# A hybrid simulation of gravitational wave production in first-order phase transitions

# (2010.00971)

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Before anything, the most important... Cool stuff always come with videos

# A message to take home...

New simulation scheme for sound-shell contribution in 1st order PT

Advantages:

1) fast (easier to explore parameter dependence);
2) Dont need to include scalar field (solve particle physics scale in a cosmo simulation);
3) Incorporate shock front easily.

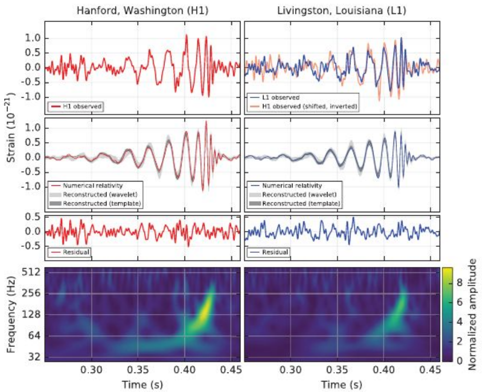
A user-friendly parametrization...

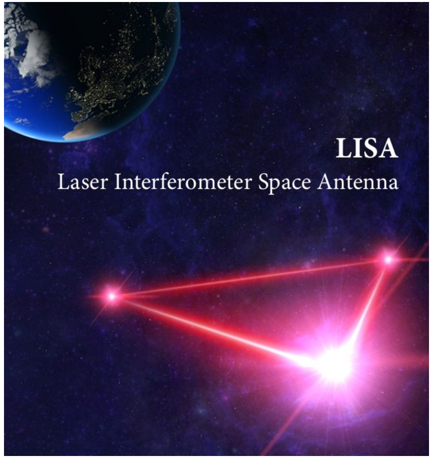
$$\Omega_{\rm GW} \propto \frac{1}{(q/q_l)^{-n_l} + (q/q_l)^{-n_m} + (q_h/q_l)^{-n_m}(q/q_h)^{-n_h}}$$

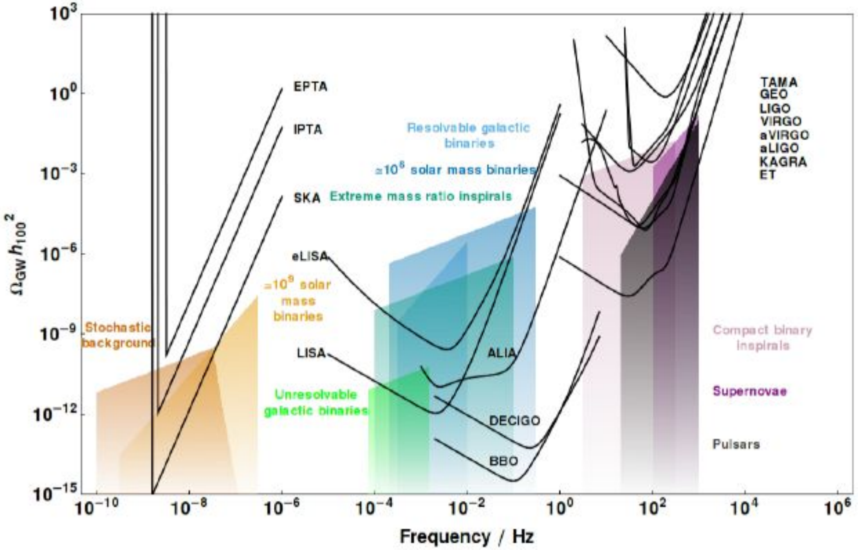
$$q_l \simeq 1, \qquad q_h \simeq 1/\xi_{\rm shell},$$

$$n_l \in [2,4]\,, \quad n_m \in [-1,0]\,, \quad n_h \in [-4,-3]\,,$$

# GW science program




LIGO
2010s (1602.03837)

LISA
2030's (1702.00786)

BBO, ET, DECIGO
? (1408.0740)

time

# GW sources

Astrophysical:

Compact objects (Neutron Stars, BH)

Cosmological (stochastic background):

1. Inflation;
2. (P)reheating;
3. Cosmic Strings.
4. 1st Order Phase Transition;

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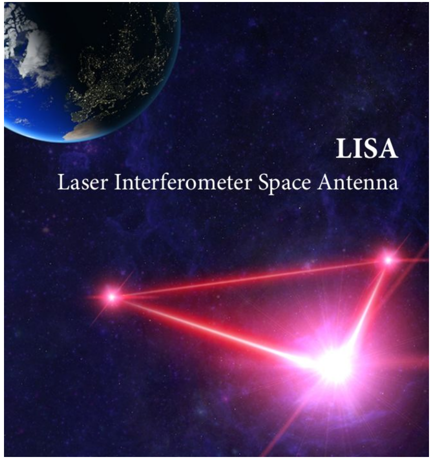
Our focus...

# Motivations for 1st Order PT

1) **LISA is flying in next decade**
2) Electroweak Baryogenesis
3) BSM physics




(1702.00786)

# Motivations for 1st Order PT

1) LISA is flying in next decade
2) **Electroweak Baryogenesis**
3) BSM physics

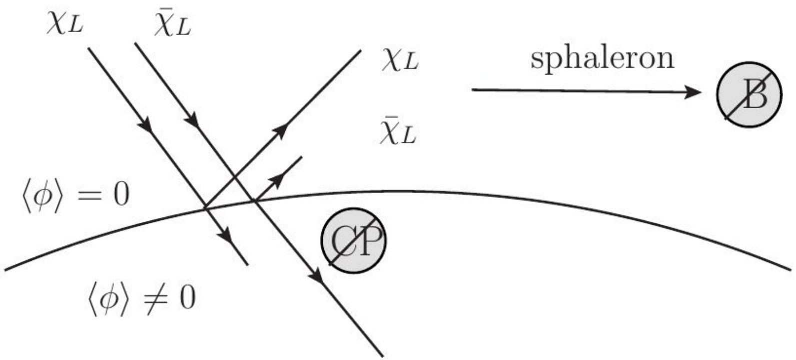
Sakharov:

A) Out of equilibrium (Bubble expanding)

B) CP violation (reflection due to interaction with Higgs wall)

C) B violation through sphaleron processes that

$$\eta \equiv \frac{n_B - \bar{n}_B}{n_\gamma} \simeq 10^{-10}$$




(1302.6713)

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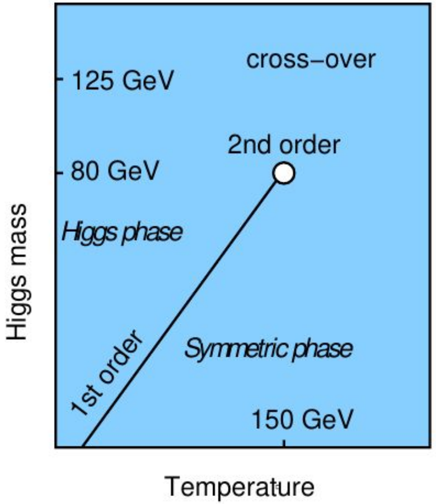
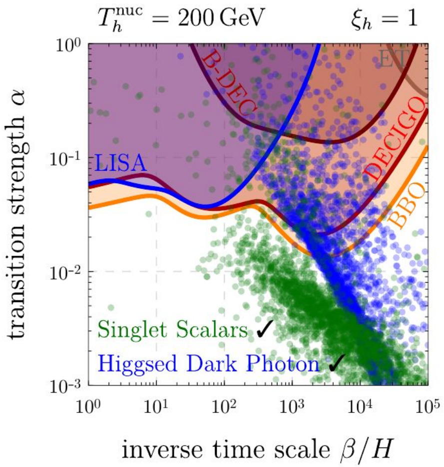
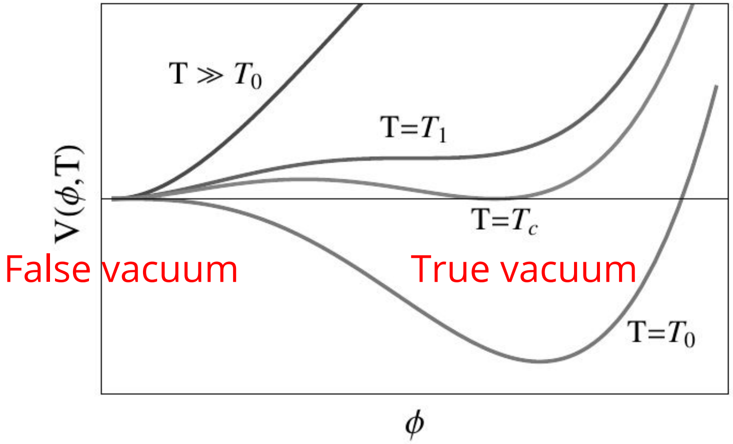



(2008.09136)




(1811.11175)

# 1st Order Phase Transition (PT)



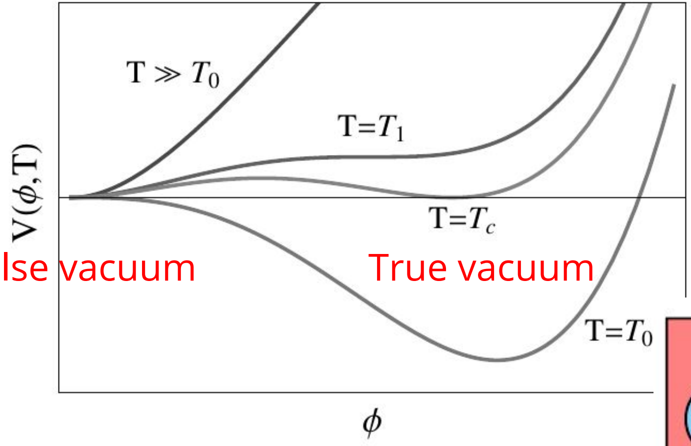

Scalar potential in thermal bath gets temperature corrections

$$V(\phi, T) = \frac{1}{2}M^2(T)\phi^2 - \frac{1}{3}\delta(T)\phi^3 + \frac{1}{4}\lambda\phi^4$$

At high temperatures we expect electroweak sym. to be restored. As temperature goes down higgs gets a vev and gives mass to particles

For a review, see Maggiore's book or (2008.09136)

