10^{-3} \text{mHz} \ u_s \left[ \frac{1}{RH_*} \right] \left[ \frac{T_*}{100 \text{GeV}} \right] \left[ \frac{g_*}{100} \right]^{1/6} \]

\[ h_0^2 \Omega_{\text{turb}} = 1.4 \times 10^{-4} u_s^5 (R H_*)^2 \left[ \frac{g_*}{100} \right]^{-1/3} \]

Turbulence: Kosowsky, Mack, Kahniashvili '01
Dolgov, Grasso, Nicolis '02

Turbulence: Caprini, Dürrer '06

Collisions: Kamionkovski, Kosowsky, Turner '93

*: quantities at the PT temperature

\( u_s \): eddy velocity (dep. on \( \alpha \))

strong PT: \( \alpha \sim 1 \), but RH. gets larger
(fewer bubbles, growing larger)

→ stronger signal, but at smaller \( f \) !!
Results in the $\Phi^6$ model

$GW \sim f^{-1.8} \Rightarrow GW \sim f^1$

(related to small bubbles!)

T. Konstandin, S.H. ‘08
Knowing the wall velocity is important:

- Strong gravitational wave signal only for $v_w > v_s$

$$h_0^2\Omega_{\text{det}} \simeq 1.2 \times 10^{-7} \kappa^2 \langle R \rangle^2 H_*^2 \left[ \frac{\alpha}{\alpha + 1} \right]^2 \left[ \frac{v_b^2}{0.24 + v_b^3} \right]^2 \left[ \frac{100}{g_*} \right]^{1/3}.$$

- Standard baryogenesis needs $v_w < v_s = 0.58$ (diffusion!)

- Sometimes the baryon asymmetry depends crucially on $v_w$

[Cline, Joyce, Kainulainen '01]
**The wall velocity:**

Friction with the plasma balances the pressure

Distinguish: supersonic vs. subsonic ($v_s^2 = 1/3$)

Standard model: $v_w \sim 0.35 - 0.45$ for low Higgs masses [Moore, Prokopec ‘95]

MSSM: $v_w \sim 0.05$ [John, Schmidt ‘00]

All other models: no detailed computations

*bubble walls can run away, i.e. approach $v_w = 1$* [Bodeker, Moore ‘09]
**How to compute the wall velocity?**

Main ingredients: pressure difference vs. plasma friction

Also important: reheating due to release of latent heat

**Microscopic description:** Moore, Prokopec ‘95

\[ \Box \phi + V'_T(\phi) + \sum \frac{dm^2}{d\phi} \int \frac{d^3p}{(2\pi)^3} 2E \delta f(p, x) = 0 \]

\[ \frac{df}{dt} = \partial_t f + \dot{x} \cdot \partial_x f + \dot{p} \cdot \partial_p f = -C[f] \]

\[
\begin{align*}
avw \frac{\mu'}{T} + v_w \frac{\delta T'}{T} + \frac{1}{3} v' + F_1 &= -\Gamma_{\mu 1} \frac{\mu}{T} - \Gamma_{T 1} \frac{\delta T}{T} \\
bv' \frac{\mu'}{T} + v_w \frac{\delta T'}{T} + \frac{1}{3} v' + F_2 &= -\Gamma_{\mu 2} \frac{\mu}{T} - \Gamma_{T 2} \frac{\delta T}{T} \\
b \frac{\delta T'}{T} + v_w v' + 0 &= -\Gamma_{v v}
\end{align*}
\]

→ Complicated set of coupled field equations and Boltzmann equations need many scattering rates

SM: \( v \sim 0.35 - 0.45 \)
**Simplified approach:** (Ignatius, Kajantie, Kurki-Suonio, Laine ‘94)

1) describe friction by a friction coefficient $\eta$

2) model the fluid by a fluid velocity and temperature

\[
\frac{d^2 \phi(x)}{dx^2} = \frac{\partial V(\phi, T)}{\partial \phi} + \frac{\phi^2}{T_s^1} v \frac{d\phi(x)}{dx} \\
(4aT^4 - T \frac{\partial V(\phi, T)}{\partial T}) \gamma^2 v = C_1 \\
(4aT^4 - T \frac{\partial V(\phi, T)}{\partial T}) \gamma^2 v^2 + P_r - V(\phi, T) + \frac{1}{2} \left( \frac{d\phi}{dx} \right)^2 = C_2
\]

