From Regular Black Holes to Experimental Constraints

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Aspects of Gravity

RTG Models of Gravity, Bielefeld, October 07, 2020

P. Nicolini, E. Spallucci, M.F. Wondrak, Phys. Lett. **B797** (2019) 134888, arXiv:1902.11242 [gr-qc].
M.F. Wondrak, M. Bleicher, Symmetry **11** (2019) 1478, arXiv:1910.08203 [gr-qc].

Setting

- General relativity
  - In agreement with precision experiments.
  - Conceptual breakdown at short scales:
    Cosmological and black hole singularities.
- String theory
  - Natural UV cutoff by smallest sensible distance,
    T-duality self-dual radius: $R^\star = \sqrt{\alpha'}$
  - In case of one compactified extra dimension:

$$R \to {R^\star}^2/R, \qquad n \leftrightarrow w$$

n: Kaluza-Klein mode number, w: winding mode number

$\rightarrow$ Restrict moduli space to $R \geq R^\star$.

Outline

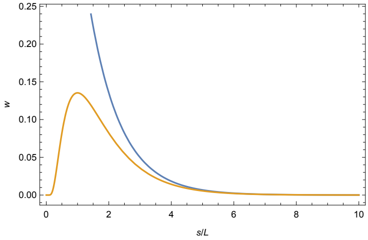
1. Propagator from Path Integral Duality and T-Duality
2. Energy Distribution
3. T-Duality Black Hole
4. Black Hole Thermodynamics
5. Conceptual Constraints on the Self-Dual Radius
6. Experimental Constraints from the Hydrogen Atom

Path Integral Duality

- World line formalism: Superposition of propagating field excitations.
- Idea: Spacetime fluctuations affect the propagation. The shorter is the proper time, the more it is hindered.
- Schwinger representation of the Feynman propagator:

$$G_{\mathrm{F}}(x,y) = \int_0^\infty \mathrm{d}s \; \underbrace{\exp\left(-m\left[s + L^2/s\right]\right)}_{w(s)} \; K(x,y;s)$$

$\Rightarrow$ effective zero-point length: $(x-y)^2 \to (x-y)^2 + {\ell_0}^2 \qquad \ell_0 = 2L.$



Padmanabhan, Phys. Rev. Lett. **78** (1997) 1854; Padmanabhan, Phys. Rev. **D57** (1998) 6206

Bosonic String Theory

Propagation of the string center-of-mass in a non-trivial vacuum.

- Closed bosonic string on a (3+1)+1 dimensional spacetime.
- Schwinger representation of the propagator. Integrate over the compact dimension.
- Require invariance under T-duality.

$\Rightarrow$ Effective 4-dim description with the same modified weight factor $w(s)$. Identify $\ell_0 = 2\pi R^\star$.

Fontanini, Spallucci, Padmanabhan, Phys. Lett. **B633** (2006) 627

Bosonic String Theory

Propagator in momentum space:

$$G(k) = -\frac{\ell_0 \, K_1\left(\ell_0 \sqrt{k^2+m^2}\right)}{\sqrt{k^2+m^2}}$$

$$\to \begin{cases} -\dfrac{1}{k^2+m^2}, & \text{for } k^2+m^2 \ll {\ell_0}^{-2} \\ -\dfrac{\sqrt{\ell_0}}{\left(k^2+m^2\right)^{3/4}} \exp\left(-\ell_0 \sqrt{k^2+m^2}\right), & \text{for } k^2+m^2 \gg {\ell_0}^{-2} \end{cases}$$

$K_\nu(x)$: Modified Bessel functions of 2nd kind.

Small momenta $\to$ standard propagator.
Large momenta $\to$ exponential suppression.

Fontanini, Spallucci, Padmanabhan, Phys. Lett. **B633** (2006) 627

Static Potential and Energy Density

Exchange of virtual massless particles between masses M and M_2.
Energy functional $W[J]$:

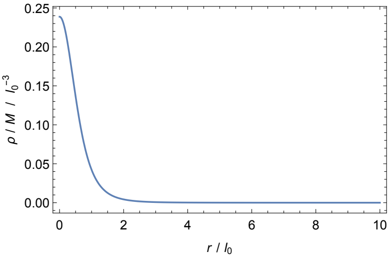
$$W[J] = -\ln Z[J] = \lim_{T\to\infty} M\, M_2\, T \int \frac{d^3k}{(2\pi)^3}\, G(k)|_{k^0=0}\, e^{i\vec{k}\cdot\vec{r}}$$

Interaction potential:

$$V(r) = -\frac{1}{M_2}\frac{W[J]}{T} = -\frac{M}{\sqrt{r^2+\ell_0{}^2}}$$

Re-interpretation in terms of standard theory: Quantum modified matter.

$$\rho(r) = \frac{3\ell_0{}^2 M}{4\pi\left(r^2+\ell_0{}^2\right)^{5/2}}$$



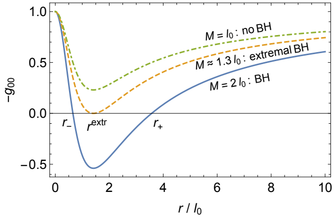
Metric

Solution to Einstein equations:

- Anisotropic fluid of energy density ρ.
- Static & spherically symmetric spacetime:

$$ds^2 = g_{00}\,\mathrm{d}t^2 + g_{rr}\,\mathrm{d}r^2 + r^2\,\mathrm{d}\Omega^2$$

$$\Rightarrow -g_{00}(r) = g_{rr}^{-1}(r) = 1 - \frac{2Mr^2}{\left(r^2+\ell_0{}^2\right)^{3/2}}$$



Properties:

- Black holes ($r_+ \geq \sqrt{2}\,\ell_0$) and horizonless matter accumulations.
- Regular solution! Repulsive de Sitter core: $\Lambda_{\mathrm{eff}} = 6M/\ell_0{}^3$.
- ℓ_0 enters non-perturbatively.
- Bardeen black hole upon replacement of UV cutoff: zero-point length $\rightarrow$ magnetic monopole charge.

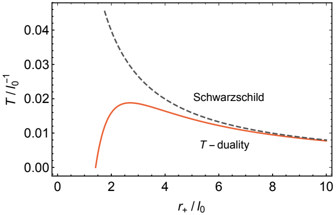
Bardeen, GR5, Tbilisi, USSR, Sept. 9–13, 1968; Ayon-Beato, Garcia, Phys. Lett. **B493** (2000) 149

Black Hole Thermodynamics

Thermodynamical properties:

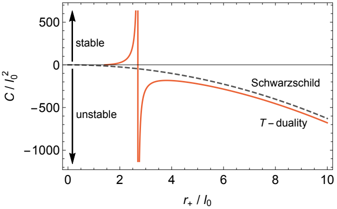
$$T = \frac{1}{4\pi\, r_+}\left(1 - \frac{3{\ell_0}^2}{r_+^2 + {\ell_0}^2}\right)$$

$$C = -\frac{2\pi\,\left(r_+^2 - 2{\ell_0}^2\right)\left(r_+^2 + {\ell_0}^2\right)^{5/2}}{r_+^5 - 7{\ell_0}^2\, r_+^3 - 2{\ell_0}^4\, r_+}$$



Implications:

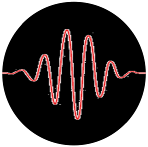
- Critical horizon radius $r^{\text{crit}} = \sqrt{\frac{7+\sqrt{57}}{2}}\,\ell_0 \approx 2.7\,\ell_0$
- Small black holes, $r^{\text{extr}} \le r_+ \le r^{crit}$, are thermodynamically stable.
- Black hole evaporation process will stop eventually with a cold remnant instead of a final explosion.



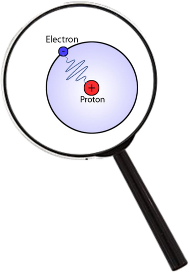
See also Nicolini, Smailagic, Spallucci, Phys. Lett. **B632** (2006) 547; Pawlowski, Stock, Phys. Rev. **D98** (2018) 106008

Determining the Self-Dual Radius ℓ_0

Theory



Experiment



Theory: UV Self-Completeness

UV self-completeness paradigm: Horizon radius of the smallest black hole equals its Compton wavelength.
Particle–black hole transition at the extremal configuration:

$$r^{\mathrm{extr}} \overset{!}{=} \lambdabar_{\mathrm{C}}\left(M^{\mathrm{extr}}\right) = \frac{4}{3\sqrt{3}} \frac{\ell_{\mathrm{P}}^2}{\ell_0}$$

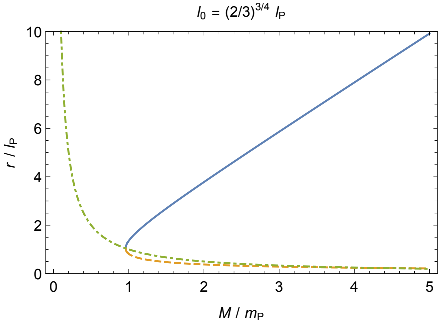
with the reduced Compton wavelength $\lambdabar_{\mathrm{C}} = \hbar / M\,\mathrm{c}$.

$$r^{\mathrm{extr}} = \frac{2^{5/4}}{3^{3/4}}\,\ell_{\mathrm{P}} \approx 1.0\,\ell_{\mathrm{P}}$$

$$M^{\mathrm{extr}} = \frac{3^{3/4}}{2^{5/4}}\,m_{\mathrm{P}} \approx 1.0\,m_{\mathrm{P}}$$

Unique value for ℓ_0:

$$\ell_0 = \frac{2^{3/4}}{3^{3/4}}\,\ell_{\mathrm{P}} \approx 0.8\,\ell_{\mathrm{P}}$$



Hydrogen Atom as a Precision System

- Hydrogen atom as a precision testbed.
  - Ground state energy.
  - Ly-α line (transition between the $2S_{1/2}$ and $1S_{1/2}$ levels).
- Attractive interaction between proton and electron mediated by virtual photons.
- Modified photon dynamics $\rightarrow$ modified Coulomb interaction.
- Constraints from comparison high-precision measurements.

