Heavy Mesonic Spectral Functions at Finite Temperature and Finite Momentum

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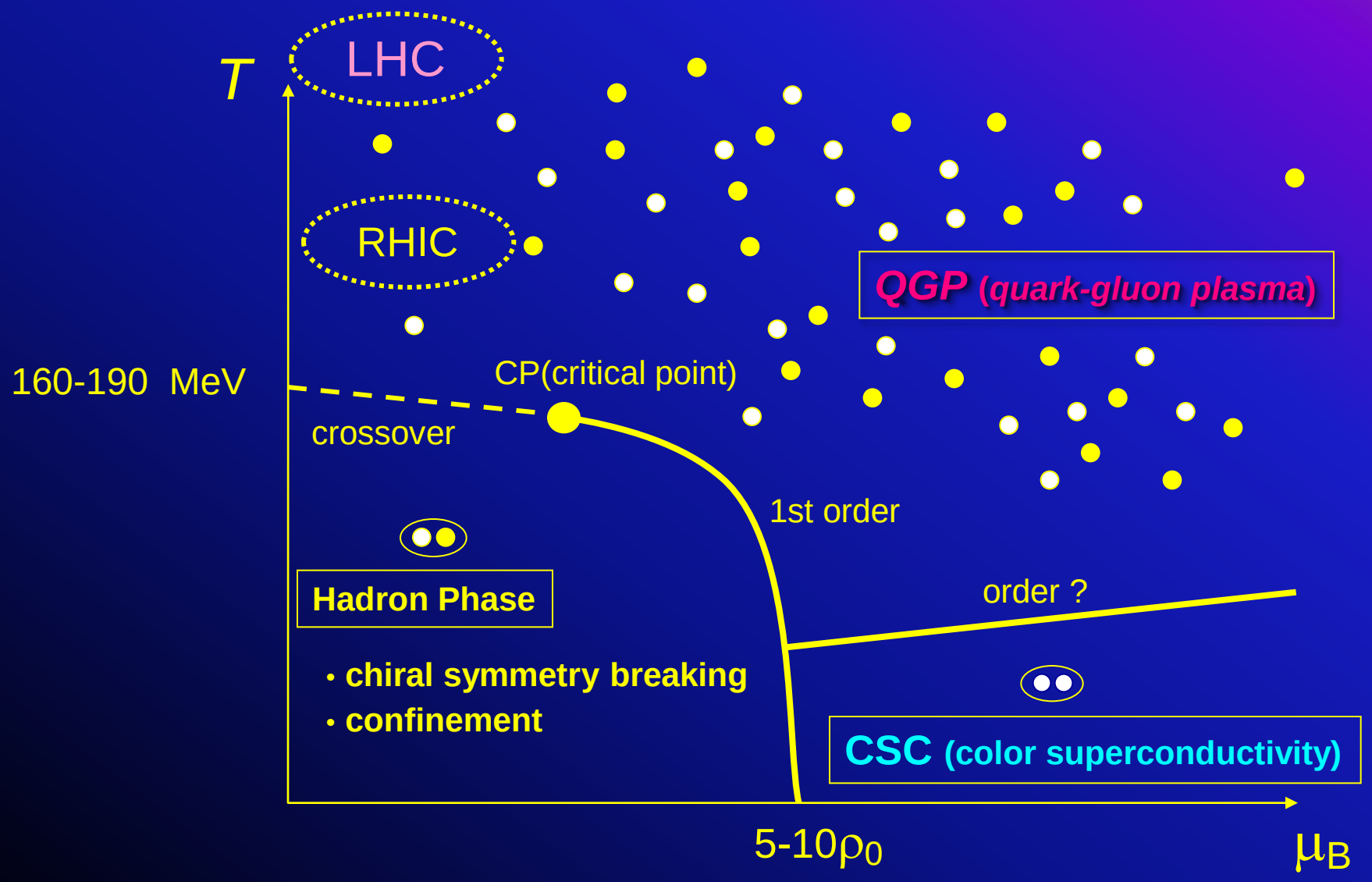
@ EMMI Workshop

QCD Phase Diagram



SPF @Finite Momentum

- At finite momentum,
  - S and PS channels:

$$\rho(\omega, \vec{p})$$

  - V and AV channels:

$$\rho_{\mu\nu}(\omega, \vec{p}) = \rho_T(\omega, \vec{p})(P_T)_{\mu\nu} + \rho_L(\omega, \vec{p})(P_L)_{\mu\nu}$$

P_T (P_L): transverse (longitudinal) projection operator

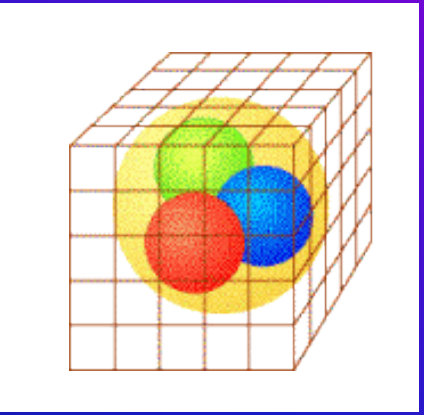
- Hadrons @ finite temperature and finite momentum,
  - potential model ?
  - collisions with gluons and quarks (*hot wind* effect)

    $\rightarrow$ p_T dependence of J/ψ yield etc.
  - dispersion relation
  - difference between ρ_T and ρ_L
  - dilepton spectra (ρ_T and ρ_L) and photon spectra (ρ_T, on-shell)

1st Principle Calculation of SPF

- What's measured on the Lattice is Imaginary Time Correlation Function $D(\tau)$

$$D(\tau) = \int \left\langle O(\tau, \vec{x})\, O'^{\dagger}(0, \vec{0}) \right\rangle d^3x$$



$D(\tau)$ and $A(\omega) \equiv A(\omega, \vec{0})$ are related by

$$D(\tau) = \int_0^{\infty} K(\tau, \omega) A(\omega)\, d\omega$$

$K(\tau,\omega)$: Known Kernel

However,

- Measured in Imaginary Time
- Measured at a Finite Number of discrete points
- Noisy Data ⟵ Monte Carlo Method

χ^2-fitting : inconclusive !

Direct Inversion: *ill-posed !*

Similar Difficulties in Many Areas

- Lattice

  - Analytic Continuation to Imaginary Time is measured
  - Measured at a Finite Number of discrete points
  - *Noisy* Data

- X-ray Diffraction Measurement in Crystallography

  - Fourier Transformed images are measured
  - Measured at a Finite Number of data points
  - *Noisy* Data

- Observational Astronomy

  - Smeared Images due to Finite Resolution are measured
  - Measured by a Finite Number of Pixels
  - *Noisy* Data

MEM

- *M*aximum *E*ntropy *M*ethod
  - a method to infer the most statistically probable image (such as A(ω)) given data, instead of solving the (ill-posed) inversion problem

$$D(\tau) = \int_0^\infty K(\tau, \omega) A(\omega)\, d\omega$$

  - Theoretical Basis: Bayes' Theorem

$$P[X|Y] = \frac{P[Y|X]P[X]}{P[Y]}$$

$P[X|Y]$: Probability of X given Y

  - In Lattice QCD

D: Lattice Data (Average, Variance, Correlation…etc.)

