cosmology in the era of gravitational wave astronomy

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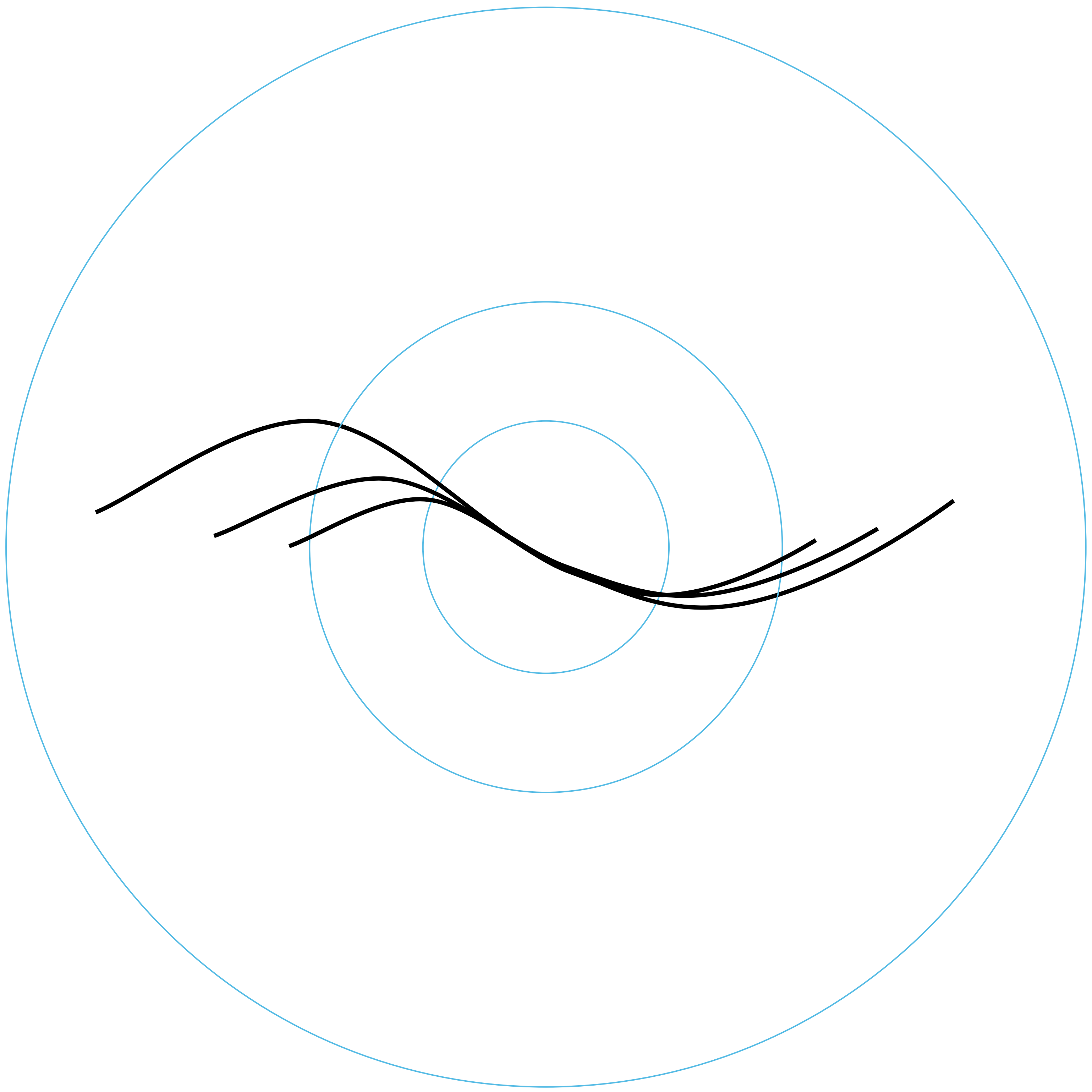
Slow contraction is a super-smoother.

-> to appear *very* soon:

Cook, Glushchenko, Ijjas, Pretorius, Steinhardt (2020): Super-smoothing through slow contraction

Cook, Davies, Ijjas, Pretorius, Steinhardt (2020): The robustness of slow contraction to cosmic initial conditions

Slow contraction leads to super-Hubble modes.



