Flavoured Leptogenesis from Nonequilibrium QFT

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in collaboration with

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Outline

- Introduction
- Closed Time Path (CTP) Formalism of Noneq. QFT
- CTP Approach to Flavoured Leptogenesis
- Numerical Results
- Conclusions and outlook
Why baryogenesis?

To explain the excess of matter over antimatter in the universe:

\[ \frac{n_B}{n_\gamma} = \left(6.1^{+0.3}_{-0.2}\right) \times 10^{-10} \]

Why is there something rather than nothing?
Leptogenesis [Fukugita, Yanagida (1986)]

- The Standard Model is extended by adding heavy right-handed Majorana neutrinos $N_i, i = 1, 2, \ldots$ (eg. see-saw models)

$$\mathcal{L} \ni -h_{ab}^* \phi \bar{\psi}_b P_R \psi_R a - Y_{ia}^* \bar{\psi}_{la}(\epsilon \phi)\dagger \psi_{Ni} - \frac{1}{2} \bar{\psi}_{Ni} M_i \psi_{Ni} + h.c.$$  

- Lepton asymmetry is generated through out-of-equilibrium $L$- and $CP$-violating Yukawa decays: $N_1 \rightarrow \ell \phi$
Nonequilibrium QFT Approach to Leptogenesis

1related / complementary aspects in:
Buchmüller, Fredenhagen (2000); De Simone, Riotto (2007);
Garny, Hohenegger, Kartavtsev, Lindner (2009 & 2010);
Anisimov, Buchmüller, Drewes, Mendizabal (2010);
Anisimov, Besak, Bödeker (2010)
Closed Time Path* (CTP) Formalism
* a.k.a. Schwinger-Keldysh Formalism [Schwinger (1961); Keldysh (1964); Calzetta, Hu (1988)]

- Usually in QFT matrix elements are computed within the in-out framework:

\[
\langle \text{out}|\hat{S}|\text{in} \rangle \quad \leftarrow \quad \text{time-ordered correlators:} \quad \langle T[\psi(x)\bar{\psi}(y)] \rangle
\]

- In leptogenesis we want to calculate expectation values in a finite density medium, e.g.

\[
j^0(x) = \langle \bar{\psi}(x)\gamma^0\psi(x) \rangle \equiv \text{tr} \left[ \hat{\rho} \bar{\psi}(x)\gamma^0\psi(x) \right]
\]

- \(\hat{\rho}\) is an (unknown) quantum density operator
In-In generating functional on a Closed Time Path:

Path-ordered correlators:

\[ iS_C(x, y) = \langle T_C[\psi(x)\bar{\psi}(y)] \rangle \]

Four propagators with respect to real time variable:

\[ iS^{++}(u, v) = iS^T(u, v) = \langle T(\psi(u)\bar{\psi}(v)) \rangle \]
\[ iS^{+-}(u, v) = iS^<(u, v) = -\langle \bar{\psi}(v)\psi(u) \rangle \]
\[ iS^{-+}(u, v) = iS^>(u, v) = \langle \psi(u)\bar{\psi}(v) \rangle \]
\[ iS^{--}(u, v) = iS^{\bar{T}}(u, v) = \langle \bar{T}(\psi(u)\bar{\psi}(v)) \rangle \]
### Kadanoff-Baym Equations

- **Generic Schwinger-Dyson equations for (CTP) propagators:**

\[
\begin{align*}
G &= G_0 + \Sigma \\
\end{align*}
\]

- **Kadanoff-Baym equations** are the \(<, >\) components:

\[
(i\partial - m)S^{<,>} - \Sigma^H \circ S^{<,>} - \Sigma^{<,>} \circ S^H = \frac{1}{2} (\Sigma^{>} \circ S^{<} - \Sigma^{<} \circ S^{>})
\]

- \((A \circ B)(u, v) \equiv \int d^4w A(u, w)B(w, v)\) denotes convolution

- Renormalization, thermal corrections (thermal masses etc.)