← 1 + 1 dimensions

3) Determine $\eta$ from fitting the to the full result by Moore and Prokopec

[with Miguel Sopena, see also Megevand, Sanchez ’09]

→ the formalism should describe situations with SM friction well

→ study models with SM friction, but different potential, e.g. phi^6 model

→ same for the MSSM

→ such a model can be used for numerical simulations of the phase transition

[work in progress with Hindmarsh, Rummukainen, Weir]
Friction in relaxation time approximation:

\[ -\frac{(m^2)'}{2E} v \beta \gamma \frac{e^{\beta \gamma (E-vp_z)}}{(e^{\beta \gamma (E-vp_z)} \pm 1)^2} + \frac{p_z}{E} \delta f' = -\frac{\delta f}{\tau}. \]

\[ \frac{d^2 \phi}{dz^2} + \frac{\partial V(\phi, T)}{\partial \phi} + \sum \frac{dm^2}{d\phi} \int \frac{d^3 p}{(2\pi)^3 2E} \tau \beta \gamma \nu \frac{(m^2)'}{2E} \frac{e^{\beta \gamma (E-vp_z)}}{(e^{\beta \gamma (E-vp_z)} \pm 1)^2} = 0. \]

Tau needs to be fixed from the microscopic calculations
momentum integral can done numerically

\[ \Rightarrow \text{In essence gives } \eta(v/T) \text{ successfully capturing the Boltzmann factor} \]
Results for $\Phi^6$ model

- for a moderately strong phase transition, the wall moves subsonic in the dim-6 model (good for baryon asymmetry, bad for GW’s)
- relative mild dependence on the friction coefficient $\eta$
- unstable bubbles for some temperatures?
Wall velocity for $m_h=125$ GeV

Find runaway behavior

Good agreement between different ways of determining tau

“Moore, Prokopec”

“Bodeker, Moore”

SH, Sopena, to appear
MSSM: S.H., M. Sopena 2011

**Input:**

<table>
<thead>
<tr>
<th>( \tan \beta )</th>
<th>( m_U^2 )</th>
<th>( v_w ) (John and Schmidt)</th>
<th>Fitted ( \eta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>-60^2</td>
<td>0.060</td>
<td>4.58</td>
</tr>
<tr>
<td></td>
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<td>0.090</td>
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</tr>
<tr>
<td></td>
<td>+60^2</td>
<td>0.155</td>
<td>2.55</td>
</tr>
</tbody>
</table>

**Output:**

**1-loop potential**

- \( m_Q = 2000 \text{ GeV} \)

**2-loop potential**

- \( m_h = 112 \text{ GeV} \)

- For \( m_h = 125 \) we get \( v_w \sim 0.03 \)
viable models for a strong first order phase transition:

► 2HDM: $m_H \sim 300$ (transport by tops)

► SM with a dim-6 Higgs potential for $M < 800$ GeV

  (EDMs similar to 2HDM)

► MSSM: light stop for the phase transition (very constrained now!)

  transport by the charginos (instead of tops)

  severe constraints from EDMs

► Singlet models (NMSSM): many possibilities

  cubic terms in the tree-level potential induce a strong phase transition

  EDM constraints somewhat relaxed (or totally absent for transitional $\varphi\Gamma$)

► Gravitational waves: considerable uncertainties ($f^{-1}$ fall off!), probably difficult to detect

what else is the LHC going to find??
extended models have a large parameter space which is typically only partially explored

take into account additional constraints from dark matter, electroweak bounds, EDMs, etc.

establish a closer link to collider physics

computation of the wall velocities in extended models

more fancy models, such as Wilson line Higgs,…

what else is the LHC going to find??
Colliders vs. cosmology: nMSSM

[Balazs, Carena, Freitas, Wagner ‘07]

**Dark matter:**

(problem: large error on neutralino mass at LHC)

**Baryogenesis:**

Presence of *light charginos* could be shown, especially at ILC

LHC can see the *Higgs signal*, but difficult to identify the different Higgs states (ILC!)

