

The Impact of Massive Neutrinos on the Matter Bispectrum

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Content

- 1) Introduction
- 2) Standard Perturbation Theory for CDM
- 3) Boltzmann Equation and
Perturbation Theory for Neutrinos
- 4) Results

Introduction

Goal calculate
density contrast:

$$\delta = \frac{\rho}{\bar{\rho}} - 1$$

$\rho \hat{=}$ energy density

$\bar{\rho} \hat{=}$ background density

e.g.:

$$\delta = \frac{\Omega_{CDM} \delta_{CDM} + \Omega_v \delta_v}{\Omega_{CDM} + \Omega_v}$$

$$\Omega_i = \frac{8\pi G}{3} \frac{\bar{\rho}_{i,0}}{H_0^2}$$

Only correlation function predictable:

Powerspectrum: $\langle \delta(\mathbf{k}_1) \delta(\mathbf{k}_2) \rangle = (2\pi)^3 \delta(\mathbf{k}_1 + \mathbf{k}_2) P(k_1)$

Bispectrum: $\langle \delta(\mathbf{k}_1) \delta(\mathbf{k}_2) \delta(\mathbf{k}_3) \rangle = (2\pi)^3 \delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) B(k_1, k_2, k_3)$

Initial conditions set by inflation:

Gaussian initial conditions $P_{\delta\Phi}(k) \propto k^{-3}$ and $B_{\delta\Phi} = 0$

Linear evolution $\longrightarrow P_i \neq P_{\delta\Phi}$ and $B_i = 0$

Non-Linear Evolution of CDM Perturbations

Fluid equations:

$$\frac{\partial \varphi_a}{\partial s} + \Omega_{ab} \varphi_b = a(s) \left[\gamma_{abc} \varphi_b(\mathbf{k}_1) \varphi_c(\mathbf{k}_2) \right]_k$$

CDM perturbations:

$$\varphi = \begin{pmatrix} \delta \\ \theta \end{pmatrix}$$

Peculiar velocity:

$$\theta = i \mathbf{k} \cdot \mathbf{u}$$

$\mathbf{u} \hat{=} \text{velocity field}$

Super conformal time:

$$ds = \frac{d\tau}{a} = \frac{dt}{a^2}$$

$$\Omega = \begin{pmatrix} 0 & a \\ \frac{3}{2} a \mathcal{H}^2 \Omega_{CDM} & a \mathcal{H} \end{pmatrix}$$

↑
Gravity

$$\mathcal{H} = \frac{1}{a} \frac{da}{d\tau} = aH \quad \gamma \hat{=} \text{vertex function}$$

$$[\]_k = \int \frac{d^3 k_1}{(2\pi)^3} \frac{d^3 k_2}{(2\pi)^3} (2\pi)^3 \delta_D(\mathbf{k} - (\mathbf{k}_1 + \mathbf{k}_2))$$

Non-Linear Evolution of CDM Perturbations

Linear solution given by Green's function:

$$\frac{\partial g_{ab}}{\partial s} + \Omega_{ac} g_{cb} = \delta_{ab} \delta(s - s')$$

Integral equation equivalent to fluid equations:

$$\varphi_a = g_{ab}(s, s_i) \varphi_{i,b} + \int_{s_i}^s ds' g_{ab}(s, s') a(s') [\gamma_{bcd} \varphi_c(s') \varphi_d(s')]_k$$

Iterative solution yields expansion in φ_i :

$$\begin{aligned} \varphi_a = & g_{ab}(s, s_i) \varphi_{i,b} \\ & + \int_{s_i}^s ds' g_{ab}(s, s') a(s') [\gamma_{bcd} g_{ce}(s', s_i) \varphi_{i,e} g_{df}(s', s_i) \varphi_{i,f}]_k + \dots \end{aligned}$$