Generation of primordial perturbations

e.g., Ijjas et al.: PRD 89 (2014)123520

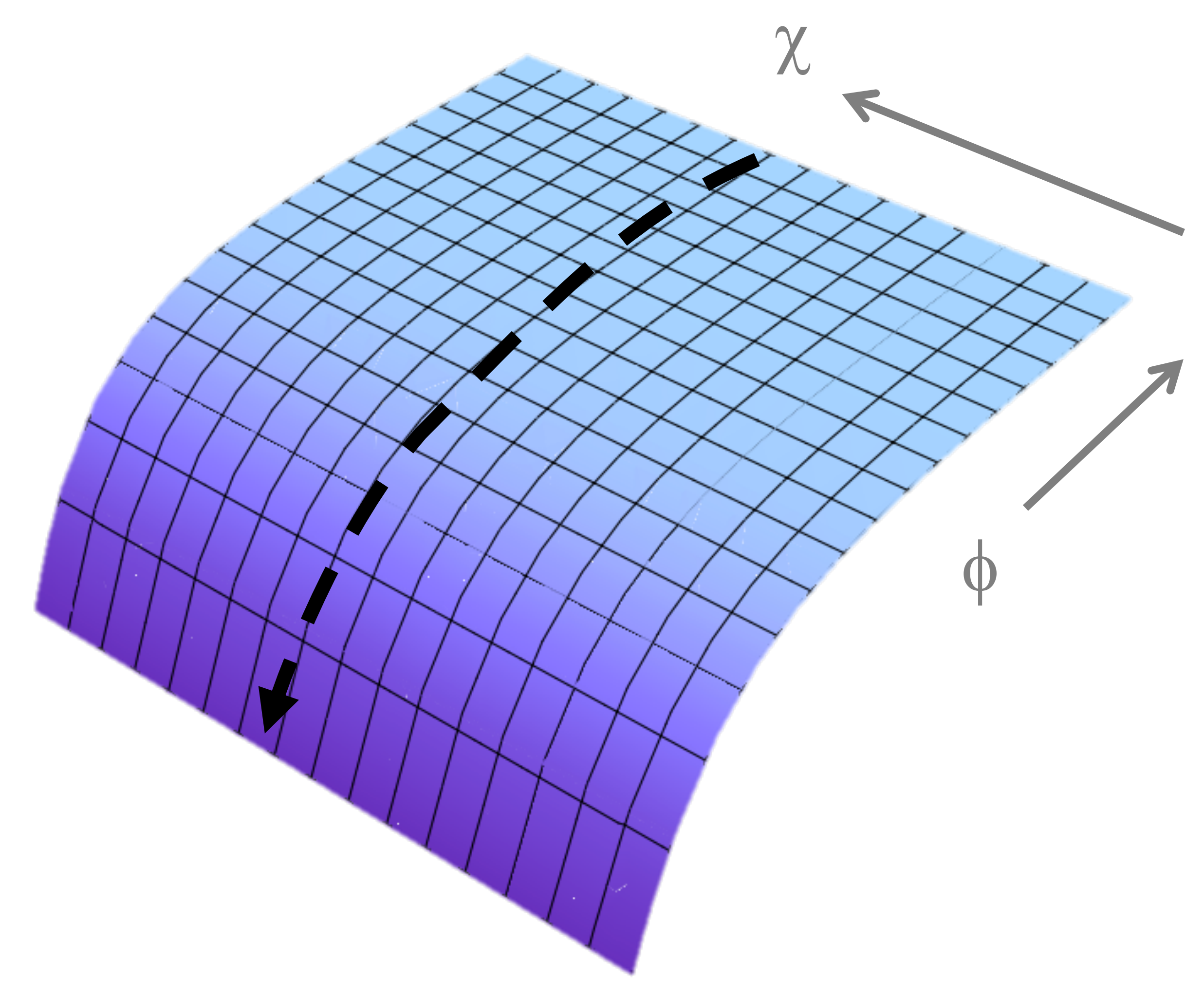
$$\mathcal{S} = \int d^4x\sqrt{-g}\left(\frac{1}{2}R - \frac{1}{2}(\partial_\mu\phi)^2 + V_0\exp(-\sqrt{2\epsilon}\phi) - \frac{1}{2}\Omega^2(\phi)(\partial_\mu\chi)^2\right)$$

FRW BACKGROUND:

$$H^2 = \frac{1}{3}\left(\frac{1}{2}\dot{\phi}^2 + \frac{1}{2}\Omega^2(\phi)\dot{\chi}^2 + V(\phi)\right),$$

$$\ddot{\phi} + 3H\dot{\phi} - \Omega\Omega_{,\phi}\,\dot{\chi}^2 + V_{,\phi} = 0,$$