H: All definitions and *prior knowledge* such as $A(\omega) \geq 0$

Bayes' Theorem → $P[A|DH] \propto P[D|AH]P[A|H]$

Ingredients of MEM

- $P[D|AH]$

  $= \chi^2$-likelihood function

  $P[D|AH] = \exp(-L)/Z_L$

- $P[A|H]$

  given by Shannon-Jaynes Entropy

$$P[A|H\alpha m] = \frac{\exp(\alpha S)}{Z_S}$$

$$S = \int \left[A(\omega) - m(\omega) - A(\omega)\log\left(\frac{A(\omega)}{m(\omega)}\right)\right] d\omega$$

$$Z_S = \int e^{\alpha S}[dA], \quad \alpha \in \mathbf{R}$$

max at $A(\omega) = m(\omega)$

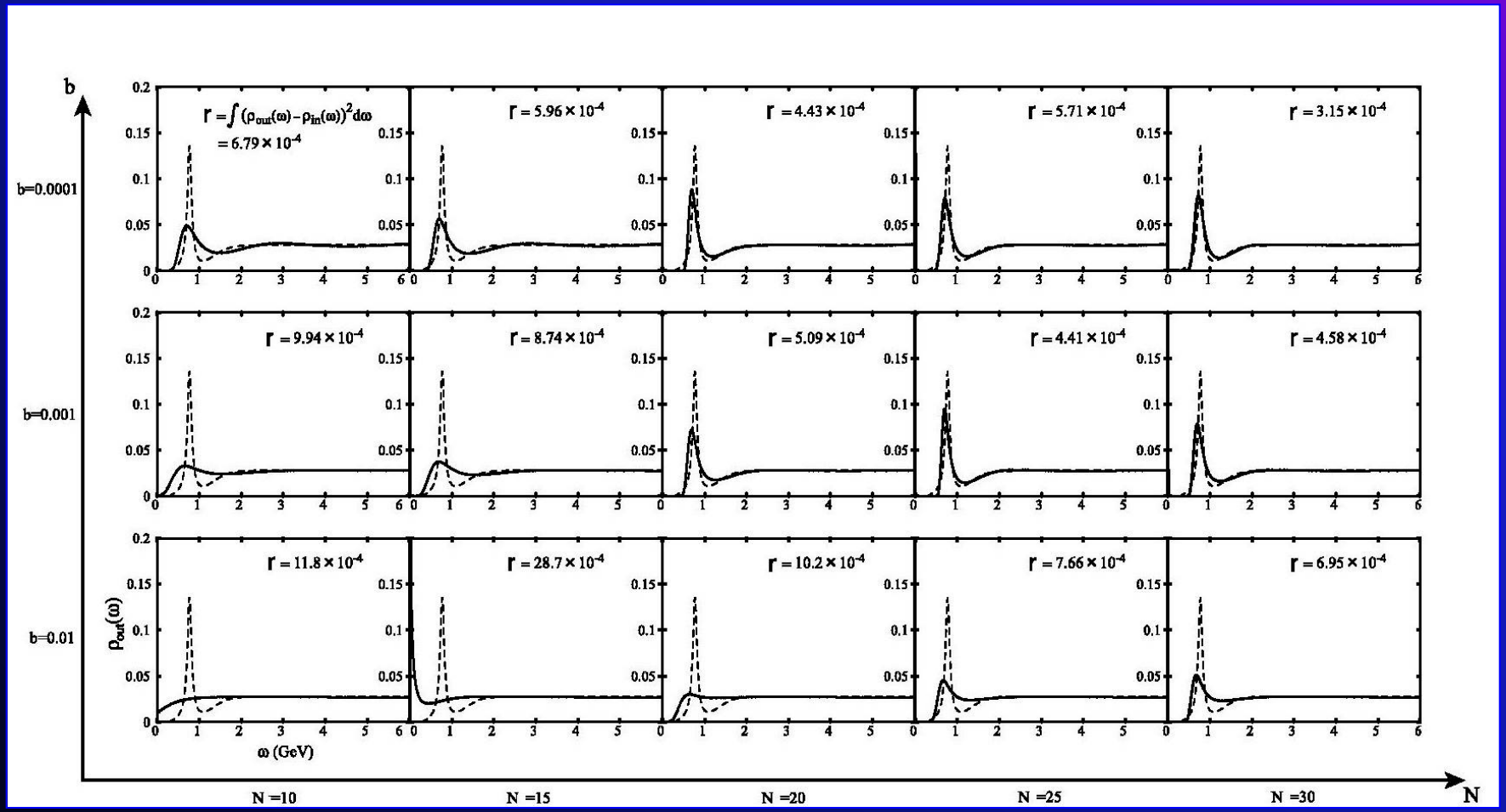
Default Model $m(\omega) \in \mathbf{R}$: Prior knowledge about $A(\omega)$

such as semi-positivity,
perturbative asymptotic value, operator renormalization etc.

For further details,
Y. Nakahara, and T. Hatsuda, and M. A., Prog. Part. Nucl. Phys. 46 (2001) 459

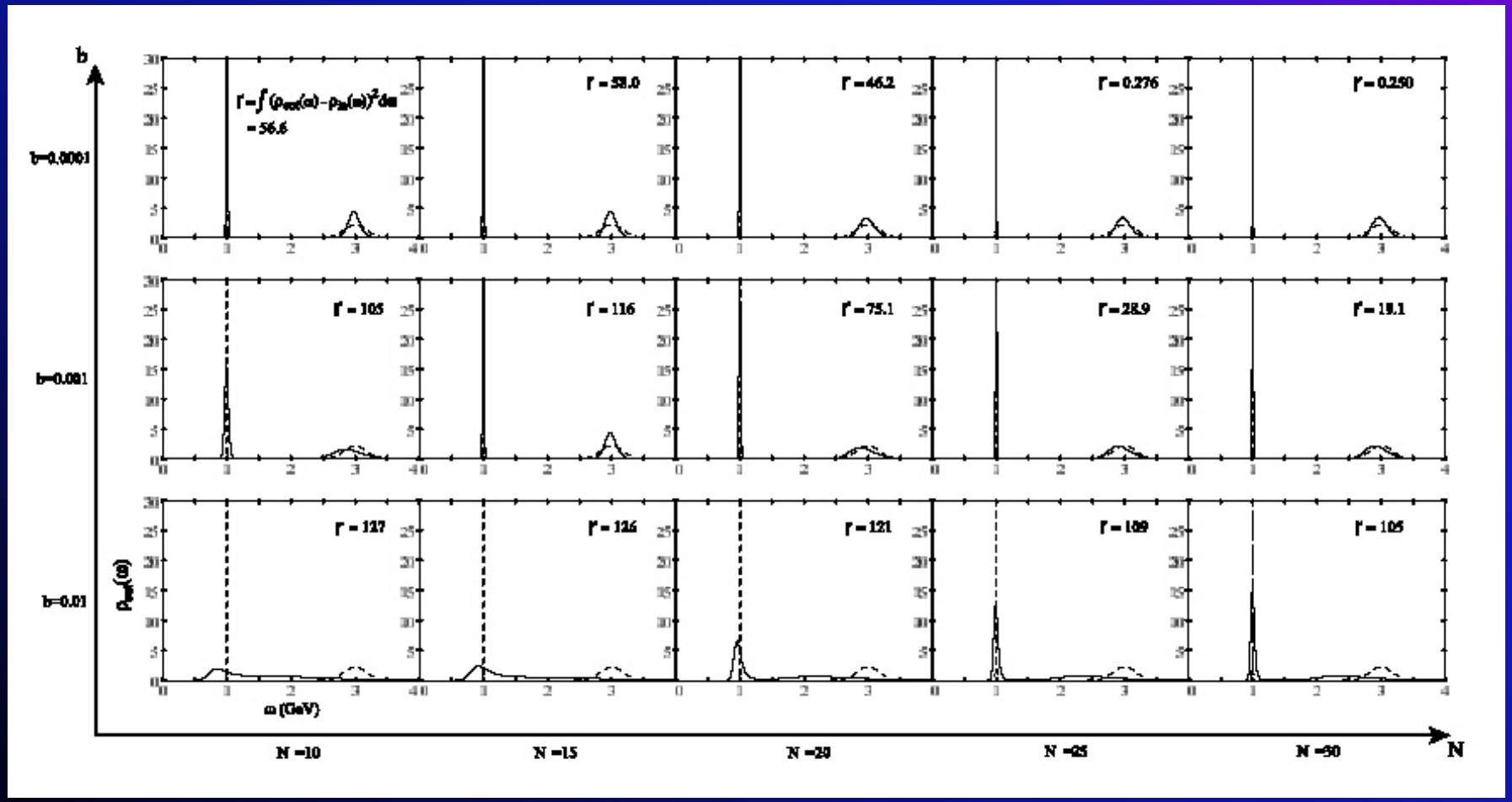
Result of Mock Data Analysis

N(# of data points)-b(noise level) dependence



Result of Mock Data Analysis (cont'd)