Neutrinos

Neutrinos are massive

Earth based experiments:

$$0.056 \text{ eV} \lesssim \sum_i m_i \lesssim 6 \text{ eV}$$



Neutrino oscillations

Lesgourgues, Pastor 2006



Tritium beta decay

Cosmology:

$$\sum_i m_i = O(1 \text{ eV})$$

Hannestad 2006

- Neutrinos become non-relativistic at late times
- Larger neutrino mass means larger neutrino density

Neutrino fraction: $\frac{\Omega_\nu}{\Omega_{CDM} + \Omega_\nu} = O(0.1)$ \longrightarrow May have significant impact

Neutrinos as a Fluid

Fluid description follows
from phase-space distribution:

$$\rho = \int d^3 q f(q)$$

$$\rho \mathbf{u} = (am)^{-1} \int d^3 q \mathbf{q} f(q)$$

Non-vanishing stress:

$$\boldsymbol{\sigma} = (am)^{-2} \int d^3 q \mathbf{q} \mathbf{q} f(q) - \rho \mathbf{u} \mathbf{u}$$

On large scales:

$$\boldsymbol{\sigma} \approx c_s^2 \delta \mathbf{1} \quad \text{Shoji\&Komatsu 2010}$$

Gravitational part:

$$\frac{3}{2} a \mathcal{H}^2 \Omega_{CDM} \delta_{CDM} + \left(\frac{3}{2} a \mathcal{H}^2 \Omega_\nu - a k^2 c_s^2 \right) \delta_\nu \longrightarrow \text{Less effective clustering on small scales}$$

$\boldsymbol{\sigma} = ? \longrightarrow$ **No fluid description valid**

Neutrino Evolution

Split distribution function into background and perturbation: $f = \bar{f} + \delta f$

Evolution given by Boltzmann equation

$$\frac{\partial \delta f}{\partial s} + i \frac{\mathbf{q} \cdot \mathbf{k}}{m} \delta f - ia^2 m \frac{3}{2} \mathcal{H}^2 \frac{\mathbf{k}}{k^2} \cdot \frac{\partial \bar{f}}{\partial \mathbf{q}} \Omega \delta = ia^2 m \frac{3}{2} \mathcal{H}^2 \left[\frac{\mathbf{k}_1}{k_1^2} \cdot \frac{\partial \delta f}{\partial \mathbf{q}} (\mathbf{k}_2) \Omega \delta (\mathbf{k}_1) \right]_k$$

Straightforward expansion as for CDM in principle possible

- Much more involved due momentum dependence
- Only interested in integral over momentum

—► Try to integrate over momentum before solving

Gilbert's Equation

At linear order distribution function determined by density contrast:

$$\delta f = g_{FS}(s, s_i) \delta f_i + \int_{s_i}^s ds' g_{FS}(s, s') ia^2 m \frac{3}{2} \mathcal{H}^2 \frac{\mathbf{k}}{k^2} \cdot \frac{\partial \bar{f}}{\partial \mathbf{q}} \Omega \delta$$

Free-streaming solution: $g_{FS}(s, s') = \exp\left(-i \frac{\mathbf{q} \cdot \mathbf{k}}{m} (s - s')\right)$

Integrating over momentum yields Gilbert's equation:

$$\delta = I + \int_{s_i}^s ds' K(s, s') \delta(s') \quad \text{Solution: } \delta = \int_{s_i}^s ds' G(s, s') I(s')$$

$$I = \int d^3 q g_{FS}(s, s_i) \delta f_i \quad K(s, s') = \frac{3}{2} \mathcal{H}^2 a^2 (s - s') \Omega \int d^3 q g_{FS}(s, s') \bar{f}$$

e.g. Bertschinger 2007

Boyanovsky et al. 2008

Gilbert's Equation

Rewrite Boltzmann equation including the non-linear term:

$$\delta f = g_{FS}(s, s_i) \delta f_i + \int_{s_i}^s ds' g_{FS}(s, s') i\alpha^2 m \frac{3}{2} \mathcal{H}^2 \frac{\mathbf{k}}{k^2} \cdot \frac{\partial \bar{f}}{\partial \mathbf{q}} \Omega \delta$$

$$+ \int_{s_i}^s ds' g_{FS}(s, s') i\alpha^2 m \frac{3}{2} \mathcal{H}^2 \left[\frac{\mathbf{k}_1}{k_1^2} \cdot \frac{\partial \delta f}{\partial \mathbf{q}}(\mathbf{k}_2) \Omega \delta(\mathbf{k}_1) \right]_k$$