$$\ddot{\chi} + \left(3H + 2\frac{\dot{\Omega}}{\Omega}\right)\dot{\chi} = 0,$$



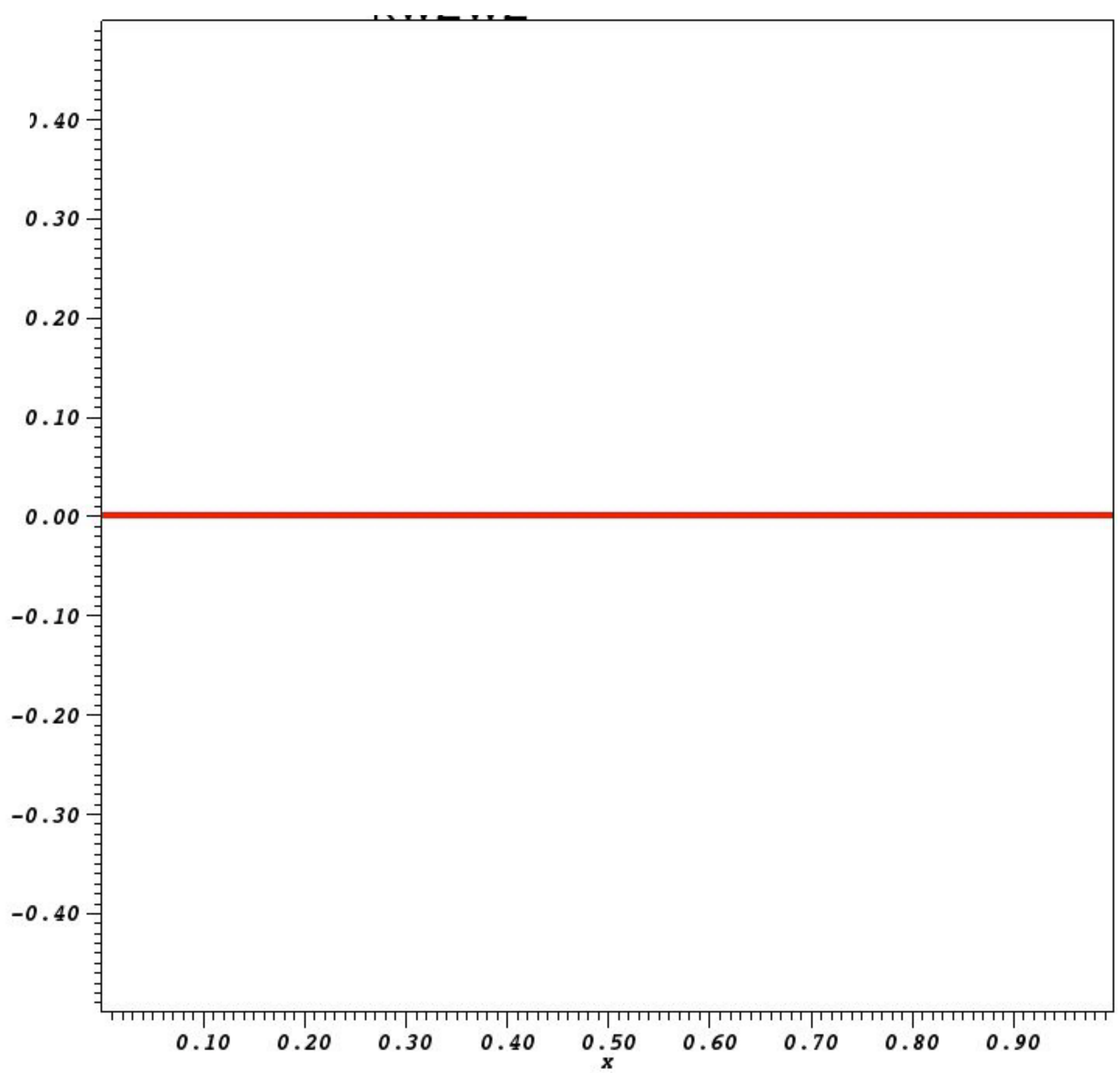
DENSITY PERTURBATIONS:

- stable background solution;
- (near) scale invariance
- local non-gaussianity: $f_{NL} = 0$

NO PRIMARY TENSOR PERTURBATIONS!

