QCD thermodynamics from the lattice

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- Symmetries of QCD at $T>0$: chiral and deconfinement transitions
- Universal aspects of the chiral transition and the transition temperature
- Deconfinement and color screening
- Equation of state
- Thermodynamics at non-zero chemical potential and fluctuations of conserved charges

Calculations with controlled discretization errors

$\Rightarrow$ Highly Improved Staggered Quark (HISQ) action

$\Rightarrow$ asqtad, p4, stout

For a recent review see P.P. arXiv:1203.5320

Final Colloquium: International Research Training Group, GRK 881, September, 12-14, 2012

Symmetries of QCD at T>0

- Chiral symmetry : $m_{u,d} \ll \Lambda$

$SU_A(2)$ symmetry $\psi \to e^{i\phi T^a \gamma_5}\psi$ $\quad \psi_{L,R} \to e^{i\phi_{L,R} T^a}\psi_{L,R}$

The vacuum (ground state) is not $\langle\bar\psi\psi\rangle = \langle\bar\psi_L\psi_R\rangle + \langle\bar\psi_R\psi_L\rangle \neq 0$

$\langle\bar\psi\psi\rangle = 0$
restored

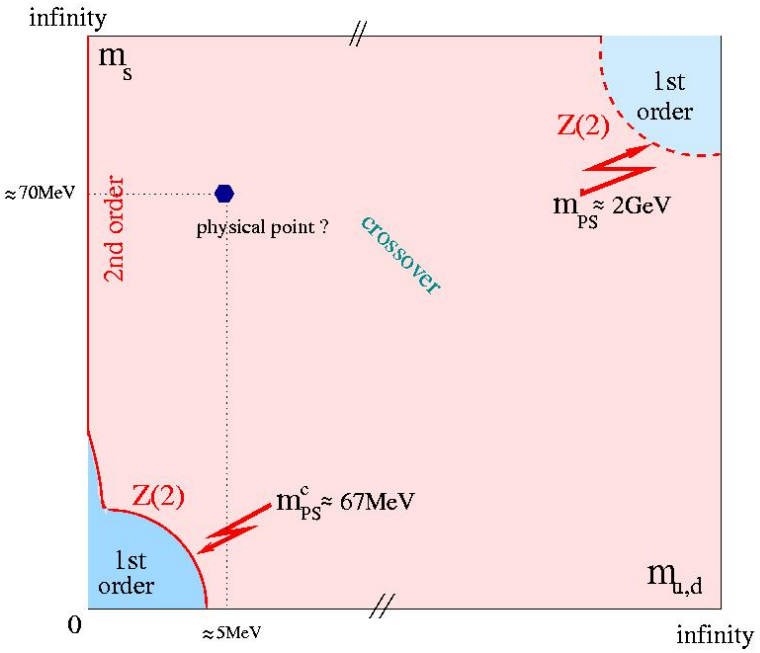
- Center (Z3) symmetry : invariance under global gauge transformation

$$A_\mu(0,\mathbf{x}) = e^{i2\pi N/3} A_\mu(1/T,\mathbf{x}), \quad N = 1,2,3$$

Exact symmetry for infinitely heavy quarks $\quad \langle L\rangle = 0$

Polyakov loop : $\quad L = \mathrm{tr}\mathcal{P}e^{ig\int_0^{1/T} d\tau A_0(\tau,\vec{x})}$

$\langle L\rangle \neq 0$
broken



LQCD calculations with staggered quarks suggest crossover, e.g. Aoki et al, Nature 443 (2006) 675

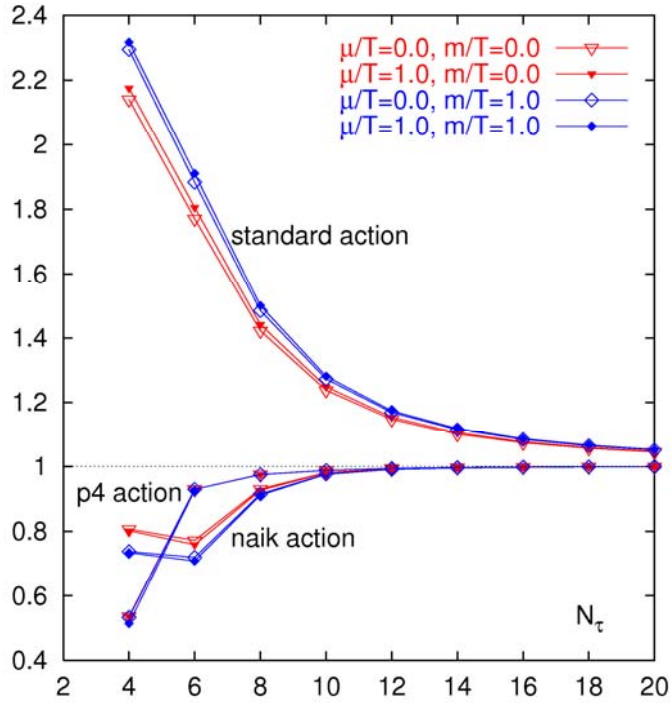
Evidence for 2nd order transition in the chiral limit
⇒universal properties of QCD transition:

$SU_A(2) \sim O(4)$
relation to spin models

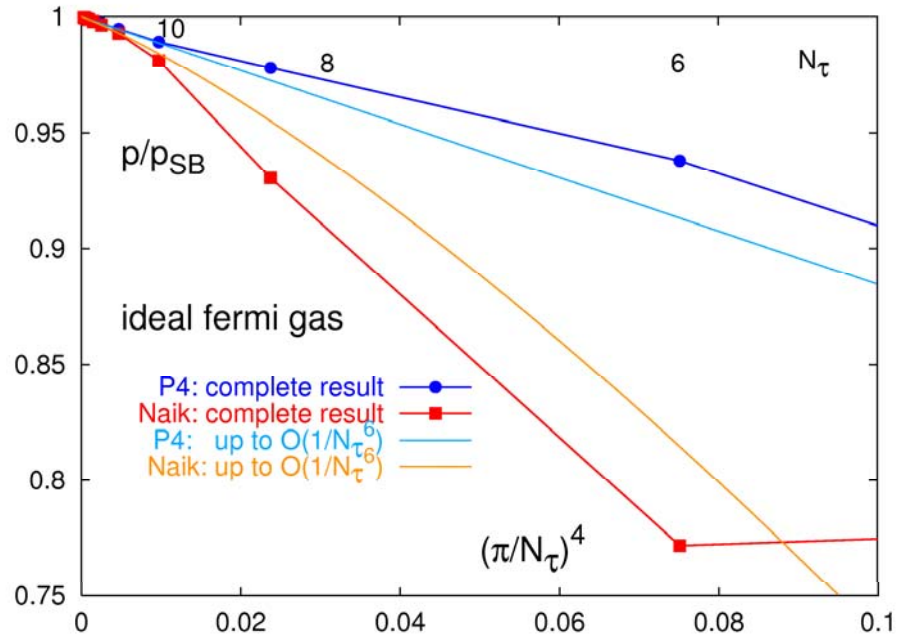
Center symmetry does not seem to play any role for physical values if the light quark masses

Why improved actions ?

1) Possibly large $\mathcal{O}((aT)^2) \sim \mathcal{O}(1/N_\tau^2)$ errors at high $T \Rightarrow$ next-to-nearest terms



$$a = \frac{1}{TN_\tau}$$



$$\frac{p(N_\tau)}{p_{cont}} = 1 - \frac{1143}{980}\left(\frac{\pi}{N_\tau}\right)^4 + \frac{73}{2079}(1+6528c_{30})\left(\frac{\pi}{N_\tau}\right)^6 + \mathcal{O}(N_\tau^{-8})$$

$c_{30} = 0$ for p4, $c_{30} = -1/48$ for Naik

$\alpha_s a^2$ corrections are smaller for p4 than for Naik

2) Flavor symmetry is broken by $\mathcal{O}(\alpha_s a^2)$ effects in the in the staggered fermion formulations $\Rightarrow$ smeared (fat) gauge fields

