A hybrid simulation of gravitational wave production in first-order phase transitions

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Before anything, the most important... Cool stuff always come with videos
A message to take home...

New simulation scheme for sound-shell contribution in 1st order PT

Advantages:
1) fast (easier to explore parameter dependence);
2) Dont need to include scalar field (solve particle physics scale in a cosmo simulation);
3) Incorporate shock front easily.

A user-friendly parametrization...

$$\Omega_{GW} \propto \frac{1}{(q/l)^{-n_l} + (q/l)^{-n_m} + (q/h)^{-n_m}(q/h)^{-n_h}}$$

$$q_l \simeq 1, \quad q_h \simeq 1/\xi_{shell},$$

$$n_l \in [2, 4], \quad n_m \in [-1, 0], \quad n_h \in [-4, -3],$$
GW science program

LIGO 2010s (1602.03837)

LISA 2030's (1702.00786)

BBO, ET, DECIGO ? (1408.0740)
GW sources

Astrophysical:

Compact objects (Neutron Stars, BH)

Cosmological (stochastic background):

1. Inflation;
2. (P)reheating;
3. Cosmic Strings.
4. 1st Order Phase Transition;
GW sources

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Cosmological (stochastic background):
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4. 1st Order Phase Transition;

Our focus...
Motivations for 1st Order PT

1) LISA is flying in next decade
2) Electroweak Baryogenesis
3) BSM physics
Motivations for 1st Order PT

1) LISA is flying in next decade
2) **Electroweak Baryogenesis**
3) BSM physics

\[ \eta \equiv \frac{n_B - \bar{n}_B}{n_\gamma} \simeq 10^{-10} \]

Sakharov:
A) Out of equilibrium (Bubble expanding)
B) CP violation (reflection due to interaction with Higgs wall)
C) B violation through sphaleron processes that

(1302.6713)
Motivations for 1st Order PT

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(2008.09136)
1st Order Phase Transition (PT)

Scalar potential in thermal bath gets temperature corrections

\[ V(\phi, T) = \frac{1}{2} M^2(T) \phi^2 - \frac{1}{3} \delta(T) \phi^3 + \frac{1}{4} \lambda \phi^4 \]

At high temperatures we expect electroweak sym. to be restored. As temperature goes down higgs gets a vev and gives mass to particles

For a review, see Maggiore's book or (2008.09136)
1st Order Phase Transition (PT)

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\[ \Gamma(t) = \Gamma_* e^{\beta(t-t_*)} \]

Disjoint regions of space can make the transition, generating bubbles at different places.
1st Order Phase Transition (PT)

The bubbles expand (a huge literature about expansion dynamics) and eventually collide.

Generates a quadrupole and GW

\[ f_{\text{peak}} \sim \text{a few} \times 10^{-6} \left( \frac{\beta}{H_*} \right) \left( \frac{T_*}{100 \text{ GeV}} \right) \text{ Hz} \]

Nucleation rate O(100)

Close to EW scale

For a review, see Maggiore's book or (2008.09136)
1st Order Phase Transition (PT)

3 sources:
- Scalar field
- Sound waves
- Turbulence

(Caprini et al, 15)
GW from 1st Order PT -- State of the art

Envelope approximation

Energy contained in a thin non-collided yet shell (fluid or scalar)

Konstandin, Huber (08)

See also Kamionkowski, Kosowsky, Turner (94) and Jinno, Takimoto (17a)
GW from 1st Order PT -- State of the art

Envelope approximation

Latter, it became clear that the sound shell contribution is larger than the envelope Enhanced by \( \left( \frac{\beta}{H_*} \right) \)

Energy contained in a thin non-collided yet shell (fluid or scalar)

Difficult for bubbles to runaway, coupling to the plasma (Bodeker and Moore, 17)

See also Kamionkowski, Kosowsky, Turner (94) and Jinno, Takimoto (17a)
GW from 1st Order PT -- State of the art

Envelope approximation

Energy contained in a thin non-collided yet shell (fluid or scalar)

Bulk flow

Sound shell model

\[
\frac{d\Omega_{\mathrm{GW}}(k)}{d \ln(k)} \sim \begin{cases} 
(kR_*)^5, & k\Delta R_*, kR_* \ll 1, \\
(kR_*)^1, & k\Delta R_* \ll 1 \ll kR_*, \\
(kR_*)^{-3}, & 1 \ll k\Delta R_*, R_*.
\end{cases}
\]

