Flavoured Leptogenesis from Nonequilibrium QFT

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in collaboration with

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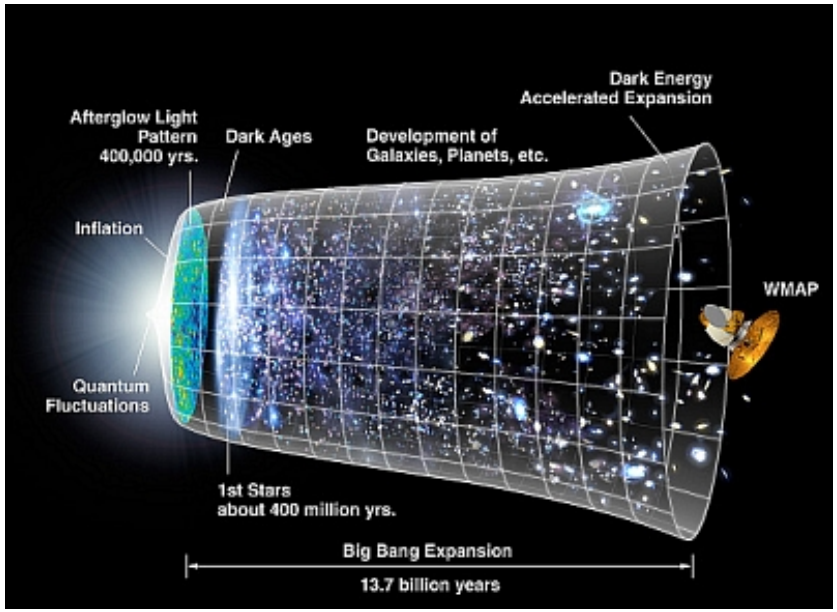
Outline

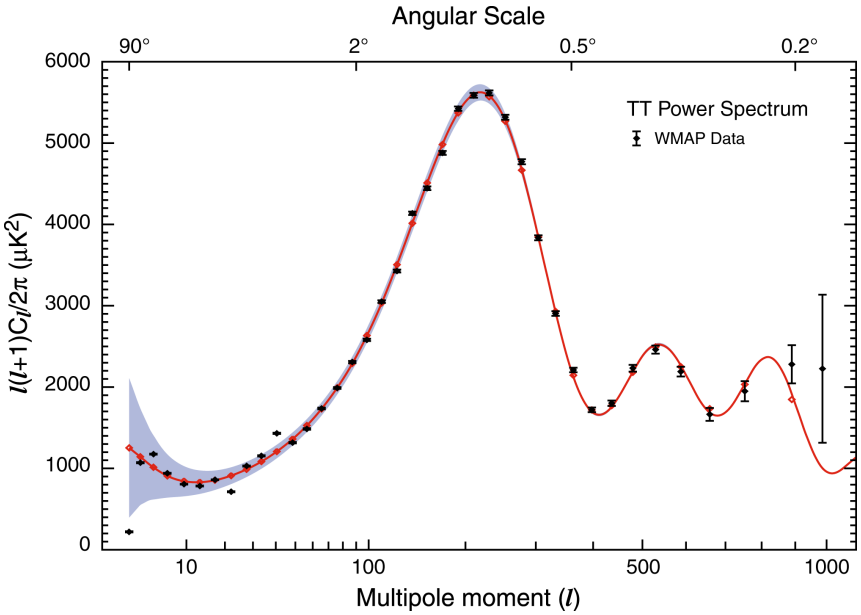
- Introduction
- Closed Time Path (CTP) Formalism of Noneq. QFT
- CTP Approach to Flavoured Leptogenesis
- Numerical Results
- Conclusions and outlook

Why baryogenesis?

To explain the excess of matter over antimatter in the universe:
[WMAP 2003]

$$\frac{n_B}{n_\gamma} = \left(6.1^{+0.3}_{-0.2}\right) \times 10^{-10}$$





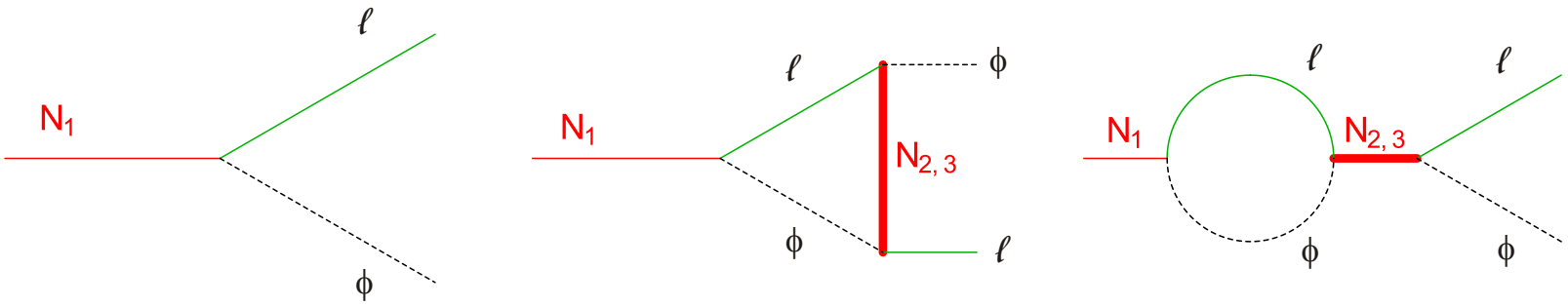
- Why is there something rather than nothing?

