# Quark number susceptibilities from resummed perturbative QCD

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In collaboration with Jens O. Andersen (Trondheim), Nan Su and Aleksi Vuorinen (Bielefeld).

# MOTIVATIONS

- Why finite $T$ and $\mu$ QCD via analytical techniques?

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- What do we intend to do here?
  1. Studying to which extent one can understand recent high precision lattice data, using weakly coupled quasiparticle picture.

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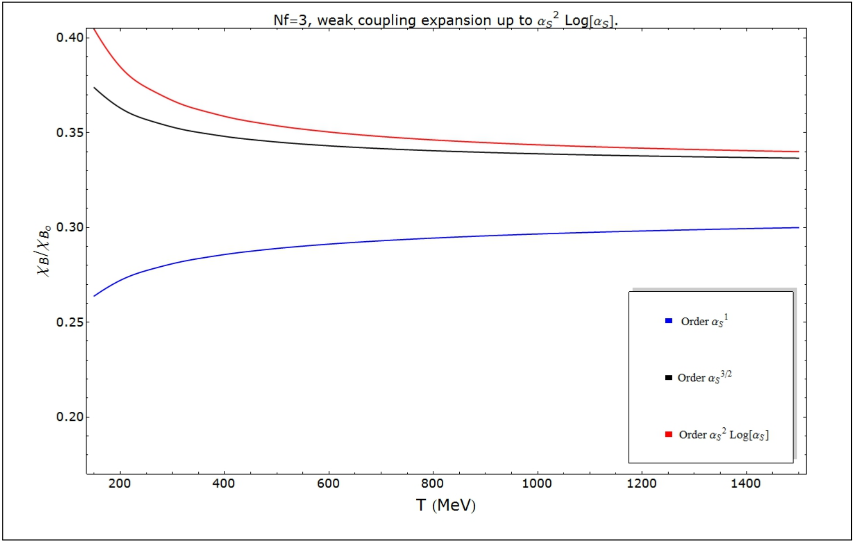
# Problem of convergence of the series



# RECALLS

- Imaginary time formalism,
- Dimensional regularization in the momentum space: $T\sum_{P_0}\int\frac{\mathrm{d}^{3-2\epsilon}\mathbf{p}}{(2\pi)^{3-2\epsilon}}$,
- Lagrangian density of QCD:

$$\begin{aligned}\mathcal{L}_{\rm QCD} &= -\frac{1}{2}\mathrm{Tr}\left(F_{\mu\nu}F^{\mu\nu}\right)+i\bar{\psi}\gamma^{\mu}D_{\mu}\psi\\ &+\mathcal{L}_{\rm gf}+\mathcal{L}_{\rm ghost}+\Delta\mathcal{L}_{\rm QCD},\end{aligned}$$

- Relation between the thermodynamic potential $\Omega$ and the diagonal quark number susceptibility $\chi$:

$$\chi\left(T\right) \doteq -\left.\frac{\partial^2\Omega\left(T,\mu\right)}{\partial\mu^2}\right|_{\mu=0}.$$

# BASIC FORMALISM

- Reorganizing the perturbative series of thermal QCD:

$$\mathcal{L} = (\mathcal{L}_{\rm QCD} + \mathcal{L}_{\rm HTL})\Big|_{g\to\sqrt{\delta}g} + \Delta\mathcal{L}_{\rm HTL},$$

- With the HTL improvement term:

$$\mathcal{L}_{\rm HTL} = -\frac{1}{2}(1-\delta)m_D^2 \mathrm{Tr}\left(F_{\mu\alpha}\left\langle \frac{y^\alpha y^\beta}{(y\cdot D)^2}\right\rangle_y F^\mu{}_\beta\right) + (1-\delta)\, i m_q^2 \bar{\psi}\gamma^\mu \left\langle \frac{y_\mu}{y\cdot D}\right\rangle_y \psi,$$

- $\delta$: formal expansion parameter, # of HTL dressed loops; $\Delta\mathcal{L}_{\rm HTL}$: HTL counterterm(s), $y^\mu$: lightlike 4-vector; $m_D/m_q$: Debye/quark thermal mass parameters,
- Adding $\mathcal{L}_{\rm HTL}$ shifts the expansion to an ideal gas of thermal quasiparticles.