See also Kamionkowski, Kosowsky, Turner (94) and Jinno, Takimoto (17a)
GW from 1st Order PT -- State of the art

Lattice simulations

Scalar Field (HEP scale) + bubble size (cosmo scale)

Huge hierarchy between those scales

Hindmarsh, Huber, Rummukainen, Weir (13,15,17)
Motivation: construct a simulation that doesn't need to solve the Higgs

Important: Higgs is only (indirectly) as a boundary condition
Our set up in a nutshell

Plasma velocity and enthalpy

Simulate (1d + spherical sym) how the velocity and enthalpy evolves

Motivation: construct a simulation that doesn't need to solve the Higgs

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Our set up in a nutshell

Plasma velocity and enthalpy
Simulate (1d + spherical sym) how the velocity and enthalpy evolves

1d simulation

Collision

Higgs hold the bubble wall
1/r decay

$w, v$

$t = t_c$
$t > t_c$

$\cdots$
Our set up in a nutshell

Embed a 1d hydro simulation (fast to run) into a 3d lattice
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Other advantages:
More bubbles -- O(2500)
Realistic nucleation (not simultaneous)
Our set up in a nutshell

Calculate the GW at each time step in the 3d lattice
A small rest for the eyes
Our set up

Now slowly and with more details...

A 5 steps calculation
Our set up

1) Run the 1d lattice simulation (spherical symmetry) for the given wall velocity and PT strength;

2) Nucleate bubbles and let them grow in a 3d lattice;

3) Calculating when each differential part of each bubble surface collide;

4) Construct a velocity grid embedding the 1d simulation;

5) Calculate GW from stress-energy tensor.
Our set up

1) Run the 1d lattice simulation (spherical symmetry) for the given wall velocity and PT strength;
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Initially higgs field sustain the profile. Scale invariant evolution (1004.4187)

Decays as $1/r$ respecting $\partial_{\mu}T^{\mu\nu} = 0$ (1905.00899)
Our set up

\[
\partial_t \left( \frac{\rho}{v} \right) + A \partial_r \left( \frac{\rho}{v} \right) + h = 0,
\]

1) Run the 1d lattice simulation (spherical symmetry) for the given wall velocity and PT strength;

Initially higgs field sustain the profile. Scale invariant evolution (1004.4187)

Decays as 1/r respecting \( \partial_\mu T^{\mu\nu} = 0 \) (1905.00899)

Need to solve shock fronts!
Intricate discretization scheme called Kurganov-Tadmor
Our set up

A detail for those that like numerical schemes

If one try to solve the shocks with a standard numerical schemes, it wont work (see Appendix A)
Our set up

1) Run the 1d lattice simulation (spherical symmetry) for the given wall velocity and PT strength;

2) Nucleate bubbles and let them grow in a 3d lattice;

\[ \Gamma \propto e^{\beta t} \]

Nucleation rate per volume parameterized by beta
Our set up

1) Run the 1d lattice simulation (spherical symmetry) for the given wall velocity and PT strength;

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3) Calculating when each differential part of each bubble surface collide;
Our set up

\[ \frac{\Delta w}{w_0} \approx \sum_{i: \text{bubbles}} \frac{\Delta w^{(i)}}{w_0}, \quad \vec{v} \approx \sum_{i: \text{bubbles}} \vec{v}^{(i)}, \]

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Our set up

\[ T^{ij}(\vec{x}) = v^i(\vec{x})v^j(\vec{x})\rho(\vec{x}) \]

\[ T_+(\vec{k}) = \sum_{i,j} \frac{T^{ij}(\vec{k})}{\sqrt{2}} \left( \theta_i(\vec{k})\theta_j(\vec{k}) - \phi_i(\vec{k})\phi_j(\vec{k}) \right) , \]

\[ T_\times(\vec{k}) = \sum_{i,j} \frac{T^{ij}(\vec{k})}{\sqrt{2}} \left( \theta_i(\vec{k})\phi_j(\vec{k}) + \phi_i(\vec{k})\phi_j(\vec{k}) \right) . \]

\[ T_{+,\times}(q, \vec{k}, t) = \sum_{t' = t_{\text{init}}}^{t} e^{iqt'} T_{+,\times}(t', \vec{k}) , \]

\[ \Omega(q, t) = C q^3 \langle T_+T_+^* + T_\times T_\times^* \rangle|_{\vec{k}| = q} . \]

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2) Nucleate bubbles and let them grow in a 3d lattice;

3) Calculating when each differential part of each bubble surface collide;

