AUTUMN WORKSHOP "ASPECTS OF GRAVITY"

HIGHER DIMENSIONAL DARK ENERGY MODEL IN SAEZ-BALLESTER SCALAR-TENSOR THEORY OF GRAVITATION

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Chapter 1- Introduction

- Gravity is the oldest force known to mankind.
- Newton's theory of gravitation is a stunningly successful theory, but it has fatal flaws.
- For centuries, astronomer used it to calculate the motions of the planets with astonishing success rate. But the world needs more consistent theory to understand the universe.
- In 1916, Einstein proposed general theory of relativity.

About General Relativity

- This theory has its own essence which gives relationship between gravitation and spacetime.

- This theory was developed on three principles,
  - **Mach Principle**
  - **Principle of Equivalence**
  - **Principle of Covariance**

- Einstein's General relativity successfully shows the gravitational phenomenon and became the basis for the model of the universe.

Why do we need Alternative Theories?

- The Big Bang theory which was based on Einstein's field equations, successfully explains phenomena of expanding universe, CMBR etc.
- But the earlier predictions about universe don't meet our expectations.
- Along with this, some mysterious result regarding the redshift from the extra galactic observations continues to contradict the theoretical explanations given by the big bang theory type of model.

Alternative Theories

- Alternative theories of gravity are physical theories which are giving satisfactory competition to GR.

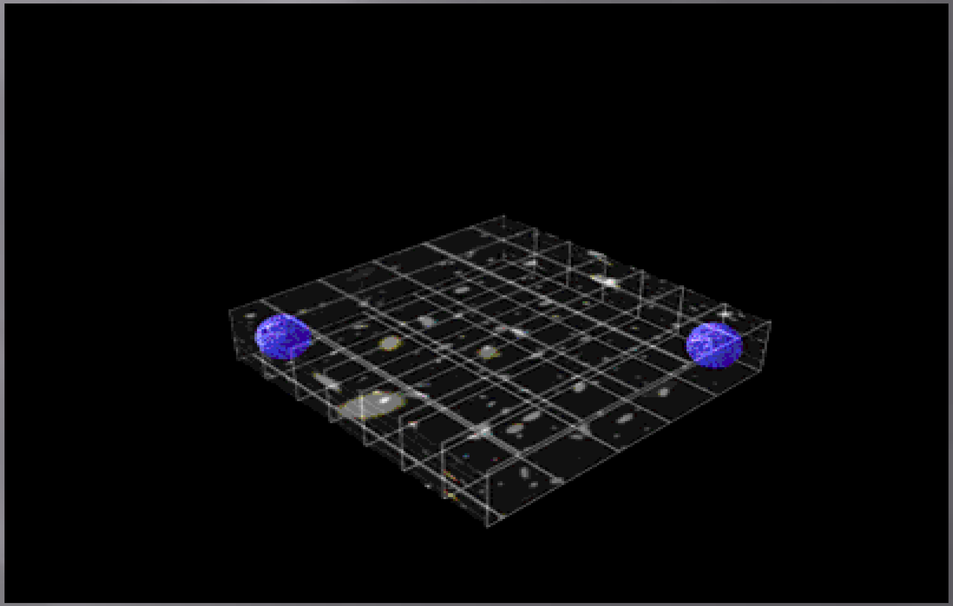
- Now-a-days, researchers are extensively studying modified theory of gravity like, Scalar-Tensor , CS, $f(T)$, $f(R,T)$, $f(R)$ *etc.*

Higher Dimensional Universe and Kaluza-Klein Theory

- The study of superstrings and supergravity theories discovered the importance of studying the universe in Higher dimensions.

- The view of Kaluza was that gravity can be combined with electromagnetism if the dimensions of the world were extending from 4 dimensions to 5.

- Klein did the geometrical unification of Gravitational and Electromagnetic field by assuming that the extra dimension was microscopically small with the size as the order of magnitude of electron.