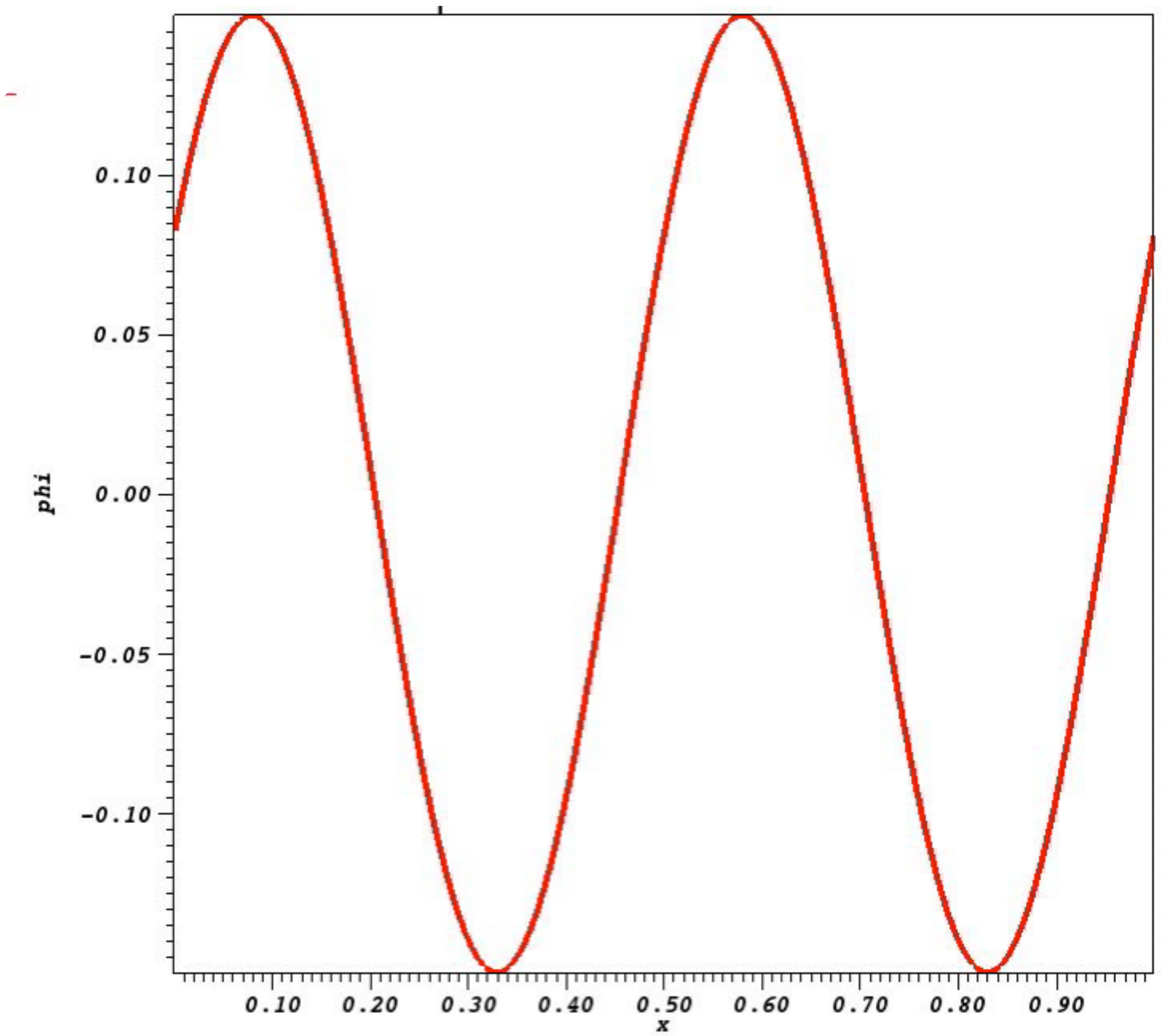
Non-perturbative analysis

Garfinkle, Ijjas, Pretorius, Steinhardt: to appear soon



Time=0

time evolution of the χ- field's kinetic energy density



Time=0

time evolution of the ϕ-field

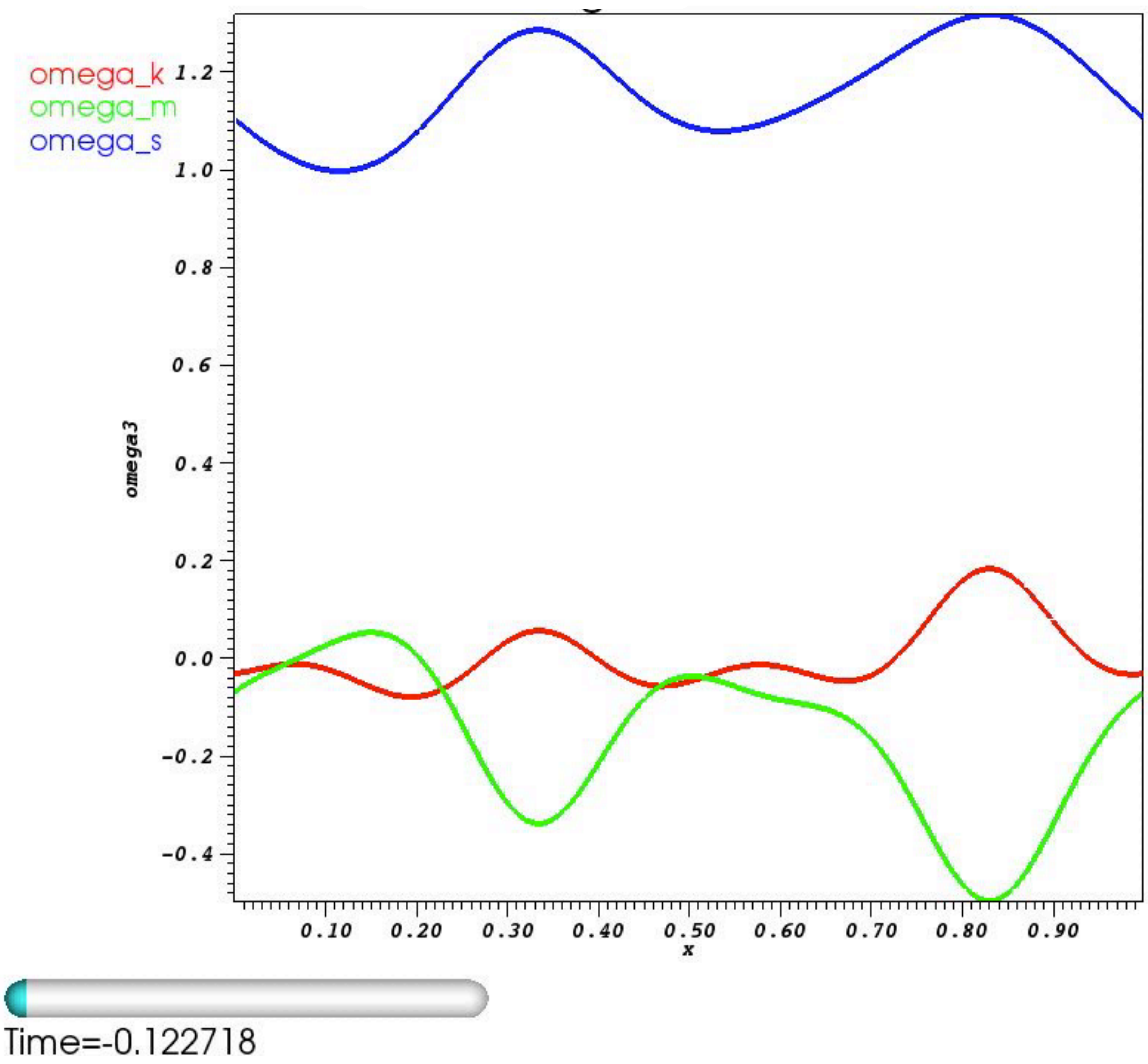
Non-perturbative analysis

Garfinkle, Ijjas, Pretorius, Steinhardt: to appear soon

shear – blue

curvature – red

scalar field matter – green