- Finite width effects

- Collision term
Approximations

Wigner representation:

\[ S(k, x) = \int d^4 r \, e^{i k \cdot r} \, S(x + \frac{r}{2}, x - \frac{r}{2}) , \]

and gradient expansion to the lowest order in

- \( x \)-derivatives: \( \partial_x S(k, x) \), \( \partial_x \Sigma(k, x) \), etc.
- coupling constants in \( \Sigma(k, x) \)

\[ \rightarrow \text{Constraint and Kinetic Equations:} \]

\[ 2 (k^0 - k \cdot \gamma^0) i \gamma^0 S^{<,>}_{\ell} - \left\{ \Sigma^H_{\ell} \gamma^0, i \gamma^0 S^{<,>}_{\ell} \right\} - \left\{ i \Sigma^<,> \gamma^0, \gamma^0 S^H_{\ell} \right\} = -\frac{1}{2} \left( i C_\ell - i C^\dagger \right) \]

\[ i \partial_\eta i \gamma^0 S^{<,>}_{\ell} - \left[ \Sigma^H_{\ell} \gamma^0, i \gamma^0 S^{<,>}_{\ell} \right] - \left[ i \Sigma^<,> \gamma^0, \gamma^0 S^H_{\ell} \right] = -\frac{1}{2} \left( i C_\ell + i C^\dagger \right) \]

- \( \eta \) is conformal time variable: \( d\eta = dt/a(t) \)
The zeroth order solutions to **Contraint Equation**:

- **Flavour covariant free propagators for \( \ell \):**

  \[
i S_{\ell ab}^{<}(k, \eta) = -2\pi \delta(k^2)k \left[ \vartheta(k_0)f^{+}_{\ell ab}(k, \eta) - \vartheta(-k_0)(1_{ab} - f^{-}_{\ell ab}(-k, \eta)) \right]
  \]

  \[
i S_{\ell ab}^{>}(k, \eta) = -2\pi \delta(k^2)k \left[ -\vartheta(k_0)(1_{ab} - f^{+}_{\ell ab}(k, \eta)) + \vartheta(-k_0)f^{-}_{\ell ab}(-k, \eta) \right]
  \]

- \( f^{\pm}_{\ell ab}(k, \eta) \) are time-dependent distribution functions

- \( a, b \) are flavour indices

- Similar (unflavoured) free propagators for \( N_1 \) and \( \phi \)
Contributions to Lepton Collision term

- Y-Yukawa interactions:

- MSM h-Yukawa and gauge interactions:
Kinetic Equation for Lepton Number Densities


Substituting free propagators into Kinetic Equation

First order equation for $n_{\ell ab}^\pm(\eta) = \int \frac{d^3k}{(2\pi)^3} f_{\ell ab}^\pm(k, \eta)$:

$$\frac{\partial \delta n_{\ell ab}^\pm}{\partial \eta} = \sum_c [\Xi_{ac}^\text{eff} \delta n_{\ell cb}^\pm - \delta n_{\ell ac}^\pm \Xi_{cb}^\text{eff}] \mp i \Delta \omega_{\ell ab}^\text{eff} \delta n_{\ell ab}^\pm$$

$$- \sum_c [W_{ac} \delta n_{\ell cb}^\pm + \delta n_{\ell ca}^\pm* W_{bc}^*] \pm S_{ab} - \Gamma_{\ell bl} (\delta n_{\ell ab}^+ + \delta n_{\ell ab}^-) - \Gamma_{\ell lab}^\text{fl}$$

- Gradients of the mixing matrices: $\Xi^\text{eff} \sim U^\dagger \partial \eta U$

- Flavor oscillations by the thermal masses: $\Delta \omega_{\ell}^\text{eff} \sim h^2 T \ , \ Y^2 T$

- Collision terms: Washout, $CP$-violating source, Flavour-blind damping, Flavour-sensitive damping
Fast Flavour-blind Gauge Interactions

- **Blue cut** ⟷ tree-level pair creation and annihilation and $1 \leftrightarrow 2$ scatterings
- Kinematically forbidden for on-shell excitations $\rightarrow \Gamma^{bl} \sim g_2^4 T$

