Frame Dependence of Quantum Corrections in Cosmology

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*A. Yu. Kamenshchik, *arXiv:1101.5047 [gr-qc], to be published*

Cosmological Motivation

Cosmological action: Inflaton non-minimally coupled to gravity

$$\text{JF: } S[g_{\mu\nu},\phi] = \int d^4x\sqrt{g}\Big(U(\phi)\,R(g) - \frac{1}{2}\,\partial_\mu\phi\partial^\mu\phi - V(\phi)\Big)$$

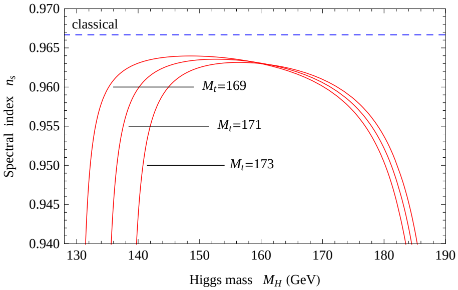
$$g_{\mu\nu}\to\tilde{g}_{\mu\nu} \Big\Downarrow \phi\to\tilde{\phi}$$

$$\text{EF: } S[\tilde{g}_{\mu\nu},\tilde{\phi}] = \int d^4x\sqrt{\tilde{g}}\Big(\frac{M_P^2}{2}\,\tilde{R} - \frac{1}{2}\,\partial_\mu\tilde{\phi}\partial^\mu\tilde{\phi} - V(\tilde{\phi})\Big)$$

- Conformal transformation:
  $g_{\mu\nu} = f(\phi)\,\tilde{g}_{\mu\nu} \Rightarrow U\sqrt{g}R = U\sqrt{\tilde{g}}\Big(f\,\tilde{R} + \frac{3}{2}f^{-1}f_{;\,\nu}f^{;\,\nu} - 3\,f_{;\nu}{}^{\nu}\Big) \Rightarrow f \equiv \frac{U_0}{U}$
- Field reparametrization:
  $\tilde{g}^{\mu\nu}\,\frac{U_0}{U}\,\frac{GU+3U'^2}{U}\phi_{,\,\nu}\phi_{,\,\mu} = \tilde{g}^{\mu\nu}\tilde{\phi}_{,\,\nu}\tilde{\phi}_{,\,\mu} \Rightarrow \Big(\frac{\partial\phi}{\partial\tilde{\phi}}\Big)^2 = \frac{U}{U_0}\,\frac{U}{GU+3U'^2} \equiv \frac{U}{U_0}\,s$
- Einstein-Hilbert term $\Rightarrow$ $U_0 \equiv \frac{M_P^2}{2}$

Multiple fields & Higgs inflation

- Standard Model Higgs inflation [1,2,3]: Quantum corrections and RG improvement crucial to predict a Higgs mass within the experimentally allowed bounds.
- $O(4)$ multiplet Φ^a instead of single field ϕ.
  Higgs inflaton $\varphi = \sqrt{\Phi^a \Phi_a}$ plus three Goldstone bosons.
- $\Rightarrow$ Important quantum contributions from Goldstone bosons.

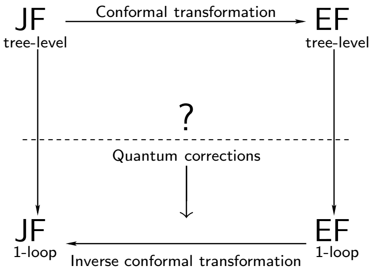


[1] F. L. Bezrukov, A. Magnin, M. Shaposhnikov, Phys. Lett. **B675** (2009) 88-92.
[2] A. De Simone, M. P. Hertzberg, F. Wilczek, Phys. Lett. **B678** (2009) 1-8.
[3] A. O. Barvinsky, A. Y. .Kamenshchik, C. Kiefer, A. A. Starobinsky, C. Steinwachs, JCAP **0912** (2009) 003.