The Highly Improved Staggered Quark (HISQ) Action

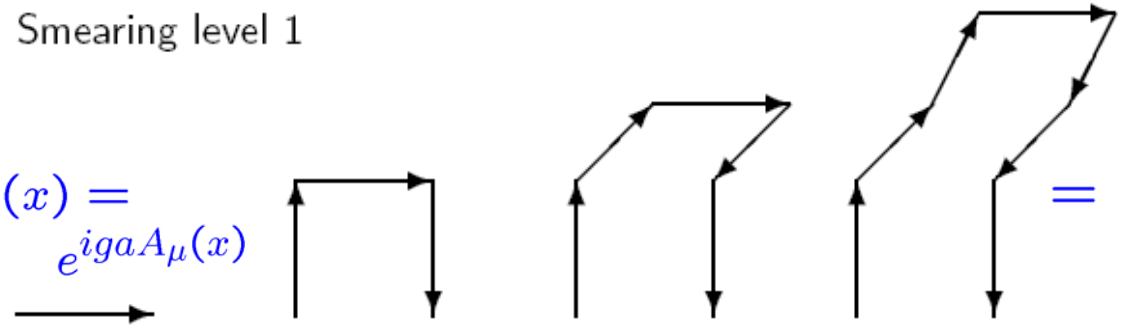
HISQ action

two levels of gauge field smearing with re-unitarization

Follana et al, PRD75 (07) 054502

Smearing level 1

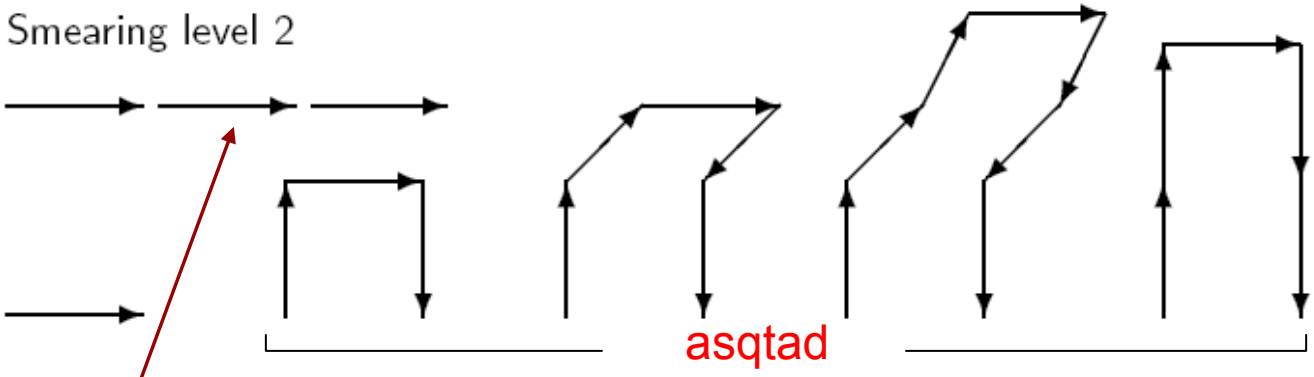
$U_\mu(x) = e^{igaA_\mu(x)}$



$= U_\mu^{fat7} \rightarrow \tilde{U}_\mu$

$= \frac{U_\mu^{fat7}}{\sqrt{U_\mu^{fat7} U_\mu^{fat7\dagger}}}$

Smearing level 2



projection onto *U(3)* improves flavor symmetry

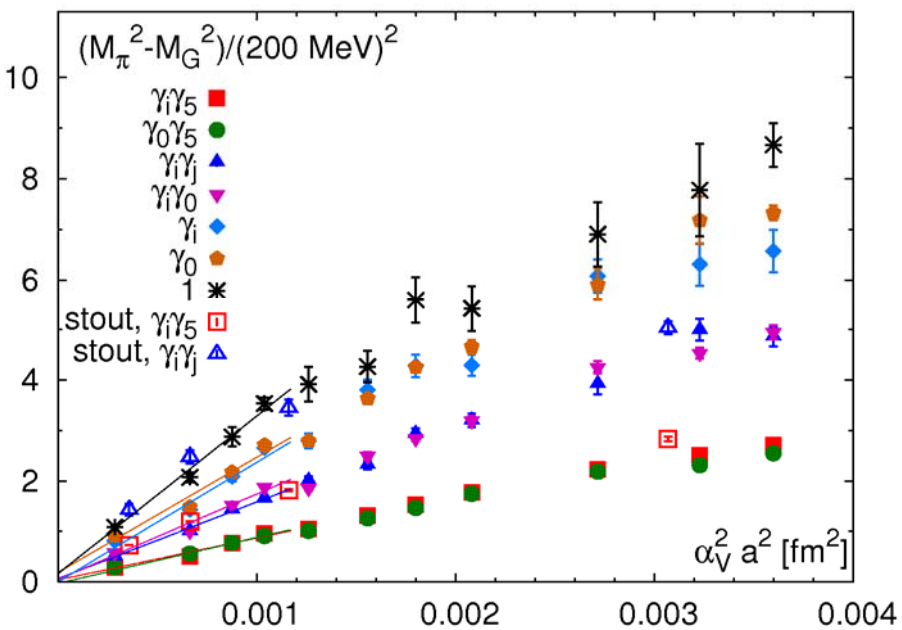
Hasefratz, arXiv:hep-lat/0211007

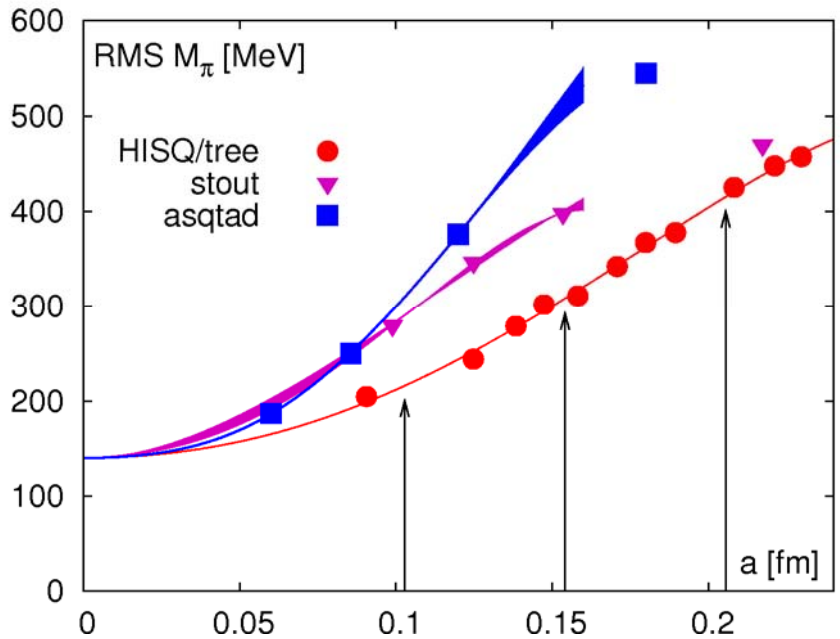
3-link (Naik) term to improve the quark dispersion relation + asqtad smearing

Taste symmetry of staggered quarks

Only one of the 16 ps mesons of the 4-flavor theory is massless in the chiral limit

$$\delta m^2_{ps_i} = M^2_{\pi_i} - M^2_G \sim \alpha_s a^2$$





$$M_\pi^{RMS} = \sqrt{\frac{1}{16}\left(M^2_{\gamma_5} + M^2_{\gamma_0\gamma_5} + 3M^2_{\gamma_i\gamma_5} + 3M^2_{\gamma_i\gamma_j} + 3M^2_{\gamma_i\gamma_0} + 3M^2_{\gamma_i} + M^2_{\gamma_0} + M^2_1\right)}$$

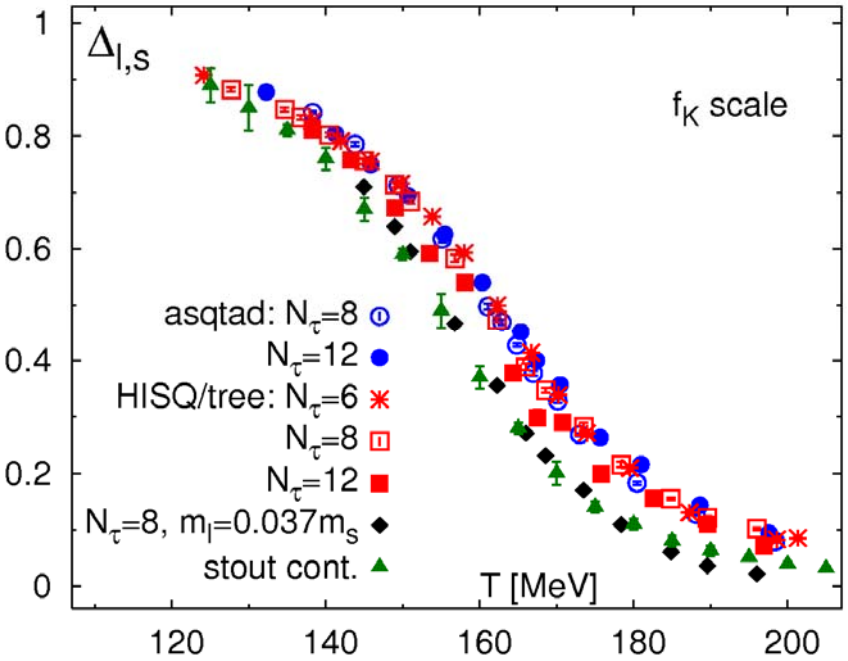
For N_τ = *6, 8* lattices the RMS pion mass is *300-400* MeV even for HISQ and even for the finest lattice (N_τ=*12*) the RMS pion mass is always larger than *200* MeV

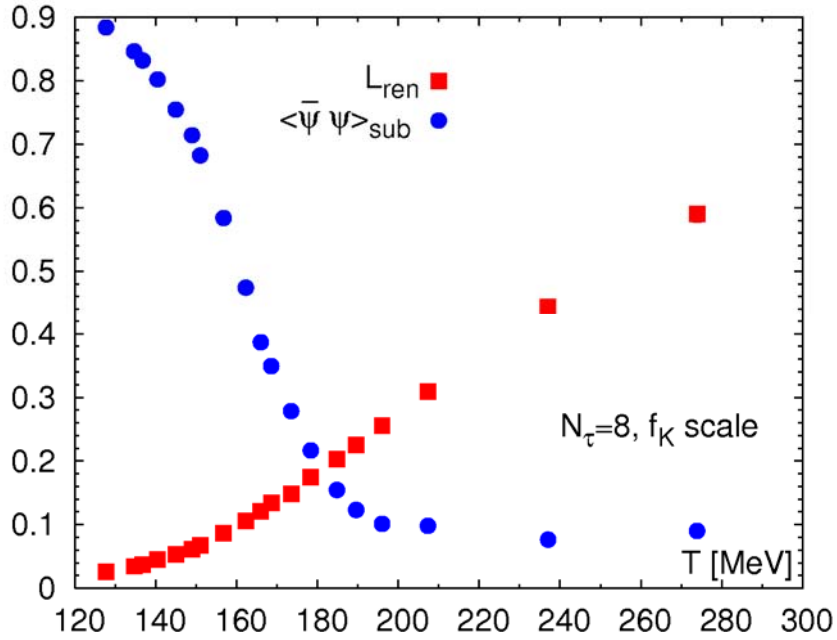
The temperature dependence of chiral condensate

Chiral condensate needs multiplicative and additive renormalization for non-zero quark mass

$$\langle\bar\psi\psi\rangle_l \Rightarrow \langle\bar\psi\psi\rangle_{sub} = \frac{\langle\bar\psi\psi\rangle_{l,T} - \frac{m_l}{m_s}\langle\bar\psi\psi\rangle_{s,T}}{\langle\bar\psi\psi\rangle_{l,T=0} - \frac{m_l}{m_s}\langle\bar\psi\psi\rangle_{s,T=0}}$$