Leptogenesis [Fukugita, Yanagida (1986)]

- The Standard Model is extended by adding heavy right-handed Majorana neutrinos N_i, $i = 1, 2, \ldots$ (eg. see-saw models)

$$\mathcal{L} \supset -h^*_{ab}\phi\,\bar{\psi}_{\ell b}P_{\mathrm{R}}\psi_{\mathrm{R}a} - Y^*_{ia}\bar{\psi}_{\ell a}(\epsilon\phi)^\dagger\psi_{Ni} - \frac{1}{2}\bar{\psi}_{Ni}M_i\psi_{Ni} + \mathrm{h.c.}$$

- Lepton asymmetry is generated through out-of-equilibrium L- and CP-violating Yukawa decays: $N_1 \to \ell\phi$



Nonequilibrium QFT Approach to Leptogenesis[1]

[1] related / complementary aspects in:
Buchmüller, Fredenhagen (2000); De Simone, Riotto (2007);
Garny, Hohenegger, Kartavtsev, Lindner (2009 & 2010);
Anisimov, Buchmüller, Drewes, Mendizabal (2010);
Anisimov, Besak, Bödeker (2010)

Closed Time Path* (CTP) Formalism

* a.k.a. Schwinger-Keldysh Formalism [Schwinger (1961); Keldysh (1964); Calzetta, Hu (1988)]

- Usually in QFT matrix elements are computed within the in-out framework:

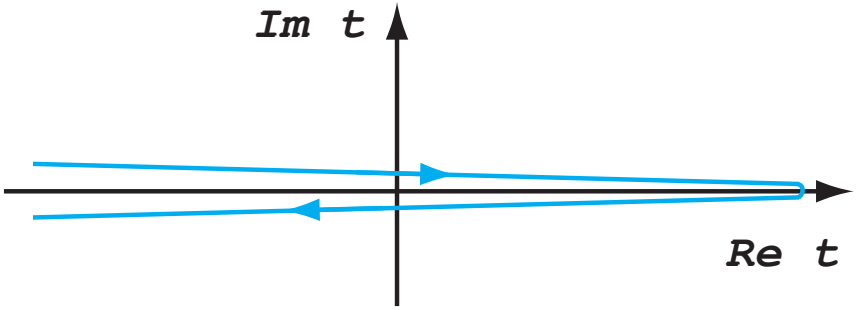
$$\langle \text{out}|\hat{S}|\text{in}\rangle \quad \longleftarrow \quad \text{time-ordered correlators:} \quad \langle T[\psi(x)\bar{\psi}(y)]\rangle$$

- In leptogenesis we want to calculate expectation values in a finite density medium, e.g.

$$j^0(x) = \langle \bar{\psi}(x)\gamma^0\psi(x)\rangle \equiv \text{tr}\left[\hat{\rho}\,\bar{\psi}(x)\gamma^0\psi(x)\right]$$

  - $\hat{\rho}$ is an (unknown) quantum density operator

$\longrightarrow$ In-In generating functional on a Closed Time Path:



Path-ordered correlators:

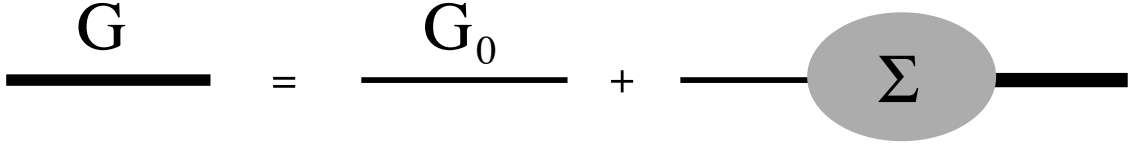
$$\mathrm{i}S_{\mathcal{C}}(x,y) = \langle T_{\mathcal{C}}[\psi(x)\bar{\psi}(y)]\rangle$$

Four propagators with respect to real time variable:

$$\begin{aligned}
\mathrm{i}S^{+-}(u,v) &= \mathrm{i}S^{<}(u,v) = -\langle\bar{\psi}(v)\psi(u)\rangle \\
\mathrm{i}S^{-+}(u,v) &= \mathrm{i}S^{>}(u,v) = \langle\psi(u)\bar{\psi}(v)\rangle \\
\mathrm{i}S^{++}(u,v) &= \mathrm{i}S^{T}(u,v) = \langle T(\psi(u)\bar{\psi}(v))\rangle \\
\mathrm{i}S^{--}(u,v) &= \mathrm{i}S^{\bar{T}}(u,v) = \langle \bar{T}(\psi(u)\bar{\psi}(v))\rangle
\end{aligned}$$

Kadanoff-Baym Equations

- Generic Schwinger-Dyson equations for (CTP) propagators:



- Kadanoff-Baym equations are the $<, >$ components:

$$(\mathrm{i}\partial\!\!\!/ - m)S^{<,>} - \Sigma^H \odot S^{<,>} - \Sigma^{<,>} \odot S^H = \frac{1}{2}\left(\Sigma^> \odot S^< - \Sigma^< \odot S^>\right)$$

  - $(A \odot B)(u,v) \equiv \int d^4w\, A(u,w)B(w,v)$ denotes convolution
  - Renormalization, thermal corrections (thermal masses etc.)
  - Finite width effects
  - Collision term

Approximations

Wigner representation:

$$S(k,x) = \int d^4r\, e^{ik\cdot r}\, S\big(x+\frac{r}{2}, x-\frac{r}{2}\big)\,,$$

and gradient expansion to the lowest order in

- x-derivatives: $\partial_x S(k,x)$, $\partial_x \Sigma(k,x)$, etc.
- coupling constants in $\Sigma(k,x)$

$\longrightarrow$ Constraint and Kinetic Equations:

$$2(k^0 - \mathbf{k}\cdot\boldsymbol{\gamma}\gamma^0)\mathrm{i}\gamma^0 S_\ell^{<,>} - \left\{\not{\Sigma}_\ell^H\gamma^0, \mathrm{i}\gamma^0 S_\ell^{<,>}\right\} - \left\{\mathrm{i}\not{\Sigma}_\ell^{<,>}\gamma^0, \gamma^0 S_\ell^H\right\} = -\frac{1}{2}\left(\mathrm{i}\mathcal{C}_\ell - \mathrm{i}\mathcal{C}_\ell^\dagger\right)$$

$$\mathrm{i}\partial_\eta \mathrm{i}\gamma^0 S_\ell^{<,>} - \left[\not{\Sigma}_\ell^H\gamma^0, \mathrm{i}\gamma^0 S_\ell^{<,>}\right] - \left[\mathrm{i}\not{\Sigma}_\ell^{<,>}\gamma^0, \gamma^0 S_\ell^H\right] = -\frac{1}{2}\left(\mathrm{i}\mathcal{C}_\ell + \mathrm{i}\mathcal{C}_\ell^\dagger\right)$$