Integrating over momentum adds a non-linear term to Gilbert's equation:

$$\delta = I + \int_{s_i}^s ds' K(s, s') \delta(s')$$

$$+ \int_{s_i}^s ds' \int d^3 q g_{FS}(s, s') i\alpha^2 m \frac{3}{2} \left[\frac{\mathbf{k}_1}{k_1^2} \cdot \frac{\partial \delta f}{\partial \mathbf{q}}(\mathbf{k}_2) \Omega \delta(\mathbf{k}_1) \right]_k$$

Gilbert's Equation

Proceeding yields

$$\begin{aligned}
 \delta = & I + \int_{s_i}^s ds' K(s, s') \delta(s') && \text{Linear term} \\
 & + \int_{s_i}^s ds' \left[\tilde{\Gamma}^{(1)} \delta(s', \mathbf{k}_2) \right]_k && \left. \begin{array}{l} \text{Second order} \\ \text{terms} \end{array} \right\} \\
 & + \int_{s_i}^s ds_1 \int_{s_i}^{s_1} ds_2 \left[\Gamma^{(2)} \delta(s_1, \mathbf{k}_1) \delta(s_2, \mathbf{k}_2) \right]_k \\
 & + \int_{s_i}^s ds_1 \int_{s_i}^{s_1} ds_2 \left[\tilde{\Gamma}^{(2)} \delta(s_1, \mathbf{k}_1) \delta(s_2, \mathbf{k}_2) \right]_k && \left. \begin{array}{l} \text{Third order} \\ \text{terms} \end{array} \right\} \\
 & + \int_{s_i}^s ds_1 \int_{s_i}^{s_1} ds_2 \int_{s_i}^{s_2} ds_3 \left[\Gamma^{(3)} \delta(s_1, \mathbf{k}_1) \delta(s_2, \mathbf{k}_2) \delta(s_3, \mathbf{k}_3) \right]_k \\
 & + \dots
 \end{aligned}$$

Bispectrum

Density contrast up to second order

$$\delta(s, \mathbf{k}; \delta f_i) = \delta^{(1)}(s, \mathbf{k}; \delta f_i) + \delta^{(2)}(s, \mathbf{k}; \delta f_i) + \dots$$

