Time-dependent features of primordial power spectrum

Effects of heavy fields on inflaton two-point correlation function

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Jinsu Kim

Institute for Theoretical Physics, Georg-August University Göttingen

with Laura Covi and Francesco Costa

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Main Objectives

1. Quantum effects of heavy fields on the inflationary two-point correlation function

2. Initial-time-dependence of the primordial power spectrum

Generic Model

Inflation Model

[fluctuations]

φ : inflaton, σ : massive scalar field, ψ : massive fermion field

[potential]

$$V = V_{\rm inf} + \frac{1}{2}M_s^2\sigma^2 + \frac{1}{4!}\lambda_\sigma\sigma^4 + \mu\varphi\sigma^2 + \lambda\varphi^2\sigma^2 + M_f\bar{\psi}\psi + Y\varphi\bar{\psi}\psi$$

$$M_s/H \gg 1\,, \qquad M_f/H \gg 1$$

Quantum Corrections

At one-loop order, there are three types of diagrams that contribute to the inflaton two-point correlation function:

[Computation:]

Schwinger-Keldysh Formalism

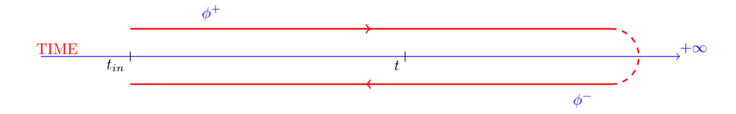
Schwinger-Keldysh Formalism

The Schwinger-Keldysh formalism:

- a.k.a. the closed-time-path formalism or in-in formalism.
- appropriate to study out-of-equilibrium quantum field theories.

Our main interest is the expectation value of an operator at a given time:

$$\langle \mathcal{O}(t) \rangle = \langle \mathrm{in} | \left[\overline{T} \exp \left(i \int_{t_i}^{t} dt' H(t') \right) \right] \mathcal{O}(t) \left[T \exp \left(-i \int_{t_i}^{t} dt' H(t') \right) \right] | \mathrm{in} \rangle \,,$$



[Schwinger, J. Math. Phys. **2**, 407 (1961)]

[Keldysh, Zh. Eksp. Teor. Fiz. **47**, 1515 (1964)]

Figure from [Covi and Dresti, arXiv:1803.02351]

Schwinger-Keldysh Formalism

Four two-point correlation function:

$$\begin{aligned} G^{++}(x,x') &= i\langle T_{\mathcal{C}}\phi^{+}(x)\phi^{+}(x')\rangle = i\langle T\phi(x)\phi(x')\rangle\,, \\ G^{+-}(x,x') &= i\langle T_{\mathcal{C}}\phi^{+}(x)\phi^{-}(x')\rangle = i\langle \phi(x')\phi(x)\rangle\,, \\ G^{-+}(x,x') &= i\langle T_{\mathcal{C}}\phi^{-}(x)\phi^{+}(x')\rangle = i\langle \phi(x)\phi(x')\rangle\,, \\ G^{--}(x,x') &= i\langle T_{\mathcal{C}}\phi^{-}(x)\phi^{-}(x')\rangle = i\langle \overline{T}\phi(x)\phi(x')\rangle\,. \end{aligned}$$

They are related through

$$G^{++}(x,x') + G^{--}(x,x') = G^{+-}(x,x') + G^{-+}(x,x')\,.$$

Schwinger-Keldysh Formalism

Convenient to go to the so-called Keldysh basis:

$$\phi^{(1)} = \frac{1}{2}(\phi^+ + \phi^-)\,, \qquad \phi^{(2)} = \phi^+ - \phi^-\,.$$

Propagators in the Keldysh basis:

$$\begin{aligned}
G^{11}(x,x') &= \frac{1}{2}\Big[G^{+-}(x,x') + G^{-+}(x,x')\Big]\,, \\
G^{12}(x,x') &= \theta(t-t')\Big[G^{-+}(x,x') - G^{+-}(x,x')\Big]\,, \\
G^{21}(x,x') &= \theta(t'-t)\Big[G^{+-}(x,x') - G^{-+}(x,x')\Big]\,, \\
G^{22}(x,x') &= 0\,.
\end{aligned}$$

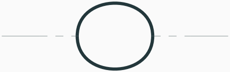
- $\rightarrow$ $F = -iG^{11}$: Hadamard (or Keldysh) propagator
- $\rightarrow$ $G^R = G^{12}$: Retarded propagator
- $\rightarrow$ $G^A = G^{21}$: Advanced propagator

[Propagators:]

F G^R G^A

Quantum Corrections in Schwinger-Keldysh Formalism

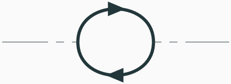
[One-loop diagrams:]


(a)


(b)


(c)


(d)


(e)


(f)


(g)

Primordial Power Spectrum

The full one-loop primordial power spectrum is given by

$$\mathcal{P}(k) = \frac{k^3}{8\pi^2 \epsilon M_{\mathrm{P}}^2} \left[P_0(k) + P_1(k)\right] .$$