- η is conformal time variable: $d\eta = dt/a(t)$

The zeroth order solutions to Contraint Equation:

- Flavour covariant free propagators for ℓ:

$$\mathrm{i}S^{<}_{\ell ab}(k,\eta) = -2\pi\delta(k^2)\not{k}\left[\vartheta(k_0)f^{+}_{\ell ab}(\mathbf{k},\eta) - \vartheta(-k_0)(\mathbb{1}_{ab} - f^{-}_{\ell ab}(-\mathbf{k},\eta))\right]$$

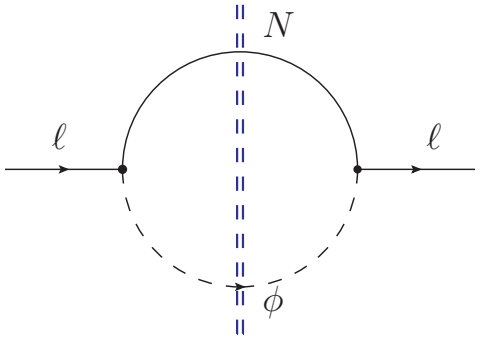
$$\mathrm{i}S^{>}_{\ell ab}(k,\eta) = -2\pi\delta(k^2)\not{k}\left[-\vartheta(k_0)(\mathbb{1}_{ab} - f^{+}_{\ell ab}(\mathbf{k},\eta)) + \vartheta(-k_0)f^{-}_{\ell ab}(-\mathbf{k},\eta)\right]$$

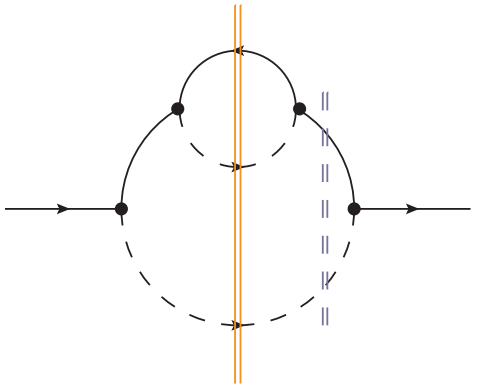
  - $f^{\pm}_{\ell ab}(\mathbf{k},\eta)$ are time-dependent distribution functions
  - a, b are flavour indices

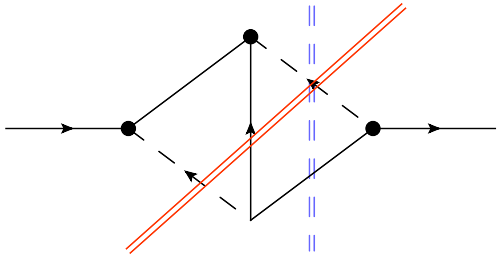
- Similar (unflavoured) free propagators for N_1 and ϕ

Contributions to Lepton Collision term

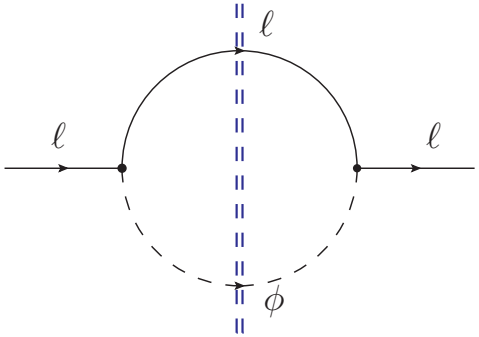
- Y-Yukawa interactions:

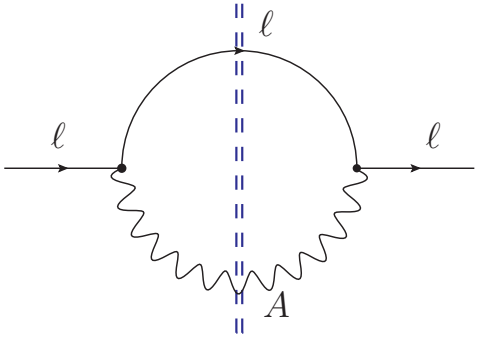






- MSM h-Yukawa and gauge interactions:





Kinetic Equation for Lepton Number Densities

[Beneke, Garbrecht, Fidler, MH, Schwaller (2010)]

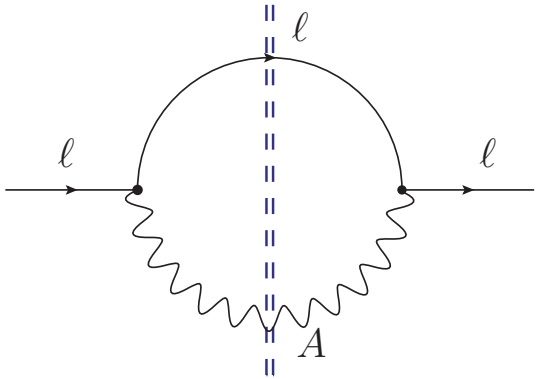
Substituting free propagators into Kinetic Equation