N(# of data points)-b(noise level) dependence



Stat. and Syst. Error Analyses in MEM

Generally,

The *Larger the Number of Data Points* and the *Lower the Noise Level* → *The Closer the Result is to the Original Image*

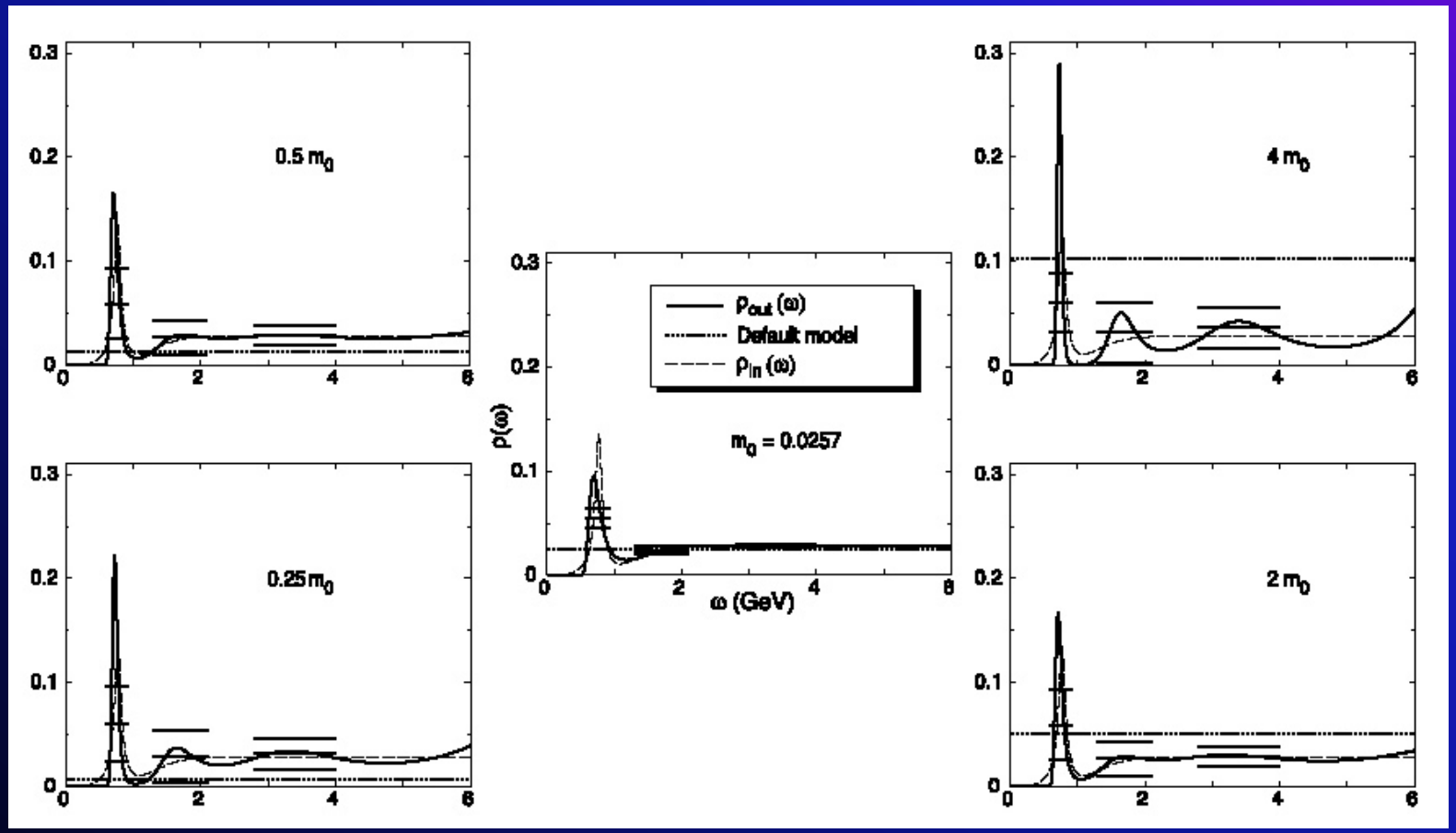
Need to do the following:

- Put Error Bars and
  Make Sure Observed Structures are Statistically Significant
  → Statistical
- Change the Number of Data Points and
  Make Sure the Result does not Change
  → Systematic

in any MEM analysis

Importance of Error Analysis 1

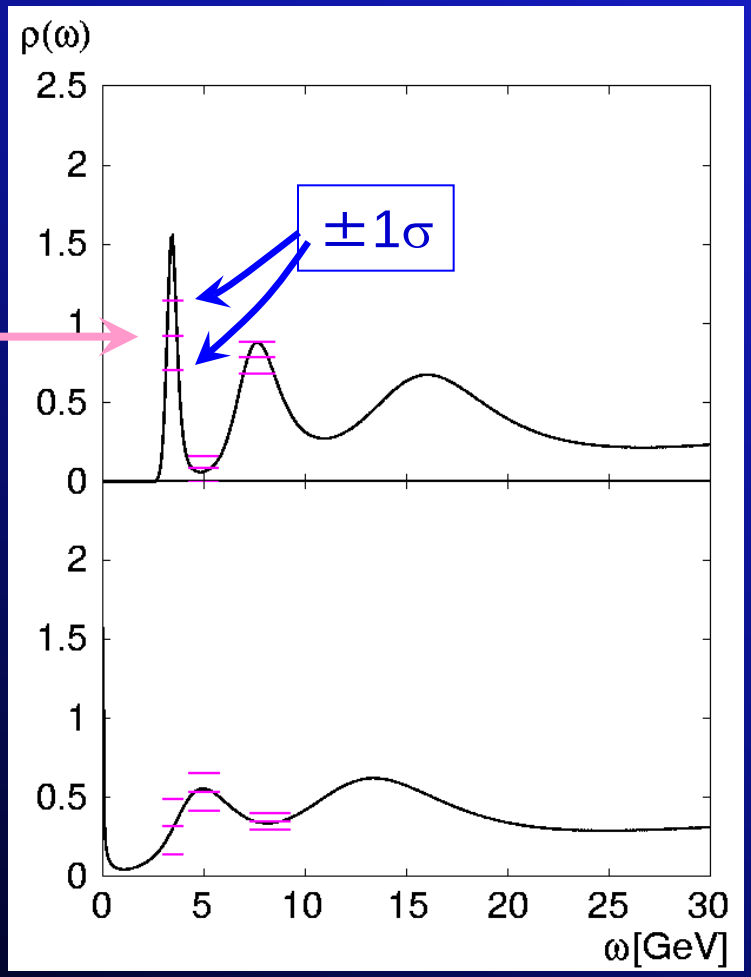
- In MEM analysis, output image itself has little significance
- Only average is of significance



Unfortunately, still many people do MEM analyses without error bars

Importance of Error Analysis 2

From our old paper



Ave.