# 1st Order Phase Transition (PT)

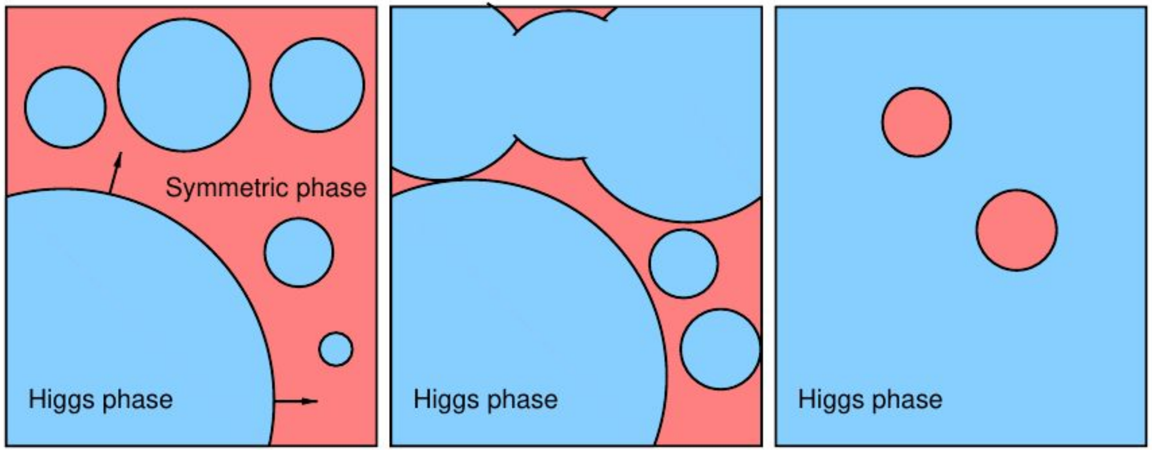



False vacuum

True vacuum

$$\Gamma(t) = \Gamma_* e^{\beta(t-t_*)}$$

Disjoint regions of space can make the transition, generating bubbles at different places

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# 1st Order Phase Transition (PT)

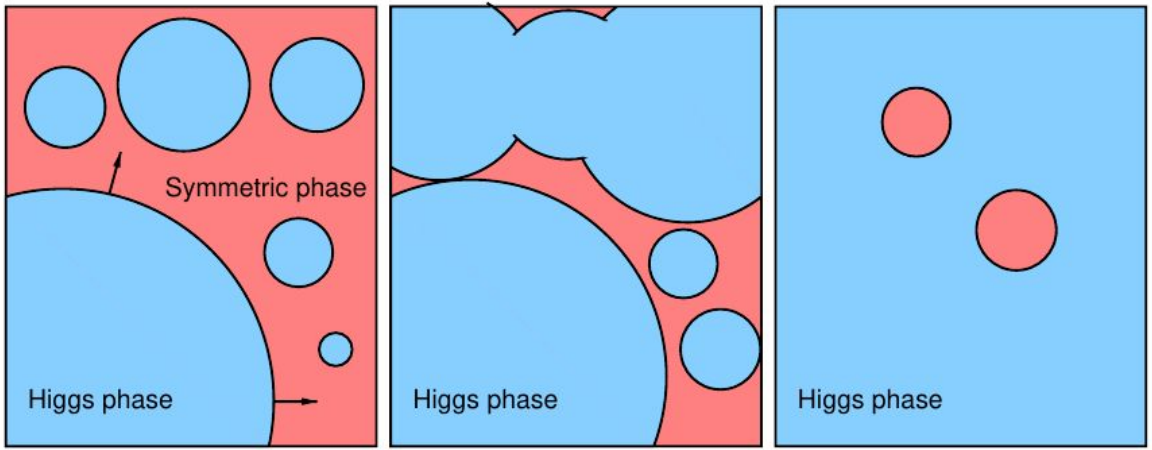
Nucleation
rate O(100)

The bubbles expand (a huge literature about expansion dynamics) and eventually collide

$$f_{\rm peak} \sim \text{a few} \times 10^{-6} \left(\frac{\beta}{H_*}\right)\left(\frac{T_*}{100\,\mathrm{GeV}}\right) \mathrm{Hz}\,.$$

Close to EW
scale

Generates a quadrupole and GW

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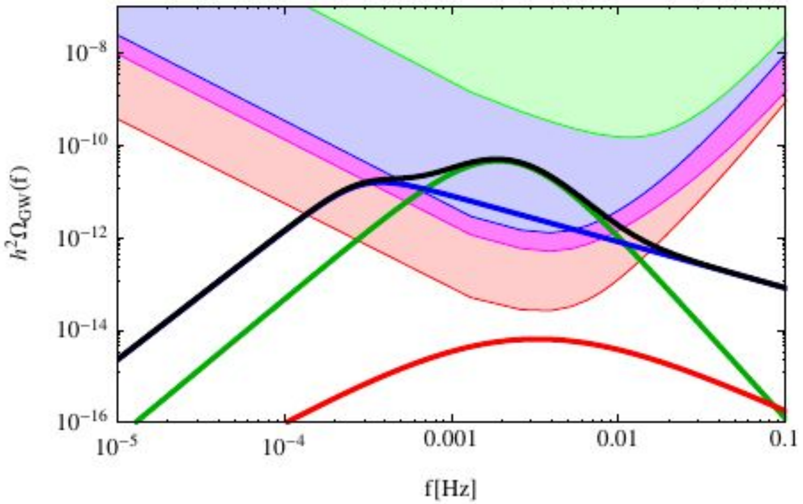


(Caprini et al, 15)

3 sources:

Scalar field

Sound waves

Turbulence

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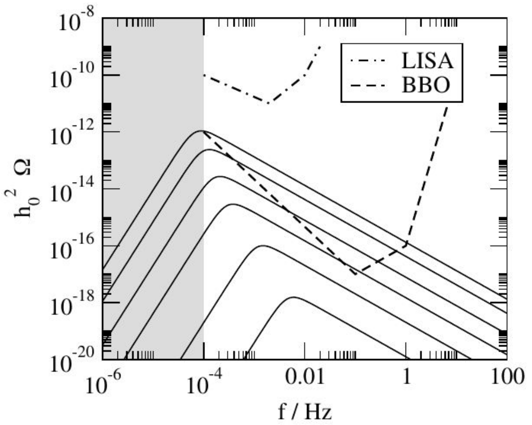
# GW from 1st Order PT -- State of the art

**Envelope approximation**




Konstandin, Huber (08)

Energy contained in a thin non-collided yet shell (fluid or scalar)

See also Kamionkowski, Kosowsky, Turner (94) and Jinno, Takimoto (17a)

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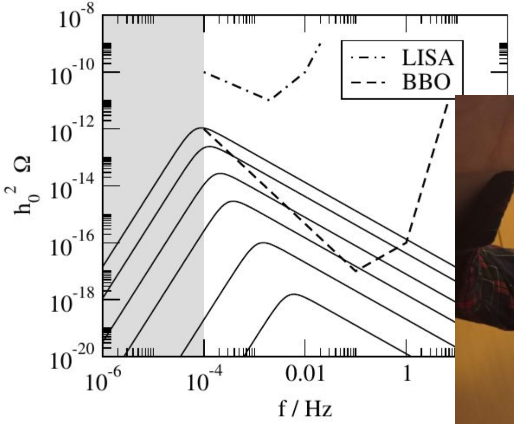


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Energy contained in a thin non-collided yet shell (fluid or scalar)

See also Kamionkowski, Kosowsky, Turner (94) and Jinno, Takimoto (17a)

Latter, it became clear that the sound shell contribution is larger than the envelope

Enhanced by (last longer) $\left(\dfrac{\beta}{H_*}\right)$

Difficult for bubbles to runaway, coupling to the plasma (Bodeker and Moore, 17)

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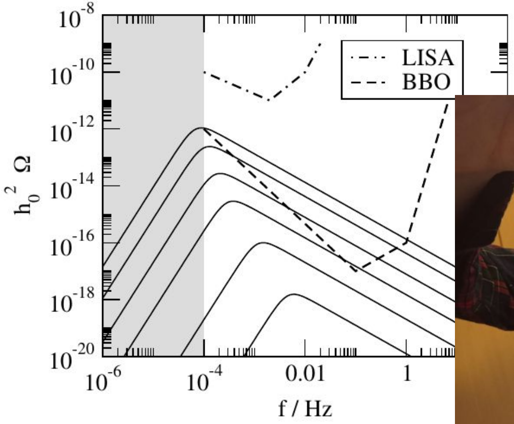
# GW from 1st Order PT -- State of the art

## Envelope approximation

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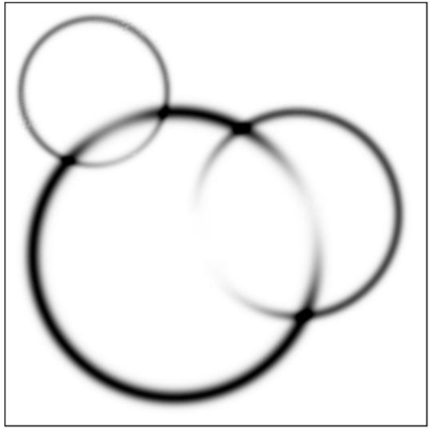
## Bulk flow

Konstandin (17) and Jinno, Takimoto (17b)



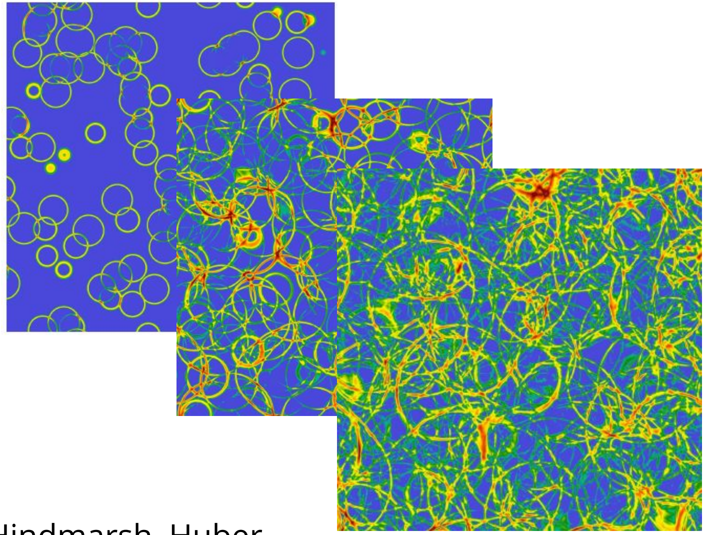
## Sound shell model

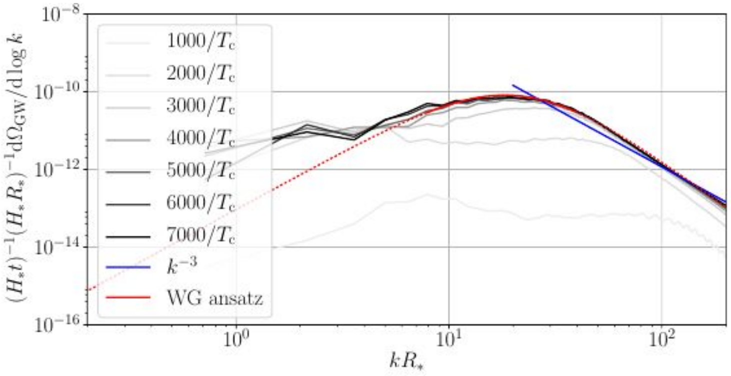
Hindmarsh (16)

$$\frac{d\Omega_{\rm GW}(k)}{d\ln(k)} \sim \begin{cases} (kR_*)^5, & k\Delta R_*, kR_* \ll 1, \\ (kR_*)^1, & k\Delta R_* \ll 1 \ll kR_*, \\ (kR_*)^{-3}, & 1 \ll k\Delta R_*, R_*. \end{cases}$$

# GW from 1st Order PT -- State of the art

**Lattice simulations**






Scalar Field (HEP scale) + bubble size (cosmo scale)