An Example of how fifth dimensions might be represented

Source: The Startup Magazine

Chapter 2

- We considered five dimensional Kaluza-Klein metric in the form in this model.

$$ds^2 = dt^2 - A^2(dx^2 + dy^2 + dz^2) - B^2 d\psi^2 \qquad (1)$$

- The field equation is given by Saez-Ballester,

$$G_{mn} - w\phi^{\eta}\left(\phi_{,m}\phi_{,n} - \frac{1}{2}g_{mn}\phi_{,l}\phi^{,l}\right) = -T_{mn} \qquad (2)$$

- And the scalar field satisfies the equation,

$$2\phi^{\eta}\phi^{;m}_{;m} + \eta\phi^{\eta-1}\phi_{,l}\phi^{,l} = 0 \qquad (3)$$

Since action I is scalar and hence equation of motion,

$$T^{mn}_{;n} = 0 \tag{4}$$

The T_{mn} fluid taken to be,

$$T^{m}_{n} = d(\rho, -p_x, -p_y, -p_z, -p_\psi) \tag{5}$$

Field Equations

For solving field equations we are assuming,

- A scalar field ϕ depends upon t only.
- Using Saez- Ballester field equation on Kaluza-Klein metric, it leads to independent field equations

$$3\frac{\dot{A}}{A^2} + 3\frac{\dot{A}}{A}\frac{\dot{B}}{B} - w\phi^{\eta}\dot{\phi}^2 = -\rho \qquad (6)$$

$$2\frac{\ddot{A}}{A} + \frac{\dot{A}^2}{A^2} + 2\frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\ddot{B}}{B} + w\phi^{\eta}\dot{\phi}^2 = p \qquad (7)$$

$$3\frac{\ddot{A}}{A} + 3\frac{\dot{A}^2}{A^2} + w\phi^{\eta}\dot{\phi}^2 = w\rho \qquad (8)$$

$$\ddot{\phi} + \dot{\phi}\left(3\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right) + \frac{\eta}{2}\frac{\dot{\phi}^2}{\phi} = 0 \qquad (9)$$

Solutions of the field equations

- The spatial volume is given by

$$V = A^3 B \tag{10}$$

As well as we define,

- Average scale factor of the Kaluza-Klein model is

$$a = (A^3 B)^{\frac{1}{4}} \tag{11}$$

- Average Hubble's Parameter is

$$H = \frac{\dot{a}}{a} = \frac{1}{4}\left(3\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right) \tag{12}$$

- The scalar expansion,

$$\theta = 3\frac{\dot{A}}{A} + \frac{\dot{B}}{B} \tag{13}$$

- Average Anisotropic parameter,

$$A_\gamma = \frac{1}{4}\sum_{i=1}^{4}\left(\frac{H_i - H}{H}\right) \tag{14}$$

- Shear Scalar,

$$\sigma^2 = \frac{1}{3}\left(3\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right)^2 \tag{15}$$

- In cosmology deceleration parameter which measures the space expansion in FRLW universe is dimensionless quantity and it is defined as,

$$q = -\frac{\ddot{a}a}{\dot{a}^2} \tag{16}$$

- On integration, the deceleration parameter yields the solution, without loss of generality,

$$a = (A^3B)^{\frac{1}{4}} = \left(t^{\beta}e^{t}\right)^{\frac{1}{4\mu}} \tag{17}$$

- Next we have assumed scalar expansion θ proportional to shear scalar σ^2 which leads to $A = B^m$

$$A = t^{\frac{\beta}{\mu(m+3)}}e^{\frac{t}{\mu(m+3)}} \tag{18}$$

$$B = t^{\frac{\beta m}{\mu(3+m)}}e^{\frac{tm}{\mu(3+m)}} \tag{19}$$

- Hence the metric is

$$ds^2 = dt^2 - t^{\frac{\beta}{\mu(m+3)}} e^{\frac{t}{\mu(m+3)}} (dx^2 + dy^2 + dz^2) - t^{\frac{\beta m}{\mu(3+m)}} e^{\frac{tm}{\mu(3+m)}} d\psi^2$$

Chapter 4- Physical Properties of Model

1. $V = \left(t^{\beta} e^{t}\right)^{\frac{1}{\mu}}$
2. $\sigma^2 = \frac{9(3m^2+1)}{2\mu^2(m+3)^2}\left[\frac{\beta}{t} + 1\right]^2$
3. $H = \frac{3(m+1)}{\mu(m+3)}\left[\frac{\beta}{t} + 1\right]$
4. $A_{\gamma} = \frac{3m^2+1}{4(m+1)^2}$
5. $\theta = \frac{12(m+1)}{\mu(m+3)}\left[\frac{\beta}{t} + 1\right]$
6. $q = -1 + \frac{4\mu\beta}{(\beta+t)^2}$