T = 1.62T_c

T = 1.70T_c

V channel (J/ψ)
N_σ = 32

Error Analysis in MEM (Statistical)

- MEM is based on Bayesian Probability Theory

  - In MEM, *Errors can be and must be assigned*
  - This procedure is *essential* in MEM Analysis

- For example, Error Bars can be put to

Average of Spectral Function in $I=[\omega_1,\omega_2]$, $\langle A_\alpha \rangle_I = \frac{1}{\omega_2-\omega_1}\int_{\omega_1}^{\omega_2} A_\alpha(\omega)d\omega$

$$\left\langle (\delta A_\alpha)^2 \right\rangle_I = \frac{1}{(\omega_2-\omega_1)^2}\int [dA]\int_{I\times I} d\omega d\omega' \delta A(\omega)\delta A(\omega') P[A\,|\,DH\alpha m]$$

$$\simeq -\frac{1}{(\omega_2-\omega_1)^2}\int_{I\times I} d\omega d\omega' \left(\frac{\delta^2 Q(A)}{\delta A(\omega)\delta A(\omega')}\right)^{-1}_{A=A_\alpha}$$

$$\delta A(\omega) = A(\omega) - A_\alpha(\omega)$$

$$Q(A) = \alpha S - L$$

$$[dA] = \prod_{l=1}^{N_\omega} \frac{dA_l}{\sqrt{A_l}}$$

Gaussian approximation

For further details,
Y. Nakahara, and T. Hatsuda, and M. A., Prog. Part. Nucl. Phys. 46 (2001) 459

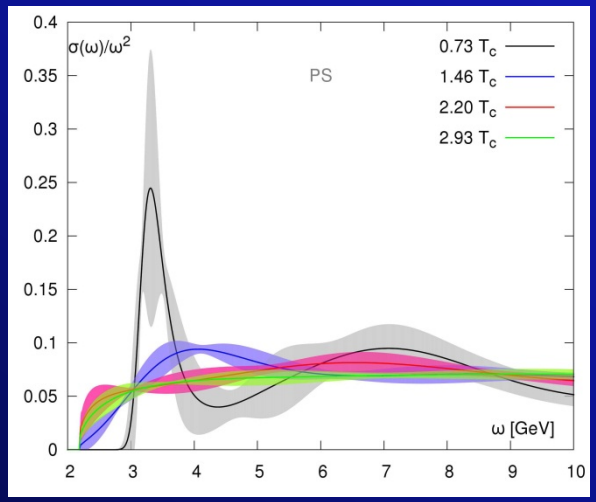
Statistical Error Analysis ≠ Jackknife

Lattice fluctuation: included in χ^2, i.e. $P[D|AH]$

On the other hand, MEM calculates $P[A|DH] \propto P[D|AH]P[A|H]$

➡ Statistical error analysis in MEM includes more than Lattice fluctuation

- Jackknife Method misses $P[A|H]$ part



Statistical Error with Jackknife Method

Ding et al. (2010)

Stat. and Syst. Error Analyses in MEM

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Need to do the following:

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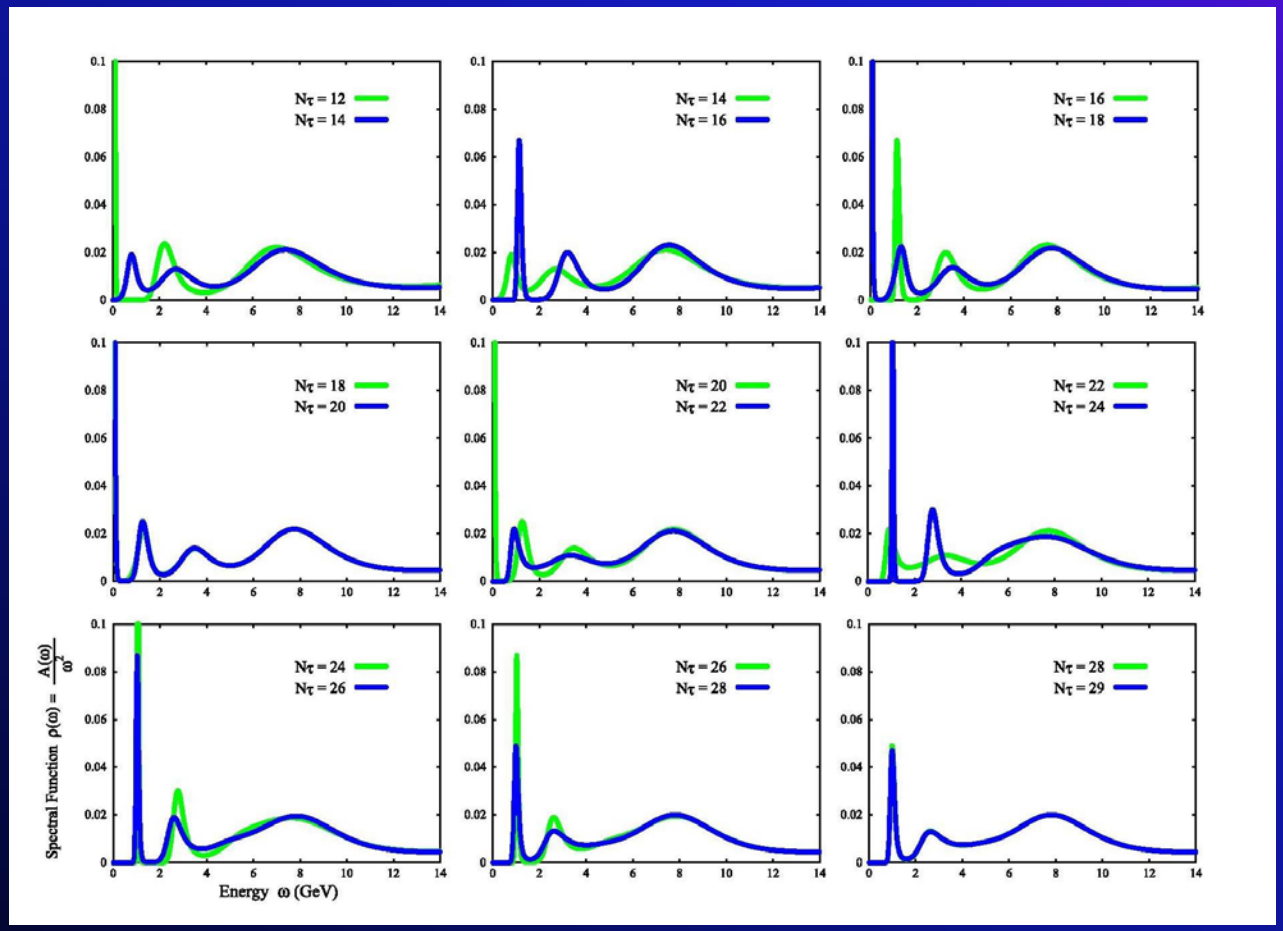
in any MEM analysis