P.P. arXiv:1203.5320





- Cut-off effects are significantly reduced when f_K is used to set the scale
- After quark mass interpolation based on *O(N)* scaling the HISQ/tree results agree with the stout continuum result !
- The deconfinement in terms of color screening sets in at temperatures higher than the chiral transition temperature

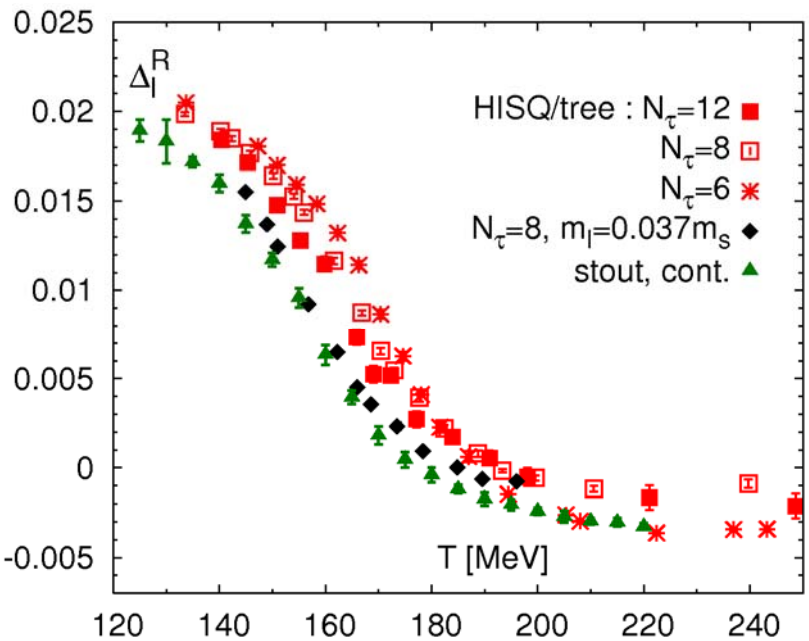
The temperature dependence of chiral condensate

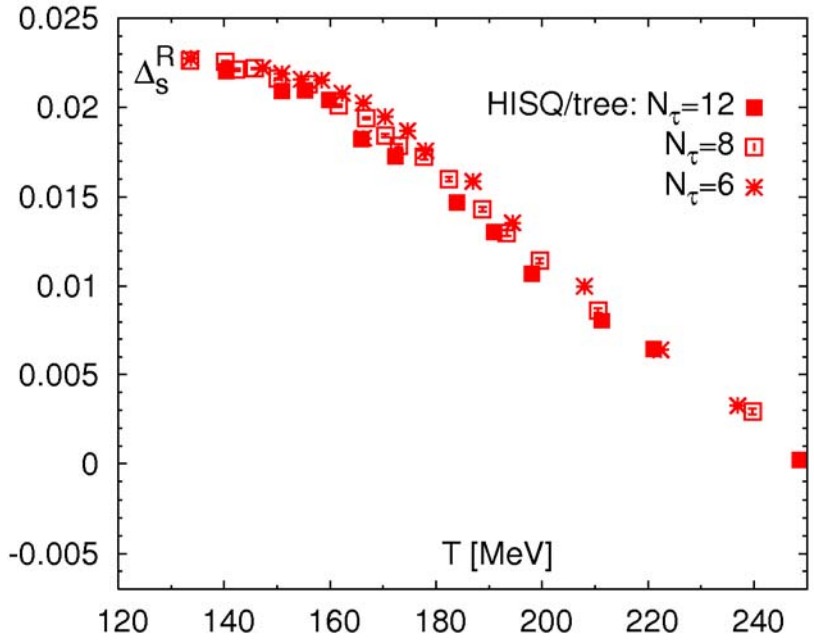
Renormalized chiral condensate introduced by Budapest-Wuppertal collaboration

$$\langle\bar{\psi}\psi\rangle_q \Rightarrow \Delta_q^R(T) = m_s r_1^4 \left(\langle\bar{\psi}\psi\rangle_{q,T} - \langle\bar{\psi}\psi\rangle_{q,T=0}\right) + d, \quad q = l, s$$

with our choice : $d = \langle\bar{\psi}\psi\rangle_{m_q=0}^{T=0}$

HotQCD : Phys. Rev. D85 (2012) 054503





• after extrapolation to the continuum limit and physical quark mass HISQ/tree calculation agree with stout results !

• strange quark condensate does not show a rapid change at the chiral crossover => strange quark do not play a role in the chiral transition

O(N) scaling and the chiral transition temperature

For sufficiently small m_l and in the vicinity of the transition temperature:

$$f(T,m_l) = -\frac{T}{V}\ln Z = f_{reg}(T,m_l) + f_s(t,h),\ t = \frac{1}{t_0}\left(\frac{T-T_c^0}{T_c^0} + \kappa\frac{\mu_q^2}{T^2}\right),\ H = \frac{m_l}{m_s}, h = \frac{H}{h_0}$$

governed by universal *O(4)* scaling $\quad M = -\frac{\partial f_s(t,h)}{\partial H} = h^{1/\delta} f_G(z),\ z = t/h^{1/\beta\delta}$

T_c^0 is critical temperature in the mass-less limit, h_0 and t_0 are scale parameters

Pseudo-critical temperatures for non-zero quark mass are defined as peaks in the response functions (susceptibilities) :

$$\chi_{m,l} = \frac{T}{V}\frac{\partial^2 \ln Z}{\partial m_l^2} \sim m_l^{1/\delta - 1} \qquad \chi_{t,l} = \frac{T}{V}\frac{\partial^2 \ln Z}{\partial m_l \partial t} \sim m_l^{\frac{\beta-1}{\beta\delta}} \qquad \chi_{t,t} = \frac{T}{V}\frac{\partial^2 \ln Z}{\partial t^2} \sim |t|^{-\alpha}$$

$$\downarrow \qquad\qquad \downarrow \qquad\qquad \downarrow$$

$$T_{m,l} = \qquad T_{t,l} = \qquad T_{t,t} = T_c^0$$

in the zero quark mass limit

$$\frac{\chi_{l,m}}{T^2} = \frac{T^2}{m_s^2}\left(\frac{1}{h_0}h^{1/\delta-1} f_\chi(z) + reg.\right)$$

universal scaling function has a peak at $z=z_p$

$$\Downarrow$$

$$T_c(H) = T_{m,l} = T_c^0 + T_c^0\frac{z_p}{z_0}H^{1/(\beta\delta)} + \ldots$$

Caveat : staggered fermions O(2)
$m_l \to 0$, $a > 0$,
proper limit $a \to 0$, before $m_l \to 0$

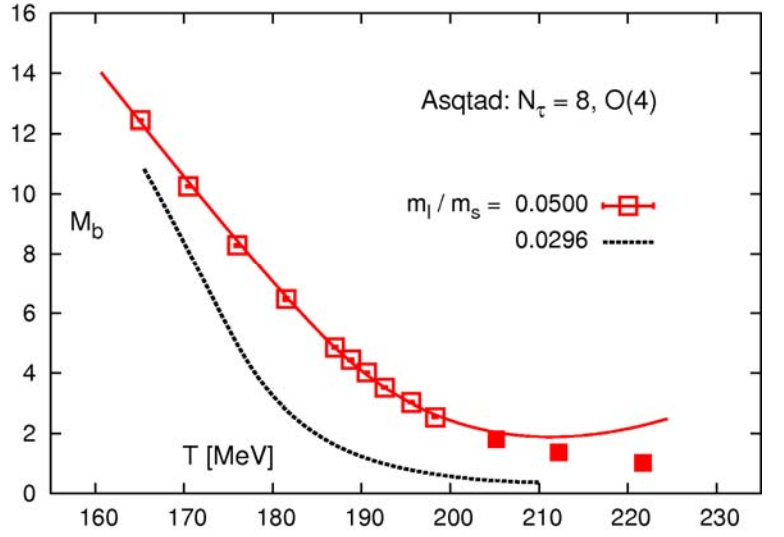
O(N) scaling and the transition temperature

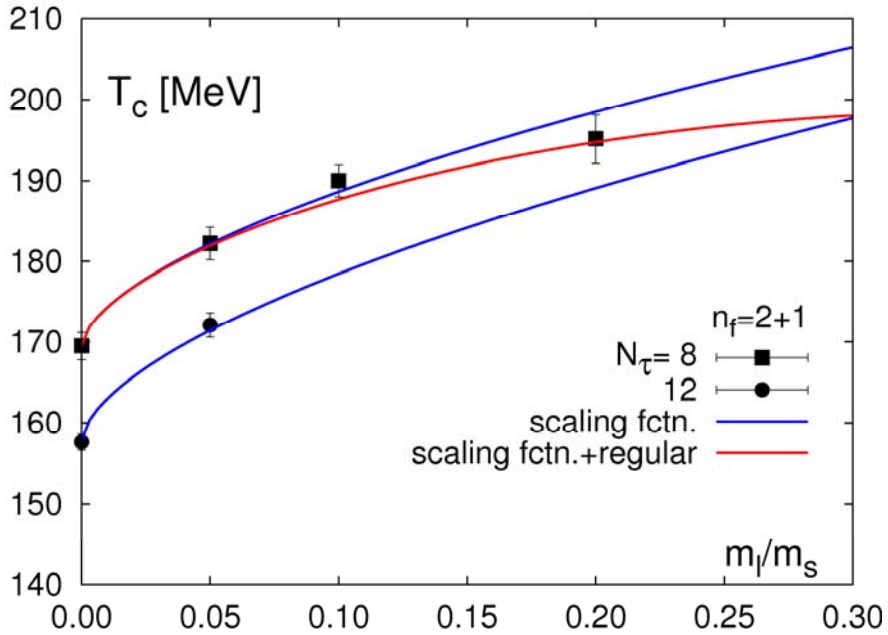
The notion of the transition temperature is only useful if it can be related to the critical temperature in the chiral limit : fit the lattice data on the chiral condensate with scaling form + simple Ansatz for the regular part