7. $$\frac{\sigma^2}{\theta^2} = \frac{1}{32}\frac{3m^2+1}{(m+1)^2}.$$

8. $$\phi = -t^{1-\frac{\beta}{\mu}}\left(\frac{t}{\mu}\right)^{\frac{\beta}{\mu}}\left|\left(1-\frac{\beta}{\mu},\frac{t}{\mu}\right)\right. + const.$$

9. $$\rho = (-1)^n w t^{n-2\frac{\beta}{\mu}} e^{-2\frac{t}{\mu}}\left(\frac{1}{\mu}\right)^{\left(\frac{\beta\eta}{\mu}+2\right)}\Im(t)F(t) - \frac{3(m+1)}{\mu^2(m+3)^2}\left[\frac{\beta}{t}+1\right]^2$$

Graphs

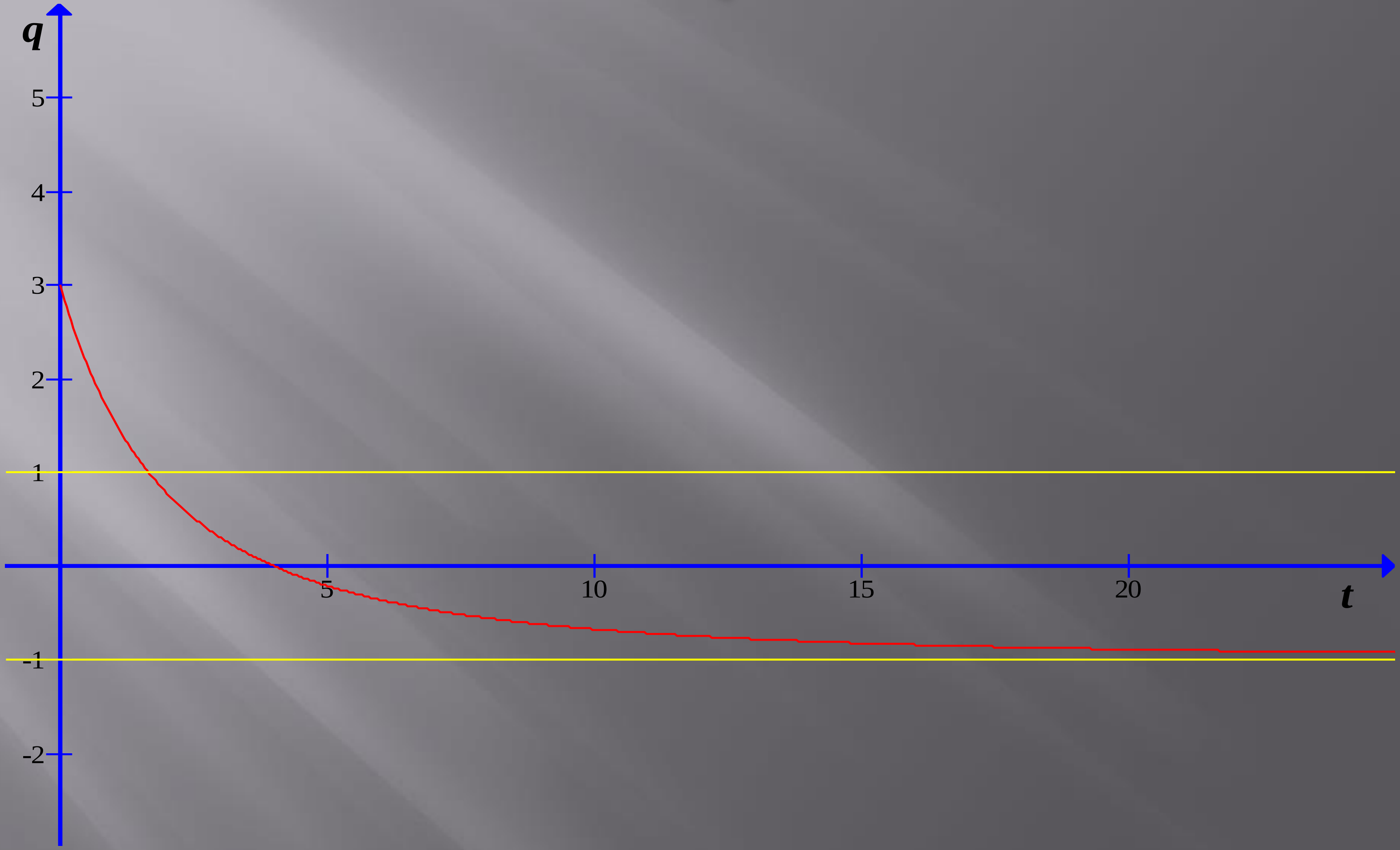


Fig1. The variation of q vs t, $\beta = 4$ and $\mu = 4$.