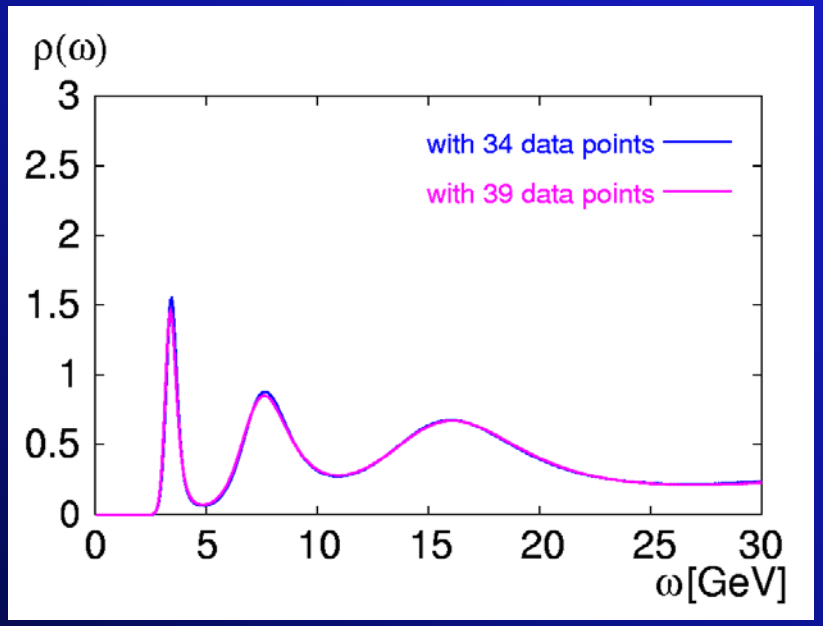
Importance of Error Analysis 3

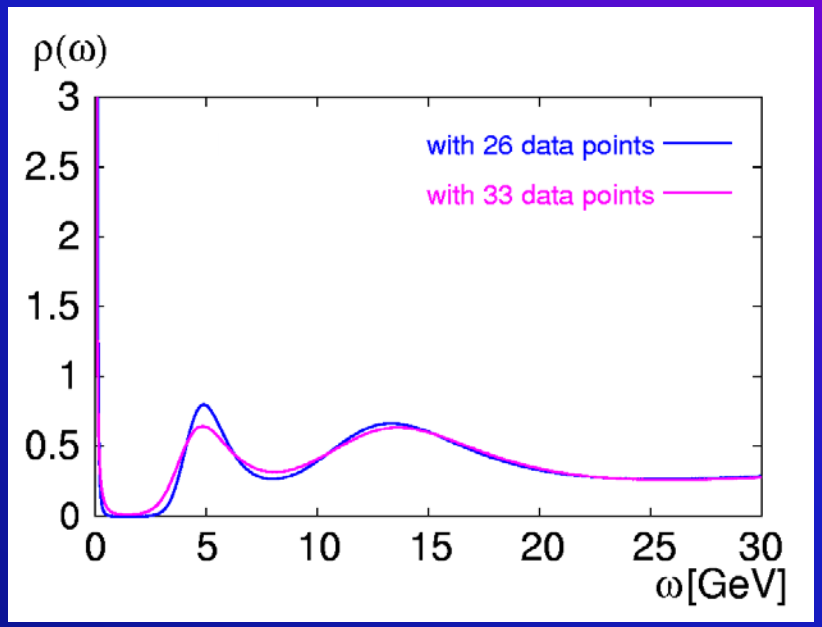


$40^3 * 30$ lattice
$\beta = 6.47$, 150 confs.
isotropic lattice ($T<T_c$)

- Necessity to use large β (=6/g^2) to have many point in τ
- Necessity to change N_{data}

Importance of Error Analysis 4





$N_\tau = 46$ ($T = 1.62T_c$)
V channel (J/ψ)
$N_\sigma = 32$

$N_\tau = 40$ ($T = 1.87T_c$)
V channel (J/ψ)
$N_\sigma = 32$

Lattice Parameters

1. Lattice Sizes
   $64^3 * N_\tau$ (previously $32^3 * N_\tau$)

2. $\beta = 7.0$, $\xi_0 = 3.5$
   $\xi = a_\sigma / a_\tau = 4.0$ (anisotropic)

3. $a_\tau = 9.75 * 10^{-3}$ fm
   $L_\sigma = 2.5$ fm

4. Standard Plaquette Action

5. Wilson Fermion

6. Heatbath : Overrelaxation
   = 1 : 4

   1000 sweeps between measurements

7. Quenched Approximation

8. Gauge Unfixed

9. Machine: PACS-CS @University of Tsukuba
   BlueGene @KEK
   ϕ @KMI Nagoya University

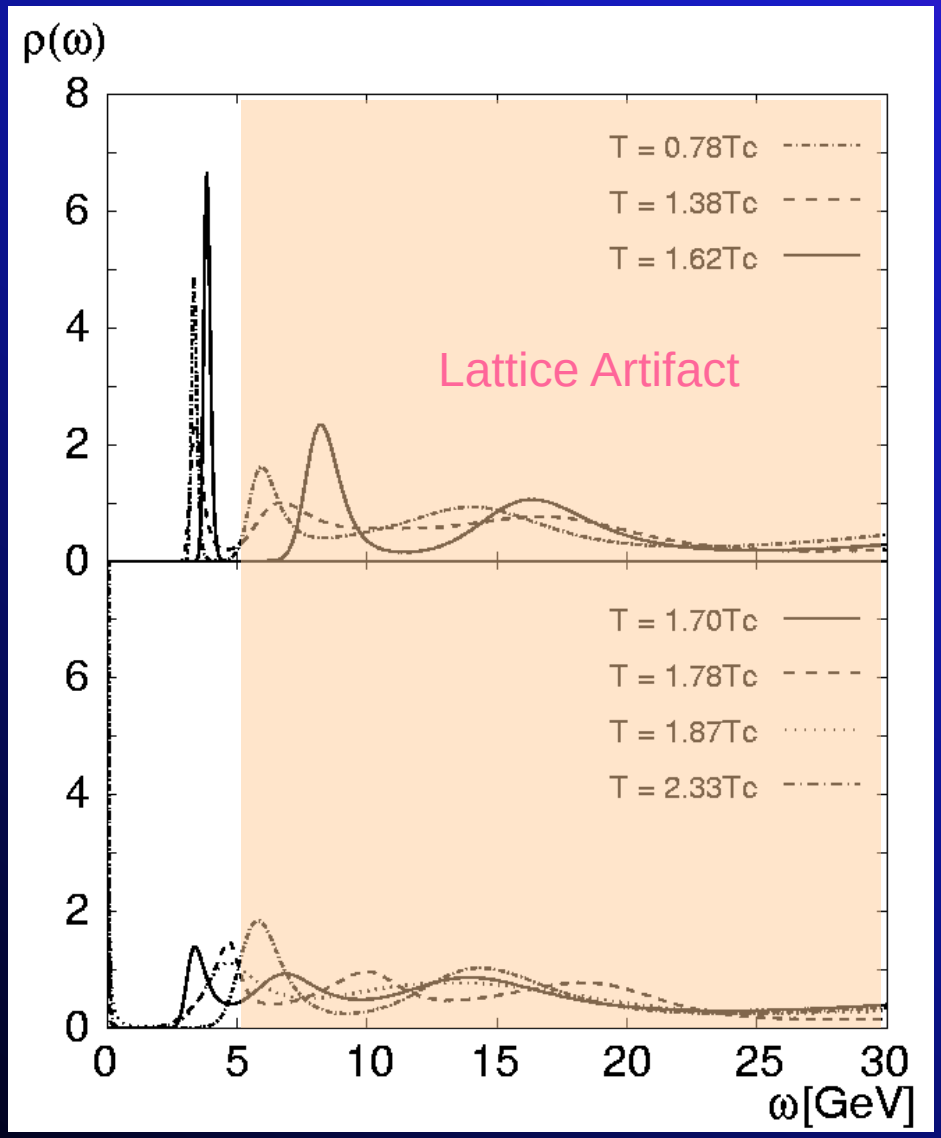
Lattice Parameters and # of Conf.