ILC could determine *crucial parameters for the phase transition* $A_\lambda$, $t_s$, $m_s$ at 10-20%

(still not sufficient to establish a strong PT)

EDMs should (probably) be seen by next generation experiments

→ predicts *new physics* at LHC

Keep in mind: model dependence!! (only an example case)

(Also the non-SUSY singlet models have been studied recently, e.g. Profumo et al.)
Electroweak phase transition

Electroweak symmetry was (probably) restored in the early universe at

\[ T > \sim 100 \text{ GeV} \]
\[ t < \sim 1 \text{ nano second after the big bang} \]
by thermal effects

How did the symmetry break?

** first order phase transition (bubbles!)
* second order phase transition
* cross over
→ does depend on particle properties at the weak scale!
→ LHC!?
Depending on the question, there are different problems to be addressed, however two are universal:

* strength of the phase transition (size of the order parameter)

* bubble dynamics (nucleation, wall velocity,...)
1) Strength of the phase transition: under control
   strong phase transition from singlets, higher-dim operators, etc.
   2-loop, lattice for the 2HDM?

2) Wall velocity: unknown in most cases
   slow walls in the MSSM
   velocities on extended models (singlets, 2HDM,...)
   effect of infrared gauge field modes

3) Baryon asymmetry: good progress
   CP violation for mixing fermions (quantum Boltzmann eqs.)
   more realistic set of Boltzmann eqs. (Yukawas, etc…)
   supersonic baryogenesis, transitional CP violation
4) Gravitational waves: lot’s of activity recently
   requires supersonic bubbles
   how to model the source correctly?
   turbulence?
   full simulations?

5) Magnetic fields
   mechanisms for their generation?
   source of cosmic magnetic fields?
   effect on the phase transition, baryogenesis?
6) Model building
   NMSSM type, extra U(1)'s, E6SSM
   extra Higgses (2HDM,...)
   extra dimensional models (gauge-Higgs, AdS/CFT)
   little Higgs models

7) Collider and other signatures
   new particles at the LHC
   can one reconstruct the potential
   signals of CP violation
Further Aims:

Produce: up to date review on the physics of the electroweak phase transition?

Think about a follow up meeting next year
Summary

viable models:

► SM with a dim-6 Higgs potential for $M<800$ GeV and $m_h<170$ GeV
  (EDMs similar to 2HDM)

► 2HDM: $m_H>\sim 300$ and $m_h<\sim 200$ GeV

► MSSM: light stop for the phase transition and a Higgs mass $<\sim 120$ GeV
  transport by the charginos (instead of tops)
  severe constraints from EDMs

► Singlet models (NMSSM): many possibilities
  cubic terms in the tree-level potential induce a strong phase transition
  EDM constraints somewhat relaxed (or totally absent for transitional $\mathcal{CP}$)

► Gravitational waves: difficult to detect

what is the LHC going to find??
Outlook

- **extended models** have a large parameter space which is typically only partially explored
- take into account **additional constraints** from dark matter, electroweak bounds, EDMs, etc.
- establish a closer link to collider physics
- computation of the **wall velocities** in extended models
- more **fancy models**, such as Wilson line Higgs,…
Results for the PT

Evaluating the 1-loop thermal potential:

- strong phase transition for $M < 850$ GeV up to $m_h \sim 170$ GeV

(LEP bound applies, $m_h > 114$ GeV)

- wall thickness $2 < L_w T_c < 16$

Bödeker, Fromme, S.H., Seniuch ‘04
Baryogenesis in the nMSSM

$\lambda$ above Landau pole preferred:
(and $\tan \beta \sim 1$)

CP violation in $t_3 e^{i q} S$ (phase in $\mu$ parameter induced, not constant along the bubble wall)

EDM constraints with 1TeV sfermions (1. & 2. generation):

S.H., Konstandin, Prokopec, Schmidt ‘06
Gravitational waves in the nMSSM

Cannot confirm the much larger results of Apreda, Maggiore, Nicolis, Riotto '01 (important to compute the bubbles shapes correctly!)

Baryogenesis: strong GW signal only for $v_b > v_s$ → no diffusion. Baryogenesis would at least look different
The baryon asymmetry

for $\text{Im}(x)=1$ and $v_w=0.01, 0.3$

$\eta_B$ increases rapidly with smaller $M$
because of the stronger PT

prediction for EDMs with M. Pospelov, A. Ritz
→ testable with next generation experiments!