Frame Dependence of Quantum Corrections

- Transition between JF and EF for $O(N)$ multiplet always possible[1].
- General $O(N)$ multiplet action in the Jordan frame

$$S^{JF}[g_{\mu\nu}, \Phi^a] = \int d^4x \sqrt{g}\Big(U(\varphi)\, R - \frac{1}{2}\, G(\varphi)\, \partial_\mu \Phi^a \partial^\mu \Phi_a - V(\varphi) \Big) \,.$$



[1] In contrast to the claim made in: D. I. Kaiser in Phys. Rev. D, **81** 084044, (2010).

Effective action & minimal operator

Divergent part of the one-loop effective action:

$$i\, W^{\text{div}}_{1-\text{loop}}[\bar{\Psi}] = -\frac{1}{2}\,\text{Tr}\ln F_{AB} + \text{Tr}\ln Q_{\mu}{}^{\nu}$$

$$\text{with } F_{AB}(\bar{\nabla}) \equiv \frac{\delta^2 S[\Psi]}{\delta\Psi^A\delta\Psi^B}\bigg|_{\Psi=\bar{\Psi}} - \frac{\delta\chi^{\mu}}{\delta\Psi^A}\, c_{\mu\nu}\, \frac{\delta\chi^{\nu}}{\delta\Psi^B} \text{ and } Q_{\mu}{}^{\nu} \equiv \frac{\delta\chi^{\mu}}{\delta\Psi^A}\, G^A_{\nu}$$

Matrix representation of F_{AB} & reduction algorithm to minimal form $\hat{F}(\mathcal{D})$ [1]:

$$I.\quad F_{AB}(\bar{\nabla}) = C_{AB}\bar{\Box} + \Gamma^{\mu}_{AB}\bar{\nabla}_{\mu} + W_{AB}$$

$$II.\quad \hat{F}(\bar{\nabla}) = (C^{-1})^{BC} F_{CA}(\bar{\nabla}) = \hat{1}\,\bar{\Box} + \hat{\Gamma}^{\mu}\bar{\nabla}_{\mu} + \hat{W}$$

$$III.\quad \hat{F}(\mathcal{D}) = \hat{\mathcal{D}}_{\mu}\hat{\mathcal{D}}^{\mu} + \hat{P} - \frac{1}{6}\,R\,\hat{1}, \text{ with } \hat{\mathcal{D}}_{\mu} \equiv \hat{1}\bar{\nabla}_{\mu} + \hat{\Gamma}_{\mu},\ [\hat{\mathcal{D}}_{\mu}, \hat{\mathcal{D}}_{\nu}] \equiv \hat{\mathcal{R}}_{\mu\nu}$$

[1] A. O. Barvinsky, G. A. Vilkovisky, Phys. Rept. **119** (1985) 1-74.

Schwinger-DeWitt Technique

Integral representation of $\hat{F}$, kernel and heat equation

$$-\frac{1}{\hat{F}} = i\int_0^\infty \mathrm{d}s\, e^{i\,s\,\hat{F}} \quad \Rightarrow \quad \ln \hat{F} = -\int_0^\infty \frac{\mathrm{d}s}{s}\, e^{i\,s\,\hat{F}}$$

$$K(x,x'|s) \equiv e^{i\,s\,\hat{F}}\delta(x,x'), \qquad i\frac{\partial}{\partial s}K(x,x'|s) = -\hat{F}\,K(x,x'|s)$$

"Proper time expansion" Ansatz & recursion relation[1]

$$K(x,x'|s) = \frac{i}{(4\pi\, i)^\omega}\frac{\mathcal{D}^{1/2}(x,x')}{s^\omega}\exp\Big[i\,\frac{\sigma(x,x')}{2s}\Big]\sum_{n=0}^{\infty}(is)^n\,\hat{a}_n(x,x'),$$

$$(n+1)\,\hat{a}_{n+1} + \sigma^\mu\nabla_\mu\,\hat{a}_{n+1} = \Delta^{1/2}\Box(\Delta^{1/2}\,\hat{a}_n) + (\hat{P} - \frac{1}{6}R\,\hat{1})\,\hat{a}_n$$