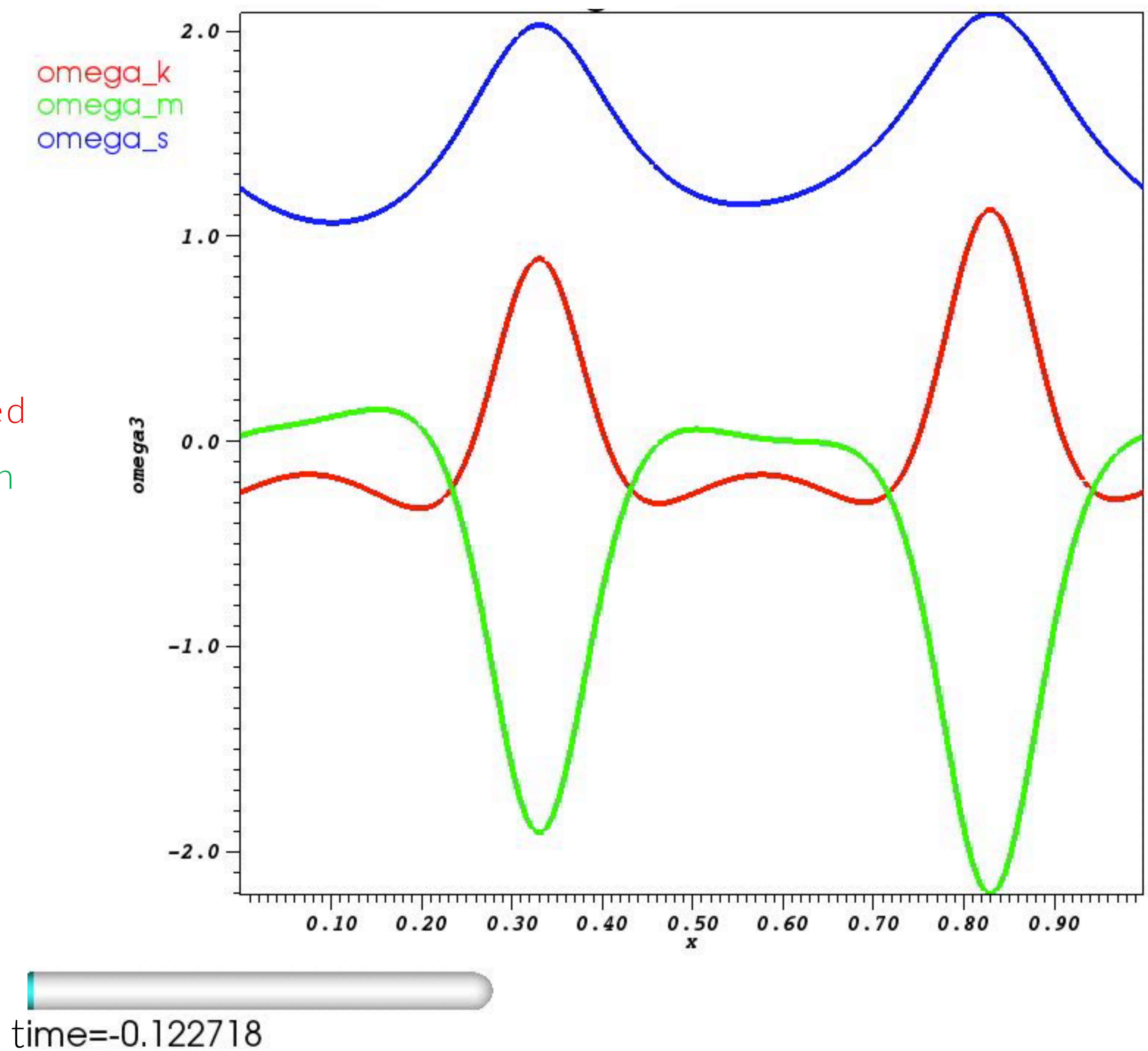
You better break that symmetry …

Garfinkle, Ijjas, Pretorius, Steinhardt: to appear soon

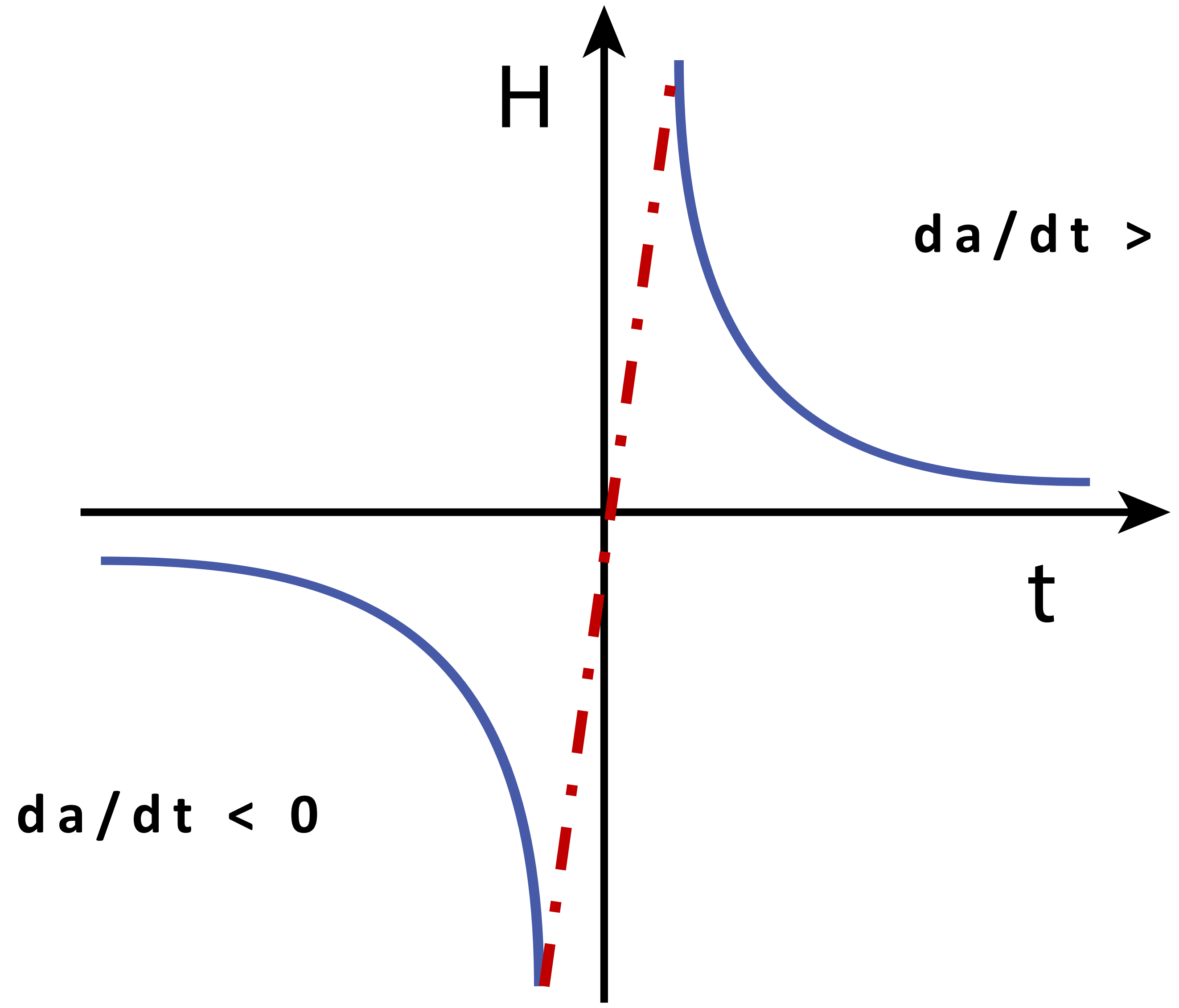
shear – blue

curvature – red

matter – green



What does it take to `bounce?'



VIOLATE NULL CONVERGENCE CONDITION:

$$R_{\alpha\beta}n^{\alpha}n^{\beta} \ngeq 0$$

singularity resolution through `braiding'

$$\mathcal{L} = \sqrt{-g}\left(\frac{1}{2}M_{\rm Pl}^2 R - \frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi + \frac{1}{\Lambda_3^3}\nabla_\mu\phi\nabla^\mu\phi\Box\phi\right)$$

- known since the 70s as Lorentz-invariant GR modification;
- rediscovered in early 2000s based on EFT considerations;
- radiatively stable below strong coupling scale $\Lambda_3 << M_{Pl}$;
- admits FRW bounce solution

But is it a `good' dynamical theory?