$\longrightarrow$ First order equation for $n^{\pm}_{\ell ab}(\eta) = \int \frac{d^3k}{(2\pi)^3} f^{\pm}_{\ell ab}(\mathbf{k},\eta)$:

$$\frac{\partial \delta n^{\pm}_{\ell ab}}{\partial \eta} = \sum_c [\Xi^{\rm eff}_{ac}\delta n^{\pm}_{\ell cb} - \delta n^{\pm}_{\ell ac}\Xi^{\rm eff}_{cb}] \mp {\rm i}\Delta\omega^{\rm eff}_{\ell ab}\delta n^{\pm}_{\ell ab}$$
$$- \sum_c [W_{ac}\delta n^{\pm}_{\ell cb} + \delta n^{\pm *}_{\ell ca} W^*_{bc}] \pm S_{ab} - \Gamma^{\rm bl}(\delta n^{+}_{\ell ab} + \delta n^{-}_{\ell ab}) - \Gamma^{\pm {\rm fl}}_{\ell ab}$$

- Gradients of the mixing matrices: $\Xi^{\rm eff} \sim U^\dagger \partial_\eta U$
- Flavor oscillations by the thermal masses: $\Delta\omega^{\rm eff}_{\ell} \sim h^2 T\,,\ Y^2 T$
- Collision terms: Washout, CP-violating source, Flavour-blind damping, Flavour-sensitive damping

Fast Flavour-blind Gauge Interactions



- Blue cut $\longleftrightarrow$ tree-level pair creation and annihilation and $1 \leftrightarrow 2$ scatterings
- Kinematically forbidden for on-shell excitations $\longrightarrow \Gamma^{\rm bl} \sim g_2^4 T$

- Force kinetic equilibrium with generalized chemical potentials $\mu^{\pm}_{ab}(\eta)$:

$$f^{\pm}_{\ell ab}(\mathbf{k},\eta) = \left(\frac{1}{\mathrm{e}^{\beta|\mathbf{k}|-\beta\mu^{\pm}}+1}\right)_{ab}$$

- Flavour and momentum dependence factorizes to first order in $\mu^{\pm}_{ab}$:

$$\delta n^{\pm}_{\ell ab} = \mu^{\pm}_{ab}\frac{T^2}{12} \qquad \Longrightarrow \qquad \delta f^{\pm}_{\ell ab}(\mathbf{k},\eta) = \delta n^{\pm}_{\ell ab}\frac{12\beta^3\mathrm{e}^{\beta|\mathbf{k}|}}{(\mathrm{e}^{\beta|\mathbf{k}|}+1)^2}$$

$\longrightarrow$ Simple interaction terms in the kinetic equation

Suppression of Flavour Oscillations

- Toy equations for flavour oscillations

$$\frac{\partial \delta n^{\pm}_{\ell ab}}{\partial \eta} = \mp \mathrm{i} \Delta\omega^{\mathrm{eff}}_{\ell ab} \delta n^{\pm}_{\ell ab} - \Gamma^{\mathrm{bl}} (\delta n^{+}_{\ell ab} + \delta n^{-}_{\ell ab})$$

- Parametrically $\Gamma^{\mathrm{bl}} \gg \Delta\omega^{\mathrm{eff}}_{\ell}$

Suppression of Flavour Oscillations

- Toy equations for flavour oscillations

$$\frac{\partial \delta n^{\pm}_{\ell ab}}{\partial \eta} = \mp \mathrm{i} \Delta\omega^{\mathrm{eff}}_{\ell ab} \delta n^{\pm}_{\ell ab} - \Gamma^{\mathrm{bl}} (\delta n^{+}_{\ell ab} + \delta n^{-}_{\ell ab})$$

- Parametrically $\Gamma^{\mathrm{bl}} \gg \Delta\omega^{\mathrm{eff}}_{\ell}$

$\longrightarrow$ Two over-damped solutions with short and long time scales:

  - Short mode with $\tau_{\mathrm{short}} = 1/(2\Gamma^{\mathrm{bl}})$ forces an effective constraint:

$$\delta n^{-}_{ab} = -\delta n^{+}_{ab}$$

  - Flavour oscillations of δn^{+}_{ab} are over-damped with a long decay time:

$$\tau_{\mathrm{damp}} \sim 2\Gamma^{\mathrm{bl}} / (\Delta\omega^{\mathrm{eff}}_{\ell ab})^2$$

Final Kinetic Equations

- Lepton asymmetries $q \equiv \delta n^+ - \delta n^-$ (L- and R-handed):

$$\frac{\partial q_{\ell ab}}{\partial \eta} = \sum_c \left[\Xi_{ac} q_{\ell cb} - q_{\ell ac}\Xi_{cb} - W_{ac} q_{\ell cb} - q_{\ell ac} W_{cb}\right] - \Gamma^{\rm fl}_{\ell ab} + 2S_{ab}$$