- Force **kinetic equilibrium** with generalized chemical potentials $\mu_{ab}^\pm(\eta)$:

$$f_{\ell ab}^\pm(k, \eta) = \left( \frac{1}{e^{\beta|k|} - e^{\beta\mu_{ab}^\pm} + 1} \right)_{ab}$$

- Flavour and momentum dependence **factorizes** to first order in $\mu_{ab}^\pm$:

$$\delta n_{\ell ab}^\pm = \mu_{ab}^\pm \frac{T^2}{12} \quad \Longrightarrow \quad \delta f_{\ell ab}^\pm(k, \eta) = \delta n_{\ell ab}^\pm \frac{12 \beta^3 e^{\beta|k|}}{(e^\beta|k| + 1)^2}$$

$\rightarrow$ Simple interaction terms in the kinetic equation
Suppression of Flavour Oscillations

- Toy equations for flavour oscillations

\[
\frac{\partial \delta n_{\ell ab}^\pm}{\partial \eta} = \mp i \Delta \omega_{\ell ab} \delta n_{\ell ab}^\pm - \Gamma^{bl}(\delta n_{\ell ab}^+ + \delta n_{\ell ab}^-)
\]

- Parametrically \( \Gamma^{bl} \gg \Delta \omega_{\ell}^{\text{eff}} \)
Suppression of Flavour Oscillations

- Toy equations for flavour oscillations

\[
\frac{\partial \delta n_{\ell ab}^\pm}{\partial \eta} = \mp i \Delta \omega_{\ell ab}^\text{eff} \delta n_{\ell ab}^\pm - \Gamma_{bl}^\text{bl} (\delta n_{\ell ab}^+ + \delta n_{\ell ab}^-)
\]

- Parametrically \( \Gamma_{bl}^\text{bl} \gg \Delta \omega_{\ell}^\text{eff} \)

→ Two over-damped solutions with short and long time scales:

- Short mode with \( \tau_{\text{short}} = 1/(2\Gamma_{bl}^\text{bl}) \) forces an effective constraint:

\[
\delta n_{ab}^- = -\delta n_{ab}^+
\]

- Flavour oscillations of \( \delta n_{ab}^+ \) are over-damped with a long decay time:

\[
\tau_{\text{damp}} \sim 2\Gamma_{bl}^\text{bl}/(\Delta \omega_{\ell ab}^\text{eff})^2
\]
Final Kinetic Equations

- Lepton asymmetries \( q \equiv \delta n^+ - \delta n^- \) (L- and R-handed):

\[
\frac{\partial q_{\ell ab}}{\partial \eta} = \sum_c \left[ \Xi_{ac} q_{\ell cb} - q_{\ell ac} \Xi_{cb} - W_{ac} q_{\ell cb} - q_{\ell ac} W_{cb} \right] - \Gamma_{\ell ab}^f + 2S_{ab}
\]

\[
\frac{\partial q_{Rab}}{\partial \eta} = -\Gamma_{Rab}^f
\]

- Majorana neutrino \( N_1 \):

\[
\frac{\partial f_{N1}(k_{\text{com}})}{\partial \eta} = -2|Y_1|^2 \frac{k_{\text{com}} \mu}{2k_{\text{com}} 0} \Sigma_N^\mu(k_{\text{com}}) \left[ f_{N1}(k_{\text{com}}) - f_{N1}^{\text{eq}}(k_{\text{com}}) \right]
\]

- Higgs field \( \phi \) remains in thermal equilibrium
Lepton Asymmetries: \[ Y_{\ell ab} = \frac{n_{\ell ab}^+ - n_{\ell ab}^-}{s} \], Fully Flavoured case

\[ h_\tau = 0.030 \]

- **dark blue:** \( Y_{\ell 11} \)
- **light blue:** \( Y_{\ell 22} \)
- **brown dotted:** \( \text{Re}[Y_{\ell 12}] \)
- **red dashed:** \( \text{Im}[Y_{\ell 12}] \)

\[ M_1 = 10^{12} \text{ GeV}, \quad M_2 = 10^{14} \text{ GeV} \]

\[ Y_{\text{Yuk}} = \begin{pmatrix} 1.4 \times 10^{-2} & 1 \times 10^{-2} \\ i \times 10^{-1} & 10^{-1} \end{pmatrix} \]