Fromme, S.H. ‘06
EDM from dim-6

x adjusted to get \( \eta = \eta_{\text{obs}} \):

Barr, Zee ‘90

Experimental bounds:

\[ d_e < 1.6 \times 10^{-27} \text{ e cm} \]
\[ d_n < 3.0 \times 10^{-26} \text{ e cm} \]

S.H., Pospelov, Ritz ‘06
1) operators which contribute to EW observables must be suppressed by $\Lambda >> M \sim \text{TeV}$, e.g.

$$\frac{1}{\Lambda^2} (H^\dagger D_{\mu} H)^2$$

with $\Lambda > 10 \text{ TeV} \rightarrow 1\%$ tuning required? \[Grojean, Servant, Wells \ '04\]

2) deviations from the SM cubic Higgs self coupling $\mu H^3$

LHC: order unity test

ILC: 20%
The baryon asymmetry

The relative phase between the Higgs vevs, $\theta$, changes along the bubble wall → phase of the top mass varies

$\theta_t = \theta / (1 + \tan^2\beta)$

top transport generates a baryon asymmetry, but $\tan\beta < 10$ (?) → only one phase, so EDMs can be predicted: here $d_n = 0.1 \times 10^{-26} - 7 \times 10^{-26}$ e cm

exp. bound: $d_n < 3.0 \times 10^{-26}$ e cm

$\eta_B$ in units of $10^{-11}$, $\varphi = 0.2$

Moretti et al. ‘07: LHC could see a triple Higgs coupling $Hhh$
The phase transition

Evaluate 1-loop thermal potential:

loops of heavy Higgses generate a cubic term

→ strong PT for
  m_H > 300 GeV
  m_h up to 200 GeV
→ PT ~ independent of Φ
→ thin walls only for very strong PT (agrees with Cline, Lemieux ’96)

missing: 2-loop analysis of the thermal potential; lattice; wall velocity

[Fromme, S.H., Senuich ’06]
Extra U(1)´s

Kang, Langacker, Li, Liu ’04

\[ W_H = h S H_d H_u + \lambda S_1 S_2 S_3 \]

Ham, Oh ’07

\[ W \approx h_t Q H_2 t_R^c + \lambda N H_1^T \epsilon H_2 \]

Strong phase transition possible
No computation of the BAU
Examples have large \( \lambda = 0.7, 0.8 \)

\[ m_{S_1 S_2}^2 \equiv |m_{S_1 S_2}^2| e^{i\gamma} \]

thin wall approximation used,
tau lepton contribution only
\[ f_{\text{turb}} = 3.4 \times 10^{-3} \text{mHz} \ u_s \left[ \frac{1}{RH_*} \right] \left[ \frac{T_*}{100 \text{GeV}} \right] \left[ \frac{g_*}{100} \right]^{1/6} \]

\[ h_0^2 \Omega_{\text{turb}} = 1.4 \times 10^{-4} u_s^5 (RH_*)^2 \left[ \frac{g_*}{100} \right]^{-1/3} \]

Turbulence: Kosowsky, Mack, Kahniashvili '01
Dolgov, Grasso, Nicolis '02
Caprini, Dürrer '06

Collisions: Kamionkovski, Kosowsky, Turner '93

*: quantities at the PT temperature
u_s: eddy velocity (dep. on \( \alpha \))

strong PT: \( \alpha \sim 1 \), but RH grows larger
(fewer bubbles, growing larger)

→ stronger signal, but at smaller \( f \)!!

discrepancy between Dol. and CD!
double peak structure?
Dynamics of the transition

At the critical temperature $T_c$ the two minima are degenerate.