Huge hierarchy between those scales

Hindmarsh, Huber, Rummukainen, Weir (13,15,17)

# Our set up in a nutshell

Motivation: construct a simulation that doesnt need to solve the Higgs

Important: Higgs is only (indirectly) as a boundary condition

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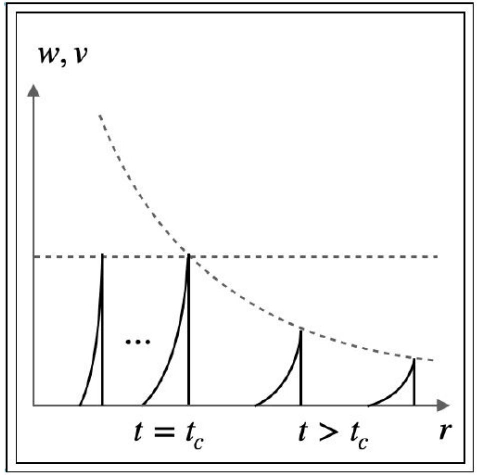
# Our set up in a nutshell

1d simulation




Plasma velocity and enthalpy

Simulate (1d + spherical sym) how the velocity and enthalpy evolves

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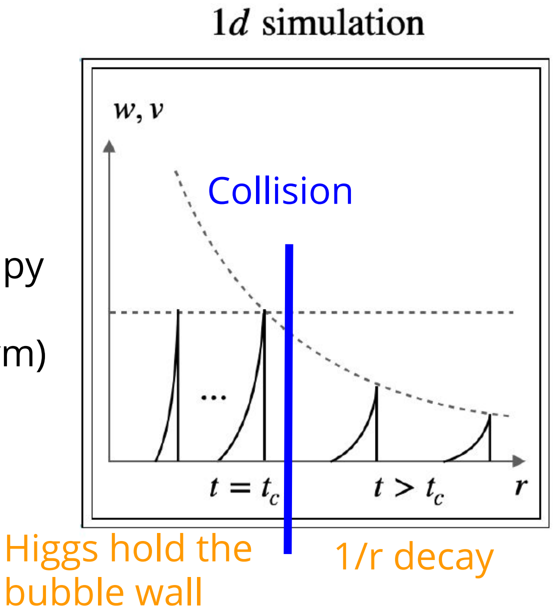
Simulate (1d + spherical sym) how the velocity and enthalpy evolves

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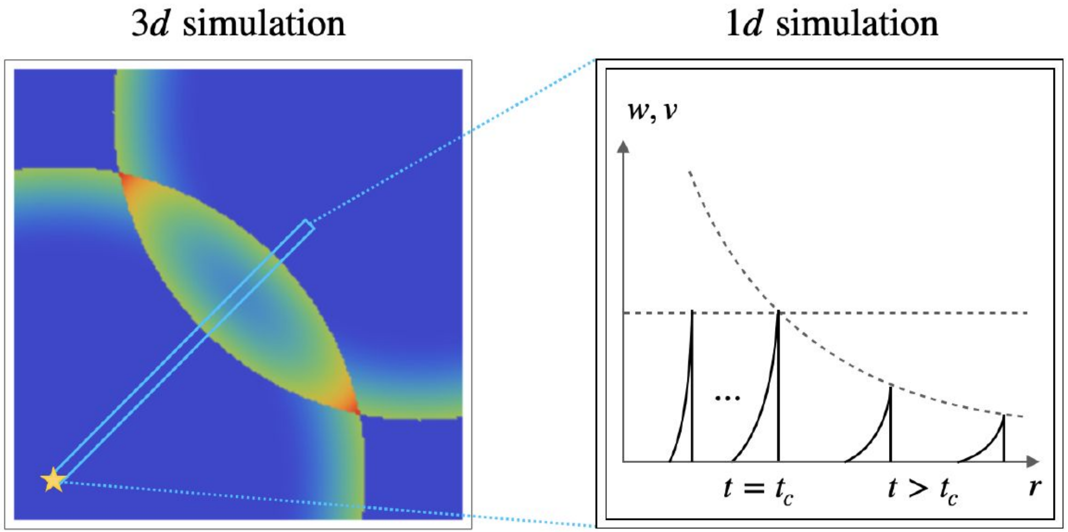


Embed a 1d hydro simulation (fast to run) into a 3d lattice

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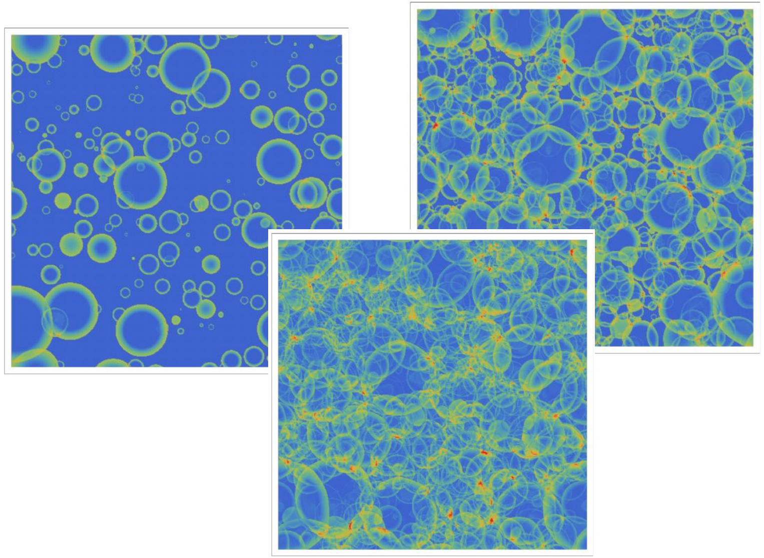

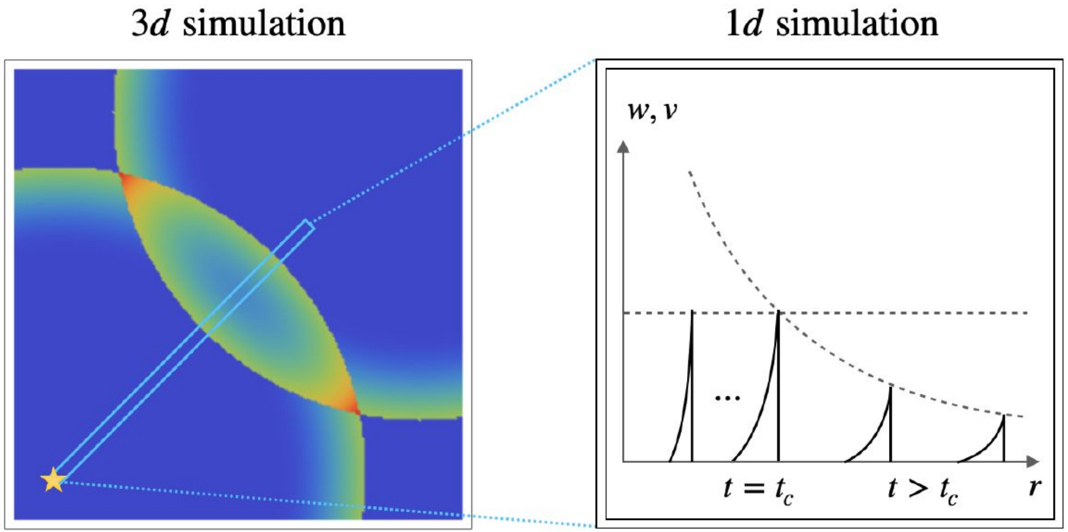


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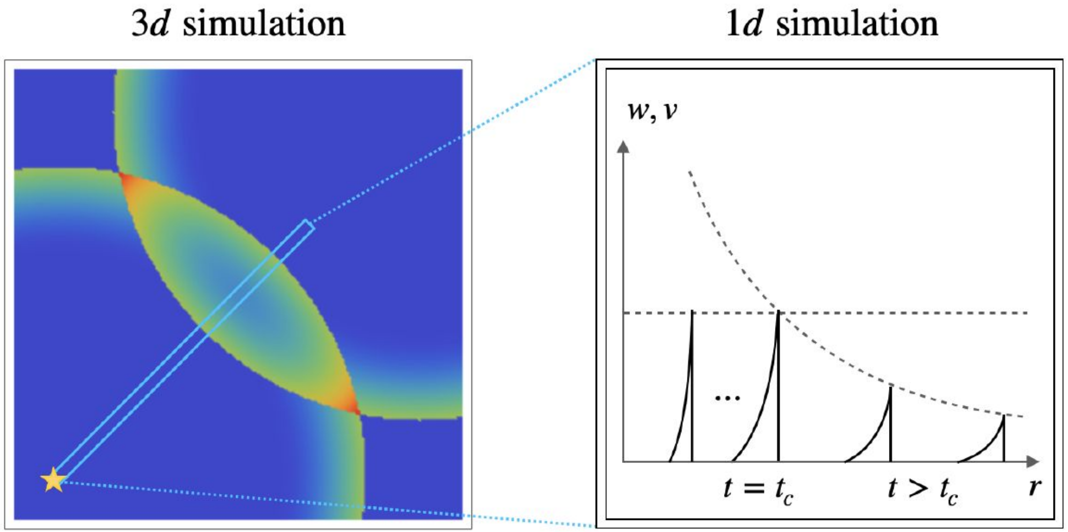


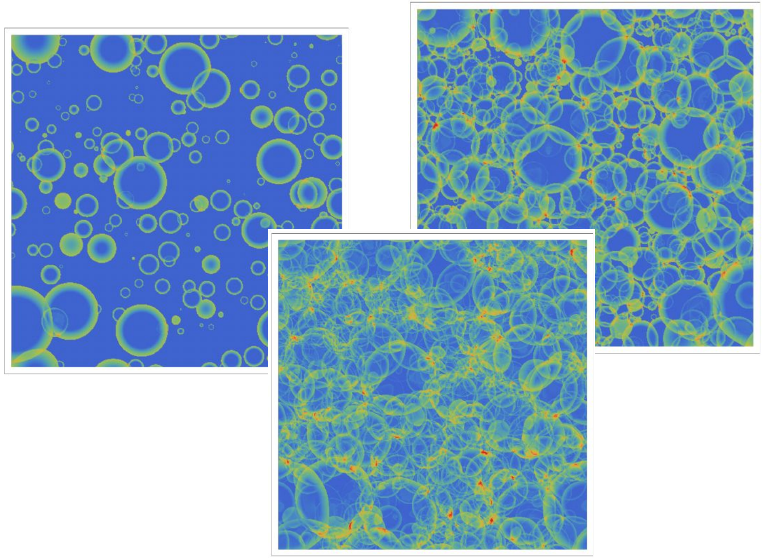

Embed a 1d hydro simulation (fast to run) into a 3d lattice

Other advantages:
More bubbles -- O(2500)
Realistic nucleation (not simultaneous)