Correction of the Energy Spectrum

- Rayleigh-Schrödinger perturbation theory.
- Correcting Hamiltonian:

$$H_{\mathrm{Td}} = V_{\mathrm{Td}} - V_0 = \frac{\alpha}{r} - \frac{\alpha}{\sqrt{r^2 + {\ell_0}^2}}$$

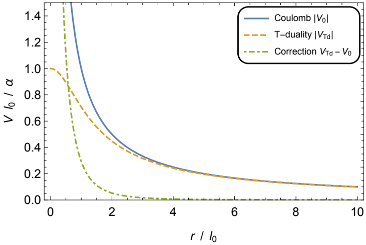
- Energy correction:

$$\Delta E_{n,l}^{\mathrm{Td}} = \left\langle nlm^{(0)} \,\middle|\, H_{\mathrm{Td}} \,\middle|\, nlm^{(0)} \right\rangle$$

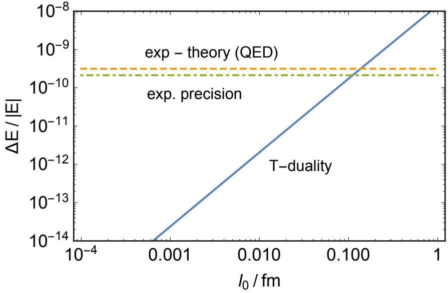
$$= \frac{2^{2+2l}\,(n-l-1)!\,\mu\alpha^2}{n^{4+2l}((n+l)!)^3}$$

$$\times \int_0^\infty \mathrm{d}y\; y^{2+2l}\, \mathrm{e}^{-2y/n} \left[L_{n+l}^{2l+1}(2y/n)\right]^2 \left(\frac{1}{y} - \frac{1}{\sqrt{y^2+x^2}}\right)$$

with $y \equiv \alpha\mu r$ and $x \equiv \alpha\lambda_0 = \alpha\mu\ell_0$.



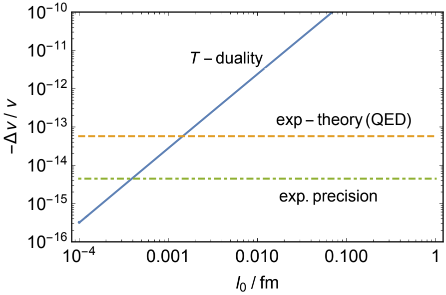
Constraint from the Hydrogen Ground State Energy



$$\Delta E_{1,0}^{\mathrm{Td}} = \mu\lambda_0^2 \left[-1 - 2\gamma + 2\ln\frac{1}{\alpha\lambda_0}\right]\alpha^4 + \mathcal{O}(\alpha^5)$$

Reference Value	Upper Bound on ℓ_0	Reference Value	Upper Bound on ℓ_0
$E_{\mathrm{exp}} - E_{\mathrm{th}}$	1.4×10^{-16} m	$\nu_{\mathrm{exp}} - \nu_{\mathrm{th}}$	1.5×10^{-18} m
ΔE_{exp}	1.1×10^{-16} m	$\Delta\nu_{\mathrm{exp}}$	3.9×10^{-19} m

Constraint from the Hydrogen Ly-α line



$$\Delta\nu^{\mathrm{Td}}_{1S-2S} = \frac{\mu\lambda_0^2}{32\pi}\left[11 + 4\ln 2 + 28\gamma - 28\ln\frac{1}{\alpha\lambda_0}\right]\alpha^4 + \mathcal{O}(\alpha^5)$$

Reference Value	Upper Bound on ℓ_0	Reference Value	Upper Bound on ℓ_0
$E_{\mathrm{exp}} - E_{\mathrm{th}}$	1.4×10^{-16} m	$\nu_{\mathrm{exp}} - \nu_{\mathrm{th}}$	1.5×10^{-18} m
ΔE_{exp}	1.1×10^{-16} m	$\Delta\nu_{\mathrm{exp}}$	3.9×10^{-19} m

Summary and Further Applications

T-duality sets a fundamental minimal length scale.

1) Which implications arise for black holes?
— UV completeness tames the singularity and evaporation process.
2) Can we fix the scale and test for it in the lab?
— From theory: around ℓ_{P}.
From precision experiments: $< 1.5 \times 10^{-18}$ m.

Further Applications:

- Connection to gravitational scattering amplitudes.
Easson, Keeler, Manton: "The classical double copy of non-singular black holes," 2007.16186 [gr-qc].
- Extension to higher dimensions and finite chemical potentials.
Pourhassan, Wani, Faizal: "Black Remnants from T-Duality," Nucl. Phys. B960 (2020) 115190.

Questions?