[1] B. S. DeWitt, "Dynamical theory of groups and fields", Gordon & Breach, New York, 1965,

Dimensional regularization & coincidence limit $x \to x'$

- World function: $\sigma(x,x') = \frac{1}{2}(\lambda_1 - \lambda_0)\int_{\lambda_0}^{\lambda_1} g_{\mu\nu} t^\mu t^\nu \propto \frac{1}{2}(\lambda_1-\lambda_0)^2$
- VanVleck determinant:
  $\mathcal{D}(x,x') = g^{1/2}(x)\Delta(x,x')g(x')^{1/2} = |\det(-\sigma_{,\mu\nu'})|$
- Dimensional regularization $\omega = \frac{d}{2} \to 2$ singles out "magical" $\hat{a}_2$ coefficient

Coincidence limit of $\hat{a}_2$ & effective action

$$\hat{a}_2(x,x) = \frac{1}{180}(R^{\alpha\beta\gamma\delta}R_{\alpha\beta\gamma\delta} - R^{\alpha\beta}R_{\alpha\beta} + \Box R)\hat{1} + \frac{1}{2}\hat{P}^2 + \frac{1}{12}\hat{\mathcal{R}}^2_{\mu\nu} + \frac{1}{6}\Box\hat{P}$$

$$W^{\rm div}_{1-{\rm loop}} = \frac{1}{32\pi^2(2-\omega)}\int {\rm d}^4x\, g^{1/2}\,{\rm tr}\,\hat{a}_2 - \frac{1}{16\pi^2(2-\omega)}\int {\rm d}^4x\, g^{1/2}\,{\rm tr}\,(a_{2\,Q})_\mu^{\ \nu}$$

- Algorithm implemented in `MathTensor`[1] package for `Mathematica` to manage the huge amount of terms in the tensor analysis.

[1] L. Parker and S.M. Christensen, "MathTensor: A System for Doing Tensor Analysis by Computer", Addison-Wesley, Redwood City, CA, 1994.

Results

- Suppression function $s = \frac{U}{GU+3U'^2} \simeq \frac{1}{6\xi}$ for $\varphi \gg \frac{M_P}{\sqrt{\xi}}$

Jordan frame quantization

$$U^{\text{JF}}_{\text{1-loop}} = V\left[-\frac{13}{3U} - \frac{7s\left(U'\right)^2}{3U^2} + \frac{4s^2\left(U'\right)^4}{U^3} - \frac{2s^2\left(U'\right)^2 U''}{U^2}\right] + V'\left[\frac{4s^2 U' U''}{U} - \frac{8s^2\left(U'\right)^3}{U^2} + \frac{8sU'}{3U}\right]$$
$$+V''\left[\frac{2s^2\left(U'\right)^2}{U} - s^2 U'' - \frac{s}{6}\right]$$

Einstein frame quantization

$$U^{\text{EF}}_{\text{1-loop}} = V\left[-\frac{s^2 G' U'}{6U} - \frac{s^2\left(U'\right)^2 U''}{U^2} + \frac{s^2\left(U'\right)^4}{2U^3} - \frac{5s\left(U'\right)^2}{6U^2} + \frac{sU''}{3U} - \frac{13}{3U}\right]$$
$$+V'\left[\frac{s^2 G'}{12} + \frac{s^2 U' U''}{2U} - \frac{s^2\left(U'\right)^3}{4U^2} + \frac{7sU'}{12U}\right] - \frac{sV''}{6}$$

Conclusion & outlook

- Transition between Jordan and Einstein frame for $O(N)$ multiplet always possible.
- Calculation of the one-loop corrections for the general model of a $O(N)$ symmetric multiplet of scalar fields non-minimally coupled to gravity.
- Quantum corrections are frame dependent
(Jordan versus Einstein frame).
- Relevant for the correct RG analysis of the Higgs Inflation model
(Inclusion of important Goldstone boson contributions in β functions).
- Information about the application range of cosmological models.
- Restoring frame independence by using a covariant formalism?