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5) Calculate GW from stress-energy tensor.
Results
Results

\[ \Omega_{GW} = \frac{w^2 \tau}{4\pi^2 \rho_{tot} M_P^2 \beta} \times Q', \]
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Simulation time
Results

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Stop growing (PT complete)

Gains power (more bubbles)

Bubble size scale

Simulation time
Results

\[ \Omega_{GW} = \frac{w^2 \tau}{4\pi^2 \rho_{\text{tot}} M_P^2 \beta} \times Q', \]

Linear Growth

Sound shell scale

Simulation time
Results

How to parametrize the result in terms of the bubble velocity and PT strength?
Results

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We wanna have something user-friendly

Give me bubble velocity and PT strength and I give you the spectra!
Results

How to parametrize the result in terms of the bubble velocity and PT strength?

Integrated spectrum gives better parameter dependence

\[ Q_{int}' = \int d\ln q \ Q'(q), \]
Results

How to parametrize the result in terms of the bubble velocity and PT strength?

Integrated spectrum gives better parameter dependence

\[ Q_{\text{int}}' \equiv \int d\ln q \ Q'(q), \]

We expect the GW spectrum to be proportional to something like

\[ Q' \propto \left( \langle w \gamma^2 v^2 \rangle / w_\infty \right)^2. \]

GW is proportional to fluid kinetic energy (v squared)

Find some quantity that resembles it
We expect the GW spectrum to be proportional to something like

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Find some quantity that resembles it

1st attempt \[ \langle w\gamma^2 v^2 \rangle_{3d} \]

3d kinetic energy
Results

We expect the GW spectrum to be proportional to something like

$$Q' \propto \left( \frac{\langle w \gamma^2 v^2 \rangle}{w_\infty} \right)^2$$

Find some quantity that resembles it

 GW spectra normalized by 3d kinetic energy

Almost no dependence on wall velocity and alpha!
Results

We expect the GW spectrum to be proportional to something like

\[ Q' \propto \left( \frac{\langle w \gamma^2 v^2 \rangle}{w_\infty} \right)^2 \]

Find some quantity that resembles it

Pretty good normalization!

\[ Q'_{\text{int}} \simeq 9 \times \xi_{\text{shell}} \times \left( \frac{\langle w \gamma^2 v^2 \rangle_{3d}}{w_\infty} \right)^2 \]

Problem: not user-friendly (needs 3d simulations)
Results

We expect the GW spectrum to be proportional to something like

\[ Q' \propto \left( \langle \gamma v^2 \rangle / w_\infty \right)^2. \]

Find some quantity that resembles it

1st attempt \[ \langle w y^2 v^2 \rangle_{3d} \]
2nd attempt \[ \langle w y^2 v^2 \rangle_{1d} \]

1d kinetic energy
We expect the GW spectrum to be proportional to something like

\[ Q' \propto \left( \langle \omega \gamma^2 v^2 \rangle / w_\infty \right)^2 \]

Find some quantity that resembles it

GW spectra normalized by 1d kinetic energy

1st attempt

\[ \langle w \gamma^2 v^2 \rangle_{3d} \]

2nd attempt

\[ \langle w \gamma^2 v^2 \rangle_{1d} \]

Good normalization!
Results

We expect the GW spectrum to be proportional to something like

\[ Q' \propto \left( \frac{\langle w \gamma^2 v^2 \rangle}{w_{\infty}} \right)^2 \]

Find some quantity that resembles it

\[ Q'_{\text{int}} \approx 12 \times \xi_{\text{shell}} \times \left( \frac{\langle w \gamma^2 v^2 \rangle_{1d}}{w_{\infty}} \right)^2 \]

Problem: not user-friendly (needs 1d simulations)
Results

We expect the GW spectrum to be proportional to something like

\[ Q' \propto \left( \frac{\langle w\gamma^2 v^2 \rangle}{w_\infty} \right)^2. \]

Find some quantity that resembles it

1st attempt \[ \langle w\gamma^2 v^2 \rangle_{3d} \]

2nd attempt \[ \langle w\gamma^2 v^2 \rangle_{1d} \]

3rd attempt

\[
\kappa \alpha = \frac{4}{\xi_w^3 w_\infty} \int d\xi \; w\gamma^2 v^2 \xi^2, \]

Very user friendly!