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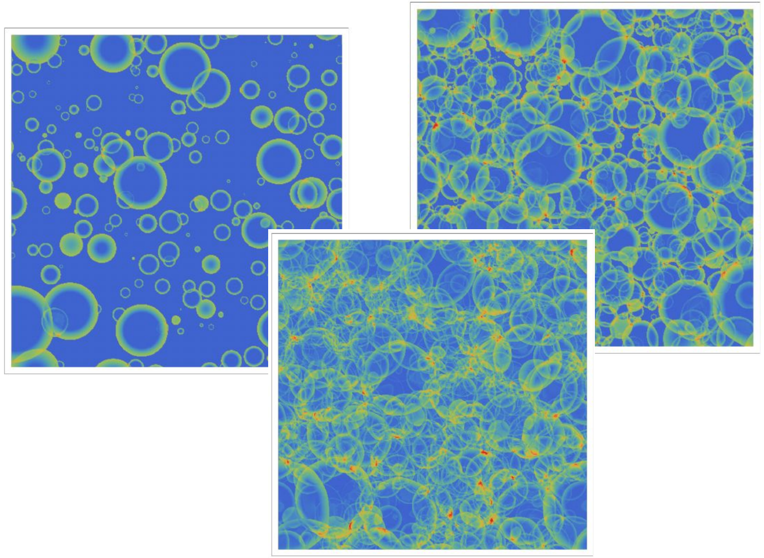

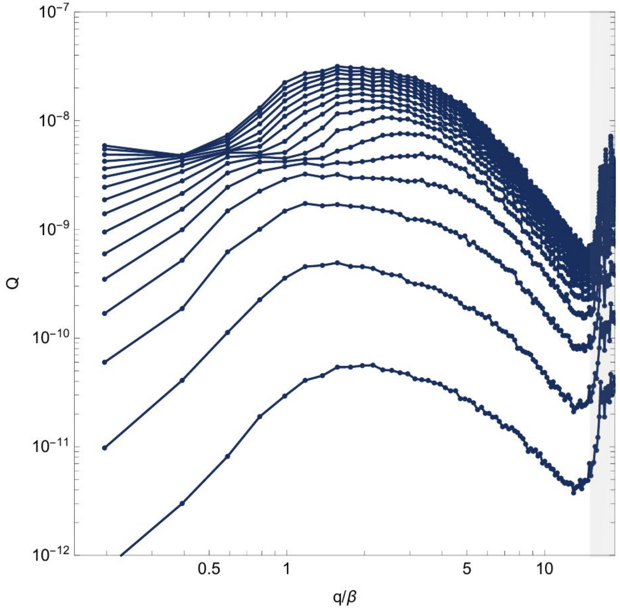


Calculate the GW at each time step in the 3d lattice

# A small rest for the eyes

# Our set up

Now slowly and with more details…

A 5 steps calculation

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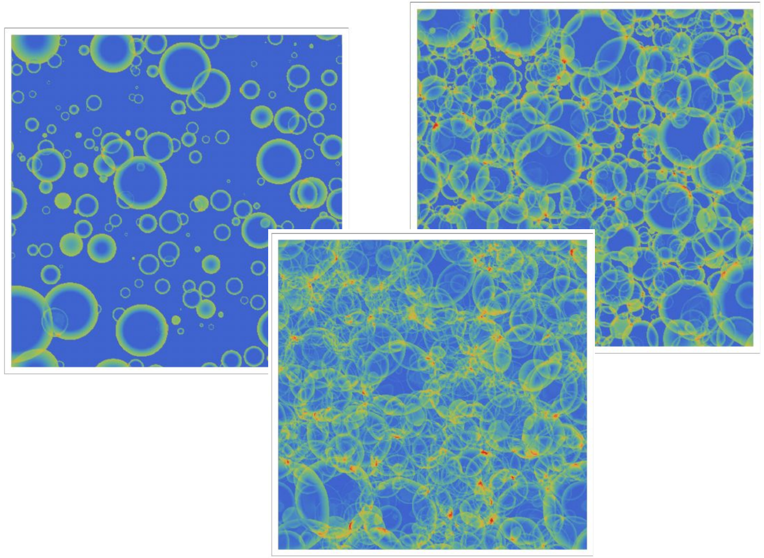
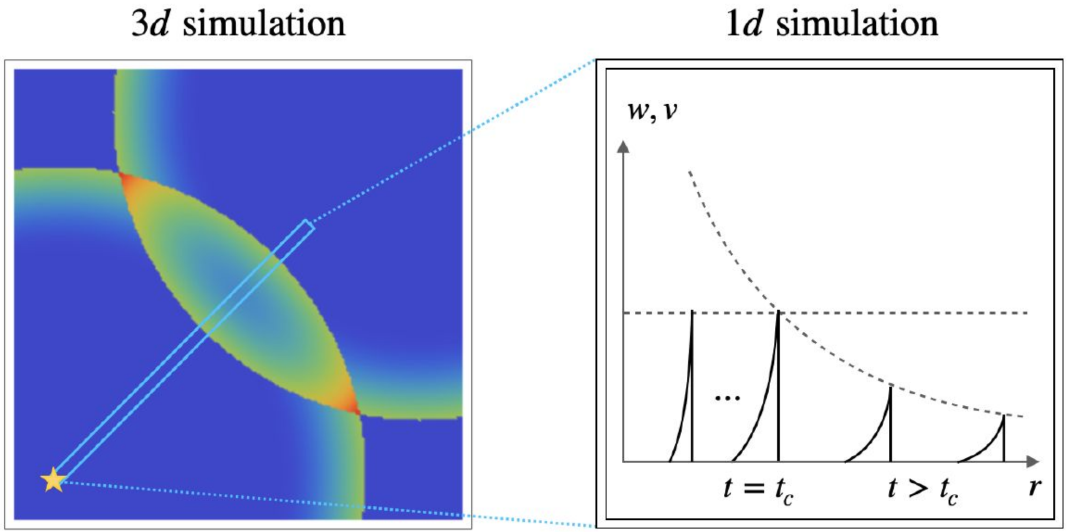
# Our set up






1) Run the 1d lattice simulation (spherical symmetry) for the given wall velocity and PT strength;

2) Nucleate bubbles and let them grow in a 3d lattice;

3) Calculating when each differential part of each bubble surface collide;

4) Construct a velocity grid embedding the 1d simulation;

5) Calculate GW from stress-energy tensor.

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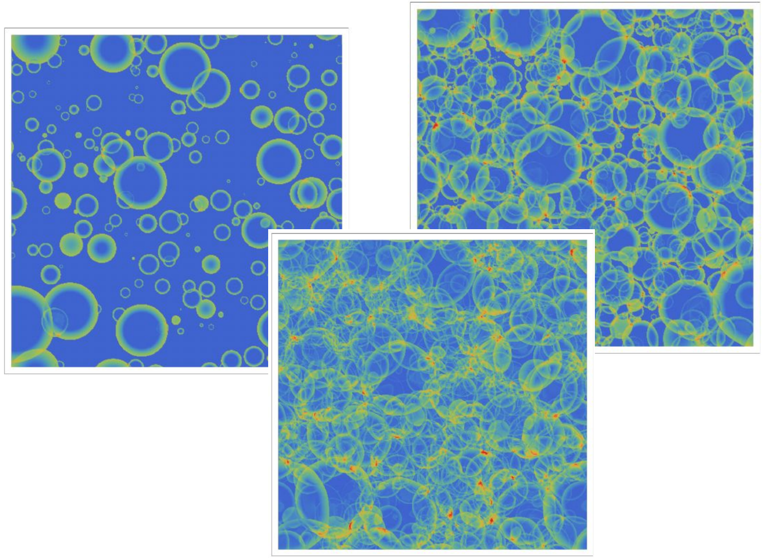

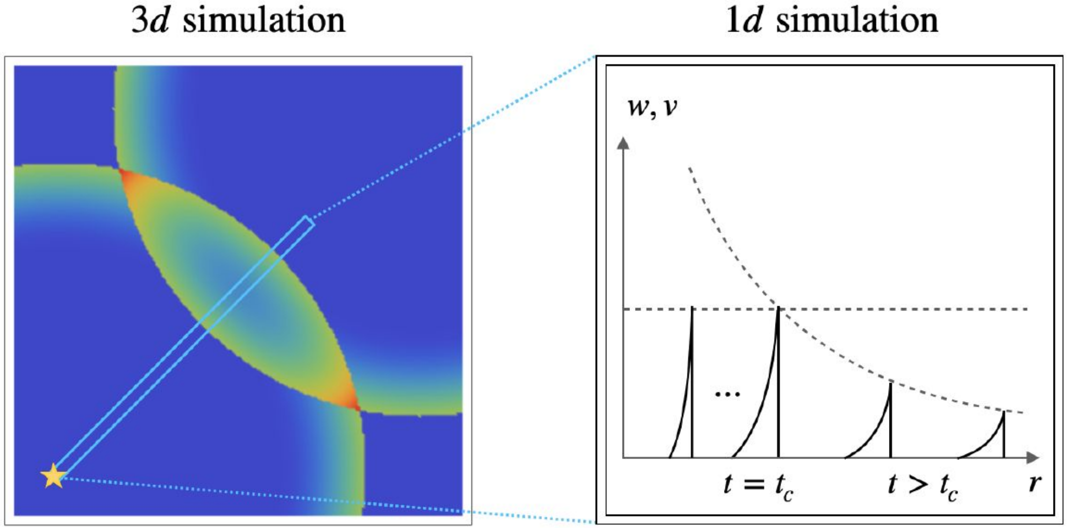


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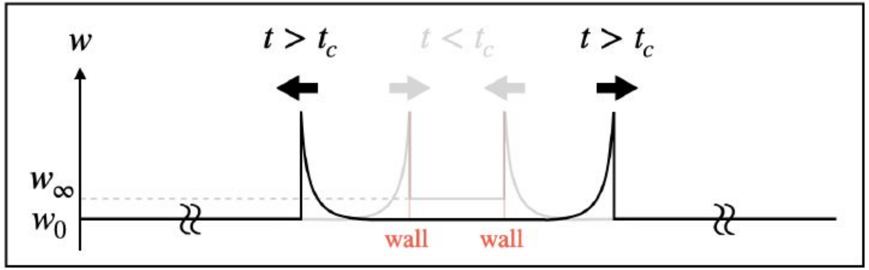

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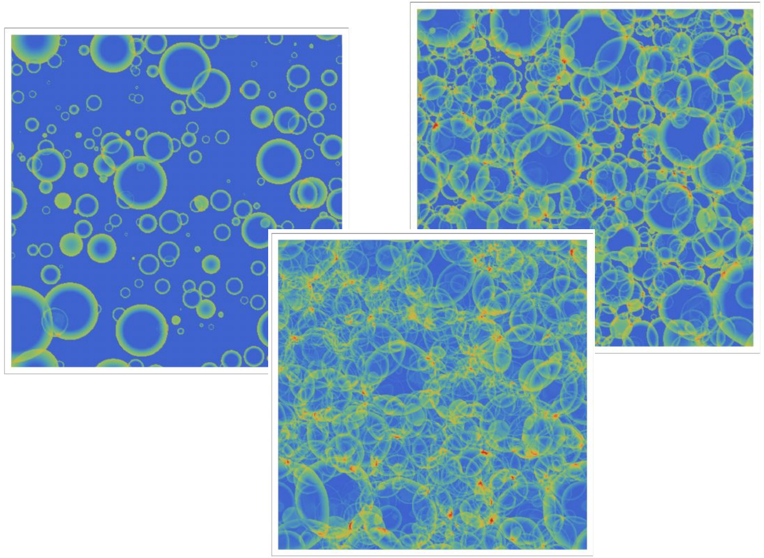