# ABOUT THE LEADING ORDER RESULT

- Prescription for the mass parameters:
  1-loop $m_D$ and $m_q$ at finite $T$ and $\mu$, i.e.

$$m_D^2 \equiv \alpha_S\,\pi\left(\frac{4N_c}{3}\,T^2+2N_f\,\frac{\mu^2}{\pi^2}\right),$$

$$m_q^2 \equiv \alpha_S\,\pi\,\frac{N_c^2-1}{4N_c}\left(T^2+\frac{\mu^2}{\pi^2}\right),$$

- Tricky sum-integrals! Integrands with powers of $\mathcal{T}_P$:

$$(\mathcal{T}_P)^m \doteq \left(\frac{\Gamma\left(\frac{3}{2}-\epsilon\right)}{\Gamma\left(\frac{3}{2}\right)\Gamma\left(1-\epsilon\right)}\right)^m\left(P_0^2\right)^m \int_0^1 \mathrm{d}c_1\,\ldots\int_0^1\mathrm{d}c_m\left\{\frac{\left(1-c_1^2\right)^{-\epsilon}}{\left(P_0^2+\mathbf{p}^2c_1^2\right)}\cdots\frac{\left(1-c_m^2\right)^{-\epsilon}}{\left(P_0^2+\mathbf{p}^2c_m^2\right)}\right\}$$

$$\equiv \left({}_2F_1\left(\tfrac{1}{2},1,\tfrac{3}{2}-\epsilon;-\tfrac{\mathbf{p}^2}{P_0^2}\right)\right)^m,$$

which look like:

$$\sum\!\!\!\!\!\!\!\!\int_{\{P\}} \left[ \frac{(P_0)^u \, (\mathcal{T}_P)^m}{(\mathbf{p}^2)^w \, (P^2)^n} \right]$$

- Possible to find analytical expressions in term of hypergeometric functions, for the first powers of $\mathcal{T}_P$ (for example in the case of $m = 1$ and $m = 2$),
- Those are needed to compute the High-T truncation, but what about the full result?
- Alternative methods: The branch cuts.

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# BRANCH CUTS TECHNIQUE

- As a first step: from a discrete summation to a contour integral,
- 'Constraints' given by the branch points, according to the integrand,
- Choice of the branch cuts,

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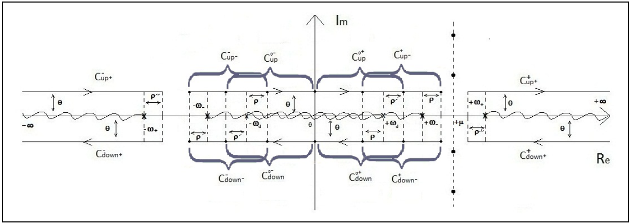
# Branch cuts



Im
Re
$-\infty$
$+\infty$
$-\omega_+$
$-\omega_-$
$-\omega_d$
$+\omega_d$
$+\omega_-$
$+\omega_+$
$+\mu$
$\theta$
$\rho$

- Then, renormalization by addition and subtraction of some relevant integrals; those contain the UV divergences without introducing any IR ones,
- Must be computable analytically, in order to extract the divergence(s),

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$$\frac{\Omega^{LO}_{\mathrm{HTL}}}{-d_A\frac{\pi^2 T^4}{45}} \equiv \left\{\frac{90}{\pi^4}\frac{d_F}{d_A}\left[\mathrm{Li}_4\left(-e^{-\beta\mu}\right)+\mathrm{Li}_4\left(-e^{+\beta\mu}\right)\right]-\frac{1}{2}\right\}$$
$$+\left\{\frac{-15}{4}\right\}\overline{m}_D^2+\left\{\frac{45}{4}\log\left(\frac{\bar{\Lambda}}{m_D}\right)\right\}\overline{m}_D^4$$
$$+\left\{\frac{-45}{2\pi^4 T^3}\left[2\,\widetilde{\mathcal{I}}_{B_T}+\widetilde{\mathcal{I}}_{B_L}\right]\right.$$
$$+\frac{d_F}{d_A}\frac{45}{\pi^4 T^3}\left[\widetilde{\mathcal{I}}^+_{F_{\widetilde{\omega}_+}}+\widetilde{\mathcal{I}}^-_{F_{\widetilde{\omega}_+}}+\widetilde{\mathcal{I}}^+_{F_{\widetilde{\omega}_-}}+\widetilde{\mathcal{I}}^-_{F_{\widetilde{\omega}_-}}\right]$$
$$\left.+\frac{45}{2\pi^5 T^4}\left[\widetilde{\mathcal{K}}_{B_{\mathrm{TL}}}\right]-\frac{d_F}{d_A}\frac{45}{\pi^5 T^4}\left[\widetilde{\mathcal{K}}^+_F+\widetilde{\mathcal{K}}^-_F\right]\right\}$$
$$+\left\{cte_1\right\}\overline{m}_D^4+\left\{-\frac{d_F}{d_A}\left(cte_2\right)\right\}\overline{m}_q^4,$$