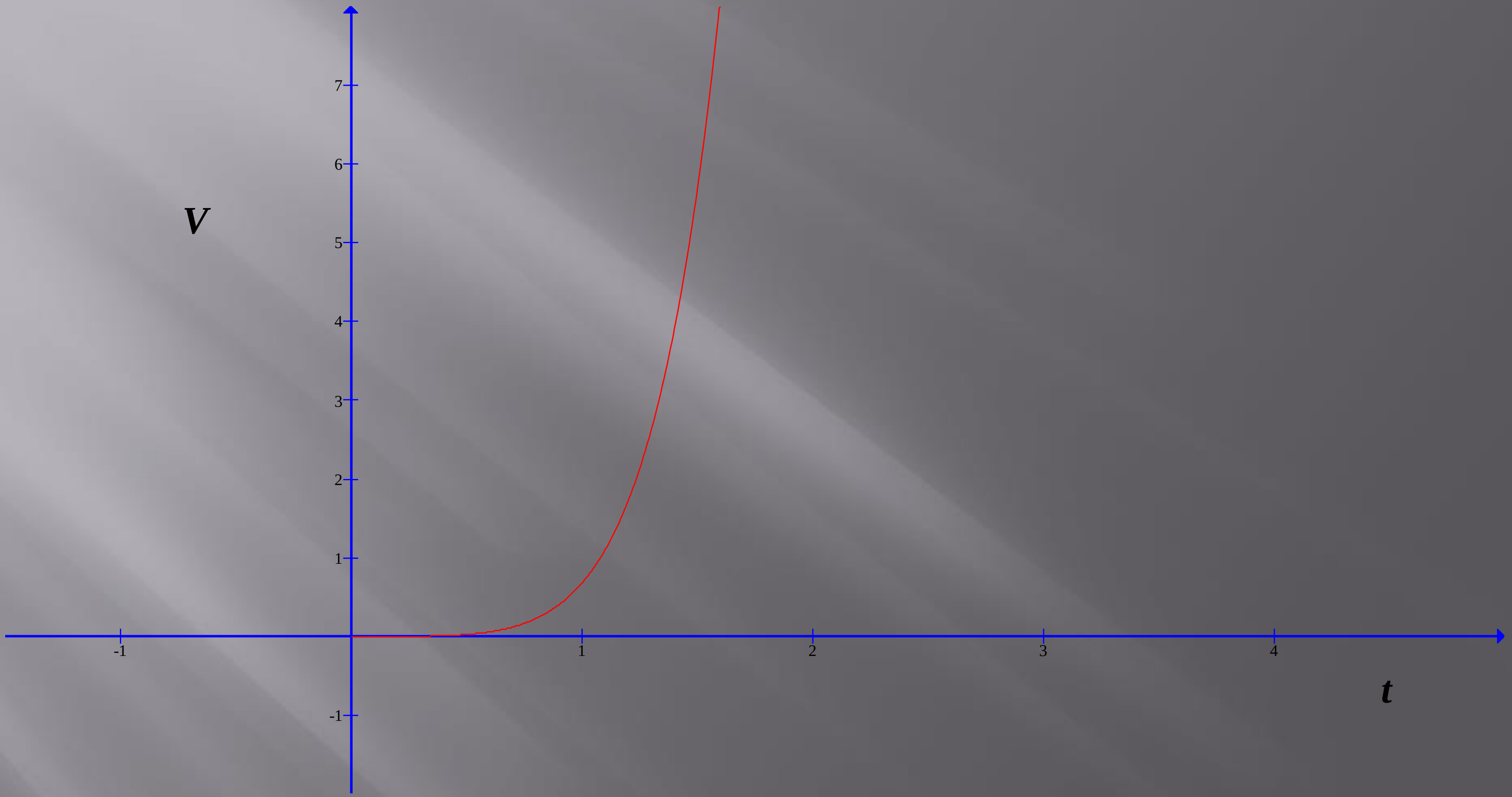


Fig2. The variation of V vs t , $\beta = 4$, $\mu = 4$

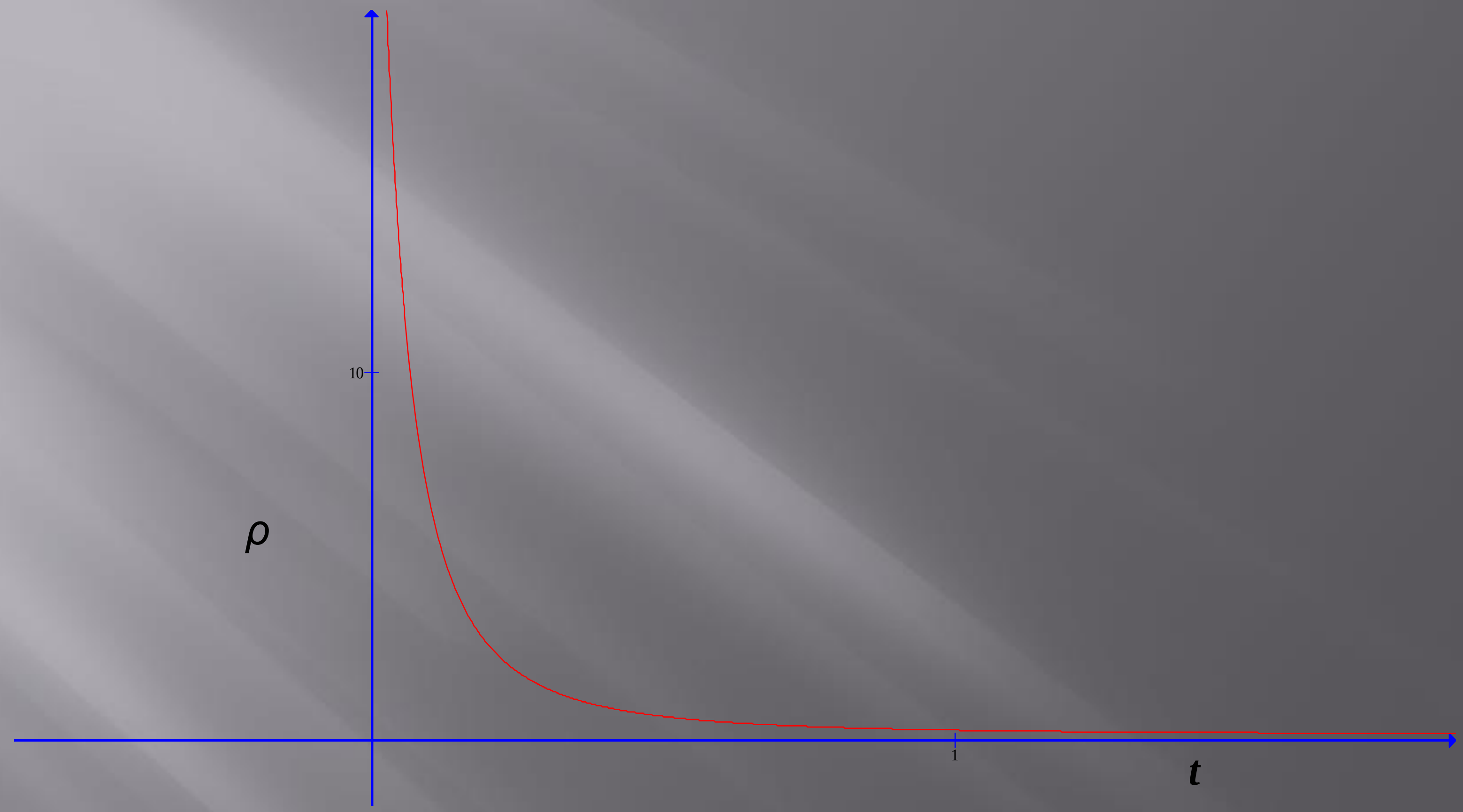


Fig3. The variation of ρ vs t

Conclusion

- In this paper, we had investigated the Kaluza-Klein cosmological model filled with perfect fluid and obeying the hybrid expansion law under the scalar-tensor theory of gravitation.
- The evolution of the universe from the phase of zero volume at the initial stage to an ever growing infinite volume as $t \to \infty$ is quite visible.
- The analysis of deceleration parameter gives a satisfactory resemblance to the early and late expansions of the actual universe shown by Riess *et al.* (1998), etc.

Conclusion Contd.

- The transition of the universe from the early decelerated expansion phase to the current accelerated expansion phase can be observed.
- Thus, our considered model can be used for describing the late time structure of the universe as the value of the deceleration parameter lies in the range $-1 < q < 0$.
- Thus, the model depicts the actual universe in an adequate way.

Thank You !