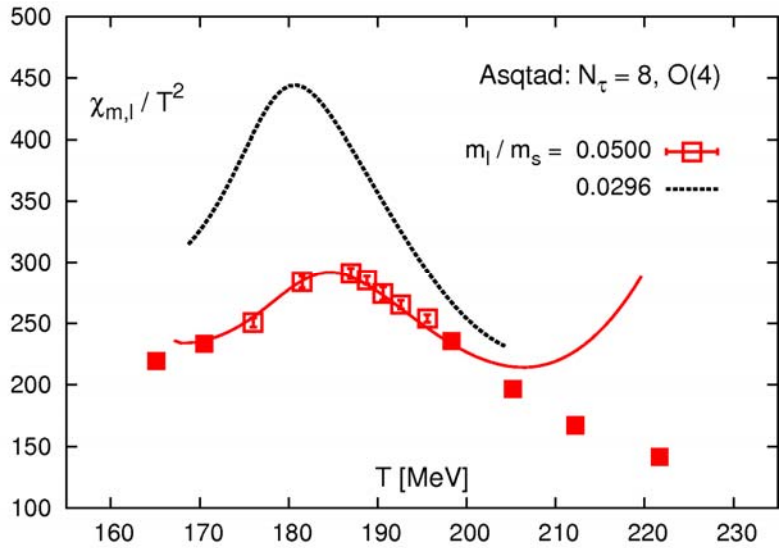
$$M_b = \frac{m_s \langle \bar{\psi}\psi \rangle_l}{T^4} = h^{1/\delta} f_G(t/h^{1/\beta\delta}) + f_{M,reg}(T,H)$$

$$f_{reg}(T,H) = (a_1(T-T_c^0) + a_2(T-T_c^0)^2 + b_1)H$$

6 parameter fit : T_c^0, t_0, h_0, a_1, a_2, b_1







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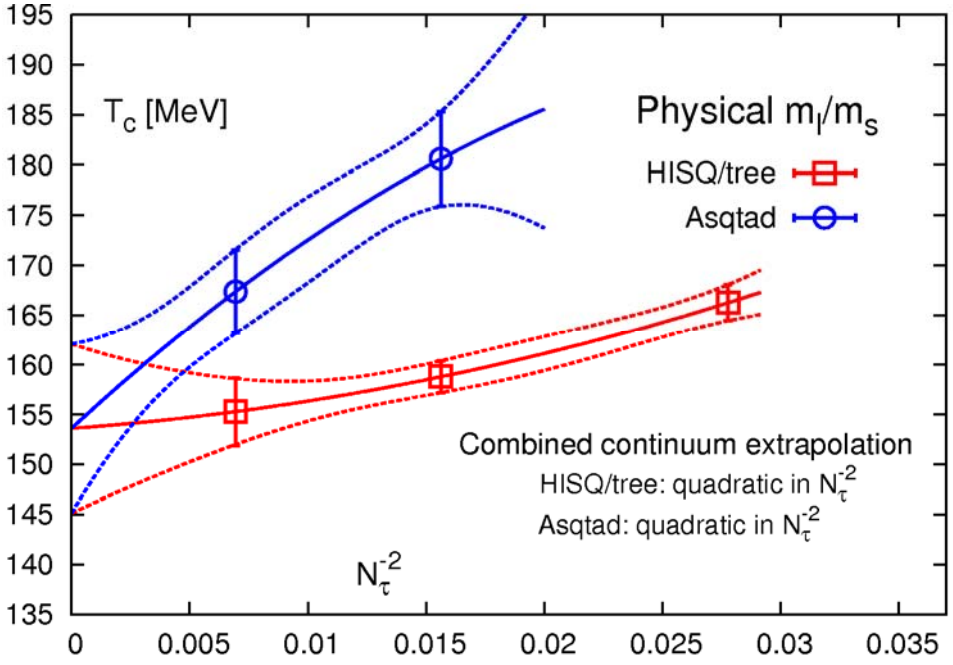
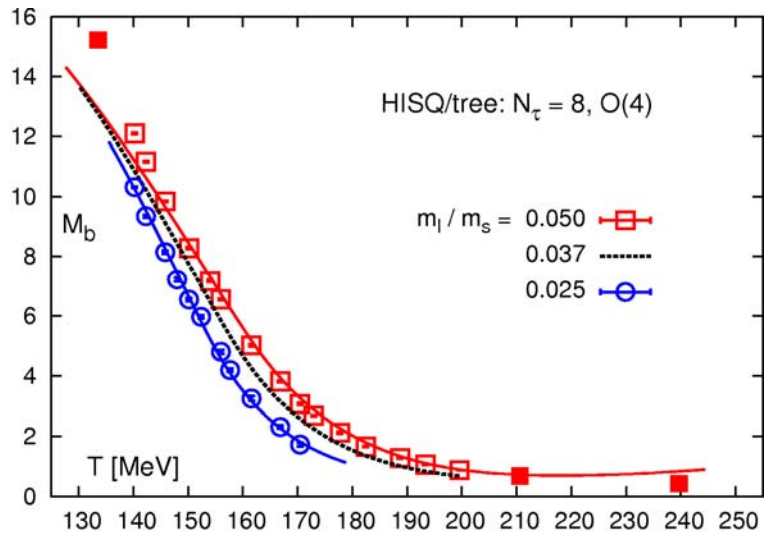
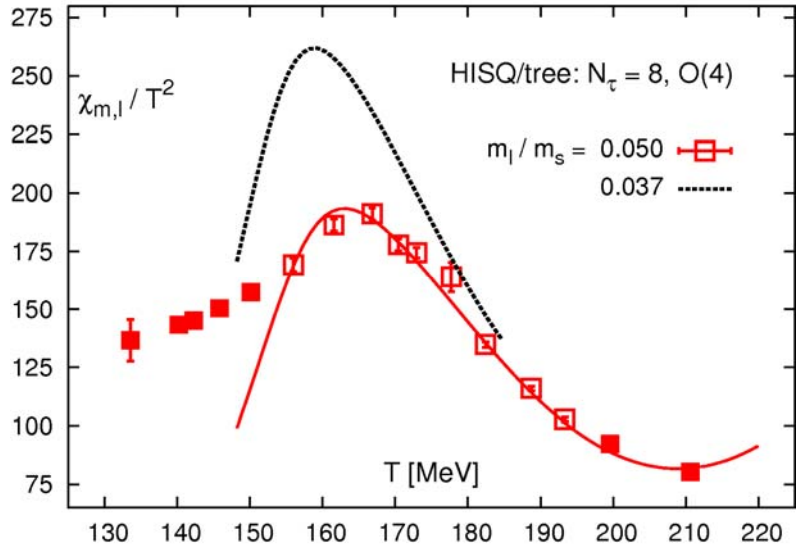
6 parameter fit : T_c^0, t_0, h_0, a_1, a_2, b_1

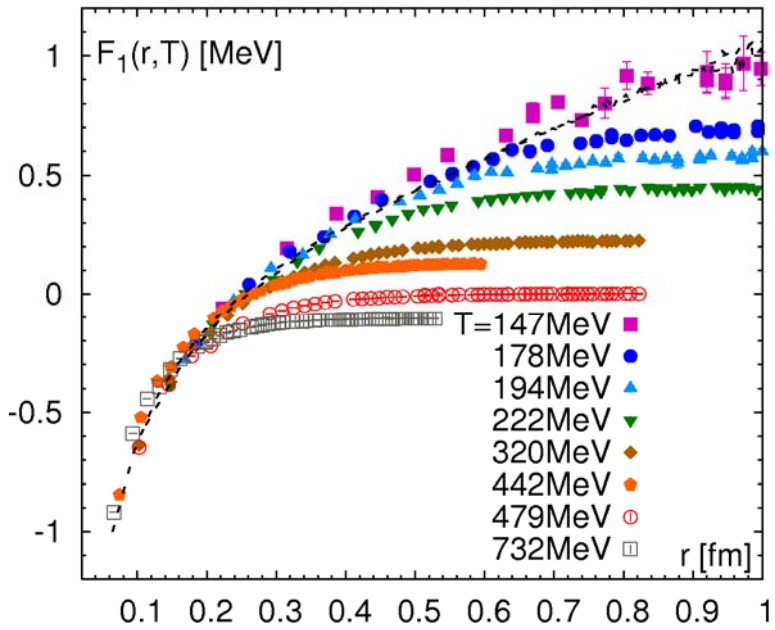
$$T_c = (154 \pm 8 \pm 1(scale))\mathrm{MeV}$$

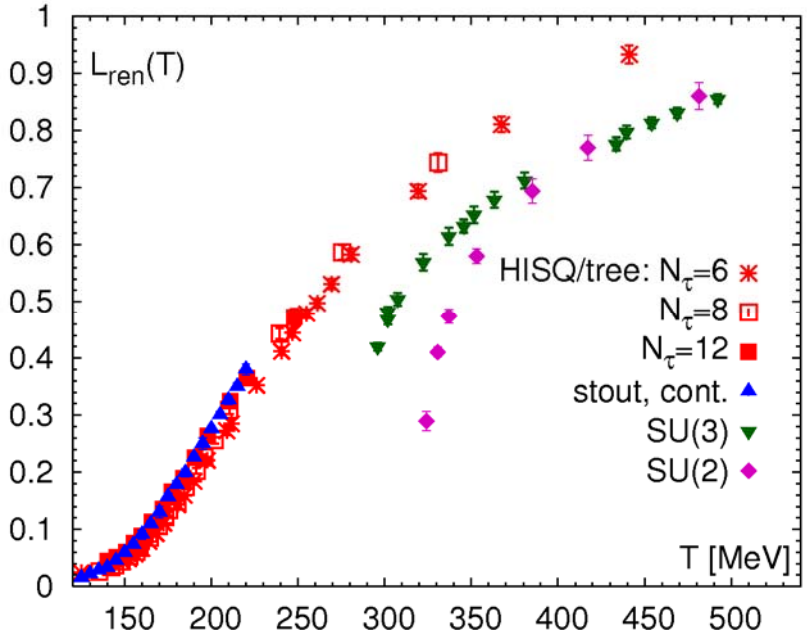






Deconfinement and color screening





free energy of static quark anti-quark pair shows Debye screening at high temperatures