- anisotropic lattice: $\xi = a_\sigma / a_\tau = 4$
- $\beta = 7.0$, $a_\tau = 9.75 \times 10^{-3}$ fm
- large spatial volume: $N_\sigma = 64$ (L_σ=2.5fm)
- P_{min} ~ 0.5 GeV (previously ~ 1 GeV)

✓ Large spatial volume: to have *small enough* P_{min} to study finite T effects
✓ Large β = small a_τ: to have *large enough* N_τ for MEM analysis

N_τ (T/T_c)	96 (0.78)	54 (1.38)	46 (1.62)	44 (1.70)	42 (1.78)	40 (1.87)	32 (2.33)
# of conf.	**260**	**400**	**400**	**400**	**400**	**400**	**400**

Our Old Result (N_σ=32) for PS channel (η_c)



PS channel

η_c ($\mathbf{p} = \mathbf{0}$) disappears between 1.62T_c and 1.70T_c

Asakawa and Hatsuda, PRL 2004

PS channel with N_σ=64

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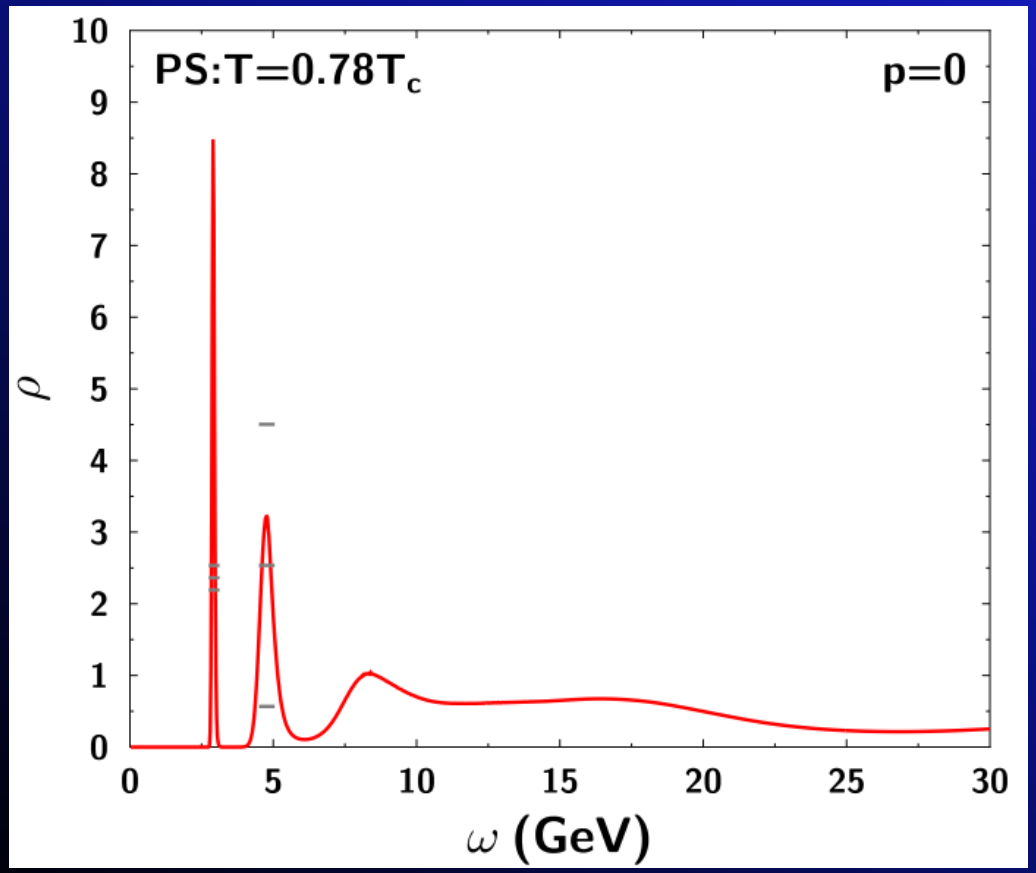

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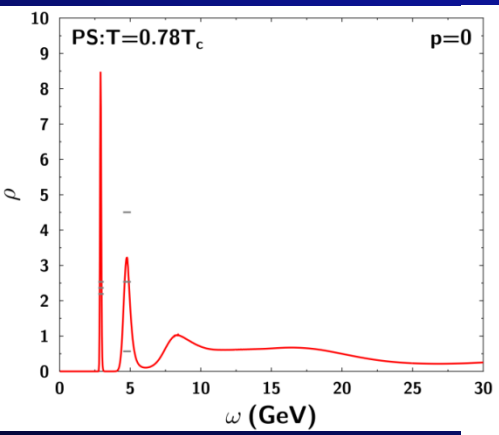

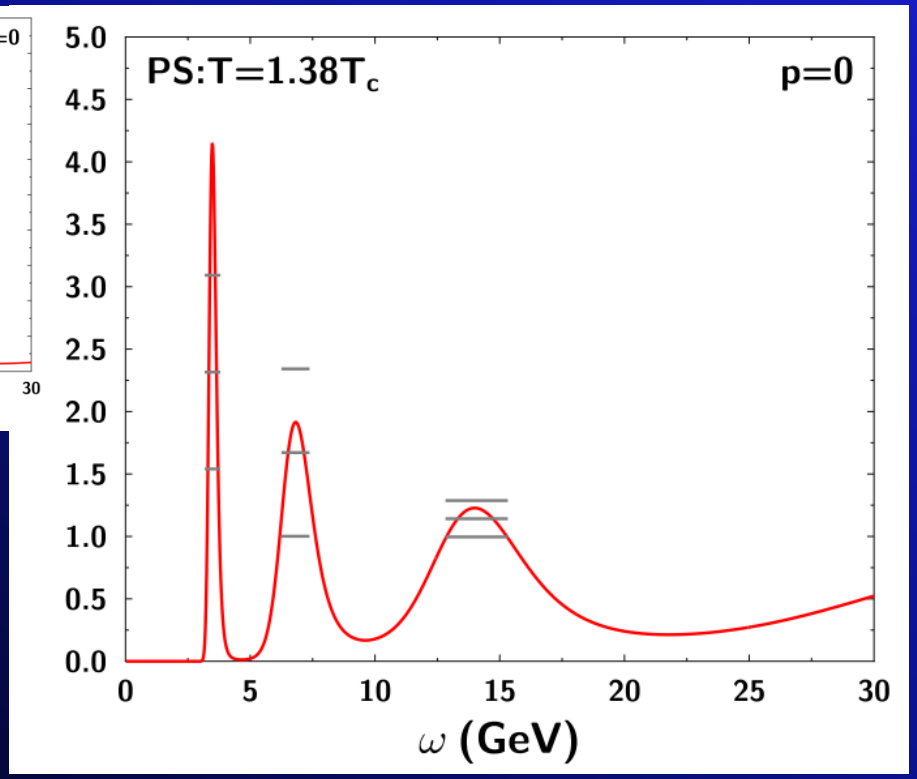

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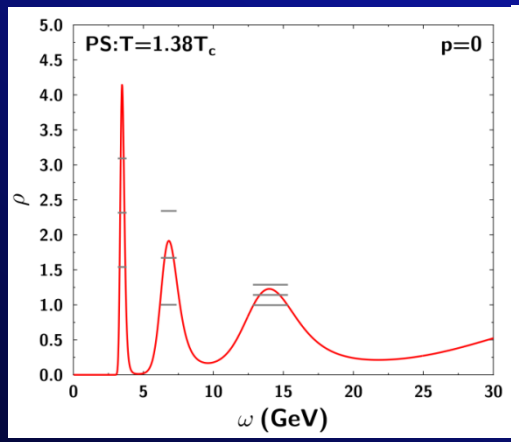