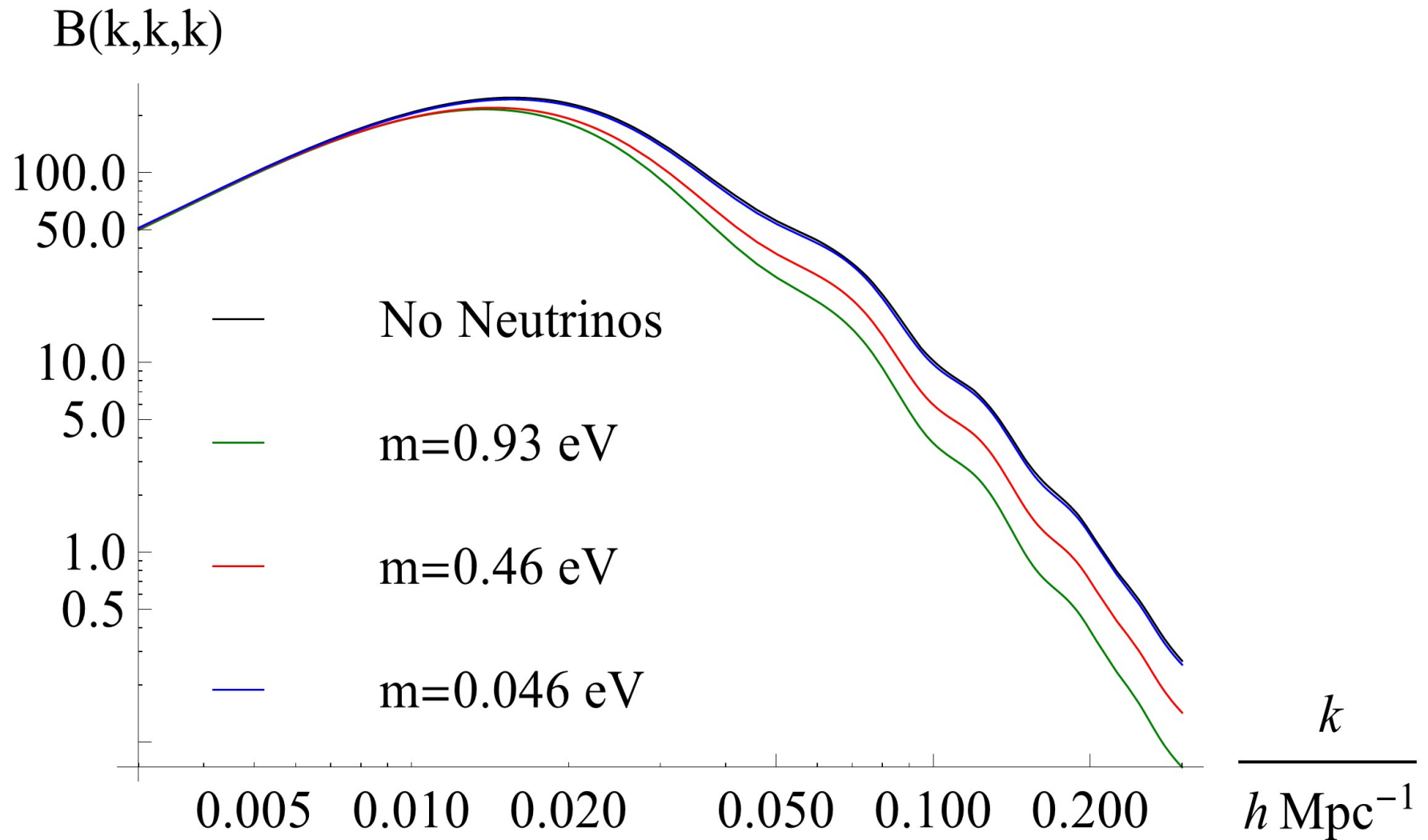
Three point function

$$\begin{aligned} \langle \delta(\mathbf{k}_1) \delta(\mathbf{k}_2) \delta(\mathbf{k}_3) \rangle &= \underbrace{\langle \delta^{(1)}(\mathbf{k}_1) \delta^{(1)}(\mathbf{k}_2) \delta^{(1)}(\mathbf{k}_3) \rangle}_{\propto B_i = 0} \\ &+ \underbrace{\langle \delta^{(2)}(\mathbf{k}_1) \delta^{(1)}(\mathbf{k}_2) \delta^{(1)}(\mathbf{k}_3) \rangle}_{= O(P_i^2)} + \text{permutations} + \dots \end{aligned}$$