Test #1: `stability' to curvature fluctuations of the linear theory

Follow evolution of gauge-invariant Mukhanov-Sasaki variable

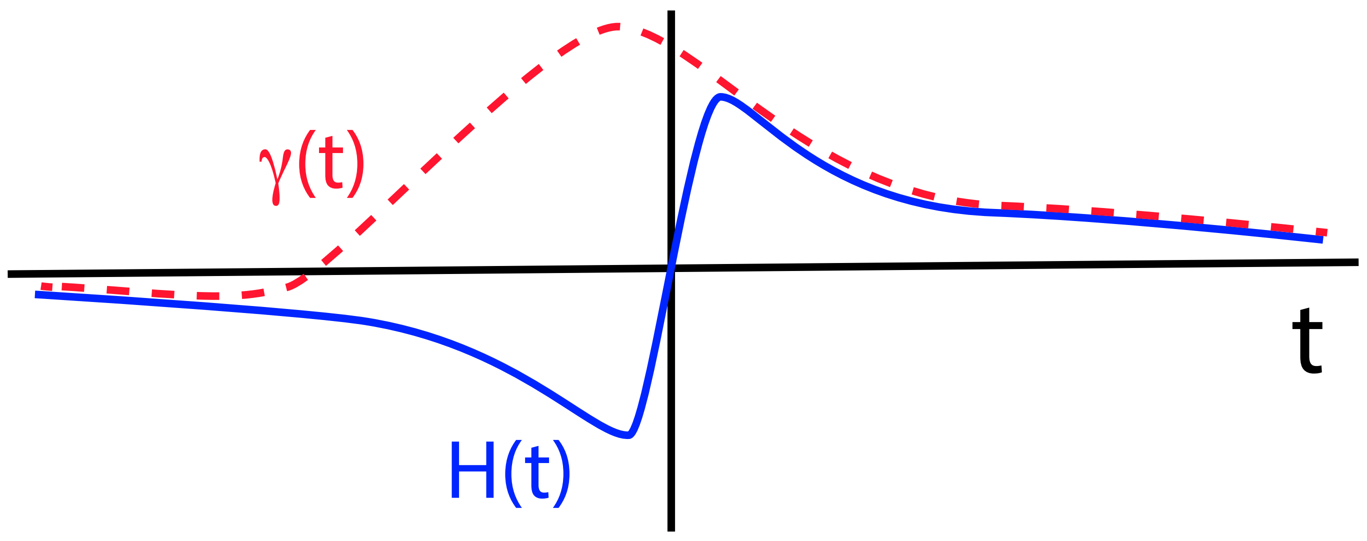
$$v \equiv \zeta - H(t)\frac{\delta\phi}{\dot\phi} \qquad \ddot v + \left(c_S^2 k^2 - \frac{\ddot z}{z}\right) v = 0$$

where $$c_S^2 \propto a(t)^{-1}\frac{\mathrm{d}}{\mathrm{d}t}\left(a(t)\frac{1}{\gamma(t)}\right) - 1$$

Ijjas, Steinhardt (2017a)

BRAIDING PARAMETER: **γ** [= H(t) in the Einstein limit]

measures interaction strength between geometry and scalar field matter



shifts problem from stably violating the null convergence condition and bounce to stably connecting to Einstein gravity BOTH before AND after the bounce

Ijjas, Steinhardt (2017b)

Test #2: check for coordinate artifacts

`instability' vs. coordinate singularity

gauge	NEWTONIAN
ODE for gauge variable	$\ddot{\Psi} + F(t,k)\dot{\Psi} + \left(m_0^2(t,k) + c_S^2(t,k)\frac{k^2}{a^2} + u_H^2(t,k)\frac{k^4}{a^4}\right)\Psi = 0$
characteristics of the ODE is determined by	see below

$$
\begin{aligned}
F(t,k) \equiv & \left(\det(P)\left(\left(H + \frac{\dot{A}_h}{A_h}\right)\left(-\dot{H} + \frac{\dot{A}_h}{A_h}H\right) - \frac{d}{dt}\left(-\dot{H} + \frac{\dot{A}_h}{A_h}H\right)\right)\right. \\
& \left. + \left(\frac{d}{dt}\ln\frac{a^3 A_h \det(P)}{(A_h H - \gamma)^2}\right)(A_h H - \gamma)^2\frac{k^2}{a^2}\right)\frac{1}{d(t,k)}\,, \\
m_0^2(t,k) \equiv & \left(2\dot{H} - H\frac{d}{dt}\ln\left(-\dot{H} + \frac{\dot{A}_h}{A_h}H\right)\right)\left(-\dot{H} + \frac{\dot{A}_h}{A_h}H\right)\frac{\det(P)}{d(t,k)}\,, \\
c_S^2(t,k) \equiv & \left(\left(-\dot{H} + \frac{\dot{A}_h}{A_h}H\right)\left(\det(P)c_\infty^2(t) + 2A_h\left(\dot{\gamma} + (A_h H - \gamma)H - \frac{d}{dt}(A_h H)\right)\right)\right. \\
& + 2(\dot{H} + H^2)(A_h H - \gamma)^2 + A_h(A_h H - \gamma)\frac{d}{dt}\left(-\dot{H} + \frac{\dot{A}_h}{A_h}H\right) - H\frac{d}{dt}(A_h H - \gamma)^2 \\
& \left. + \left(A_h(A_h H - \gamma)(-\dot{H} + \frac{\dot{A}_h}{A_h}H) + H(A_h H - \gamma)^2\right)\frac{d}{dt}\ln\det(P)\right)\frac{1}{d(t,k)}\,, \\
u_H^2(t,k) \equiv & \frac{1}{d(t,k)}(A_h H - \gamma)^2 c_\infty^2(t)\,, \\
d(t,k) \equiv & \det(P)\left(-\dot{H} + \frac{\dot{A}_h}{A_h}H\right) + (A_h H - \gamma)^2\frac{k^2}{a^2}\,, \\
c_\infty^2(t) \equiv & \frac{2\dot{A}_h\gamma + (A_h H - \gamma)\gamma - A_h\dot{\gamma}}{\det(P)}\,, \\
\det(P) \equiv & A_h\rho_K + 3(A_h H - \gamma)^2
\end{aligned}
$$