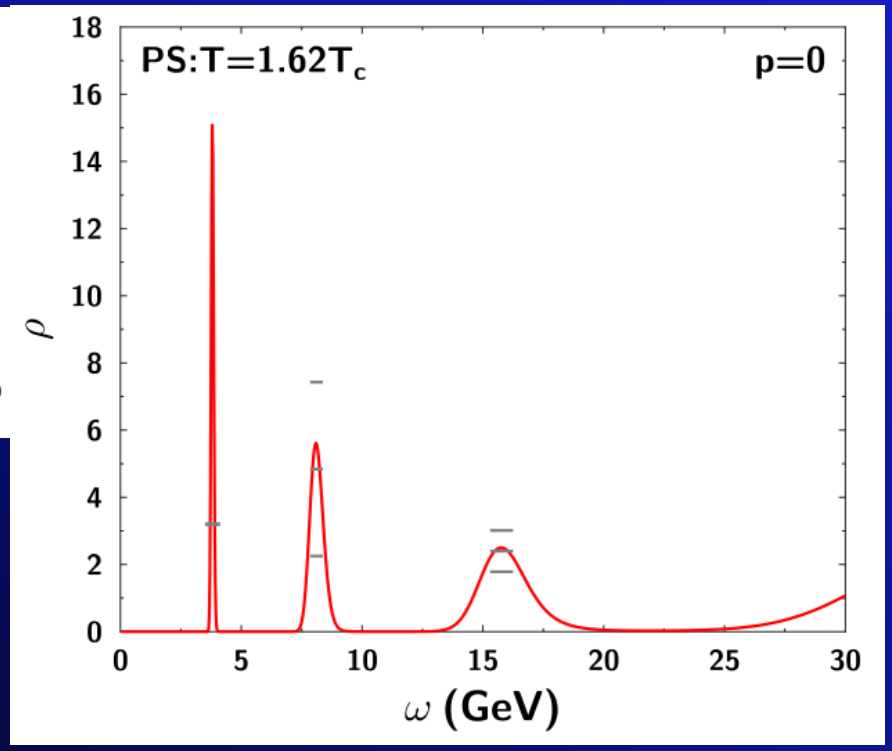

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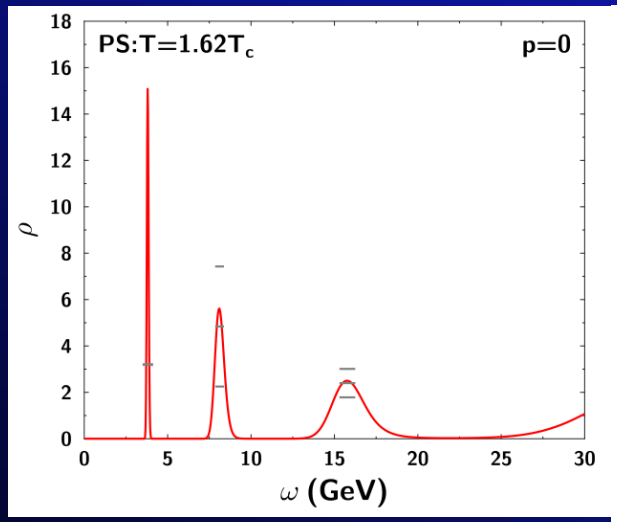

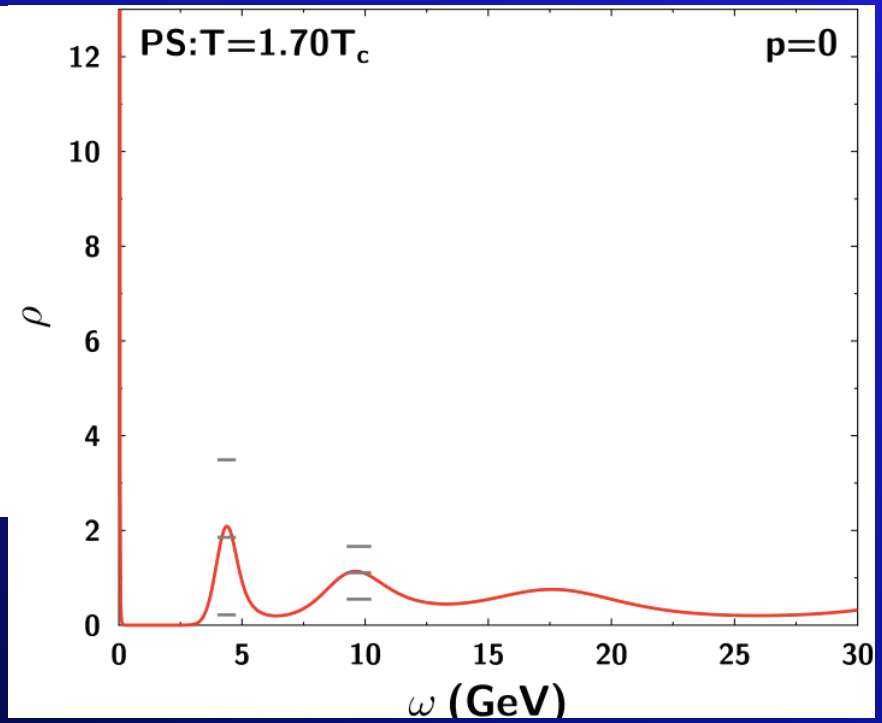

η_c melts between 1.62T_c and 1.70T_c

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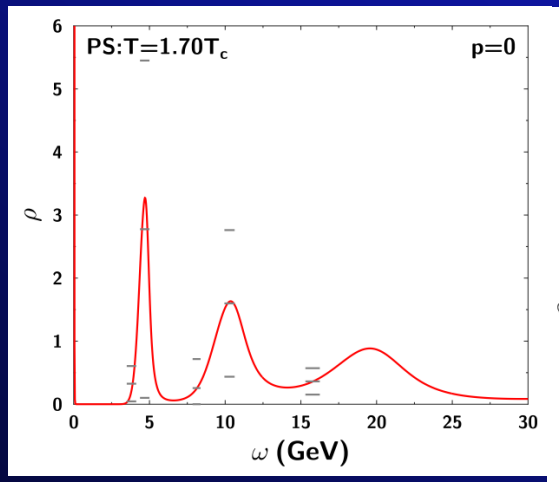