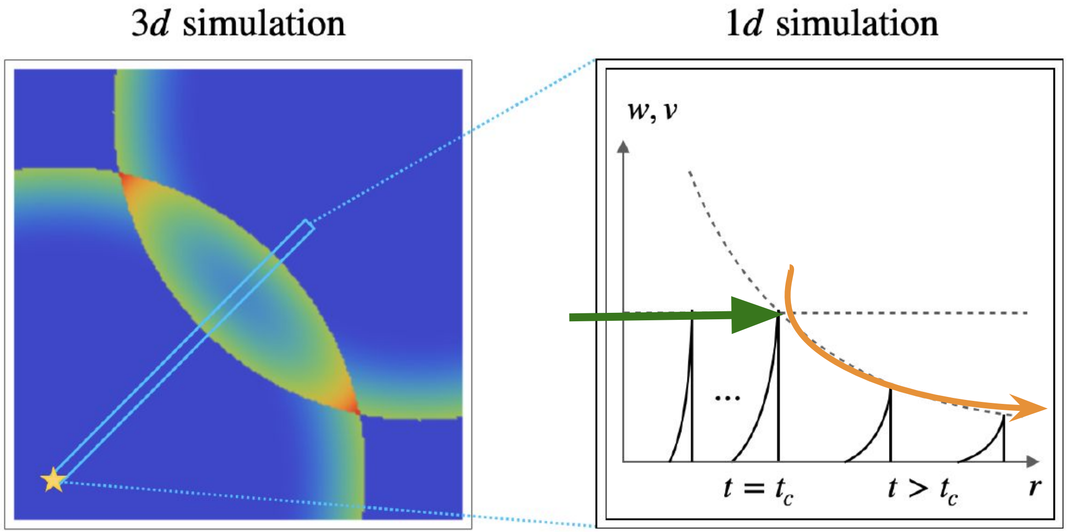


1) Run the 1d lattice simulation (spherical symmetry) for the given wall velocity and PT strength;

Initially higgs field sustain the profile. Scale invariant evolution (1004.4187)

Decays as 1/r respecting $\partial_\mu T^{\mu\nu} = 0$ (1905.00899)

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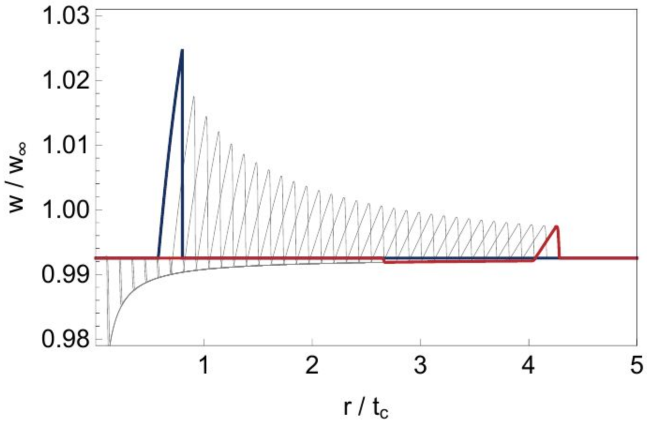
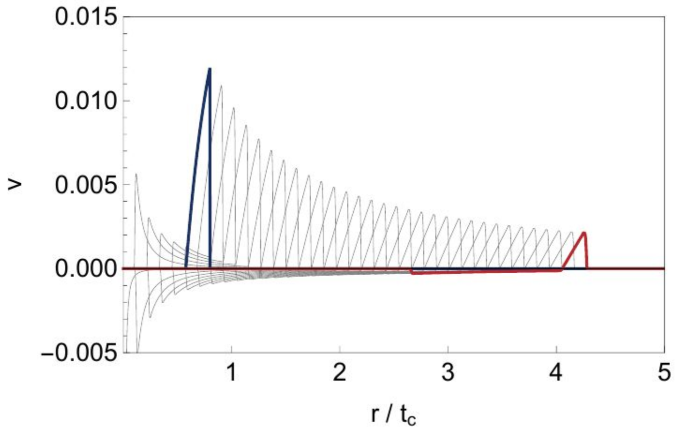
# Our set up




$$\partial_t \begin{pmatrix} \rho \\ v \end{pmatrix} + A\, \partial_r \begin{pmatrix} \rho \\ v \end{pmatrix} + h = 0,$$

Need to solve shock fronts!
Intricate discretization scheme called Kurganov-Tadmor

1) Run the 1d lattice simulation (spherical symmetry) for the given wall velocity and PT strength;

Initially higgs field sustain the profile. Scale invariant evolution (1004.4187)

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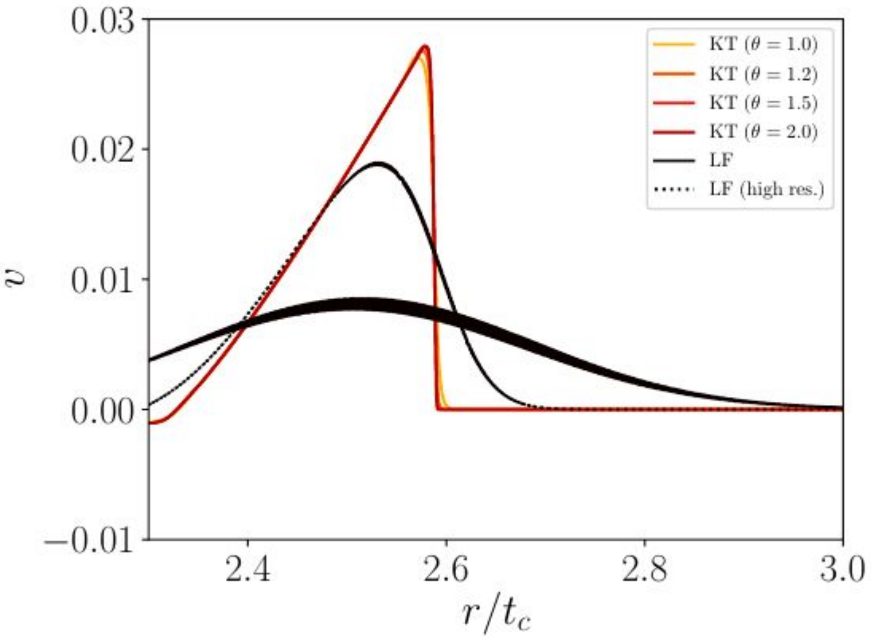
# Our set up

A detail for those that like numerical schemes

If one try to solve the shocks with a standard numerical schemes, it wont work (see Appendix A)



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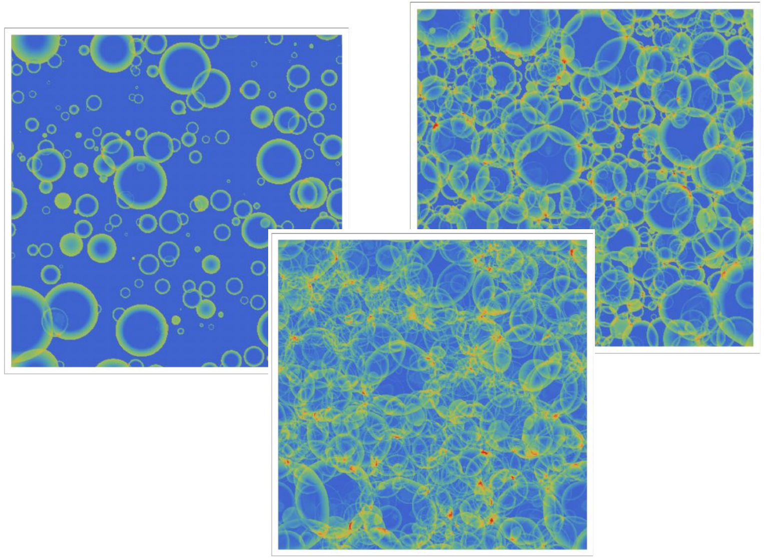

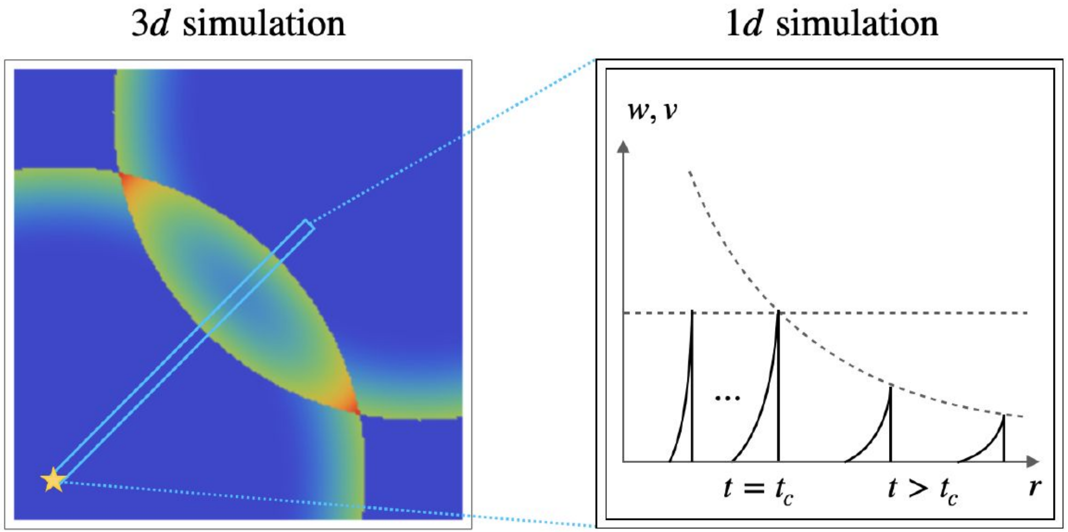


1) Run the 1d lattice simulation (spherical symmetry) for the given wall velocity and PT strength;