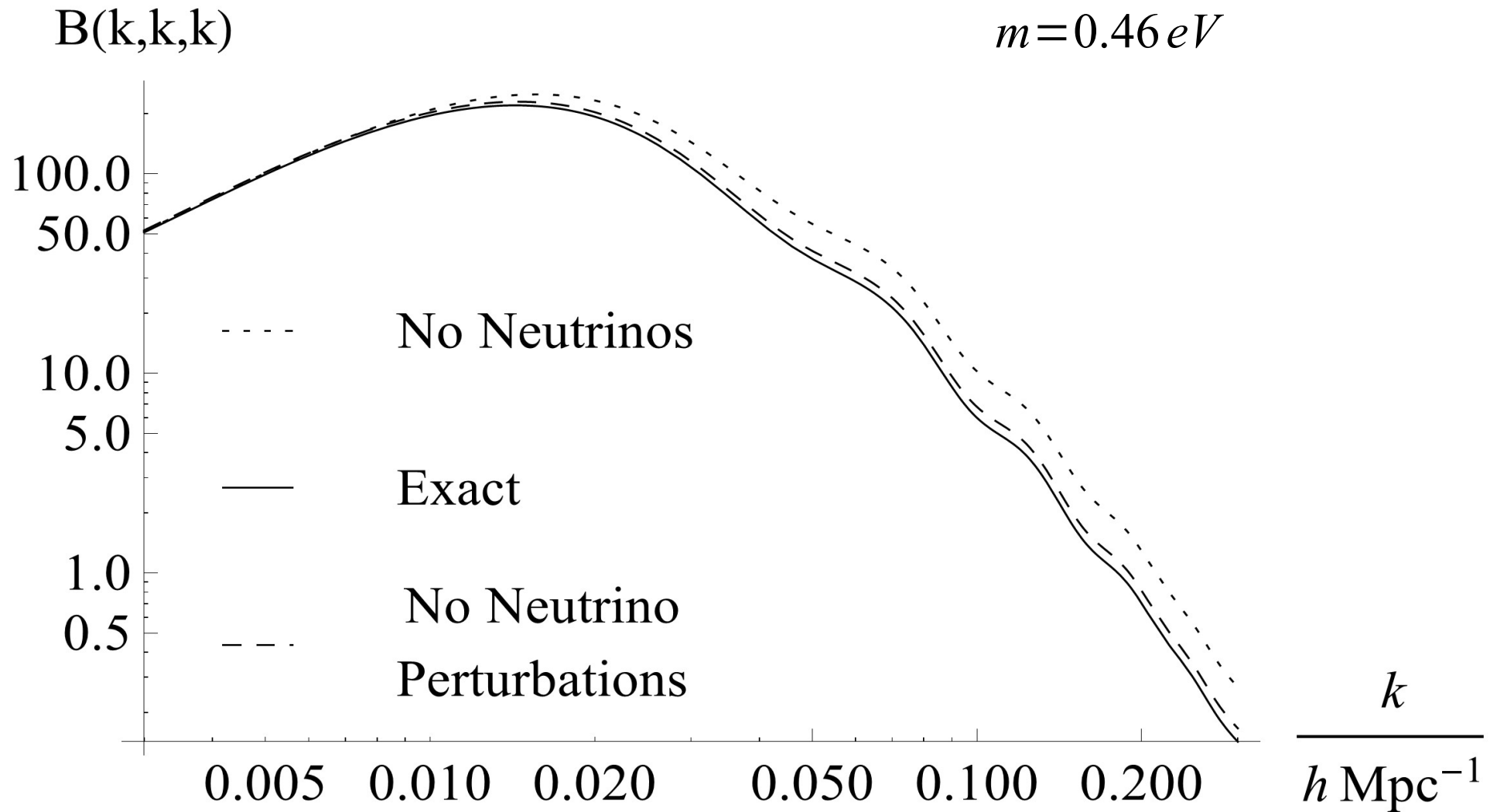
Bispectrum at lowest non-trivial order

$$B(k_1, k_2, k_3) (2\pi)^3 \delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) = \langle \delta^{(2)}(\mathbf{k}_1) \delta^{(1)}(\mathbf{k}_2) \delta^{(1)}(\mathbf{k}_3) \rangle + \text{permutations}$$