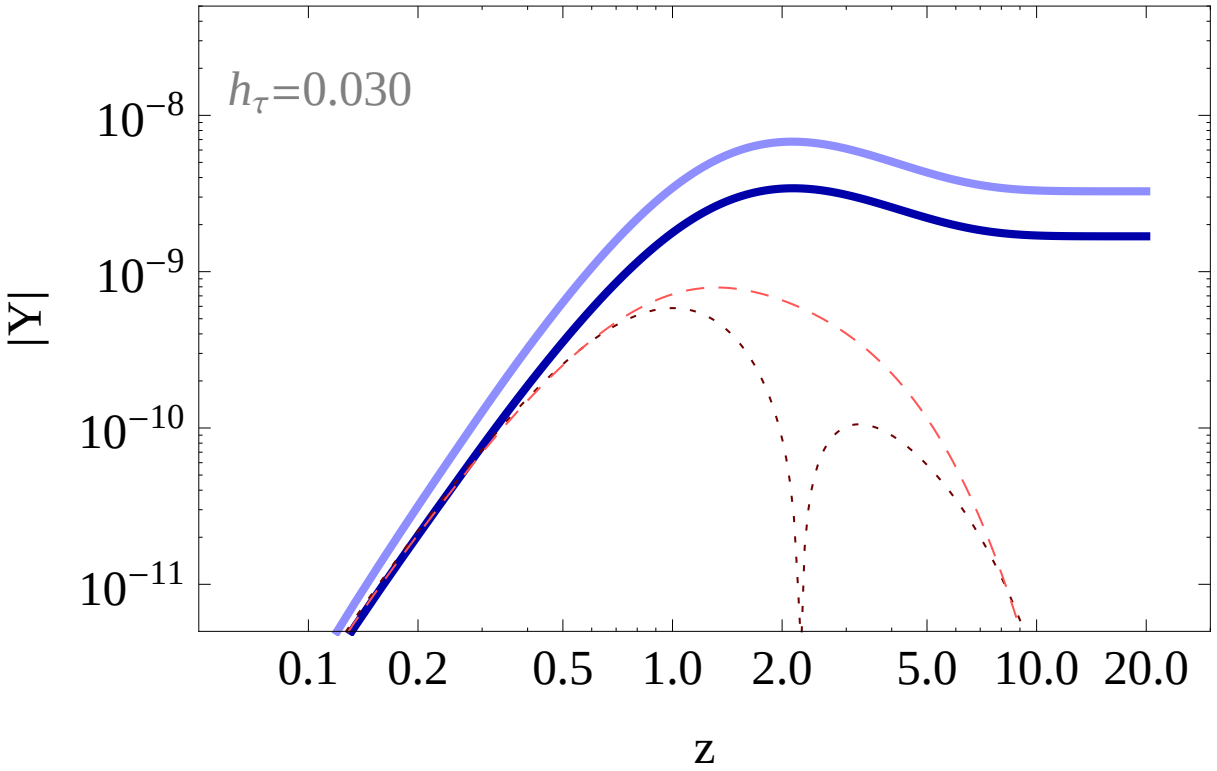
$$\frac{\partial q_{{\rm R}ab}}{\partial \eta} = -\Gamma^{\rm fl}_{{\rm R}ab}$$

- Majorana neutrino N_1:

$$\frac{\partial f_{N1}(\mathbf{k}_{\rm com})}{\partial \eta} = -2|Y_1|^2 \frac{k_{{\rm com}\,\mu}}{2k_{{\rm com}\,0}} \Sigma^\mu_N(k_{\rm com}) \left[f_{N1}(\mathbf{k}_{\rm com}) - f^{\rm eq}_{N1}(\mathbf{k}_{\rm com})\right]$$

- Higgs field ϕ remains in thermal equilibrium

Lepton Asymmetries: $Y_{\ell ab} = \frac{n^+_{\ell ab} - n^-_{\ell ab}}{s}$, Fully Flavoured case



dark blue: $Y_{\ell 11}$

light blue: $Y_{\ell 22}$

brown dotted: $\mathrm{Re}[Y_{\ell 12}]$

red dashed: $\mathrm{Im}[Y_{\ell 12}]$

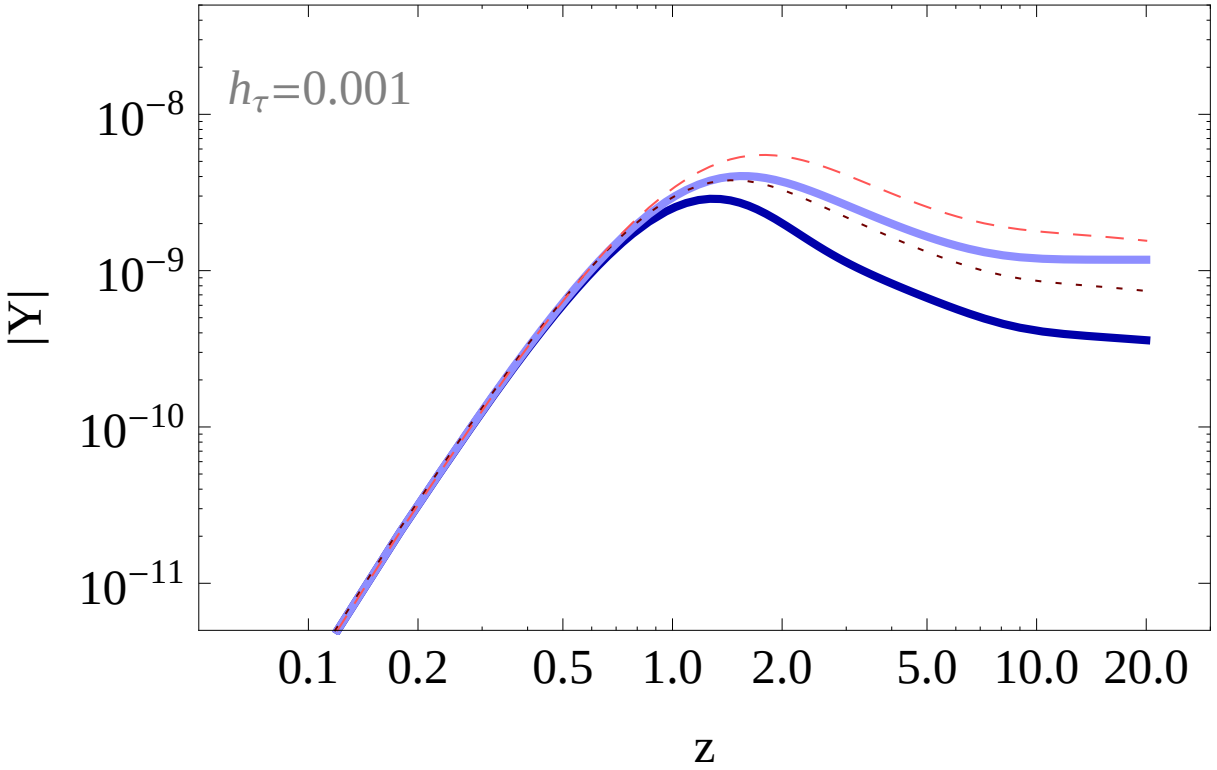
$M_1 = 10^{12}$ GeV, $M_2 = 10^{14}$ GeV

$$Y_{\mathrm{Yuk}} = \begin{pmatrix} 1.4 \times 10^{-2} & 1 \times 10^{-2} \\ \mathrm{i} \times 10^{-1} & 10^{-1} \end{pmatrix}$$