Pure glue ≠ QCD !

$$L = \mathrm{tr}\mathcal{P}e^{ig\int_0^{1/T} d\tau A_0(\tau,\vec{x})} \Rightarrow L_{ren} = \exp(-F_Q(T)/T)$$

$$F_1(r) = -\frac{4}{3}\frac{\alpha_s}{r}\exp(-m_D r) + 2F_Q(T), \;\; m_D \sim T$$

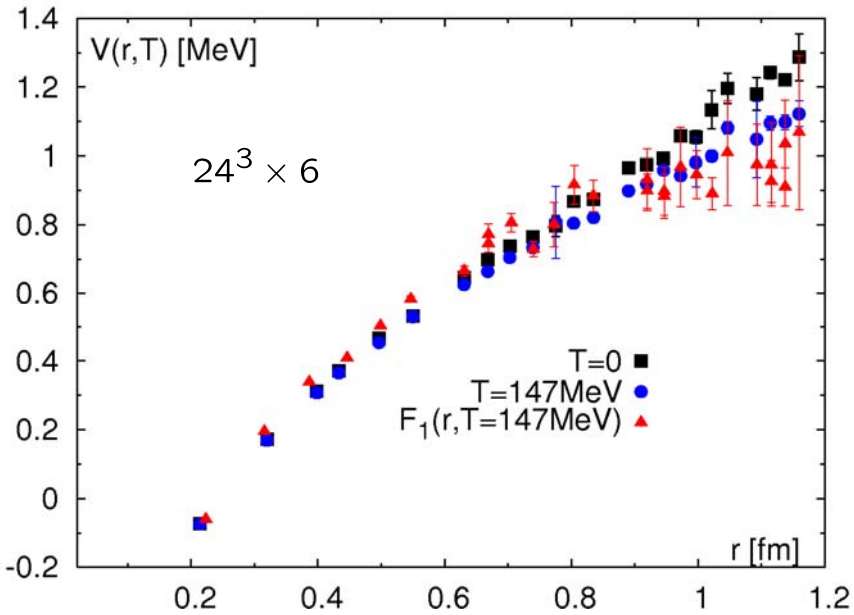
$$F_Q(T) \simeq \Lambda_{QCD} - C_F \alpha_s m_D$$

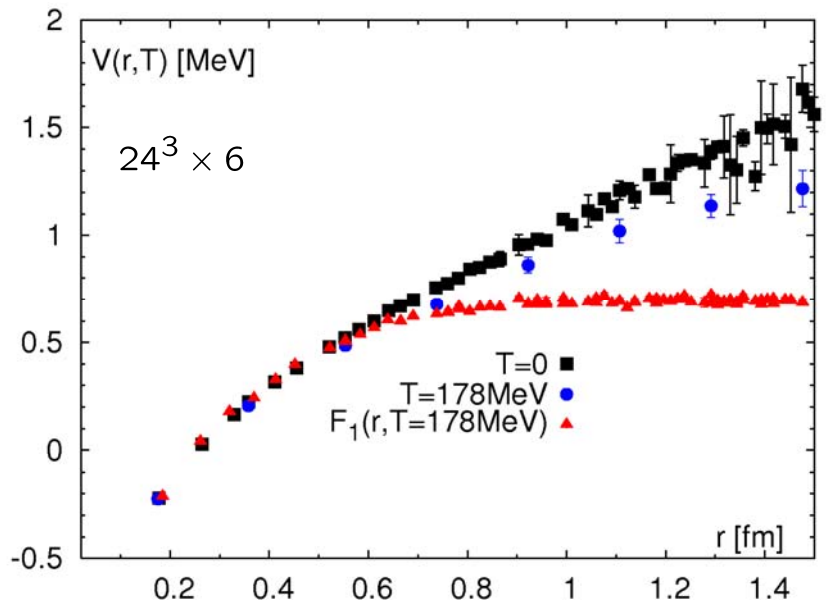
infinite in the pure glue theory or large in the “hadronic” phase ~600MeV

Decreases in the deconfined phase

melting of bound states of heavy quarks => quarkonium suppression at RHIC: $r_{bound} > 1/m_D$

Extracting the potential at T>0

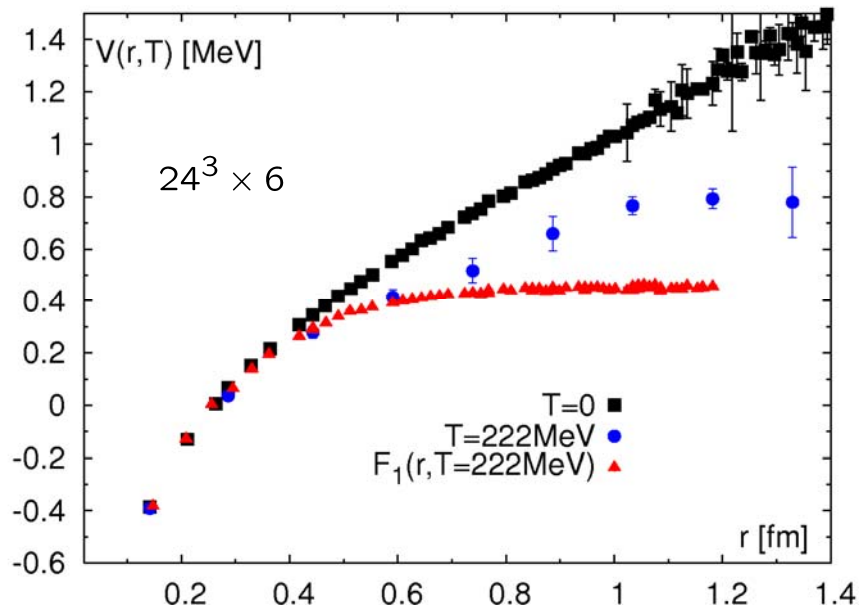




- Calculation of the Wilson loops at *T>0* + single state dominance => static quark potential *V(r,T)*

- for the lowest temperature the potential is the same as at *T=0* and agrees with the singlet free energy

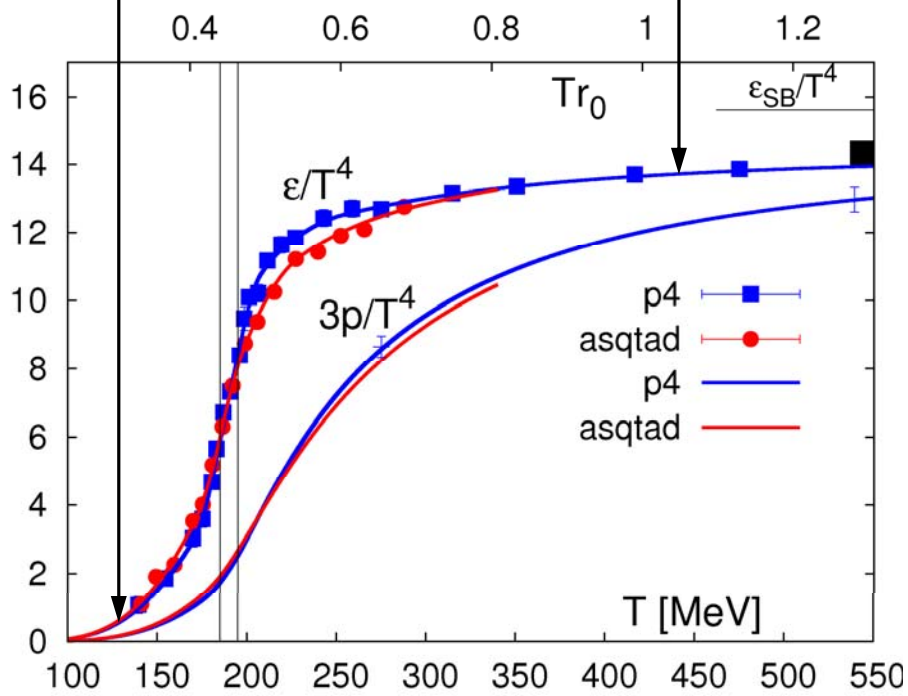
- for *150* MeV < *T* < *200* MeV the potential only slightly differs from the *T=0* potential and much larger than the singlet free energy, only for *T>200* MeV it is screened



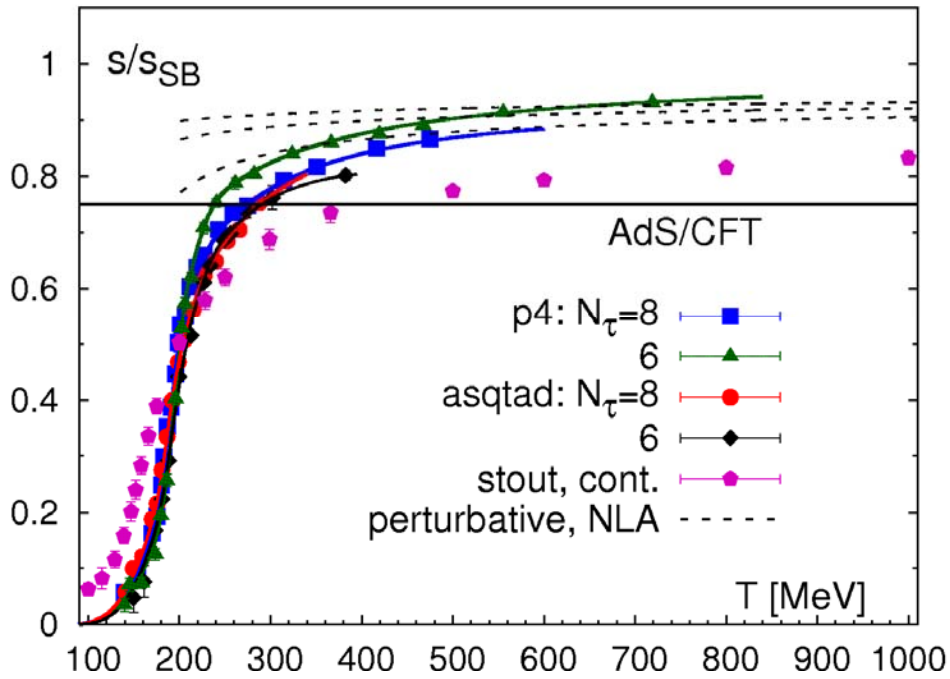
Equation of state

meson gas = 3 light d.o.f.