Bubble nucleation starts at $T < T_c$ with a rate

$$\Gamma = A T^4 e^{-S_3/T},$$

where the bubble energy is

$$S_3 = 4\pi \int dr \, r^2 \left[ \frac{1}{2} \left( \frac{d\varphi}{dr} \right)^2 + V(\varphi, T) \right]$$

The bubble configuration follows from

(with appropriate BC’s)

This bounce solution is a saddle point, not a minimum

→ difficult to compute for multi field models (one field: shooting)

For a algorithm see Konstandin, S.H. ‘06
Key parameters of the phase transition: $\Phi^6$ model, $m_h=120$ GeV

Compute as function of temperature: bubble configurations $\rightarrow E$

\[ V(H) = -\mu^2 |H|^2 + \lambda |H|^4 + \frac{1}{M^2} |H|^6 \]

nucleation rate $\Gamma \sim \exp(-E)$

bubble distribution $\rightarrow R$
The wall velocity:

Friction with the plasma balances the pressure

Distinguish: supersonic vs. subsonic ($v_s^2=1/3$)

Standard model: $v_b \sim -0.35 - 0.45$ for low Higgs masses [Moore, Prokopec ‘95]

MSSM: $v_b \sim 0.05$ [John, Schmidt ‘00]

All other models: no detailed computations

For very strong phase transitions: bubbles become supersonic, velocity dominated by hydrodynamics (neglect friction) [Steinhardt ‘82]

\[ v_b(\alpha) = \frac{1/\sqrt{3} + \sqrt{\alpha^2 + 2\alpha/3}}{1 + \alpha}. \]  

(for sufficiently large $\alpha >$ few %?)

When does this fail ??

**Recently: can the walls run away? [Bodeker, Moore ‘09]**
1) operators which contribute to EW observables must be suppressed by $\Lambda \gg M_{\sim \text{TeV}}$, e.g.

$$\frac{1}{\Lambda^2} (H^\dagger D_{\mu} H)^2$$

with $\Lambda > 10 \text{ TeV} \rightarrow 1\%$ tuning required?

Grojean, Servant, Wells ‘04

2) deviations from the SM cubic Higgs self coupling $\mu H^3$

LHC: order unity test

ILC: 20%
How to compute the wall velocity?

Main ingredients: pressure difference vs. plasma friction
Also important: reheating due to release of latent heat

Microscopic description: Moore, Prokopec ‘95

\[ \square \phi + V'_T(\phi) + \sum \frac{dm^2}{d\phi} \int \frac{d^3p}{(2\pi)^3} \frac{\delta f(p, x)}{2E} = 0 \]

\[ f = \frac{1}{1 + \exp \frac{E - E_\delta T/T - p_z v - \mu}{T}} \]

(fluid ansatz)

\[ F_1 = -\frac{v_w \ln 2 (m^2)'}{9\zeta_3 T^2}, \quad F_2 = -\frac{v_w \zeta_2 (m^2)'}{42\zeta_4 T^2} \]

(force terms)

\[ F = \dot{p}_z = -\partial_z E(z, p_z) \]

→ Complicated set of coupled field equations
   and Boltzmann equations
   need many scattering rates, infrared gauge fields??

SM: \( v \sim 0.35 - 0.45 \)
Simplified approach: (Ignatius, Kajantie, Kurki-Suonio, Laine ‘94)

1) describe friction by a friction coefficient $1/\Gamma$

2) Model the fluid by a fluid velocity and temperature

\[
\phi''(x) = \frac{\partial V}{\partial \phi} + \frac{\nu \gamma}{\Gamma} \phi'(x)
\]

\[
(4aT^4 - T \frac{\partial V}{\partial T}) \gamma^2 v = \text{const.}
\]

\[
(4aT^4 - T \frac{\partial V}{\partial T}) \gamma^2 v^2 + aT^4 + \frac{1}{2} \phi'(x)^2 - V = \text{const.}
\]

3) Determine $\Gamma$ from fitting the to the full result by Moore and Prokopec
   (with Miguel Sopena)

   We find: a good fit with a universal $\Gamma$ is possible

   → the formalism should describe situations with SM friction well
   → study models with SM friction, but different potential, e.g. phi^6 model

   see also Megevand, Sanchez ‘09
After understanding the phase transition:

What can we learn from it?
The baryon asymmetry
antimatter?
Classic: The MSSM

strong PT from stop loops
→ right-handed stop mass below $m_{\text{top}}$
  left-handed stop mass above 1 TeV
  to obtain $m_h \sim 115$ GeV [Carena et al. '96]

CP violation from varying chargino mixing:
$$\theta(z) = \arg (M_2 \mu^*) f(z)$$
$$m = \begin{pmatrix} M_2 & gH_2 \\ gH_1 & \mu_c \end{pmatrix}$$
resonant enhancement of $\eta$ for $M_2 \sim \mu$
wall velocity $\sim 0.05$ [John, Schmidt '00]
large phases $> 0.2$ required
→ 1st and 2nd generation squarks
  heavy to keep 1-loop EDMs small

