Scale-dependent non-Gaussianity

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C.Byrnes, SN, G.Tasinato, D. Wands (arXiv:0911.2780)
C.Byrnes, M. Gerstenlauer, SN, G.Tasinato, D. Wands (arXiv:1007.4277)

Non-Gaussianity can tell us much about inflation

- Single field slow roll: no detectable NG
- Observable NG generated by non-minimal models: multiple fields, non-canonical dynamics ...
- Details of NG statistics depend on the model, can discriminate between different scenarios with large NG
- Topical: constraints get significantly tighter with Planck

The simplest case: local non-Gaussianity

$$\zeta(\mathbf{k}) = \zeta_{\rm G}(\mathbf{k}) + \frac{3}{5} f_{\rm NL} \, (\zeta_{\rm G} \star \zeta_{\rm G})(\mathbf{k}) + \frac{9}{25} g_{\rm NL} \, (\zeta_{\rm G} \star \zeta_{\rm G} \star \zeta_{\rm G})(\mathbf{k}) + \ldots$$

$$\mathrm{CMB} \, + \, \mathrm{LSS} : |f_{\rm NL}| \lesssim 10^2 \, , |g_{\rm NL}| \lesssim 10^6$$ [1]

[1] Komatsu et. al. 10; Desjacques, Seljak 09; Fergusson, Regan, Shellard 11

The simplest case: local non-Gaussianity

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- Compare with the δN expression:

$$\zeta(\mathbf{k}) = N'(t_k)\delta\sigma(\mathbf{k}) + \frac{1}{2}N''(t_k)(\delta\sigma\star\delta\sigma)(\mathbf{k}) + \frac{1}{6}N'''(t_k)(\delta\sigma\star\delta\sigma\star\delta\sigma)(\mathbf{k})$$

$$f_{\rm NL} = \frac{5}{6}\frac{N''(t_k)}{N'^2(t_k)} \qquad g_{\rm NL} = \frac{25}{54}\frac{N'''(t_k)}{N'^3(t_k)}$$

- $f_{\rm NL}$ and $g_{\rm NL}$ scale-dependent in general:

$$n_{f_{\rm NL}} = \frac{{\rm d}\ln f_{\rm NL}}{{\rm d}\ln k} \sim \mathcal{O}(\epsilon) \qquad n_{g_{\rm NL}} = \frac{{\rm d}\ln g_{\rm NL}}{{\rm d}\ln k} \sim \mathcal{O}(\epsilon)$$

- Analogous to the scale-dependence of the spectrum

Local NG $\longrightarrow$ Quasi-local NG

- Replace the local Ansatz by

$$\zeta(\mathbf{k}) = \zeta_{\rm G}(\mathbf{k}) + \frac{3}{5} f_{\rm NL}(k)\,(\zeta_{\rm G}\star\zeta_{\rm G})(\mathbf{k}) + \frac{9}{25} g_{\rm NL}(k)\,(\zeta_{\rm G}\star\zeta_{\rm G}\star\zeta_{\rm G})(\mathbf{k}) + \ldots$$

$$n_{f_{\rm NL}} = \frac{{\rm d}\ln f_{\rm NL}(k)}{{\rm d}\ln k} \qquad n_{g_{\rm NL}} = \frac{{\rm d}\ln g_{\rm NL}(k)}{{\rm d}\ln k}$$

- Multiple fields: $f_{\rm NL}(k)\zeta^2 \longrightarrow f^{ab}(k)\zeta^a\zeta^b$ etc.
- Describes generic models with canonical slow roll dynamics during inflation, $n_{f_{\rm NL}}$ and $n_{g_{\rm NL}}$ easily calculable [2]

[2] Byrnes,SN,Tasinato,Wands 09; Byrnes, Gerstenlauer, SN, Tasinato, Wands 10

3-and 4-point functions

$$
\begin{aligned}
\langle \zeta_{\mathbf{k}_1}\zeta_{\mathbf{k}_2}\zeta_{\mathbf{k}_3}\rangle &= (2\pi)^3\delta(\sum_{i=1}^{3}\mathbf{k}_i)\frac{6}{5}f_{\rm NL}(k_1,k_2,k_3)\left(\ P(k_1)P(k_2)P(k_3)+{\rm p.}\ \right)\\
\langle \zeta_{\mathbf{k}_1}\zeta_{\mathbf{k}_2}\zeta_{\mathbf{k}_3}\zeta_{\mathbf{k}_4}\rangle &= (2\pi)^3\delta(\sum_{i=1}^{4}\mathbf{k}_i)\Big[\ \tau_{\rm NL}(k_1,k_2,k_3,k_4,k_{13})\left(\ P(k_1)P(k_2)P(|\mathbf{k}_1+\mathbf{k}_3|)+{\rm p.}\ \right)\\
&\quad +\frac{54}{25}g_{\rm NL}(k_1,k_2,k_3,k_4)\left(\ P(k_1)P(k_2)P(k_3)+{\rm p.}\ \right)\ \Big]
\end{aligned}
$$