free gas of quarks and gluons = 18 quark+18 anti-quarks +16 gluons =52 mass-less d.o.f



Bazavov et al (HotQCD), PRD 80 (09) 14504



P.P. arXiv:1203.5320

- rapid change in the number of degrees of freedom at *T=160-200*MeV: deconfinement
- deviation from ideal gas limit is about 10-20% at high *T* consistent with the perturbative result
- discrepancies between stout and p4 (asqtad) calculations
- energy density at the chiral transition temperature $\varepsilon(T_c=154MeV)$=240 MeV/fm^3 :

Lattice results on trace anomaly

$$\frac{\Theta^{\mu\mu}(T)}{T^4} = \frac{\epsilon - 3p}{T^4} = R_\beta \left\{ \langle S_G \rangle_0 - \langle S_G \rangle_T \right\} - R_\beta R_m \left\{ 2m_q(\langle \bar{q}q \rangle_0 - \langle \bar{q}q \rangle) + m_s(\langle \bar{s}s \rangle_0 - \langle \bar{s}s \rangle_T) \right\}$$

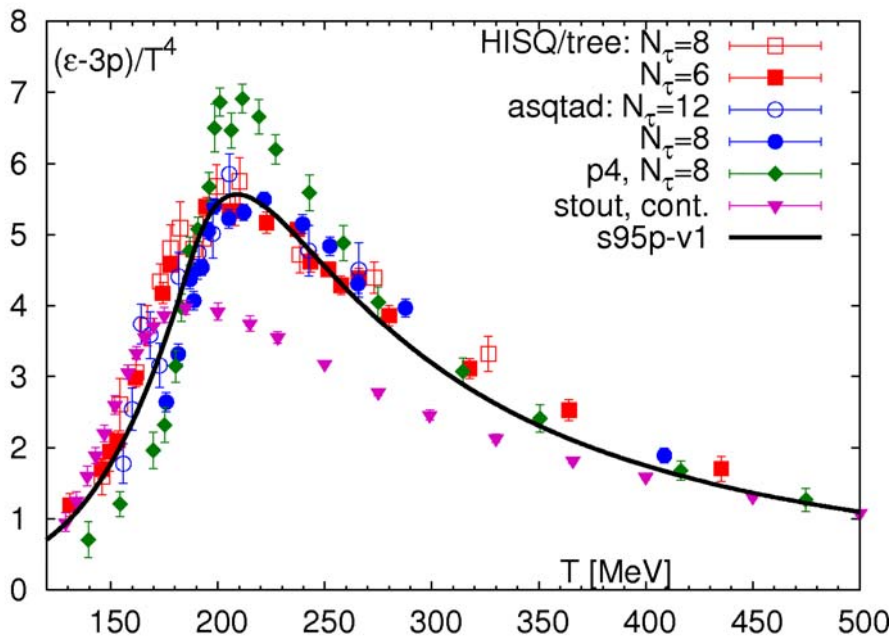
$$\frac{p(T)}{T^4} - \frac{p(T_0)}{T_0^4} = \int_{T_0}^{T} dT' \frac{\Theta^{\mu\mu}(T')}{T'^5}$$

$$s(T) = (\epsilon + p)/T$$

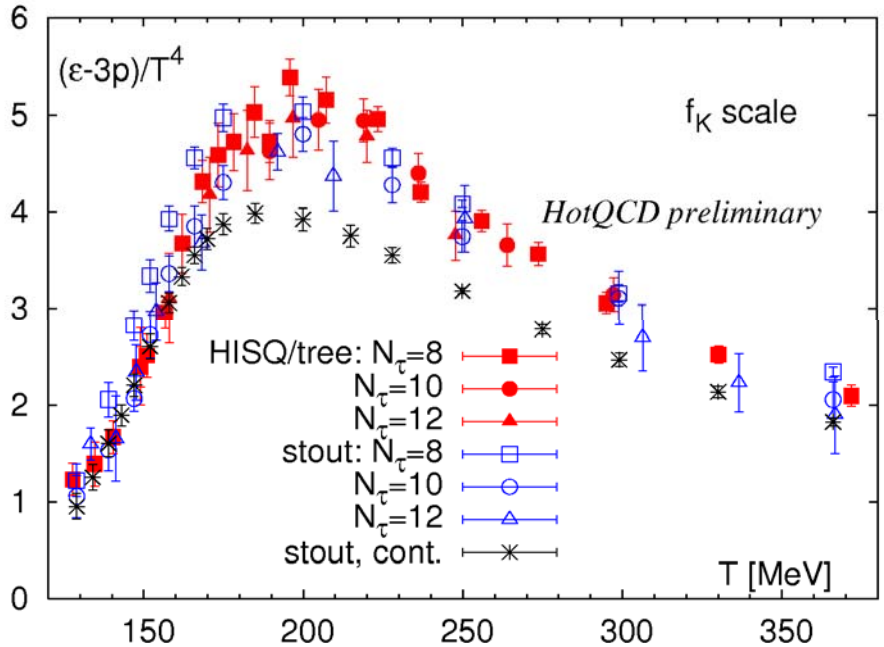
$$R_\beta(\beta) = -\frac{d\beta}{da}, \quad \beta = 6/g^2$$

$$R_m = \frac{1}{m_q(\beta)} \frac{dm_q(\beta)}{d\beta}$$

Bazavov, Lattice 2011, arXiv:1201.5345v1



PP, Lattice 2012



- HISQ and stout data are qualitatively similar but stout < HISQ
- No significant disagreement at high (*T>300* MeV) and low (*T<180* MeV) temperatures
- Subtleties in the continuum extrapolations for HISQ

QCD thermodynamics at non-zero chemical potential

Taylor expansion :

$$\frac{p(T,\mu_B,\mu_Q,\mu_S)}{T^4} = \sum_{i,j,k} \frac{1}{i!j!k!l!} \chi_{ijk}^{BQS} \cdot \left(\frac{\mu_B}{T}\right)^i \cdot \left(\frac{\mu_Q}{T}\right)^j \cdot \left(\frac{\mu_Q}{T}\right)^k$$ hadronic

$$\frac{p(T,\mu_u,\mu_d,\mu_s)}{T^4} = \sum_{i,j,k} \frac{1}{i!j!k!} \chi_{ijk}^{uds} \cdot \left(\frac{\mu_u}{T}\right)^i \cdot \left(\frac{\mu_d}{T}\right)^j \cdot \left(\frac{\mu_s}{T}\right)^k$$ quark

$$\chi_{ijk}^{abc} = T^{i+j+k} \frac{\partial^i}{\partial \mu_a^i} \frac{\partial^j}{\partial \mu_b^j} \frac{\partial^k}{\partial \mu_c^k} \frac{1}{VT^3} \ln Z(T,V,\mu_a,\mu_b,\mu_c)|_{\mu_a=\mu_b=\mu_c=0}$$

Taylor expansion coefficients give the fluctuations and correlations of conserved charges, e.g.

$$\chi_2^X = \chi_X = \frac{1}{VT^3}(\langle X^2\rangle - \langle X\rangle^2) \qquad \chi_{11}^{XY} = \frac{1}{VT^3}(\langle XY\rangle - \langle X\rangle\langle Y\rangle)$$

Computation of Taylor expansion coefficients reduces to calculating the product of inverse fermion matrix with different source vectors **=> can be done effectively on GPUs**

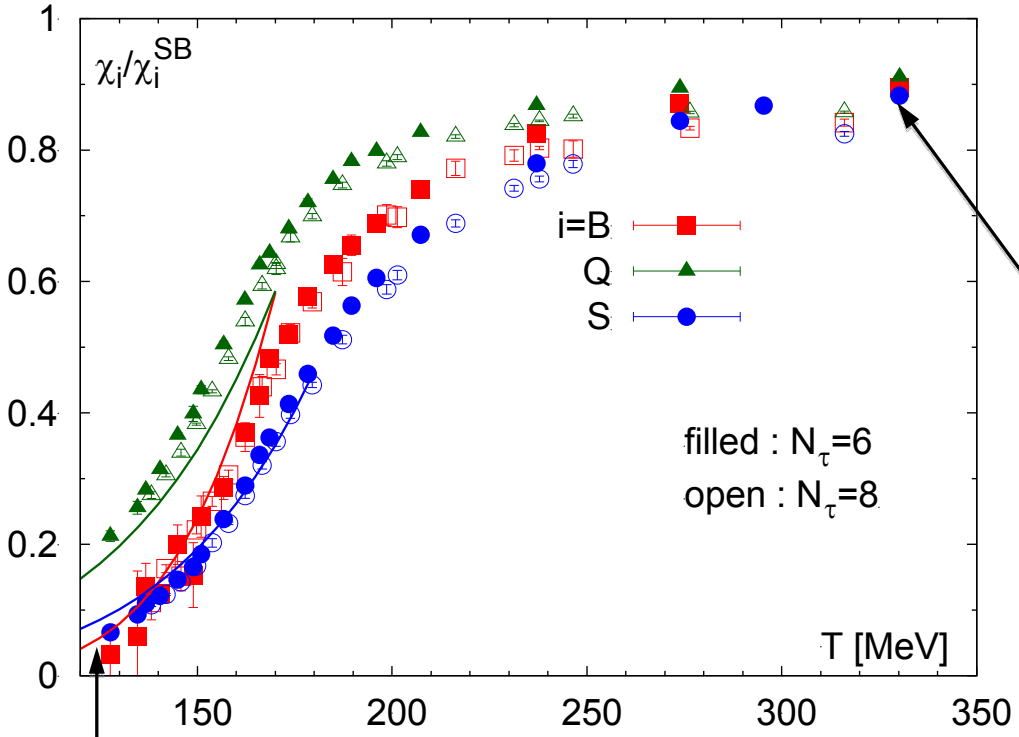
BNL-Bielefeld talk by Wagner tomorrow

Deconfinement : fluctuations of conserved charges

$$\chi_B^{SB} = \frac{1}{VT^3}(\langle B^2\rangle - \langle B\rangle^2)$$ baryon number

