Gauge-invariant correlators at high temperature: Challenges for perturbation theory

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Examples

QCD: $\int_X e^{iK\cdot X} \langle (\bar\psi\gamma^\mu\psi)(X)(\bar\psi\gamma_\mu\psi)(0)\rangle_T$

$\Rightarrow$ diffusion coefficient, electrical conductivity

$\int_X e^{iK\cdot X} \langle T^{\mu\nu}(X) T_{\rho\sigma}(0)\rangle_T \;\times\; \mathbb{P}^{\rho\sigma}_{\mu\nu}$

$\Rightarrow$ shear and bulk viscosities

$\mathrm{Tr}\langle W(\tau,\mathbf{x})\rangle_T$

$\Rightarrow$ heavy quark potential

$\int_\tau e^{i\omega_n\tau}\,\mathrm{Tr}\langle U_0\, gE^i(\tau,\mathbf{x})\, U_0\, gE_i(0,\mathbf{x})\, U_0\rangle_T$

$\Rightarrow$ heavy quark kinetic equilibration rate

$\int_X e^{iK\cdot X} \langle (\bar\psi\gamma^i gE_i\psi)(X)(\bar\psi\gamma^j gE_j\psi)(0)\rangle_T$

$\Rightarrow$ heavy quark chemical equilibration rate

EW: $\int_X e^{iK\cdot X} \langle (\tilde\phi^\dagger a_L \ell)(X)\; (\bar\ell a_R \tilde\phi)(0)\rangle_T$

$\Rightarrow$ right-handed neutrino production rate

Time orderings and kinematics

Start by computing correlators in imaginary time:

$$\tau \in \left(0, \frac{1}{T}\right) \ , \quad K = (\omega_n, \mathbf{k}) \ .$$

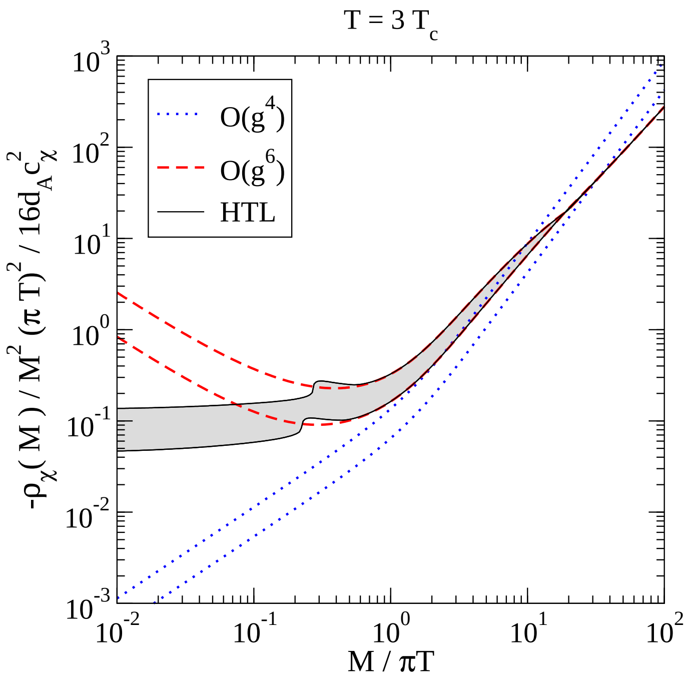
Subsequently carry out analytic continuation,

$$\begin{aligned} G_R(\omega, \mathbf{k}) &= G_E(\omega_n \to -i[\omega + i0^+], \mathbf{k}) \ , \\ \rho(\omega, \mathbf{k}) &= \operatorname{Im} G_R(\omega, \mathbf{k}) \ , \end{aligned}$$

and from these can also determine G_T, $G_<$, $G_>$, ...

The momentum could be time-like ($\mathcal{K} = (\omega, \mathbf{0})$, for transport coefficients), space-like ($\mathcal{K} = (0, \mathbf{k})$, for susceptibilities and screening), or on-shell ($\mathcal{K}^2 = M^2$, for single particle production).

The basic conceptual challenge (even for $g \equiv \sqrt{4\pi\alpha} \ll 1$):



Loop expansion may work for $M \gg \pi T$ (“hard” regime) but breaks down for $\pi T \gg M$ (“soft” regime).

Useful physics from the hard regime $\omega \equiv M \gg \pi T$?

In lattice QCD, a Euclidean correlator $G_E(\tau)$ is measured. Most physics is Minkowskian, so an analytic continuation is needed.

Analytic continuation is mathematically justified only if $G_E(\tau)$ is continuous;[1] so any divergence at $\tau \ll \frac{1}{T}$ needs to be subtracted.