2) Nucleate bubbles and let them grow in a 3d lattice;

$$\Gamma \propto e^{\beta t}$$

Nucleation rate per volume parameterized by beta

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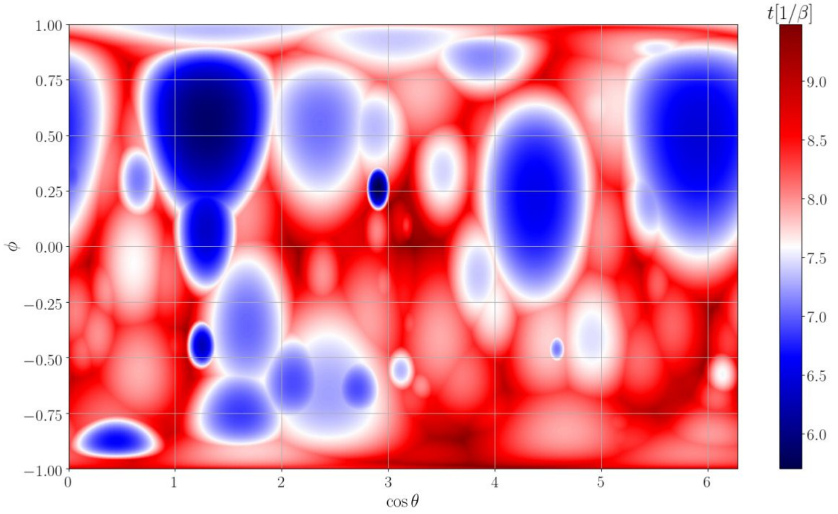
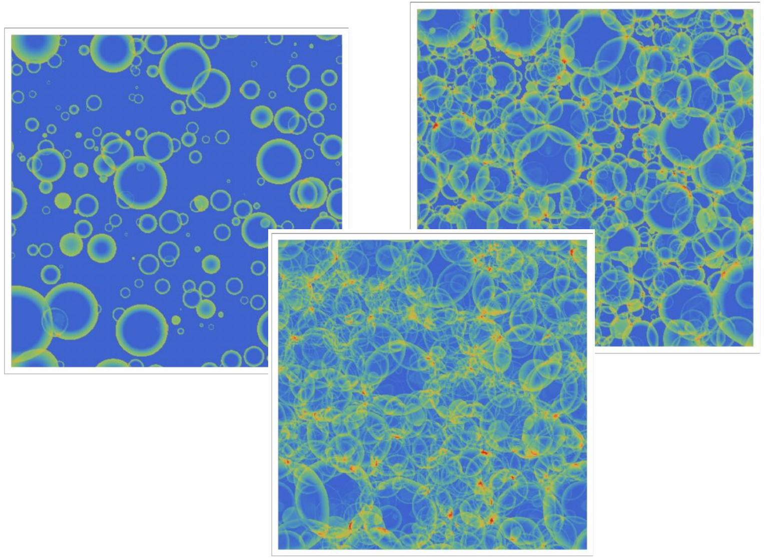
# Our set up



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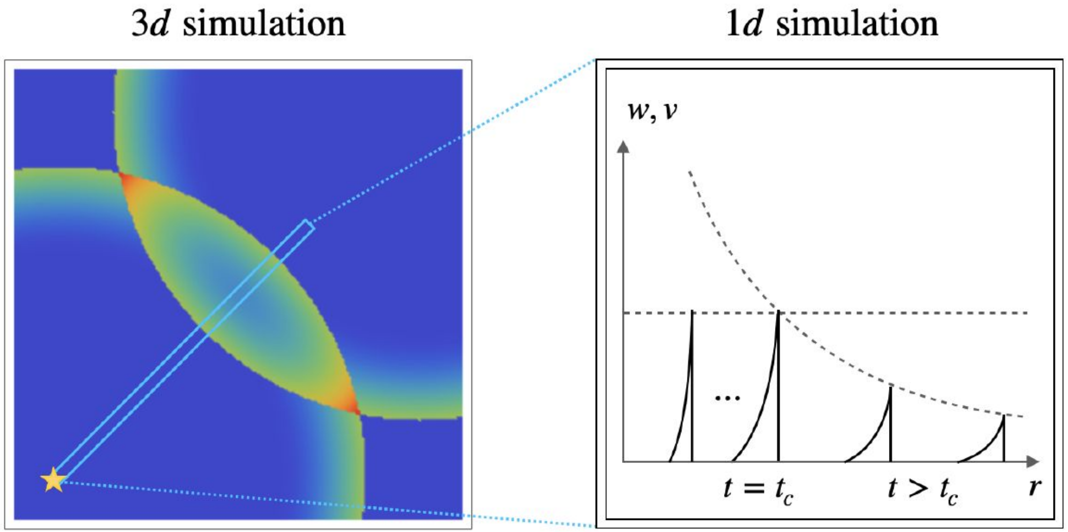


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3) Calculating when each differential part of each bubble surface collide;

4) Construct a velocity grid embedding the 1d simulation;

$$\frac{\Delta w}{w_0} \simeq \sum_{i:\text{bubbles}} \frac{\Delta w^{(i)}}{w_0}, \qquad \vec{v} \simeq \sum_{i:\text{bubbles}} \vec{v}^{(i)},$$

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# Our set up

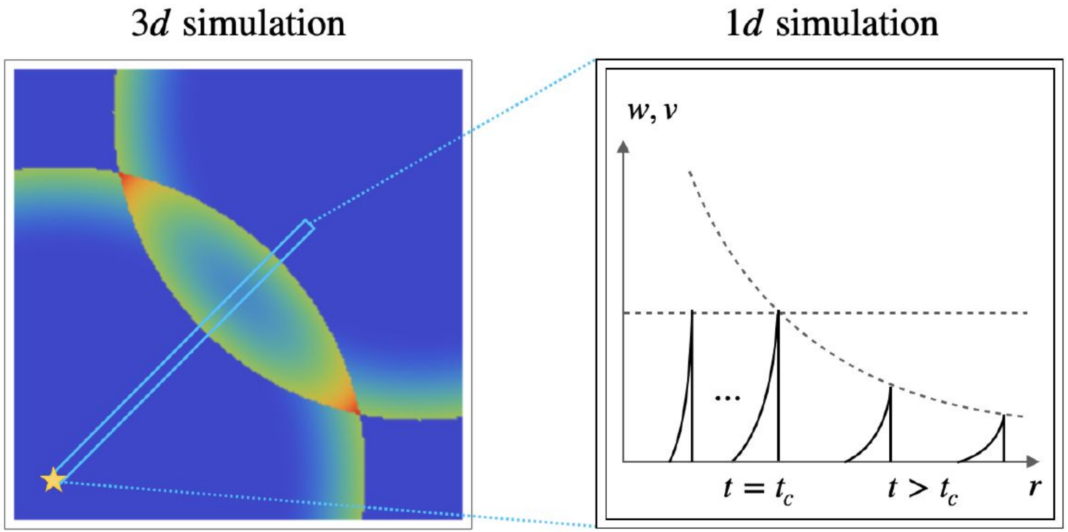
$$T^{ij}(\vec{x}) = v^i(\vec{x})v^j(\vec{x})\rho(\vec{x})$$

$$T_+(\vec{k}) = \sum_{i,j} \frac{T^{ij}(\vec{k})}{\sqrt{2}} \left(\theta_i(\vec{k})\theta_j(\vec{k}) - \phi_i(\vec{k})\phi_j(\vec{k})\right) ,$$

$$T_\times(\vec{k}) = \sum_{i,j} \frac{T^{ij}(\vec{k})}{\sqrt{2}} \left(\theta_i(\vec{k})\phi_j(\vec{k}) + \theta_i(\vec{k})\phi_j(\vec{k})\right) .$$

$$T_{+,\times}(q,\vec{k},t) = \sum_{t'=t_{\rm init}}^{t} e^{iqt'} T_{+,\times}(t',\vec{k}) ,$$

$$\Omega(q,t) = C\, q^3 \langle T_+ T_+^* + T_\times T_\times^* \rangle|_{|\vec{k}|=q} .$$

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# Results

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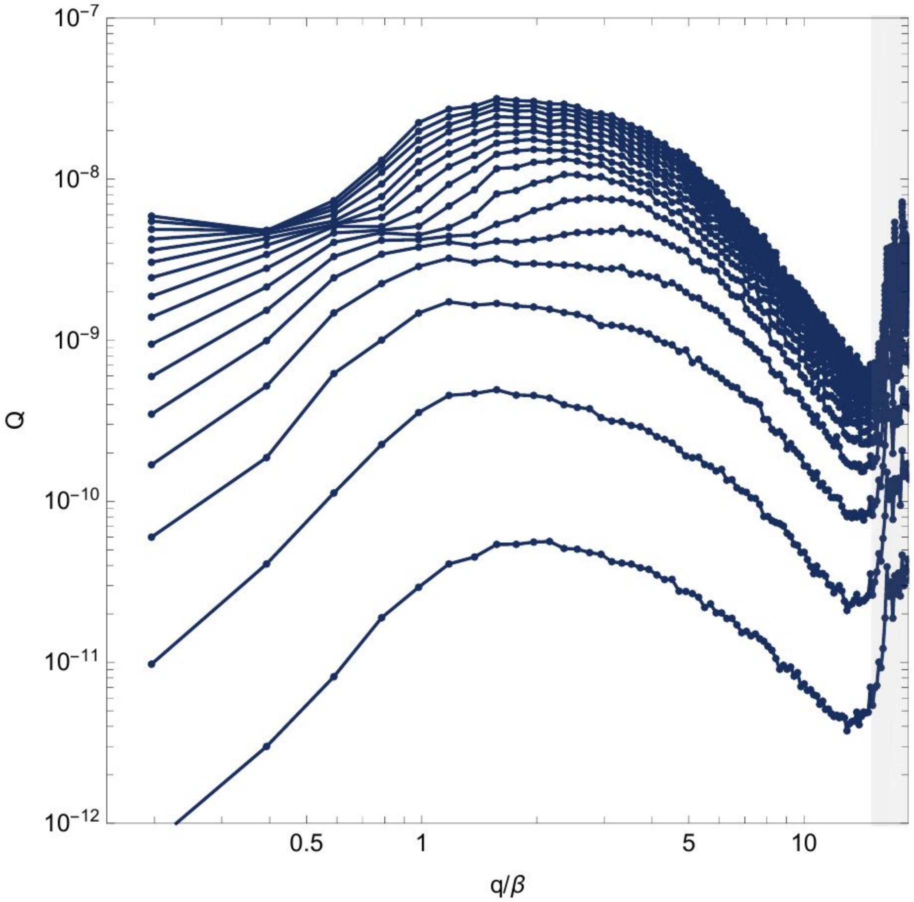
$$\Omega_{\rm GW} = \frac{w^2\,\tau}{4\pi^2\rho_{\rm tot}M_P^2\beta} \times Q',$$