- $Y_{\ell 12} = Y^*_{\ell 21}$ strongly suppressed before freeze-out at $z = \frac{M_1}{T} \approx 10$

  $\longrightarrow$ Flavour off-diagonals $Y_{\ell 12,21}$ can be negleted

Lepton Asymmetries: $Y_{\ell ab} = \frac{n^+_{\ell ab} - n^-_{\ell ab}}{s}$, Unflavoured case



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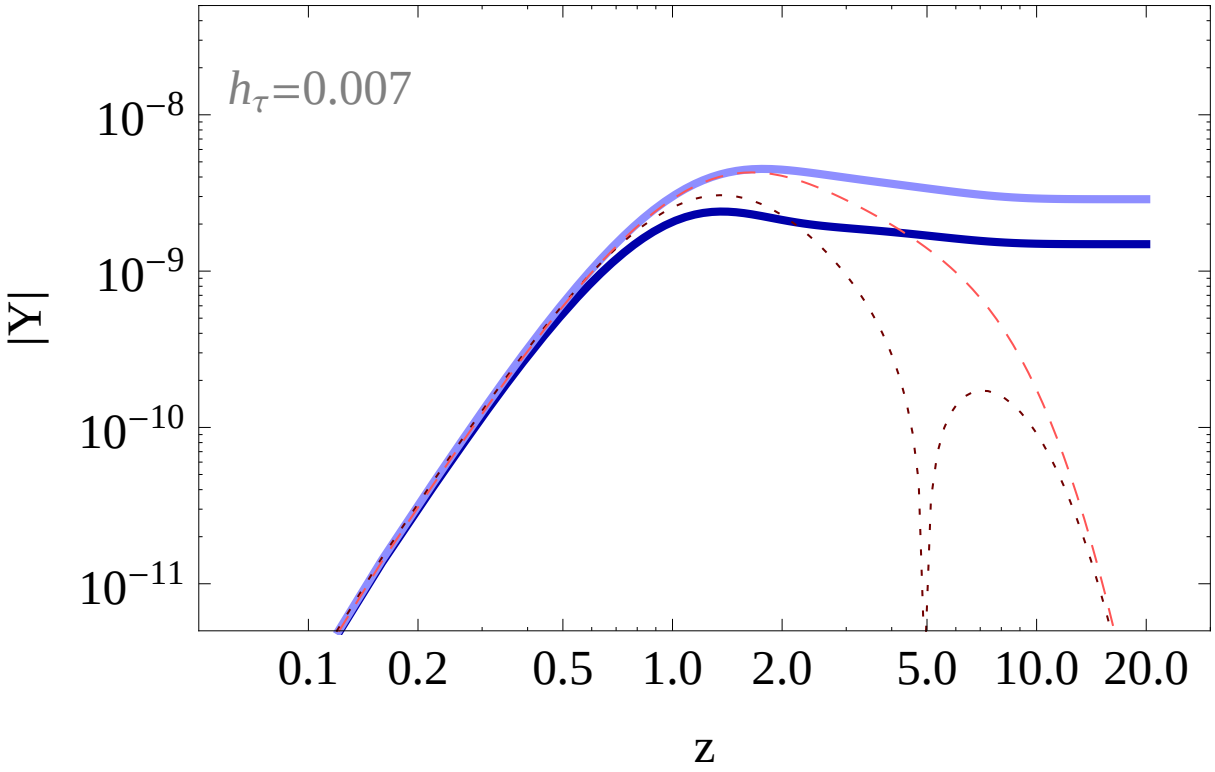
$M_1 = 10^{12}$ GeV, $M_2 = 10^{14}$ GeV

$$Y_{\mathrm{Yuk}} = \begin{pmatrix} 1.4 \times 10^{-2} & 1 \times 10^{-2} \\ \mathrm{i} \times 10^{-1} & 10^{-1} \end{pmatrix}$$

- $Y_{\ell 12} = Y^*_{\ell 21}$ decay away long after freeze-out at $z \approx 10$

  $\longrightarrow$ Flavour damping by $\Gamma^{\mathrm{fl}}_\ell$ can be neglected

Lepton Asymmetries: $Y_{\ell ab} = \frac{n^+_{\ell ab} - n^-_{\ell ab}}{s}$, Intermediate regime