\( Y_{\ell 12} = Y_{\ell 21}^* \) strongly suppressed before freeze-out at \( z = \frac{M_1}{T} \approx 10 \)

\[ \rightarrow \text{Flavour off-diagonals } Y_{\ell 12,21} \text{ can be neglected} \]
Lepton Asymmetries: \( Y_{lab} = \frac{n_{lab}^+ - n_{lab}^-}{s} \), Unflavoured case

![Graph showing lepton asymmetries](image)

- **dark blue**: \( Y_{\ell 11} \)
- **light blue**: \( Y_{\ell 22} \)
- **brown dotted**: \( \text{Re}[Y_{\ell 12}] \)
- **red dashed**: \( \text{Im}[Y_{\ell 12}] \)

\[ M_1 = 10^{12} \text{ GeV}, \; M_2 = 10^{14} \text{ GeV} \]

\[ Y_{\text{Yuk}} = \begin{pmatrix} 1.4 \times 10^{-2} & 1 \times 10^{-2} \\ i \times 10^{-1} & 10^{-1} \end{pmatrix} \]

- \( Y_{\ell 12} = Y_{\ell 21}^* \) decay away long after freeze-out at \( z \approx 10 \)

\( \rightarrow \) Flavour damping by \( \Gamma_{\ell}^{\text{fl}} \) can be neglected
Lepton Asymmetries: $Y_{\ell_{ab}} = \frac{n_{\ell_{ab}}^+ - n_{\ell_{ab}}^-}{s}$, Intermediate regime

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- dark blue: $Y_{\ell_{11}}$
- light blue: $Y_{\ell_{22}}$
- brown dotted: $\text{Re}[Y_{\ell_{12}}]$
- red dashed: $\text{Im}[Y_{\ell_{12}}]$

$M_1 = 10^{12} \text{ GeV}, M_2 = 10^{14} \text{ GeV}$

$Y_{\text{Yuk}} = \begin{pmatrix} 1.4 \times 10^{-2} \\ i \times 10^{-1} \\ 1 \times 10^{-2} \end{pmatrix}$

- Full kinetic equations need to be solved!
Total Lepton Asymmetry as a function of $\Gamma^\text{fl}_\ell$

- Scale $M_{1,2}$ and the couplings $Y_{\text{Yuk}}$ such that $\Gamma^\text{fl}_\ell$ is varying while the washout and source terms remain constant.

![Graph showing the relationship between $\text{scal} + M_1$ and the total lepton asymmetry.](image)

- **Fully flavoured**
- **Full kinetic equations**
- **Unflavoured**
Conclusions

- First principle description of leptogenesis within the CTP framework
  - RIS subtraction procedure not required
- Simple kinetic equations for lepton number densities, including
  - Quantum statistical corrections in loops and external states
    - Sizable for weak washout
- Flavour effects
  - Flavour oscillations are overdamped by fast gauge interactions
  - Full flavoured equations are needed between fully flavoured and unflavoured regimes
Outlook

- Systematic inclusion of thermal effects
  - Finite widths
  - Thermal masses
- Spectator processes
- Resonant leptogenesis
  - Flavour coherence effects between Neutrinos $N_i$
Washout contribution

\[ \frac{1}{2} \text{tr} \int \frac{d^3k}{(2\pi)^3} \int_0^\infty \frac{dk_0}{2\pi} C_{\ell ab}^Y = - \sum_c W_{ac} \delta n_{cb}^+ \]

with

\[ W_{ac} = \frac{1}{2} Y_{1a}^* Y_{1c} \int \frac{d^3k}{(2\pi)^3 2|k|} \frac{d^3k'}{(2\pi)^3 2\sqrt{k'^2 + (a(\eta)M_1)^2}} \frac{d^3k''}{(2\pi)^3 2|k''|} (2\pi)^4 \delta^4(k' - k - k'') \]

\[ \times 2k \cdot k' \left( f_{N1}(k') + f_\phi(k'') \right) \frac{12\beta^3 e^{\beta|k|}}{(e^{\beta|k|} + 1)^2} \]