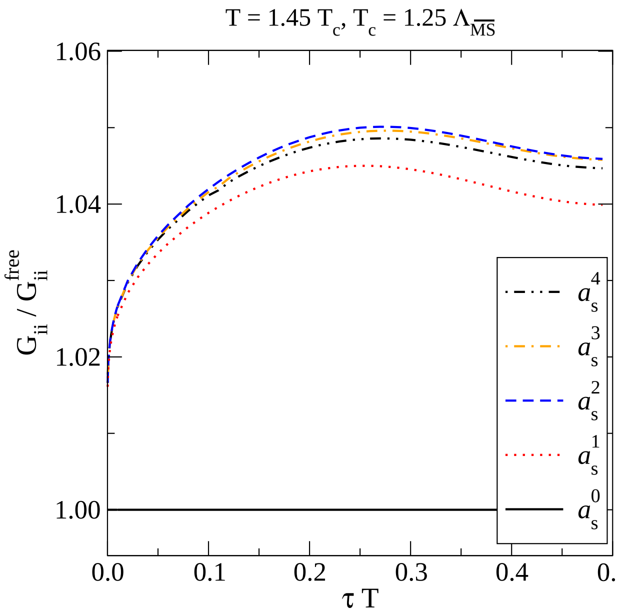
But divergences can only arise from $\omega \gg T$,

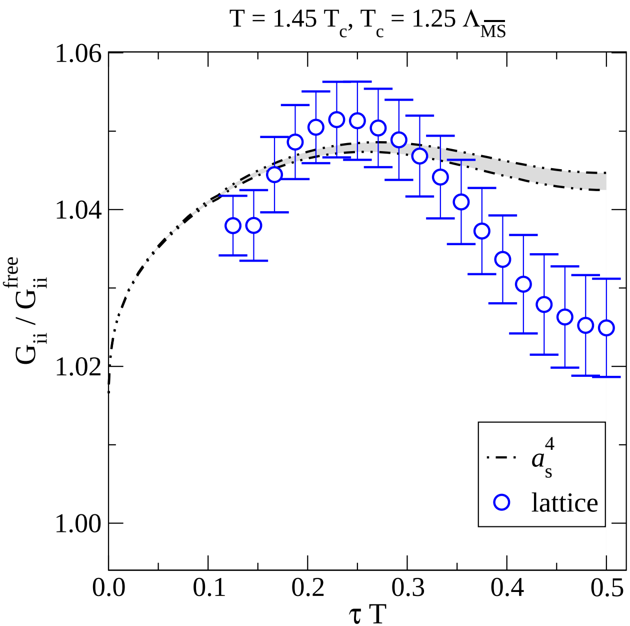
$$G_E(\tau) = \int_0^\infty \frac{\mathrm{d}\omega}{\pi} \rho(\omega) \underbrace{\frac{\cosh\left(\frac{1}{2T} - \tau\right)\omega}{\sinh\frac{\omega}{2T}}}_{\approx\, e^{-\tau\omega}}$$

and can therefore be computed within the loop expansion.

[1] G. Cuniberti, E. De Micheli and G.A. Viano, *Reconstructing the thermal Green functions at real times from those at imaginary times,* Commun. Math. Phys. 216 (2001) 59 [cond-mat/0109175].

Example for vector current:[2,3] (similar results needed for others)





[2] Analysis from: Y. Burnier and M. Laine, *Towards flavour diffusion coefficient and electrical conductivity without ultraviolet ...* , Eur. Phys. J. C 72 (2012) 1902 [1201.1994].

[3] $\mathcal{O}(a_s^4)$ result from: P.A. Baikov, K.G. Chetyrkin and J.H. Kühn, *$R(s)$ and hadronic τ-Decays in Order α_s^4: technical aspects,* Nucl. Phys. Proc. Suppl. 189 (2009) 49 [0906.2987].

In the EW case want to avoid lattice, so what can be done?

Right-handed neutrinos can (and must) be added to the SM:

$$\mathcal{L} \quad = \quad \mathcal{L}_{\mathsf{SM}} + \frac{1}{2}\bar{\tilde{N}}[i\not{\partial} - M]\tilde{N} - [h_\nu\,\bar{\ell}\,a_R\,\tilde{\phi}\,\tilde{N} + \mathrm{H.c.}]\;,$$

$$\ldots \quad \Rightarrow \quad \frac{\mathrm{d}N}{\mathrm{d}^4\mathcal{X}\mathrm{d}^3\mathbf{k}} = \frac{|h_\nu|^2}{k^0}\mathrm{Tr}\{\not{\mathcal{K}}\;\rho(\mathcal{K})\}\;.$$

Here the spectral function ρ corresponds to the correlator

$$\Sigma_E(K) \equiv \int_X e^{iK\cdot X}\langle(\tilde{\phi}^\dagger a_L \ell)(X)\;(\bar{\ell}a_R\tilde{\phi})(0)\rangle_T\;.$$

(Flavour indices have been suppressed.)