- Same scale-dependence for all shape-preserving variations, e.g.

$$
\frac{\partial \ln f_{\rm NL}(k\alpha_1,k\alpha_2,k\alpha_3)}{\partial \ln k} = n_{f_{\rm NL}} \quad \text{independent on } \alpha_i
$$

- Need 3 new parameters: $n_{f_{\rm NL}}, n_{g_{\rm NL}}, n_{\tau_{\rm NL}}$

Scale-dependence probes inflationary physics

- General expressions for $n_{f_{\rm NL}}, n_{g_{\rm NL}}, n_{\tau_{\rm NL}}$ can be derived, depend on slow roll parameters and N', N'', N'''
- A new way to discriminate between different models [2]
- Planck already gives interesting constraints

$$\Delta n_{f_{\rm NL}} \sim 0.1 \quad \text{for} \quad f_{\rm NL} = 50 \qquad [3]$$

- CMBpol could do a factor of 2 better
- LSS studies expected to have even higher sensitivities

[2] Byrnes,SN,Tasinato,Wands 09; Byrnes, Gerstenlauer, SN, Tasinato, Wands 10

[3] Sefusatti, Liguori, Yadav, Jackson, Pajer 09

Example I: scale-dependence from interactions

- Consider interacting curvatons: $V = \frac{1}{2}m^2\sigma^2 + \lambda\sigma^n$
- Level of NG determined by $r_{\rm dec} = \frac{\rho_\sigma}{\rho_{\rm total}}\Big|_{t_{\rm dec}}$ and $s = \frac{\lambda\sigma_*^{n-2}}{m^2}$

$$f_{\rm NL} = \frac{5}{6}\frac{N''}{N'^2} = \frac{5}{3}\frac{f(s)}{r_{\rm dec}}\,, \qquad g_{\rm NL} = \frac{25}{54}\frac{N'''}{N'^3} = \frac{50}{27}\frac{g(s)}{r_{\rm dec}}$$

- Interactions generate scale-dependence [2]

$$\begin{aligned} n_{f_{\rm NL}} &= \frac{N'}{N''}\frac{V'''}{3H^2} \propto \frac{\eta_\sigma}{f(s)} \\ n_{g_{\rm NL}} &= \frac{2f_{\rm NL}^2}{g_{\rm NL}}n_{f_{\rm NL}} + \frac{N'}{N'''}\frac{V''''}{3H^2} \propto \frac{\eta_\sigma}{g(s)} \end{aligned}$$

- Possible to have $|n_{f_{\rm NL}}|, |n_{g_{\rm NL}}| \gg \eta_\sigma$ [4],[5]

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[2] Byrnes,SN,Tasinato,Wands 09; Byrnes, Gerstenlauer, SN, Tasinato, Wands 10

[4] Enqvist,Byrnes,Takahashi 10

[5] Enqvist,Byrnes,Nurmi,Takahashi (in progress)

Example II: scale-dependence from multiple field effects

- Consider a simple toy model (e.g. Gaussian inflaton + free curvaton):

$$\zeta(x) = \zeta_{\phi,\mathrm{G}}(x) + \zeta_{\sigma,\mathrm{G}}(x) + f_\sigma \zeta_{\sigma,\mathrm{G}}(x)^2$$

- $f_\sigma =$ const. but f_{NL} scale-dependent

$$f_{\mathrm{NL}}(k) = \frac{5B(k,k,k)}{18P_\zeta(k)^2} = \frac{5f_\sigma P_\sigma(k)^2}{3P_\zeta(k)^2} \propto k^{2(n_\sigma - n_\zeta)}$$

- In general, both multiple field effects and interactions generate scale-dependence

Conclusions

- Non-Gaussianity described by constant amplitudes only in special cases
- $f_{\rm NL}, g_{\rm NL}, \tau_{\rm NL}$ scale-dependent in general
- Need 3 new parameters $n_{f_{\rm NL}}, n_{g_{\rm NL}}, n_{\tau_{\rm NL}}$, depend on details of inflationary physics
- Easy to compute
- New observables, can distinguish between different models with large NG
- Planck could constrain $n_{f_{\rm NL}}$