The kinetic energy of the initial fluid profile, that can be calculated from some fits (see 2004.06995)
Henrique Rubira

Results

We expect the GW spectrum to be proportional to something like

\[ Q' \propto \left( \langle w \gamma^2 v^2 \rangle / w_\infty \right)^2 \]

Find some quantity that resembles it

1st attempt \[ \langle w \gamma^2 v^2 \rangle_{3d} \]

2nd attempt \[ \langle w \gamma^2 v^2 \rangle_{1d} \]

3rd attempt

\[ \kappa \alpha = \frac{4}{\xi_w^3 w_\infty} \int d\xi \ w \gamma^2 v^2 \xi^2, \]

Bad normalization!

Very user friendly!
Henrique Rubira

Results

We expect the GW spectrum to be proportional to something like

\[ Q' \propto \left( \langle w\gamma^2 v^2 \rangle / w_\infty \right)^2. \]

Find some quantity that resembles it

1st attempt
\[ \langle w\gamma^2 v^2 \rangle_{3d} \]

2nd attempt
\[ \langle w\gamma^2 v^2 \rangle_{1d} \]

3rd attempt
\[ \kappa \alpha = \frac{4}{\xi_w^3 w_\infty} \int d\xi \ w\gamma^2 v^2 \xi^2, \]

But we can relate \( \kappa \alpha \) to \( \langle w\gamma^2 v^2 \rangle_{1d} \)

\[ Q'_\text{int} \simeq 12 \times \xi_{\text{shell}} \times \left( \langle w\gamma^2 v^2 \rangle_{1d} / w_\infty \right)^2. \]
Results

Ok, now I have the amplitude of the spectrum. What if I want the full-shape?
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Ok, now I have the amplitude of the spectrum. What if I want the full-shape?

\[
\frac{Q'}{\xi_{\text{shell}} \times \left( \frac{w^2 v^2}{v_{3d}} \right)_{\infty}} \propto \frac{1}{(q/q_i)^{-n_i} + (q/q_i)^{-n_m} + (q_h/q_i)^{-n_m} (q/q_h)^{-n_h}}
\]

\[
\approx \begin{cases} 
(q/q_i)^{n_i} & (q \ll q_i) \\
(q/q_i)^{n_m} & (q_i \ll q \ll q_h) \\
(q_h/q_i)^{n_m} (q/q_h)^{n_h} & (q_h \ll q)
\end{cases}
\]

\[n_m \in [-1, 0]\]
Ok, now I have the amplitude of the spectrum. What if I want the full-shape?

$$\frac{Q'}{\xi_{\text{shell}} \times (\langle w\gamma^2 v^2 \rangle_{3d}/w_\infty)^2} \propto \frac{1}{(q/q_l)^{-n_l} + (q/q_l)^{-n_m} + (q_h/q_l)^{-n_m} (q/q_h)^{-n_h}}$$

Low freq.

$$q_l \simeq 1,$$

Typical bubble size at collision!

$$n_l \in [2, 4],$$

$$n_m \in [-1, 0],$$

A double power law
Ok, now I have the amplitude of the spectrum. What if I want the full-shape?

\[
\frac{Q'}{\xi_{\text{shell}} \times (\langle w^2 v^2 \rangle_{3d}/w_\infty)^2} \propto \frac{1}{(q/q_l)^{-n_l} + (q/q_l)^{-n_m} + (q_h/q_l)^{-n_m} (q/q_h)^{-n_h}}
\]

Low freq.

\[q_l \approx 1, \quad n_l \in [2, 4],\]

Typical bubble size at collision!

High freq.

\[q_h \approx 1/\xi_{\text{shell}}, \quad n_h \in [-4, -3]\]

A double power law

peak at sound shell scale
Comparison to other works

When we compare to scalar field simulations, we have found:

- Similar scaling
- IR peak shifted to lower freq. (Realistic nucleation)
- Factor ~ 2 in overall amplitude (More bubbles)
Conclusions

New simulation scheme (free of scalar field scale) to calculate sound-shell contribution

\[ Q'_\text{int} \simeq 9 \times \xi_{\text{shell}} \times \left( \frac{\langle w \gamma^2 v^2 \rangle_{3d}}{w_\infty} \right)^2 \]

Parametrize as

\[ \Omega_{GW} \propto \frac{1}{(q/q_l)^{-n_l} + (q/q_l)^{-n_m} + (q_h/q_l)^{-n_m} (q/q_h)^{-n_h}} \]

\[ q_l \simeq 1, \quad q_h \simeq \frac{1}{\xi_{\text{shell}}} \]

\[ n_l \in [2, 4], \quad n_m \in [-1, 0], \quad n_h \in [-4, -3], \]