$P_0(k) = F_\phi(k,\tau,\tau)$: the tree-level two-point correlation function

One-loop correction:

$$\begin{aligned} P_1(k) \quad = \quad & \int_{\tau_i}^{\tau} d\tau_1 \int_{\tau_i}^{\tau} d\tau_2 \, \left[-iG_\phi^R(k,\tau,\tau_1)\right] \left[-iG_\phi^R(k,\tau,\tau_2)\right] A_{GG} \\ & + \int_{\tau_i}^{\tau} d\tau_1 \int_{\tau_i}^{\tau} d\tau_2 \, \left[-iG_\phi^R(k,\tau,\tau_1)\right] \left[F_\phi(k,\tau,\tau_2)\right] A_{GF} , \end{aligned}$$

τ_i : initial (conformal) time $\qquad$ τ : horizon-crossing time

UV Behaviour

The UV divergences are contained in A_{GF}:

$$\left(Y^2 - \frac{\lambda}{2}\right)\Lambda^2 + \left(\frac{\mu^2}{2} + \frac{\lambda M_s^2}{2}\right)\ln\frac{\Lambda}{M_s} - 3Y^2 M_f^2 \ln\frac{\Lambda}{M_f}\,.$$

If there are d_s (d_f) scalar (fermion) degrees of freedom which independently contribute to the one-loop diagrams:

$$\left(Y^2 d_f - \frac{\lambda}{2} d_s\right)\Lambda^2 + d_s\left(\frac{\mu^2}{2} + \frac{\lambda M_s^2}{2}\right)\ln\frac{\Lambda}{M_s} - 3Y^2 M_f^2 d_f \ln\frac{\Lambda}{M_f}\,.$$

Application

Hybrid Inflation

We apply our results to the supersymmetric hybrid inflation model

$$V_{\rm inf} \simeq \lambda_h^2 M_G^4 + M_{SB}^4 + \frac{\lambda_h^4 M_G^4}{16\pi^2} \ln\left(\frac{\lambda_h^2 \phi^2}{2\mu_\Lambda^2}\right) ,$$

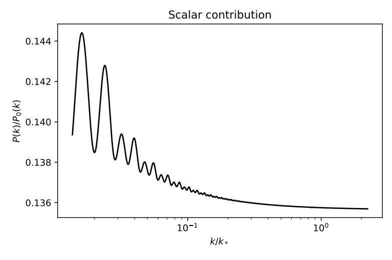
The interaction terms between the inflaton and the heavy scalar and fermion fields are described by,

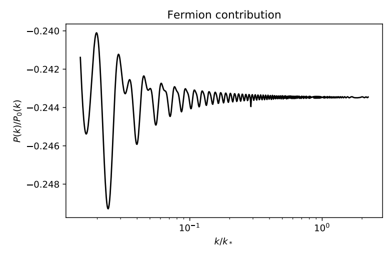
$$\mathcal{L}_{\rm int} \ni -\frac{\lambda_h^2}{4}\Phi^2\Sigma_i^2 + \frac{\lambda_h}{\sqrt{2}}\Phi\overline{\chi}\chi\,, \quad (i = 1, 2, 3, 4)$$

[Buchmüller, Covi, and Delepine, Phys. Lett. B **491**, 183 (2000)]

Hybrid Inflation

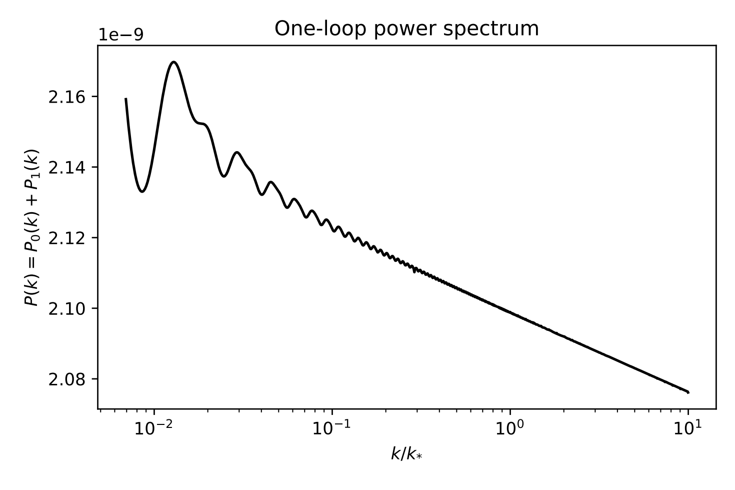
[One-loop contributions:]





$$\left|\frac{P_1}{P_0}\right| \sim \mathcal{O}(10^{-1})$$

[Full one-loop primordial power spectrum:]



Summary

Summary

1. Quantum effects of heavy fields on the inflationary two-point correlation function
    - Schwinger-Keldysh formalism
    - Hybrid inflation $\sim \mathcal{O}(10^{-1})$

2. Initial-time-dependence of the primordial power spectrum
    - Oscillatory behaviour in primordial power spectrum

Thank you