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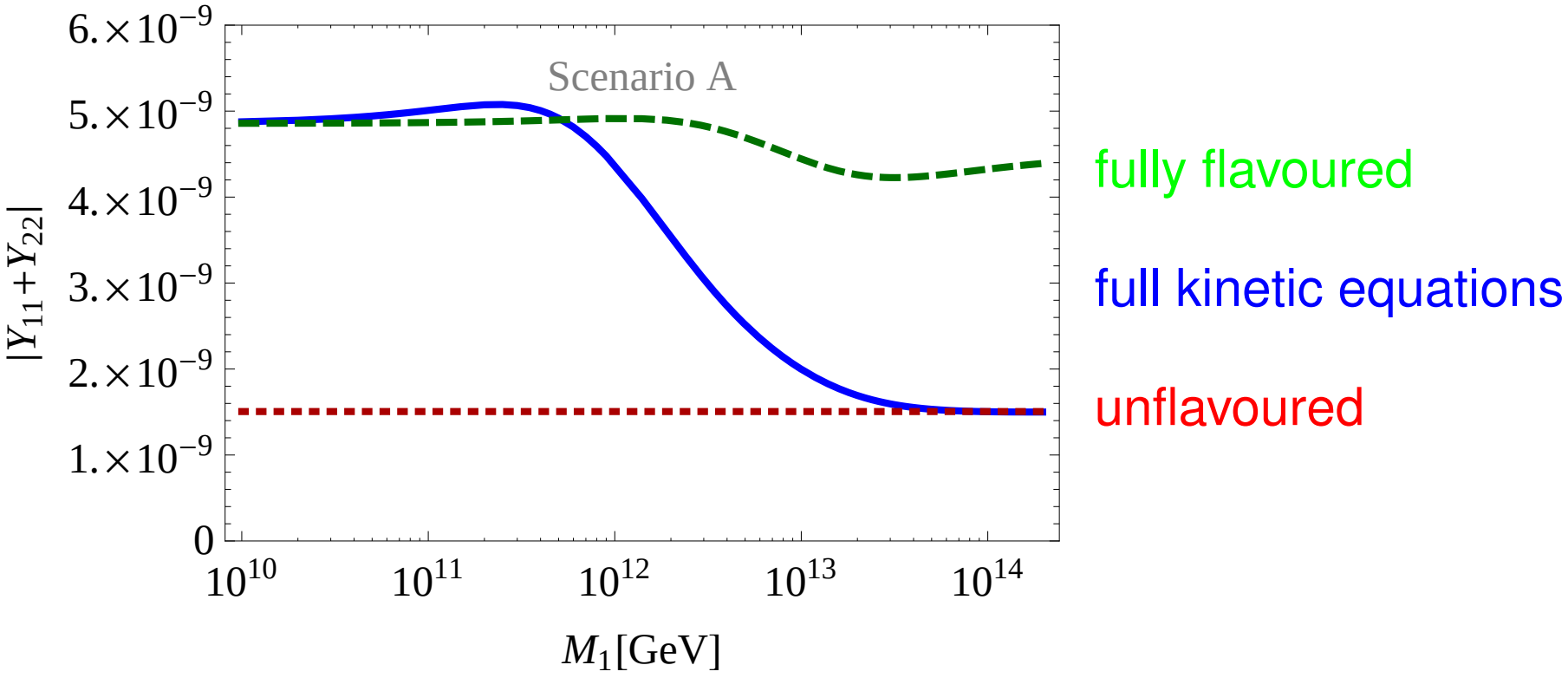
$M_1 = 10^{12}$ GeV, $M_2 = 10^{14}$ GeV

$$Y_{\mathrm{Yuk}} = \begin{pmatrix} 1.4 \times 10^{-2} & 1 \times 10^{-2} \\ \mathrm{i} \times 10^{-1} & 10^{-1} \end{pmatrix}$$

- Full kinetic equations need to be solved!

Total Lepton Asymmetry as a function of $\Gamma_\ell^{\rm fl}$

- Scale $M_{1,2}$ and the couplings $Y_{\rm Yuk}$ such that $\Gamma_\ell^{\rm fl}$ is varying while the washout and source terms remain constant



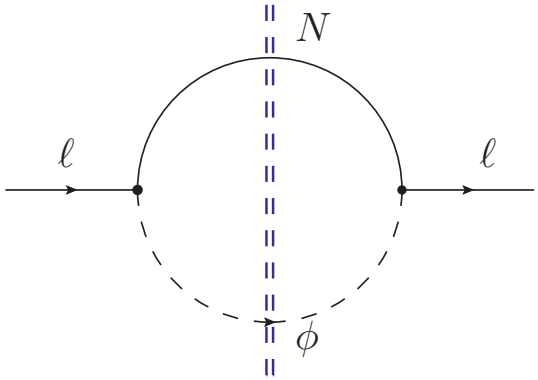
Conclusions

- First principle description of leptogenesis within the CTP framework
  - RIS subtraction procedure not required
- Simple kinetic equations for lepton number densities, including
- Quantum statistical corrections in loops and external states
  - Sizable for weak washout
- Flavour effects
  - Flavour oscillations are overdamped by fast gauge interactions
  - Full flavoured equations are needed between fully flavoured and unflavoured regimes

Outlook

- Systematic inclusion of thermal effects
  - Finite widths
  - Thermal masses
- Spectator processes
- Resonant leptogenesis
  - Flavour coherence effects between Neutrinos N_i

Washout contribution



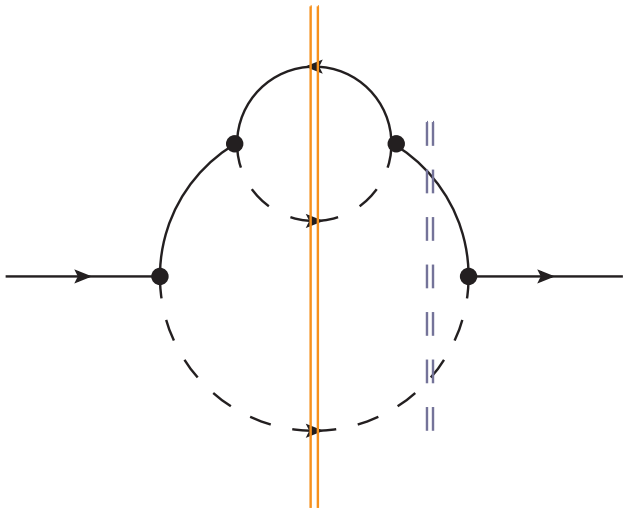
- Blue cut $\longleftrightarrow$ tree-level decays and inverse decays $N_1 \leftrightarrow \ell\phi$

$$\frac{1}{2}\,\mathrm{tr}\int \frac{d^3k}{(2\pi)^3}\int_0^\infty \frac{dk_0}{2\pi}\,\mathcal{C}^Y_{\ell ab} = -\sum_c W_{ac}\delta n^+_{\ell cb}$$

with

$$W_{ac} = \frac{1}{2}\,Y^*_{1a}Y_{1c}\int \frac{d^3k}{(2\pi)^3 2|\mathbf{k}|}\,\frac{d^3k'}{(2\pi)^3 2\sqrt{\mathbf{k}'^2 + (a(\eta)M_1)^2}}\,\frac{d^3k''}{(2\pi)^3 2|\mathbf{k}''|}\,(2\pi)^4\delta^4(k'-k-k'')$$
$$\times\, 2k\cdot k'\,\left(f_{N1}(\mathbf{k}') + f_\phi(\mathbf{k}'')\right)\,\frac{12\beta^3\,\mathrm{e}^{\beta|\mathbf{k}|}}{(\mathrm{e}^{\beta|\mathbf{k}|}+1)^2}$$