$$\chi_Q^{SB} = \frac{1}{VT^3}(\langle Q^2\rangle - \langle Q\rangle^2)$$ electric charge

$$\chi_S^{SB} = \frac{1}{VT^3}(\langle S^2\rangle - \langle S\rangle^2)$$ strangeness



conserved charges are carried by massive hadrons

Ideal gas of massless quarks :

$$\chi_B^{SB} = \frac{1}{3} \qquad \chi_Q^{SB} = \frac{2}{3}$$

$$\chi_B^{SB} = 1$$

conserved charges carried by light quarks

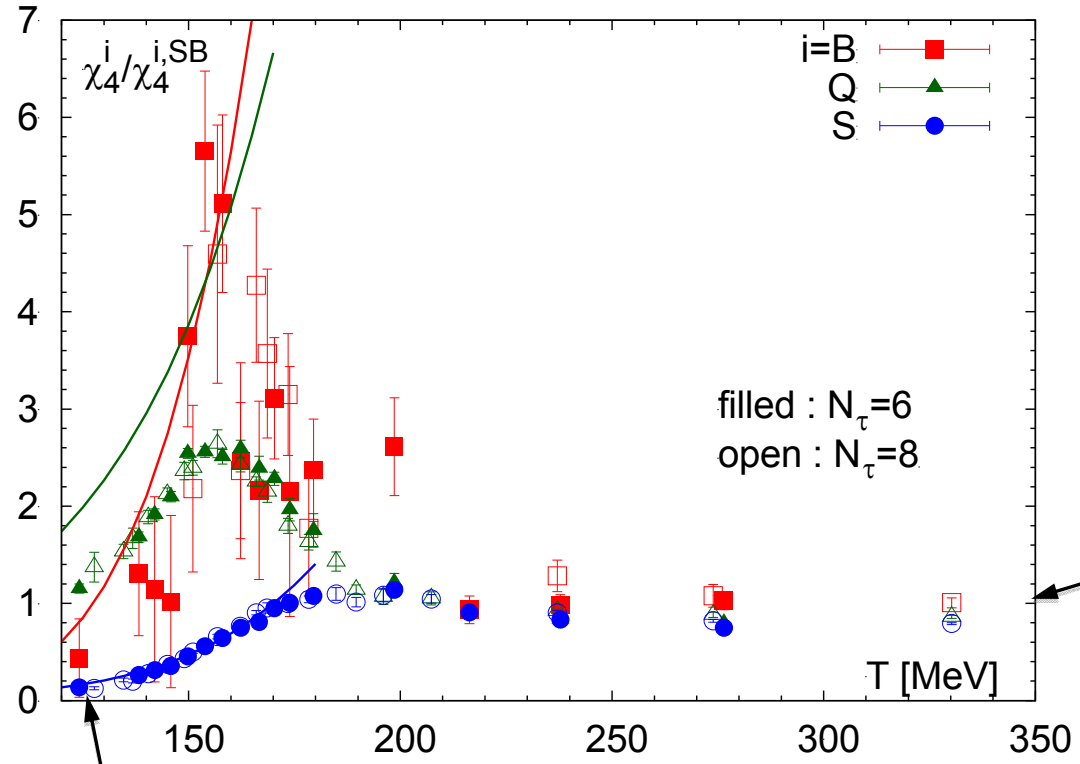
HotQCD: arXiv:1203.0784

Deconfinement : fluctuations of conserved charges

$$\chi_4^B = \frac{1}{VT^3}(\langle B^4\rangle - 3\langle B^2\rangle^2)$$ baryon number

$$\chi_4^Q = \frac{1}{VT^3}(\langle Q^4\rangle - 3\langle Q^2\rangle^2)$$ electric charge

$$\chi_4^S = \frac{1}{VT^3}(\langle S^4\rangle - 3\langle S^2\rangle^2)$$ strangeness



Ideal gas of massless quarks :

$$\chi_4^B{}_{SB} = \frac{2}{9\pi^2} \qquad \chi_4^Q{}_{SB} = \frac{4}{3\pi^2}$$

$$\chi_4^S{}_{SB} = \frac{6}{\pi^2}$$

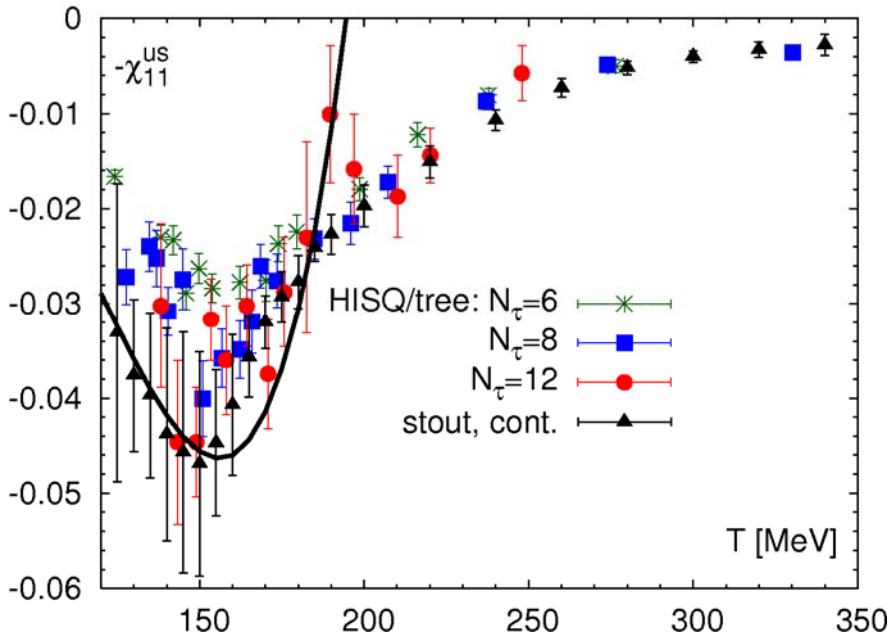
conserved charges carried by light quarks

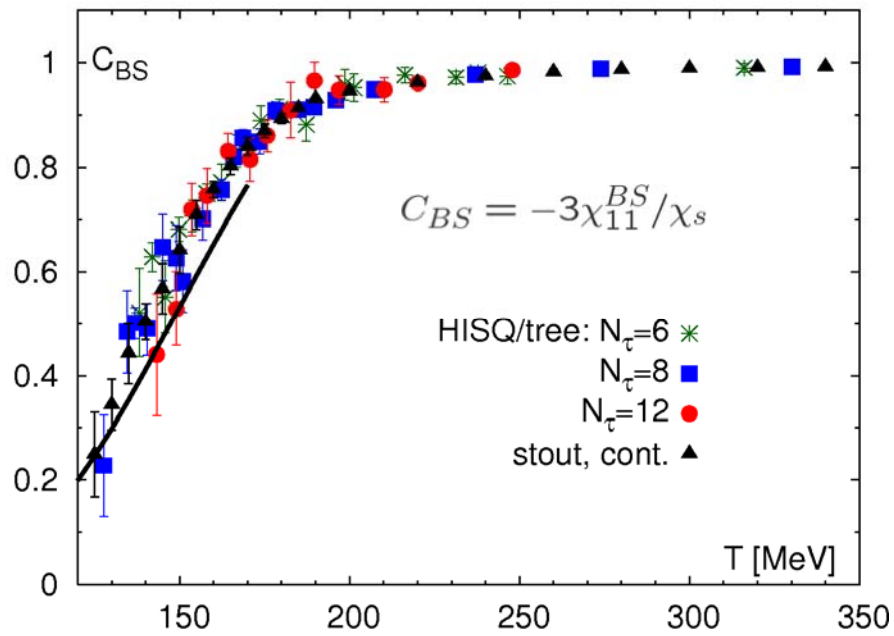
conserved charges are carried by massive hadrons

BNL-Bielefeld :
talk by Wagner tomorrow

Large cutoff effects for the 4th order charge fluctuations ?