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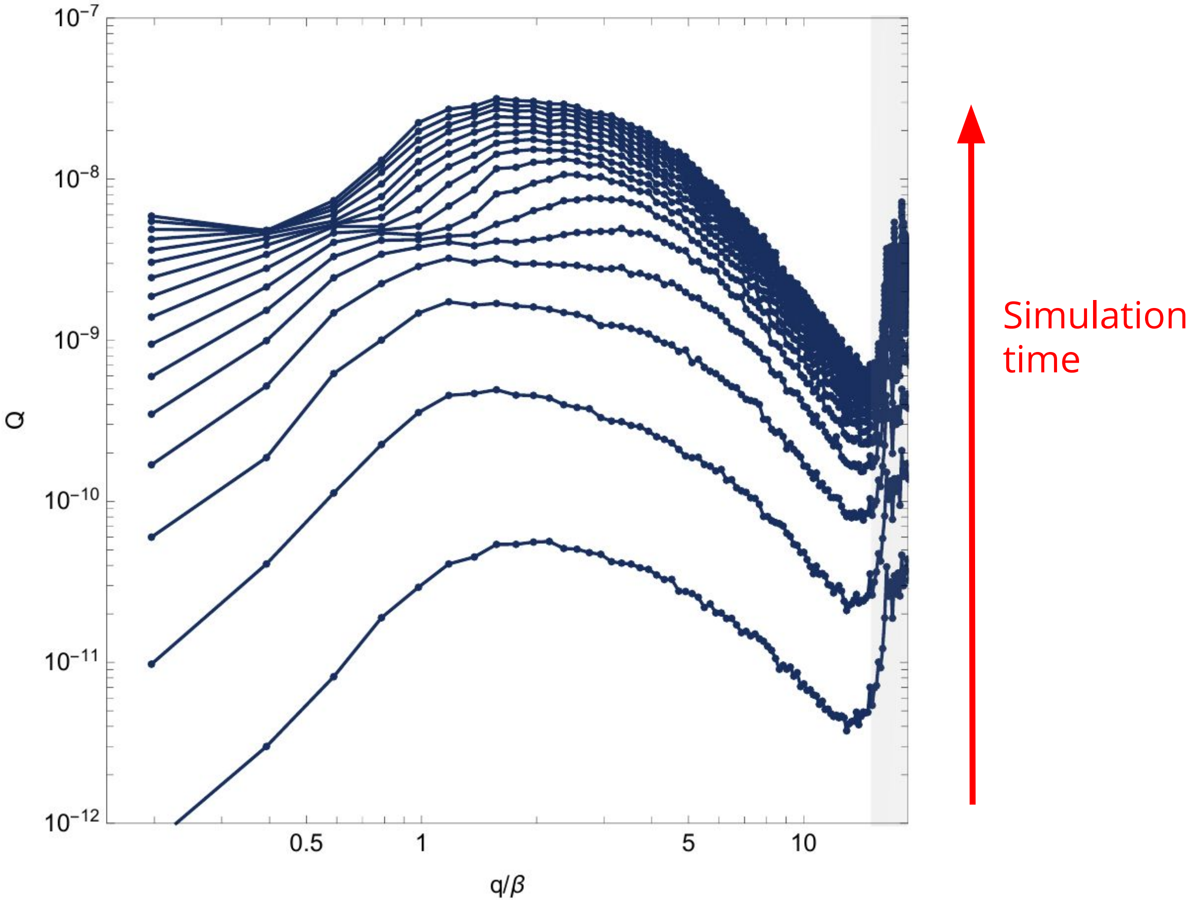

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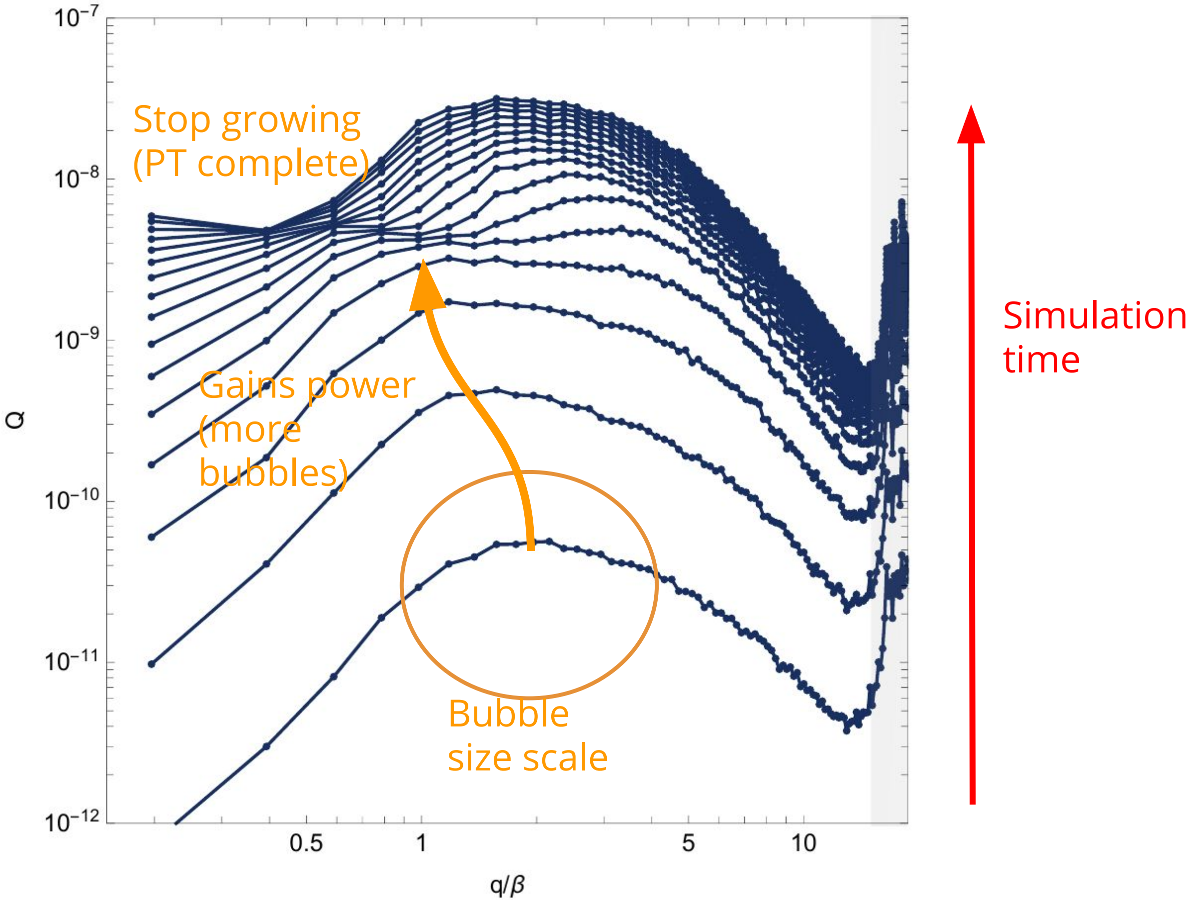

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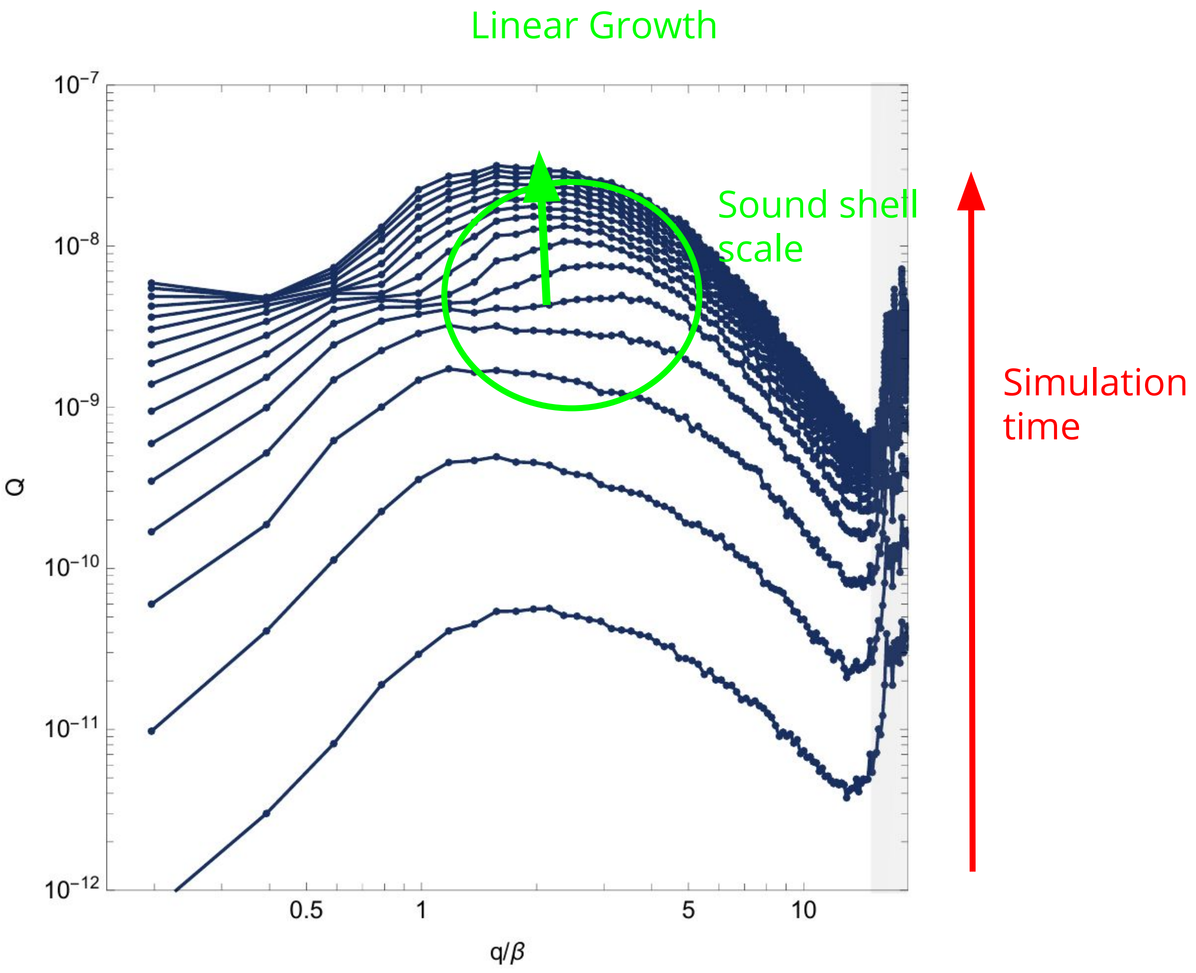

# Results

How to parametrize the result in terms of the bubble velocity and PT strength?

# Results

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We wanna have something user-friendly

Give me bubble velocity and PT strength
and I give you the spectra!

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Integrated spectrum gives better parameter dependence

$$Q'_{\rm int} \equiv \int d\ln q \; Q'(q) \,,$$

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Integrated spectrum gives better parameter dependence

$$Q'_{\rm int} \equiv \int d\ln q \; Q'(q) \,,$$

GW is proportional to fluid kinetic energy (v squared)

We expect the GW spectrum to be proportional to something like

$$Q' \propto (\langle w\gamma^2 v^2\rangle / w_\infty)^2 .$$

Find some quantity that resembles it

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Henrique Rubira

1st attempt $\langle w\gamma^2 v^2\rangle_{3\mathrm{d}}$

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3d kinetic energy

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GW spectra normalized by 3d kinetic energy

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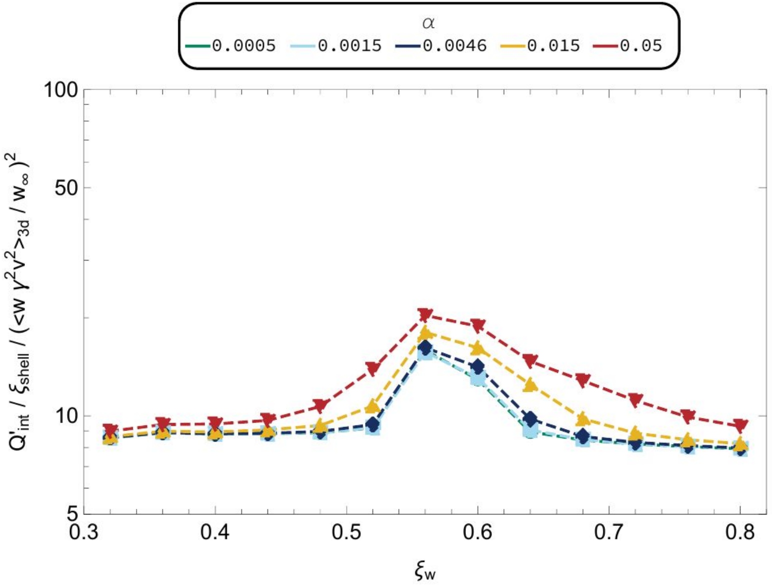


Almost no dependence on wall velocity and alpha!