WELL-BEHAVED at γ - crossing

Ijjas (2018)

Test #3: 'Mode stability' (a.k.a. local well-posedness)

Study PDE structure of the `braided' system

verify that arbitrarily small wavelength mode fluctuations do NOT grow to large amplitudes on arbitrarily small timescales

the generalized harmonic formulation: $\Box x^\mu = J^\mu(x^\alpha)$

linearized Horndeski gravity in the generalized harmonic formulation:

$$A(t)\ddot{\mathbf{v}}(t,x^m) = \sum_{m,n=1}^{3} B^{mn}(t)\frac{\partial^2 \mathbf{v}}{\partial x^m \partial x^n}(t,x^m) + \sum_{m=1}^{3} D^m(t)\frac{\partial \dot{\mathbf{v}}}{\partial x^m}(t,x^m)$$
$$+ \sum_{m=1}^{3} E^m(t)\frac{\partial \mathbf{v}}{\partial x^m}(t,x^m) + F(t)\dot{\mathbf{v}}(t,x^m) + M(t)\mathbf{v}(t,x^m)$$

where

$$A(t) = \begin{pmatrix} A_h & A_{h\pi} \\ 0 & A_\pi \end{pmatrix}, \quad B^{mn}(t) = \begin{pmatrix} B_h^{mn} & B_{h\pi}^{mn} \\ 0 & B_\pi^{mn} \end{pmatrix}, \quad D^m(t) = \begin{pmatrix} 0 & D_{h\pi}^m \\ 0 & 0 \end{pmatrix}$$

and

$$\mathbf{v} \equiv (h_{00}, h_{0x}, h_{0y}, h_{0z}, h_{xx}, h_{xy}, h_{xz}, h_{yy}, h_{yz}, h_{zz}, \pi)^{\mathrm{T}} \in \mathbb{R}^{11}$$

Ijjas, Pretorius, Steinhardt (2019)

Test #3: 'Mode stability' cont'd

`frozen coefficient approximation'

move to first-order system: $\partial_t u = i\mathcal{P}(ik_m)u$ with $u = (|k|v, -i\dot{v})^{\mathrm{T}} \in \mathbb{R}^{22}$

Principal symbol: $$\mathcal{P}^0 = |k| \begin{pmatrix} 0 & \mathbb{I}_{11} \\ \hat{B}^{mn}\tilde{k}_m\tilde{k}_n & \hat{D}^m\tilde{k}_m \end{pmatrix}$$

Conditions for weak hyperbolicity: all eigenvalues of P^0 must be real

$$\lambda^{\pm} = \pm\sqrt{(-\bar{g}_{00})\bar{g}^{mn}\tilde{k}_m\tilde{k}_n}\,, \qquad c_S^{\pm} = \pm\sqrt{A_\pi^{-1}B_\pi^{mn}\tilde{k}_m\tilde{k}_n}$$

Conditions for strong hyperbolicity: there must be a complete set of eigenvectors

two EVs associated with the scalar field: $\mathbf{s}^{\pm} = (v_{tt}^{\pm}, \dots, v_{zz}^{\pm}, 1/c_S^{\pm}, w_{tt}^{\pm}, \dots, w_{zz}^{\pm}, 1)$,

$$v_{\mu\nu}^{\pm} = \frac{c_S^{\pm} A_{h\pi}^{\mu\nu} + (1/c_S^{\mp})\, B_{h\pi}^{\mu\nu} - D_{h\pi}^{\mu\nu}}{c_S^2\, \bar{g}^{00} + \bar{g}^{mn}\tilde{k}_m\tilde{k}_n}\,, \qquad w_{\mu\nu}^{\pm} = \frac{c_S^2 A_{h\pi}^{\mu\nu} - B_{h\pi}^{\mu\nu} - c_S^{\mp} D_{h\pi}^{\mu\nu}}{c_S^2\, \bar{g}^{00} + \bar{g}^{mn}\tilde{k}_m\tilde{k}_n}$$

Ijjas, Pretorius, Steinhardt (2019)

Take-home message:

There exist Horndeski theories that are linearly well-posed around relevant cosmological backgrounds

Necessary conditions for non-perturbative numerical analysis is met.

STAY TUNED!