→ “Split SUSY + light stop”

Konstandin, Prokopec, Schmidt, Seco '05

$v_w=0.05$, $M_2=200$ GeV, maximal phase

$\eta = 0.9 \times 10^{-10}$

obs: $\eta = 0.9 \times 10^{-10}$

similar but somewhat more optimistic
results in Carena, Quiros, Seco, Wagner '02
Cirigliano, Profumo, Ramsey-Musolf '06
→ scenario is tightly constrained!
SM + higher-dim. operators

\[ V(H) = -\mu^2 |H|^2 + \lambda |H|^4 + \frac{1}{M^2} |H|^6 \]

maybe related to strong dynamics at the TeV scale, such as technicolor or gravity?

two parameters, \((\lambda, M) \leftrightarrow (m_h, M)\)

\(\lambda\) can be negative \(\rightarrow\) bump because of \(|H|^4\) and \(|H|^6\): \(M < \sim 800 \text{ GeV}\)

**CP violation:**

\[ \frac{x}{M^2} (H^\dagger H) H t^c q \]

contributes to the top mass: \(m_t = y H + \frac{x}{M^2} (H^\dagger H) H\)

induces a varying phase in \(m_t\) if \(xy^*\) is complex, with \(\theta \sim \frac{|H|^2 \text{Im}(x)}{M^2 y}\)

Can produce the baryon asymmetry without violating EDM bounds

Zhang ‘93
Grojean, Servant, Wells ‘04

Zhang, Lee, Whisnant, Young ‘94

Bödeker, Fromme, S.H., Seniuch ‘04
S.H., Pospelov, Ritz ‘06
Strong phase transition

Singlet model without discrete symmetries

\[ W = \lambda S H_1 H_2 + \frac{k}{3} S^3 + \mu H_1 H_2 + r S \]

nMSSM

\[ W_{nMSSM} = \lambda \hat{S} \hat{H}_1 \cdot \hat{H}_2 + \frac{m_{12}^2}{\lambda} \hat{S} \]

S.H., Schmidt ‘00

Menon, Morrissey, Wagner ‘04

S.H., Konstandin, Prokopec, Schmidt ‘06
**MSSM + “singlets”**

Singlets models contain cubic (SHH) terms at tree-level → stronger PT

Also new sources of CP violation

**Model building problems:** domain walls vs. destabilization of the weak scale

Which model to take?

$Z_3$ symmetry (NMSSM)

$Z_{5,7}$ R-symmetries (nMSSM)

Extra U(1)'s (ESSM, ...)

Fat Higgs...

Pietroni ’92

Davies, Froggatt, Moorhouse ’96

S.H., Schmidt ’98

Bastero-Gil, Hugonie, King, Roy, Vespati ’00

Kang, Langacker, Li, Liu ’04

Menon, Morrissey, Wagner ’04

S.H., Konstandin, Prokopec, Schmidt ‘06

Balazs, Carena, Freitas, Wagner ‘07

(Profumo, Ramsey-Musolf, Shaughnessy ‘07)

Carena, Shaw, Wagner ‘11

Computation of bubble profiles?

Konstandin, S.H. ’06

Problem with 1-loop EDM’s remains!
Is there antimatter in the universe?

We can “easily“ produce antimatter in particle colliders

Is there natural antimatter?

1) Direct search: balloon experiments

BESS has detected over 2000 antiprotons (well explained by particle collisions)

But: in 10 million helium nuclei there was not a single antihelium

→ there is almost no antimatter in our cosmic neighbourhood

BESS, first flight 1993

June 2006: satellite mission PAMELA
2) Indirect search for gamma rays from annihilation at the boundaries of matter-antimatter domains

Even anti-galaxies or clusters would not be completely separated!

domains of antimatter do not fit the observed gamma ray spectrum

→ there is virtually no antimatter in the universe!

Cohen, De Rujula, Glashow, astro-ph/9707087

Similar: gamma rays from colliding cluster, e.g. bullet cluster: no antimatter at the scale of tens of Mpc
[Steigman arXiv:0808.1122]