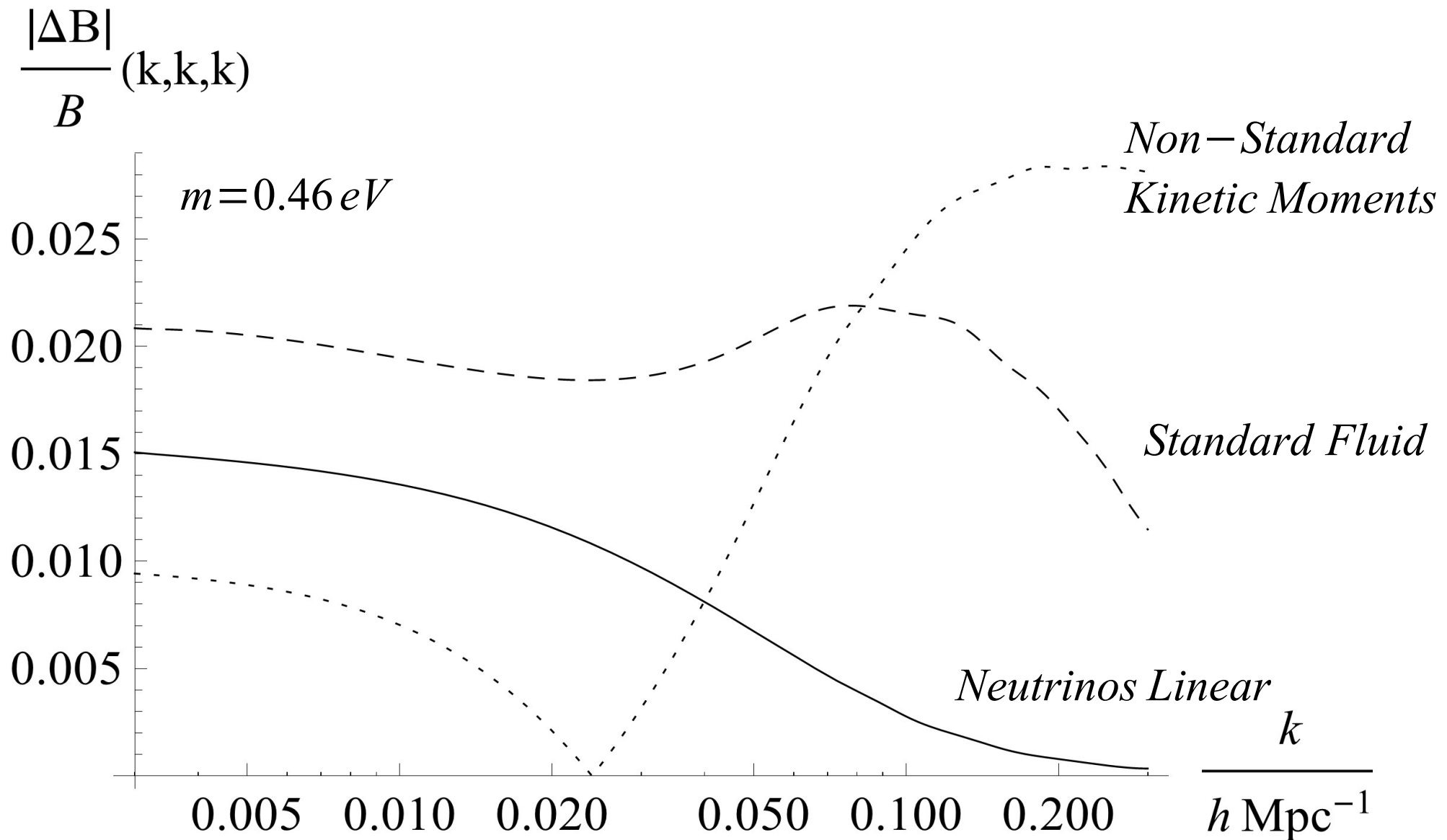
Bispectrum



Bispectrum



Conclusion



Summary and Outlook

- Neutrinos included into perturbation theory
- Bispectrum at second order calculated
 - Neutrino perturbations cannot be ignored
 - Neglecting non-linear neutrino terms gives error up to $O(10^{-2})$
 - Fluid like description gives errors at $O(10^{-2})$
- For higher orders k -integration necessary
- Use fluid like description on large scales and linear neutrinos on small scales?

Questions?