Difficult case: very early universe

Soft regime $\pi T \gg M \gg 100$ GeV:

A. Anisimov, D. Besak and D. Bödeker, *Thermal production of relativistic Majorana neutrinos: Strong enhancement by multiple soft scattering,* JCAP 03 (2011) 042 [1012.3784];
D. Besak and D. Bödeker, *Thermal production of ultrarelativistic right-handed neutrinos: Complete leading-order results,* JCAP 03 (2012) 029 [1202.1288].

Techniques ("LPM") similar to those in the QCD computation

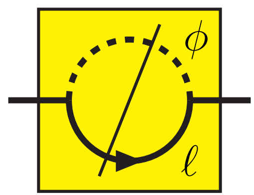
P.B. Arnold, G.D. Moore and L.G. Yaffe, *Photon emission from ultrarelativistic plasmas,* JHEP 11 (2001) 057 [hep-ph/0109064].

Concepts

"tree-level": give particles ("asymptotic") thermal masses and compute rate from the lowest-order kinematically allowed diagram.

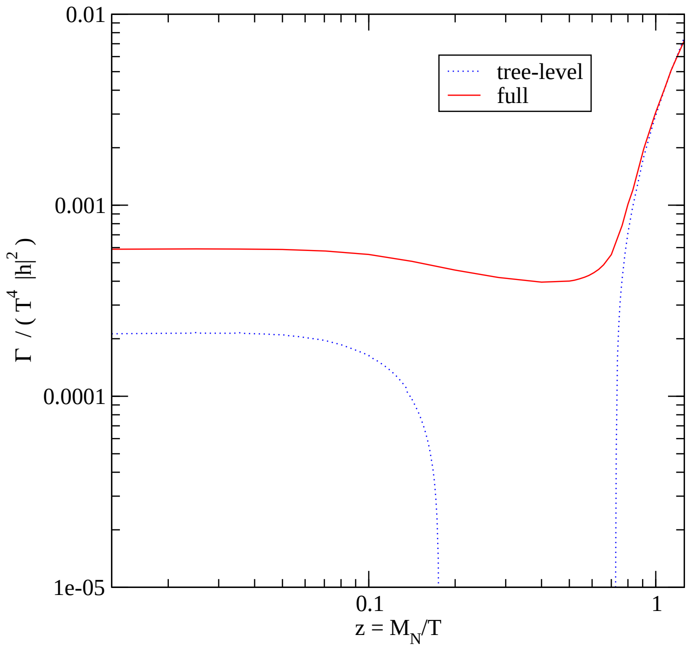
"consistent LO": include all processes which contribute at the same order in *coupling constants* (not necessarily at the same order in the loop expansion).

$$\frac{\mathrm{d}N}{\mathrm{d}^4\mathcal{X}\,\mathrm{d}^3\mathbf{k}} \propto N$$



N

Difference of “tree-level” and “consistent LO” is substantial:



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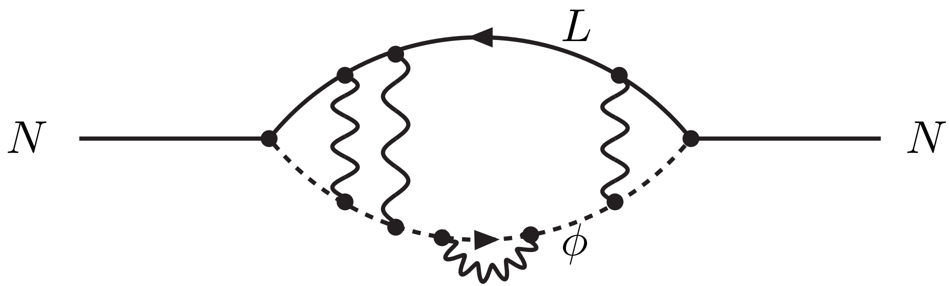
Why is this challenging?

From the plasma viewpoint it is a "**soft**" situation, with light-cone physics playing an important role: loop expansion breaks down, and needs to be resummed to all orders.



At the moment it isn't clear whether NLO is doable in practice, and whether it's small (NLO is only suppressed by $\mathcal{O}(\sqrt{\alpha})$).

Simpler case: late universe

Hard regime, $M \gg \pi T \gg 100$ GeV:

A. Salvio, P. Lodone and A. Strumia, *Towards leptogenesis at NLO: the right-handed neutrino interaction rate*, JHEP 08 (2011) 116 [1106.2814].

"*Previous partial results are extremely complicated because only some NLO effects have been computed, missing the great simplification that happens when including all NLO corrections...*"

"*In practice this means that 'gauge scatterings' and 'higgs scatterings' must be removed from codes for leptogenesis...*"

Why is this true?