Correlations of conserved charges





P.P. arXiv:1203.5320

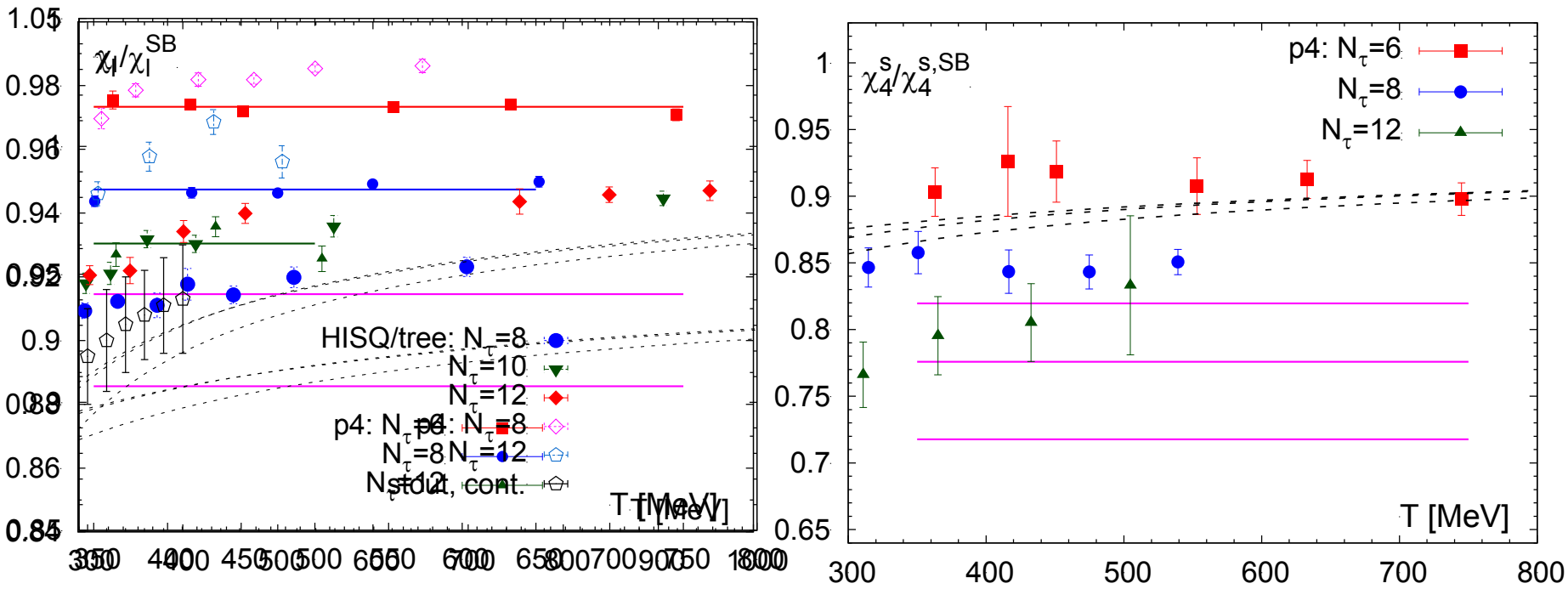
- Correlations between strange and light quarks at low *T* are due to the fact that strange hadrons contain both strange and light quarks but very small at high *T (>250* MeV*)*
=> weakly interacting quark gas

- For baryon-strangeness correlations HISQ results are close to the physical HRG result, at *T>250* MeV these correlations are very close to the ideal gas value

- The transition region where degrees of freedom change from hadronic to quark-like is broad ~ 50 MeV

Quark number fluctuations at high T

At high temperatures quark number fluctuations can be described by weak coupling approach due to asymptotic freedom of QCD



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Dashed lines : Weak coupling calculations based on EQCD (Vuorinen et al)

- Lattice results converge as the continuum limit is approached
- Good agreement between lattice and the weak coupling approach for 2nd order quark number fluctuations
- For 4th order the weak coupling result is above the continuum estimate from lattice

Fluctuations and freeze-out conditions in RHIC

LQCD

$$\chi_1^X = \frac{1}{VT^3}\langle N_X\rangle$$

$$\chi_2^X = \frac{1}{VT^3}\langle(\delta N_X)^2\rangle$$

$$\chi_3^X = \frac{1}{VT^3}\langle(\delta N_X)^3\rangle$$

Even-by-event fluctuations at RHIC

$$M_X = \langle N_X\rangle$$ mean

$$\sigma_X^2 = \langle(\delta N_X)^2\rangle$$ variance

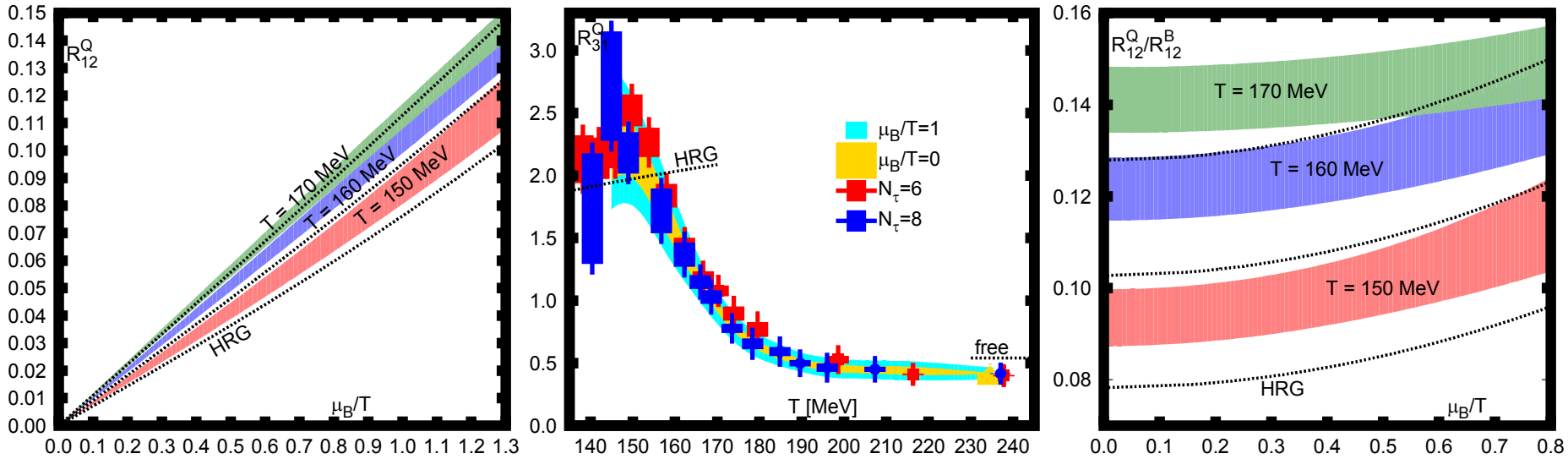
$$S_X = \langle(\delta N_X)^3\rangle/\sigma_X^3$$ skew-ness

$$N_X = X - \bar{X}, \quad \delta N_X = N_X - \langle N_X\rangle$$

The link :
Volume indep. ratios

$$R_{12}^X = M_X/\sigma_X^2$$

$$R_{31}^X = S_X \cdot \sigma_X^3/M_X$$



BNL-Bielefeld arXiv:1208.1220

Matching the ratios to the experiment freeze-out parameters T^f and μ_B^f can be determined

Summary

• Lattice QCD show that at high temperatures strongly interacting matter undergoes a transition to a new state QGP characterized by deconfinement and chiral symmetry restoration

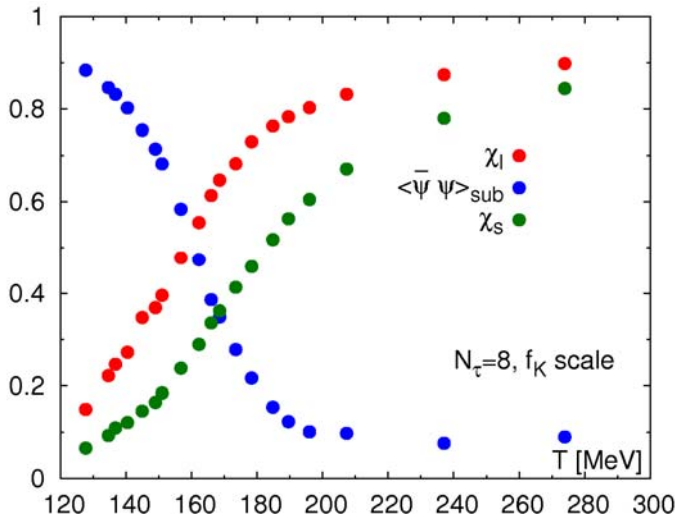
• We see evidence that provide evidence that the relevant degrees of freedom are quarks and gluons; lattice results agree well with perturbative calculations, while at low T thermodynamics can be understood in terms of hadron resonance gas The deconfinement transition can understood as transition from hadron resonance gas to quark gluon gas it is gradual and analagous to ionized gas – plasma transition (implications for sQGP and early thermalization at RHIC ?)

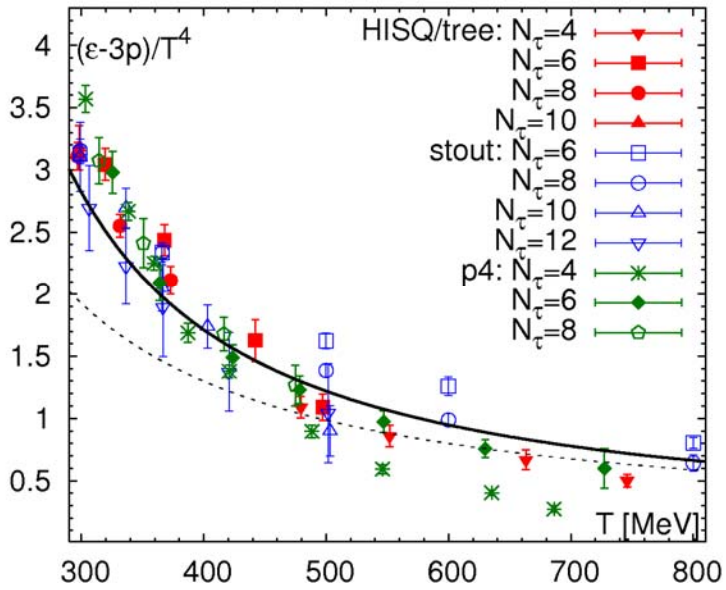
• The chiral aspects of the transition are very similar to the transition in spin system in external magnetic fields: it is governed by universal scaling. Different calculations with improved staggered actions agree in the continuum limit resulting in a chiral transition temperature (154 ± 9) MeV

•Color screening can be seen at temperatures $T>200$ MeV

•There are unresolved discrepancies in the EoS calculations that are related to differences in the trace anomaly for 200 MeV $< T < 300$ MeV
=> reliable continuum extrapolations for HISQ

Back-up:





Deconfinement transition in terms of light quark number fluctuations coincides with
the chiral transition => no
deconfinement transition temperature can be defined