Blue cut \[\leftrightarrow\] tree-level decays and inverse decays \[N_1 \leftrightarrow \ell \phi\]
**CP-violating Source $S_{ab}$: Wave-function Contribution**

- **Orange cut** ⟷ Interference between two $s$-channel scatterings
- **Blue cut** ⟷ Interference between loop and tree-level decays

\[
S_{ab}^{\text{wf}} = i \sum_c \left[ Y_1^* a Y_1^* c Y_2 Y_{2b} - Y_2^* a Y_2^* c Y_1 Y_{1b} \right] \\
\times \left( -\frac{M_1}{M_2} \right) \int \frac{d^3 k'}{(2\pi)^3 2 \sqrt{k'^2} + (a(\eta)M_1)^2} \frac{\Sigma_N^\mu(k') \Sigma_N^\mu(k')}{g_w} \delta f_{N1}(k')
\]

where the **thermal decay rate** is

\[
\Sigma^\mu_N(k) = g_w \int \frac{d^3 p}{(2\pi)^3 2 |p|} \frac{d^3 q}{(2\pi)^3 2 |q|} (2\pi)^4 \delta^4(k - p - q) p^\mu \left( 1 - f_{\ell}^{\text{eq}}(p) + f_{\phi}^{\text{eq}}(q) \right)
\]
**CP-violating Source $S_{ab}$: Vertex Contribution**

- Orange cut $\longleftrightarrow$ Interference between $s$- and $t$-channel scatterings
- Blue cut $\longleftrightarrow$ Interference between loop and tree-level decays

In the strongly hierarchical case, $M_1 \ll M_2$:

$$S_{ab}^V = \frac{1}{2} S_{ab}^{wf} \quad \implies \quad S_{ab} = \frac{3}{2} S_{ab}^{wf}$$

$$\sum_{N}^{\mu}(k) \xrightarrow{M_{N} \gg T} g_{w} \frac{k^{\mu}}{16\pi}$$ recover standard approximation
Flavour-sensitive MSM Yukawa Interactions

- **Blue cut** ←→ tree-level pair creation and annihilation and $1 \leftrightarrow 2$ scatterings
- Kinematically forbidden for on-shell excitations

\[
\Gamma_{\ell \ell_{ab}}^{\text{fl}} = \Gamma_{\ell \ell_{ab}}^{\text{an}} \left( [h^\dagger h]_{ac} q_{\ell cb} + q_{\ell ac}^\dagger [h^\dagger h]_{cb} - h_{ac}^\dagger q_{Rcd} h_{db} - h_{ad}^\dagger q_{Rde}^\dagger h_{cb} \right) \\
+ \Gamma_{\ell \ell_{ab}}^{\text{sc}} \left( [h^\dagger h]_{ac} q_{\ell cb} + q_{\ell ac}^\dagger [h^\dagger h]_{cb} - h_{ac}^\dagger q_{Rcd} h_{db} - h_{ad}^\dagger q_{Rde}^\dagger h_{cb} \right)
\]

- Example: 2 flavours, charged lepton basis

\[
\Gamma_{\ell}^{\text{fl}} = (\Gamma_{\ell}^{\text{an}} + \Gamma_{\ell}^{\text{sc}}) h_T^2 \left[ \left( \begin{array}{cc} 1 & 0 \\ 0 & 0 \end{array} \right) q_\ell + q_\ell \left( \begin{array}{cc} 1 & 0 \\ 0 & 0 \end{array} \right) - 2 \left( \begin{array}{cc} q_{R11} & 0 \\ 0 & 0 \end{array} \right) \right]
\]

- $\Gamma_{\ell}^{\text{an,sc}} \sim g_2^2 T$ need to be calculated including (thermal) finite width corrections for $\ell$ and $\phi$ propagators