# HIGH-T TRUNCATION VS WEAK COUPLING EXPANSION

- Scalar sum-integrals expanded in power of $m_D/T$ and $m_q/T$; then truncated at, and including the order $m^4_{D/q}$,
- The mass parameters are then treated as $\sim O(\alpha_S^{1/2})$,
- Therefore, this truncation is suitable to compare our result with the known weak coupling expansion up to $\alpha_S^2 \log(\alpha_S)$,
- Note that the full result includes higher order contributions, to all order beyond this truncation.

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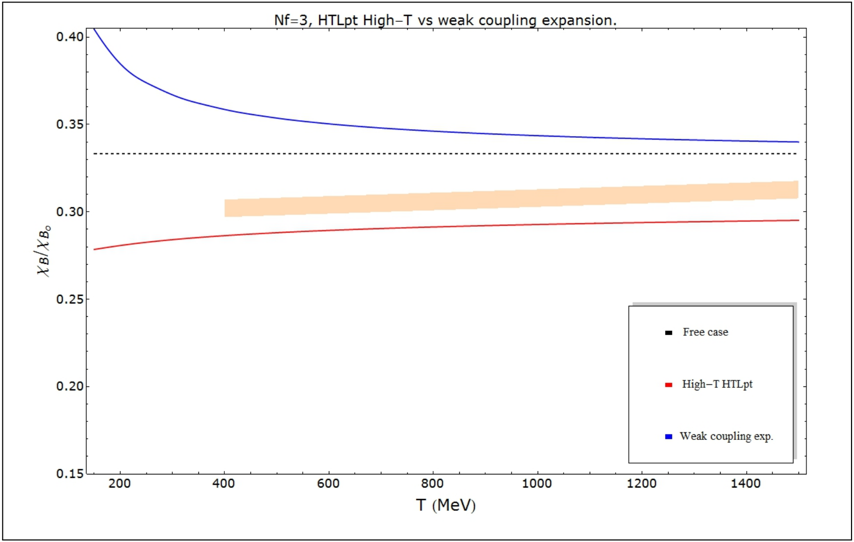
# High-T truncation vs weak coupling expansion



# BASIC FORMALISM

- Separation of the pressure into different contributions, according to their typical momentum scales:

$$p_{\mathrm{QCD}} \equiv p_{\mathrm{hard}} + p_{\mathrm{soft}},$$

- $p_{\mathrm{hard}}$: From the hard modes ($\propto 2\pi T$), computable via strict perturbative expansion in the 4D theory,
- $p_{\mathrm{soft}}$: From the soft modes ($\propto gT$), computable via the effective 3D Yang-Mills plus adjoint Higgs theory (EQCD),
- Lagrangian density of EQCD:

$$\mathcal{L}_{\mathrm{EQCD}} = \frac{1}{2}\,\mathrm{Tr}\,F_{ij}^2 + \mathrm{Tr}\,[D_i, A_0]^2 + m_{\mathrm{E}}^2 \mathrm{Tr}\,A_0^2 + \frac{ig^3}{3\pi^2}\sum_f \mu_f\,\mathrm{Tr}\,A_0^3 + \delta\mathcal{L}_{\mathrm{E}}.$$

# ABOUT THE 4-LOOP RESULT

- Already known 4-loop result,
- New resummation scheme, applied to finite $\mu$ case,
- Effect of the strange quark mass.