CP-violating Source S_{ab}: Wave-function Contribution



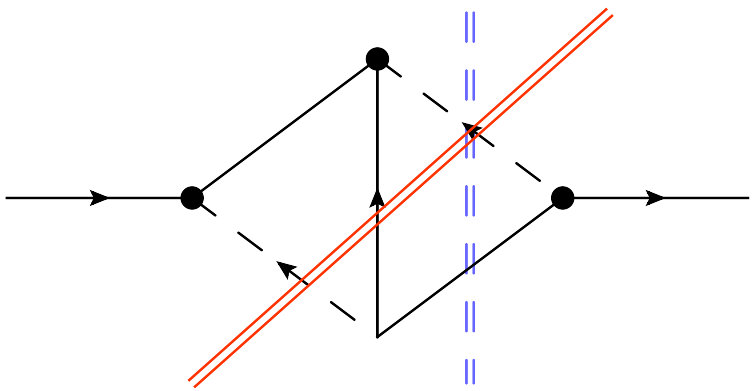
- Orange cut $\longleftrightarrow$ Interference between two s-channel scatterings
- Blue cut $\longleftrightarrow$ Interference between loop and tree-level decays

$$S_{ab}^{\rm wf} = {\rm i} \sum_c \left[Y_{1a}^* Y_{1c}^* Y_{2c} Y_{2b} - Y_{2a}^* Y_{2c}^* Y_{1c} Y_{1b} \right]$$
$$\times \left(-\frac{M_1}{M_2} \right) \int \frac{d^3 k'}{(2\pi)^3 2\sqrt{\mathbf{k}'^2 + (a(\eta) M_1)^2}} \frac{\Sigma_{N\mu}(\mathbf{k}') \Sigma_N^\mu(\mathbf{k}')}{g_w} \delta f_{N1}(\mathbf{k}')$$

where the thermal decay rate is

$$\Sigma_N^\mu(k) = g_w \int \frac{d^3 p}{(2\pi)^3 2|\mathbf{p}|} \frac{d^3 q}{(2\pi)^3 2|\mathbf{q}|} (2\pi)^4 \delta^4(k - p - q)\, p^\mu \left(1 - f_\ell^{\rm eq}(\mathbf{p}) + f_\phi^{\rm eq}(\mathbf{q}) \right)$$

CP-violating Source S_{ab}: Vertex Contribution



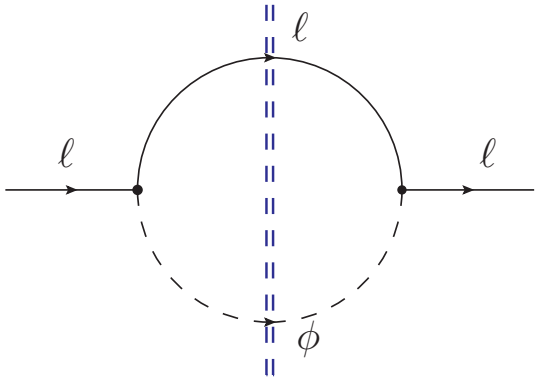
- Orange cut $\longleftrightarrow$ Interference between s- and t-channel scatterings
- Blue cut $\longleftrightarrow$ Interference between loop and tree-level decays

- In the strongly hierarchical case, $M_1 \ll M_2$:

$$S_{ab}^{\mathrm{V}} = \frac{1}{2} S_{ab}^{\mathrm{wf}} \qquad \Longrightarrow \qquad S_{ab} = \frac{3}{2} S_{ab}^{\mathrm{wf}}$$

- $\Sigma_N^{\mu}(k) \xrightarrow{M_N \gg T} g_w \frac{k^{\mu}}{16\pi}$ recover standard approximation

Flavour-sensitive MSM Yukawa Interactions



- Blue cut $\longleftrightarrow$ tree-level pair creation and annihilation and $1 \leftrightarrow 2$ scatterings
- Kinematically forbidden for on-shell excitations

$$\begin{aligned}\Gamma^{\rm fl}_{\ell ab} =& \Gamma^{\rm an}\left([h^\dagger h]_{ac} q_{\ell cb} + q^\dagger_{\ell ac}[h^\dagger h]_{cb} - h^\dagger_{ac} q_{{\rm R}cd} h_{db} - h^\dagger_{ad} q^\dagger_{{\rm R}dc} h_{cb}\right)\\ &+\Gamma^{\rm sc}\left([h^\dagger h]_{ac} q_{\ell cb} + q^\dagger_{\ell ac}[h^\dagger h]_{cb} - h^\dagger_{ac} q_{{\rm R}cd} h_{db} - h^\dagger_{ad} q^\dagger_{{\rm R}dc} h_{cb}\right)\end{aligned}$$

- Example: 2 flavours, charged lepton basis

$$\Gamma^{\rm fl}_\ell = (\Gamma^{\rm an} + \Gamma^{\rm sc})\, h_\tau^2 \left[\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} q_\ell + q_\ell \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} - 2\begin{pmatrix} q_{\rm R11} & 0 \\ 0 & 0 \end{pmatrix}\right]$$

- $\Gamma^{\rm an,sc} \sim g_2^2 T$ need to be calculated including (thermal) finite width corrections for ℓ and ϕ propagators