Henrique Rubira

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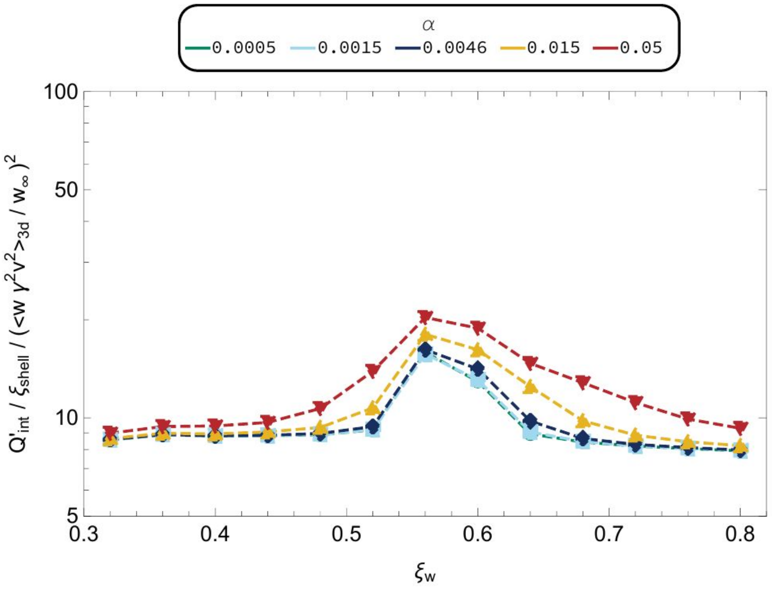


Pretty good normalization!

$$Q'_{\rm int} \simeq 9 \times \xi_{\rm shell} \times (\langle w\gamma^2 v^2\rangle_{3d}/w_\infty)^2$$

Problem: not user-friendly (needs 3d simulations)

# Results

1st attempt $\langle w\gamma^2 v^2\rangle_{3\text{d}}$

2nd attempt $\langle w\gamma^2 v^2\rangle_{1\text{d}}$

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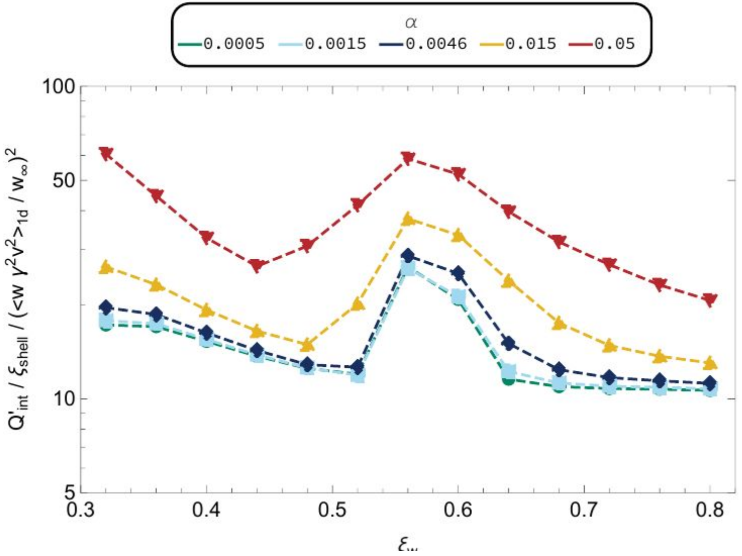


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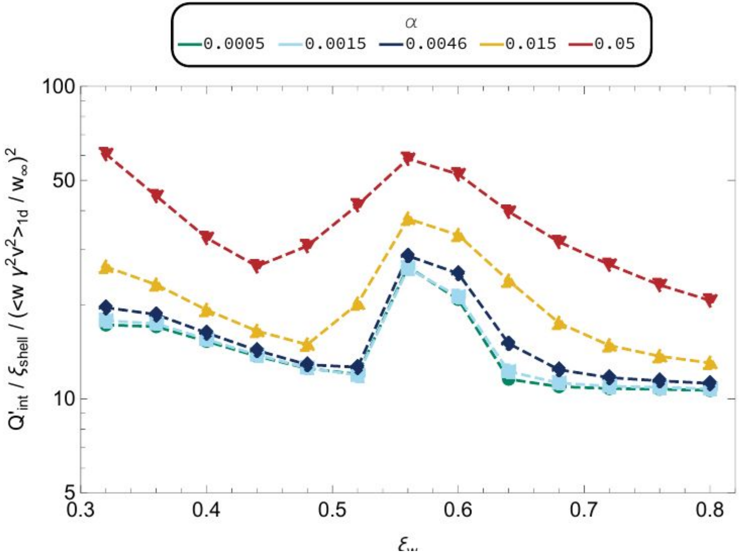


good normalization!

$$Q'_{\mathrm{int}} \simeq 12 \times \xi_{\mathrm{shell}} \times (\langle w\gamma^2 v^2\rangle_{1\mathrm{d}} / w_\infty)^2.$$

Problem: not user-friendly (needs 1d simulations)

Henrique Rubira

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1st attempt $\langle w\gamma^2 v^2\rangle_{\rm 3d}$

2nd attempt $\langle w\gamma^2 v^2\rangle_{\rm 1d}$

3rd attempt $\kappa\alpha = \dfrac{4}{\xi_w^3 w_\infty}\displaystyle\int d\xi \; w\gamma^2 v^2 \xi^2,$

Very user friendly!

The kinetic energy of the initial fluid profile, that can be calculated from some fits (see 2004.06995)

Henrique Rubira

# Results

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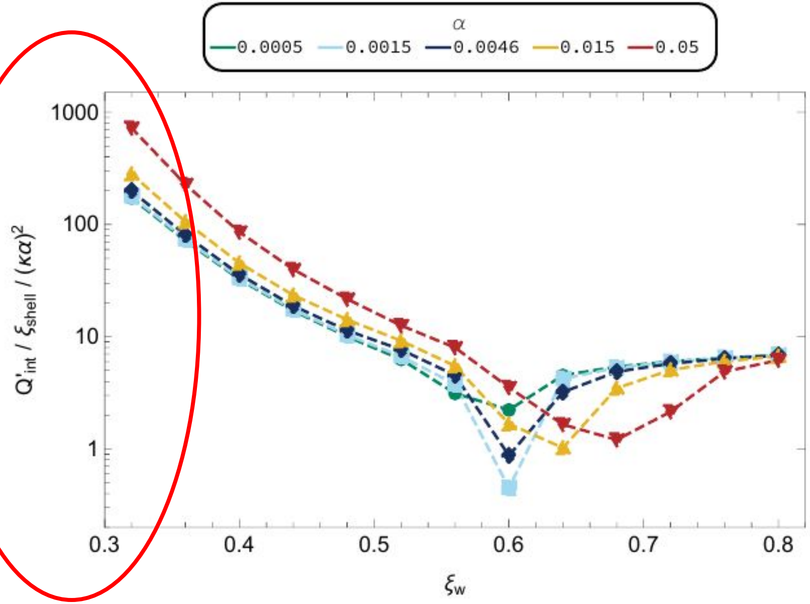

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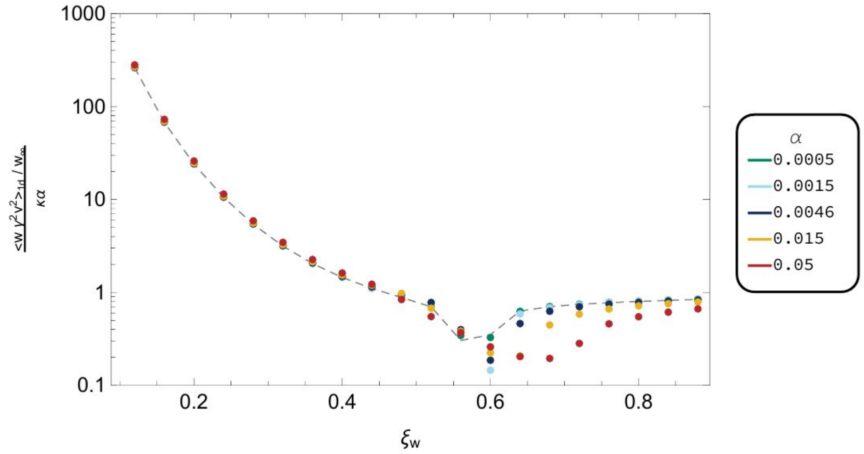
But we can relate $\kappa\alpha$ to $\langle w\gamma^2 v^2\rangle_{1\mathrm{d}}$ pretty easily




$$\boxed{Q'_{\mathrm{int}} \simeq 12 \times \xi_{\mathrm{shell}} \times (\langle w\gamma^2 v^2\rangle_{1\mathrm{d}} / w_\infty)^2.}$$

# Results

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A double power law

$$\frac{Q'}{\xi_{\rm shell} \times (\langle w\gamma^2 v^2\rangle_{\rm 3d}/w_\infty)^2} \propto \frac{1}{(q/q_l)^{-n_l} + (q/q_l)^{-n_m} + (q_h/q_l)^{-n_m}(q/q_h)^{-n_h}}$$

$$\simeq \begin{cases} (q/q_l)^{n_l} & (q \ll q_l) \\ (q/q_l)^{n_m} & (q_l \ll q \ll q_h) \\ (q_h/q_l)^{n_m}(q/q_h)^{n_h} & (q_h \ll q) \end{cases} .$$

$$n_m \in [-1, 0]$$

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Low freq.

$\boxed{q_l \simeq 1,}$

$\boxed{n_l \in [2,4]\,,}$

Typical bubble size at collision!

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High freq.

$$\boxed{q_h \simeq 1/\xi_{\rm shell}}$$

$$\boxed{n_h \in [-4, -3]}$$

peak at sound shell scale

# Comparison to other works

When we compare to scalar field simulations, we have found:

- Similar scaling
- IR peak shifted to lower freq. (Realistic nucleation)
- Factor ~ 2 in overall amplitude (More bubbles)

# Conclusions

New simulation scheme (free of scalar field scale) to calculate sound-shell contribution

$$Q'_{\rm int} \simeq 9 \times \xi_{\rm shell} \times (\langle w\gamma^2 v^2\rangle_{\rm 3d}/w_\infty)^2$$

Sound shell size

Kinetic energy

Parametrize as

$$\Omega_{\rm GW} \propto \frac{1}{(q/q_l)^{-n_l} + (q/q_l)^{-n_m} + (q_h/q_l)^{-n_m}(q/q_h)^{-n_h}}$$

$$q_l \simeq 1, \qquad q_h \simeq 1/\xi_{\rm shell},$$

$$n_l \in [2,4]\,, \quad n_m \in [-1,0]\,, \quad n_h \in [-4,-3]\,,$$