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# NEW RESUMMATION SCHEME

- Amount to let all of the DR parameters unexpanded,
- Resums to all order a certain class of diagrams,
- Substantially reduce the renormalization scale dependance.

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# EFFECT OF THE STRANGE QUARK MASS

- rescaling of massive and massless results for the pressure, by appropriate ratio of Stefan-Boltzmann law,
- As the quark contribution to the pressure converge faster than the series as a whole, a rescaling by an appropriate ratio of first order correction to the Stefan-Boltzmann law would not quite modify the story ($\sim 5\%$),
- Hence, good phenomenological way to guess the effect of the presence of a non-vanishing mass for the strange quark.

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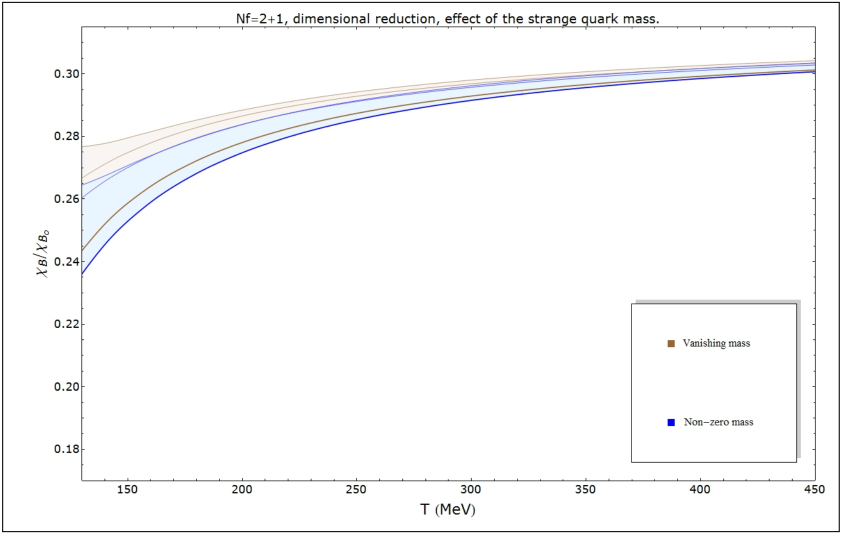
# Effect of the strange quark mass in DR



- Renormalization scale chosen to be an optimal scale;
  1-loop corrections to the electric screening mass vanish;
  In practise, $\Lambda_{N_f=2} \approx 1.29 \times 2\pi T$ and $\Lambda_{N_f=3} \approx 1.44 \times 2\pi T$,
- $\Lambda_{\overline{MS}} = 180 - 200$ MeV for DR, and $\Lambda_{\overline{MS}} \approx 140 - 160$ MeV for HTLpt, respectively for $N_f = 2$ and $N_f = 3$.