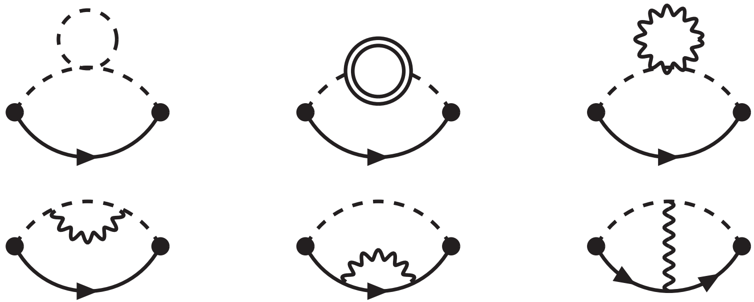
From the viewpoint of the plasma, $\mathcal{K}^2 = M^2$ is a **hard** scale. This means that we are in an "ultraviolet" regime, and can make use of the Operator Product Expansion (OPE):[4]

$$\mathrm{Tr}\{\not{\mathcal{K}}\ \rho(\mathcal{K})\} \sim f^{(2)}_{T=0}(\mathcal{K}^2) + f^{(0)}(\mathcal{K}^2; \bar{\mu}^2)\langle\phi^\dagger\phi\rangle_T + \mathcal{O}\left(\frac{T^4}{\mathcal{K}^2}\right) .$$

The absence of IR divergences can be shown up to 4-loop level; thermal corrections are small, and everything is under control!

[4] S. Caron-Huot, *Asymptotics of thermal spectral functions*, Phys. Rev. D 79 (2009) 125009 [0903.3958].

Things can be checked through a NLO computation.[5]



5 M. Laine and Y. Schröder, *Thermal right-handed neutrino production rate in the non-relativistic regime*, JHEP 02 (2012) 068 [1112.1205].

Complete self-energy:

$$
\begin{aligned}
\mathcal{Z}_\nu\,\Sigma_E(K) \quad &= \quad a_L i\not{K}\, a_R \bigg\{ \frac{1}{(4\pi)^2}\Big(\frac{1}{\epsilon} + \ln\frac{\bar\mu^2}{K^2} + 2\Big) \\
&+ \frac{|h_t|^2 N_{\rm c}}{(4\pi)^4}\Big(\frac{1}{2\epsilon^2} - \frac{3}{4\epsilon} - \frac{1}{2}\ln^2\frac{\bar\mu^2}{K^2} - \frac{7}{2}\ln\frac{\bar\mu^2}{K^2} - \frac{57}{8}\Big) \\
&+ \frac{g_1^2 + 3g_2^2}{(4\pi)^4}\Big(-\frac{3}{8\epsilon^2} + \frac{17}{16\epsilon} + \frac{3}{8}\ln^2\frac{\bar\mu^2}{K^2} + \frac{29}{8}\ln\frac{\bar\mu^2}{K^2} + \frac{275}{32} - 3\zeta(3)\Big) \\
&+ \Big[1 + \frac{6\lambda}{(4\pi)^2}\Big(\ln\frac{\bar\mu^2}{K^2} + 1\Big)\Big]\,\frac{\mathcal{Z}_m\langle\phi^\dagger\phi\rangle_T}{K^2} \;+\; \mathcal{O}\Big(g^4, \frac{T^4}{K^4}\Big)\bigg\}\,, \\
\mathcal{Z}_m\langle\phi^\dagger\phi\rangle_T \quad &= \quad \frac{T^2}{6} - \frac{T^2}{2\pi}\sqrt{\frac{m_H^2}{T^2} - \frac{g_1^2 m_{D1} + 3g_2^2 m_{D2}}{16\pi T}} \qquad \boxed{m_H^2 \sim g^2 T^2} \\
&+ \frac{T^2}{48\pi^2}\bigg\{-6\lambda\Big[\ln\Big(\frac{\bar\mu e^{\gamma_E}}{4\pi T}\Big) - 3\Big] - |h_t|^2 N_{\rm c}\ln\Big(\frac{\bar\mu e^{\gamma_E}}{8\pi T}\Big) \\
&+ \frac{3(g_1^2 + 3g_2^2)}{4}\Big[\ln\Big(\frac{\bar\mu e^{\gamma_E}}{4\pi T}\Big) - \frac{2}{3} - 2\gamma_E - 2\frac{\zeta'(-1)}{\zeta(-1)} + 4\ln\Big(\frac{2\pi T}{m_H}\Big)\Big]\bigg\} + \ldots
\end{aligned}
$$

Summary

Perturbation theory at high temperatures is faced with both conceptual challenges (slow convergence, need for resummations) and technical challenges (there is no Lorentz symmetry because the plasma defines a rest frame).

Nevertheless there are situations, both in QCD and in the EW theory, where high-order loop computations are useful, either on their own or when combined with lattice data.

So, it's worth its while to solve the technical challenges as well.