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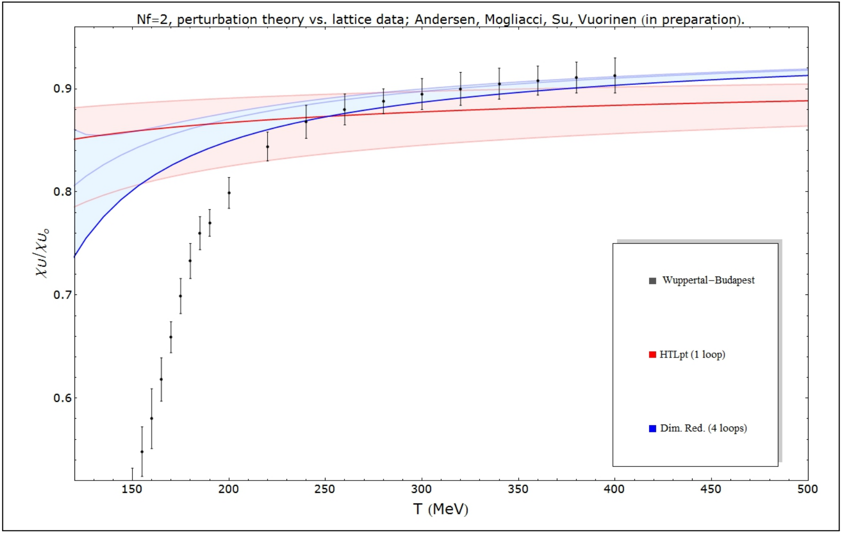
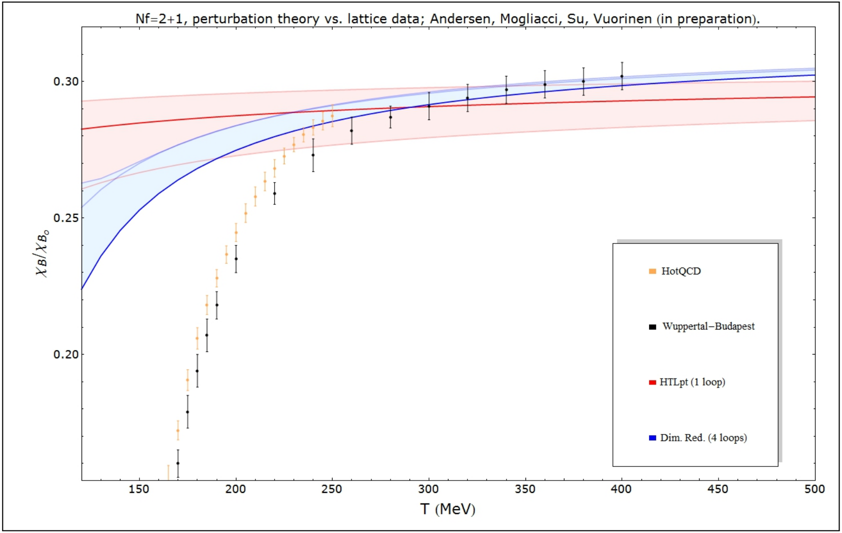
# $N_f = 2$ PERTURBATION THEORY VS LATTICE DATA

# $N_f = 2$ perturbation theory vs Lattice data



# $N_f = 2+1$ PERTURBATION THEORY VS LATTICE DATA

# $N_f = 2 + 1$ perturbation theory vs Lattice data



- Computation of $\chi$ in QCD via two different resummation schemes,
- The results are consistent with recent lattice data down to $T \approx 250 - 300$ MeV!,
- HTLpt results are from the high T expansion of the free quasiparticle gas (1-loop) seem to account for a next to leading order computation; full computation underway
- References for previous investigation of the QNS: [A. Vuorinen, 2003],
- References for the DR resummation scheme: [M. Laine, Y. Schroder, 2006],
- References for the Lattice data: [S. Borsanyi et al., A. Bazavov et al., 2012],
- For recent 3-loop HTLpt work: [J. O. Andersen, L. E. Leganger, M. Strickland, and Nan Su, 2011],

# Backup

$$\widetilde{\mathcal{I}}_{B_{T,L}} \doteq \widetilde{\mathcal{I}}_{B_{T,L}}(T,\mu) \equiv \int_0^\infty \mathrm{d}k\ k^2\ \log\left(1 - e^{-\beta\omega_{\mathrm{T,L}}}\right)\ ,$$

$$\widetilde{\mathcal{I}}^{\pm}_{F_{\omega_\pm}} \doteq \widetilde{\mathcal{I}}^{\pm}_{F_{\omega_\pm}}(T,\mu) \equiv \int_0^\infty \mathrm{d}k\ k^2 \log\left(1 + e^{-\beta[\omega_\pm \pm \mu]}\right)\ ,$$

$$\widetilde{\mathcal{K}}_{B_{\mathrm{TL}}} \doteq \widetilde{\mathcal{K}}_{B_{\mathrm{TL}}}(T,\mu) \equiv \int_0^\infty \mathrm{d}\omega\ \frac{1}{e^{\beta\omega} - 1} \int_\omega^\infty \mathrm{d}k\ k^2\left(2\phi_{\mathrm{T}} - \phi_{\mathrm{L}}\right),$$

$$\widetilde{\mathcal{K}}^{\pm}_{F} \doteq \widetilde{\mathcal{K}}^{\pm}_{F}(T,\mu) \equiv \int_0^\infty \mathrm{d}\omega\ \frac{1}{e^{\beta[\omega\pm\mu]} + 1} \int_\omega^\infty \mathrm{d}k\ k^2\theta_{\mathrm{q}}\ ,$$