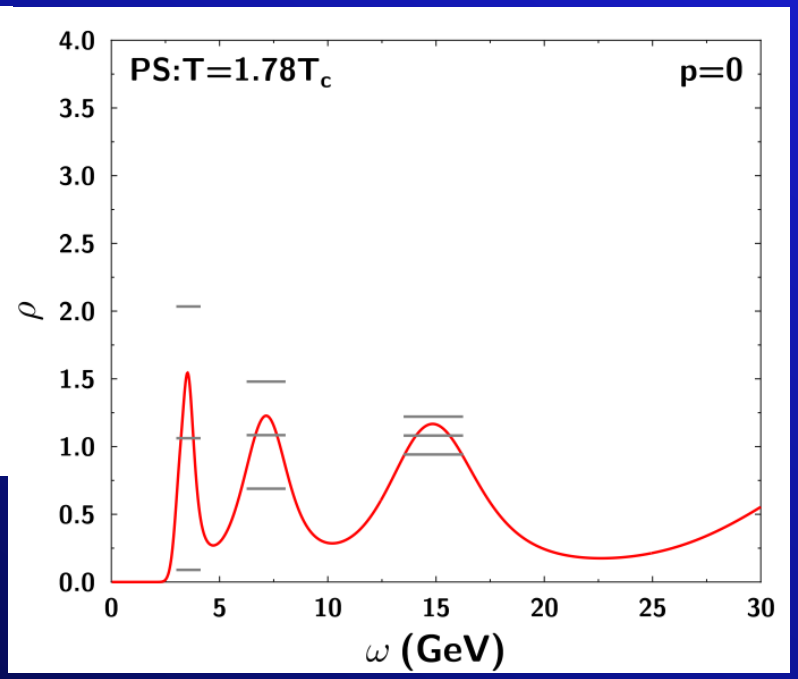

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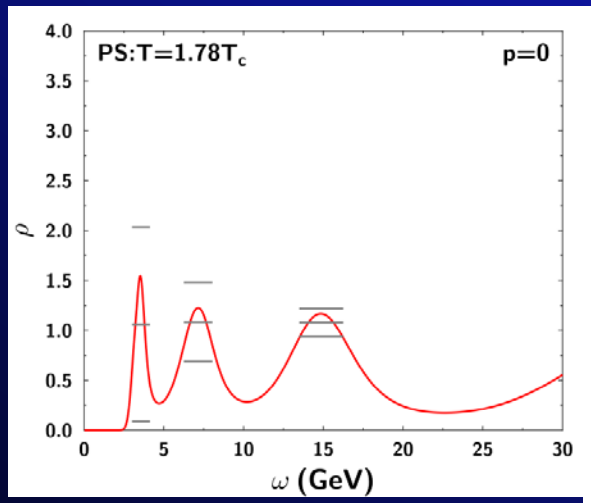
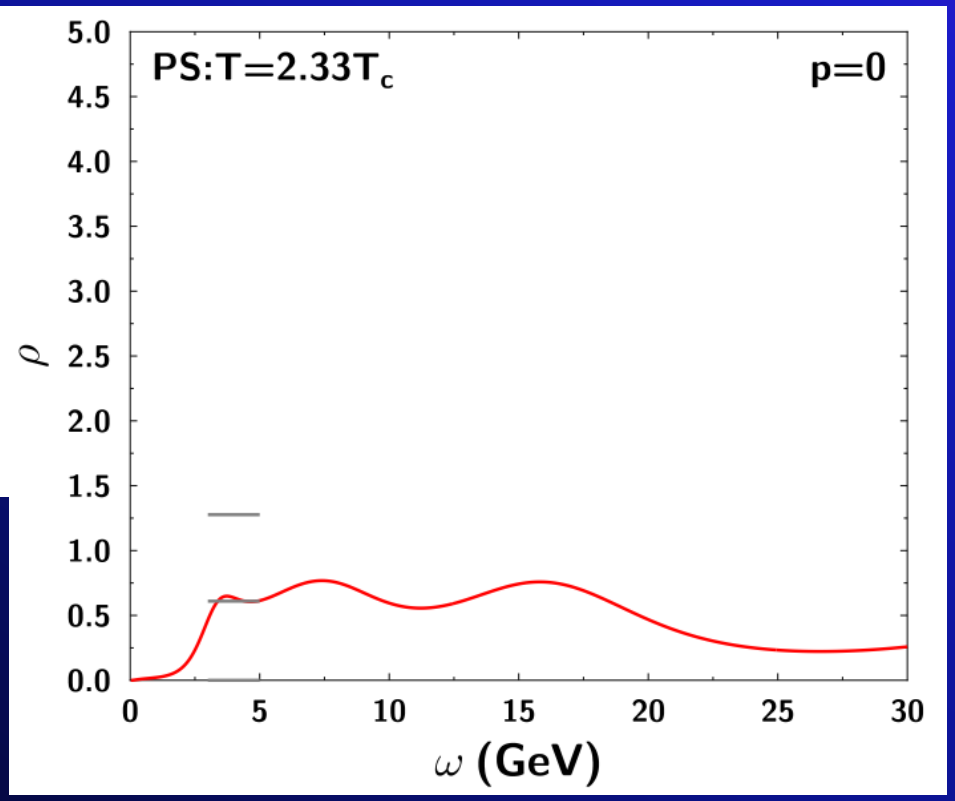
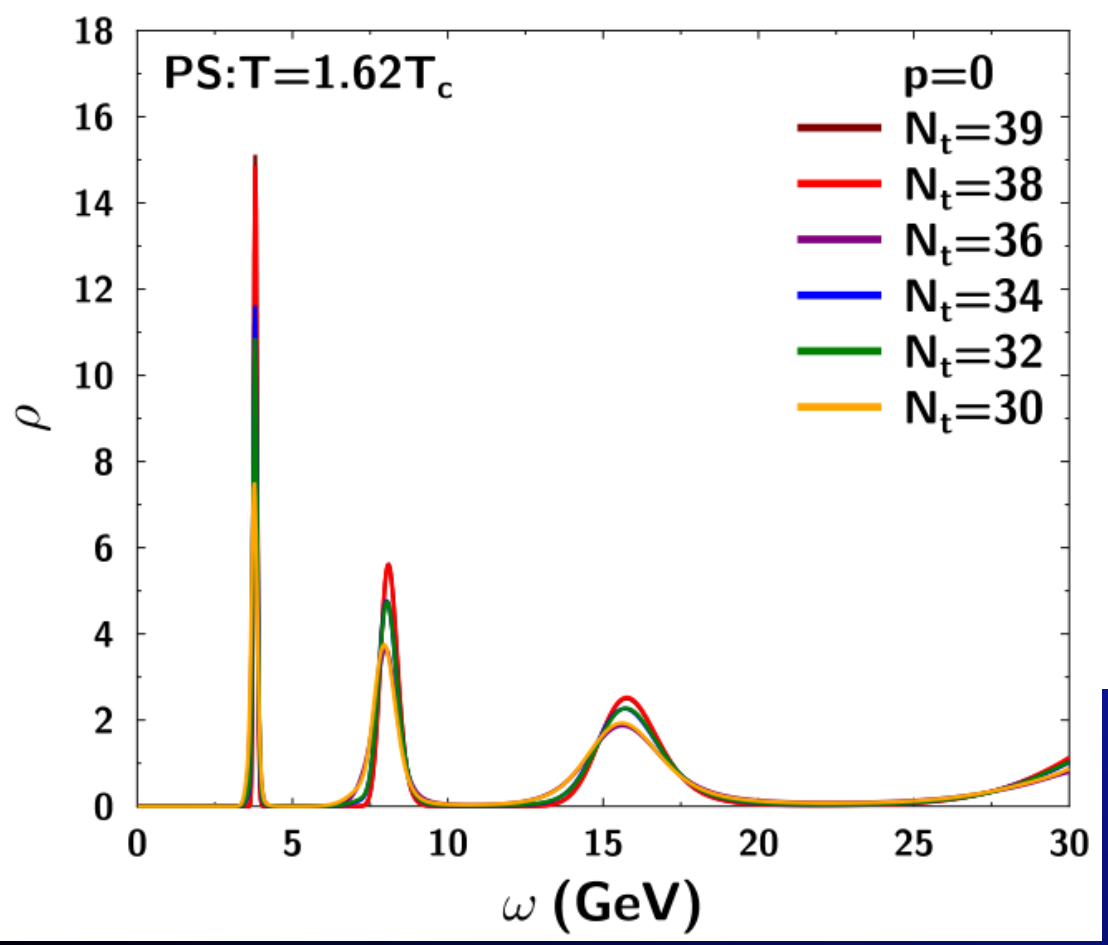


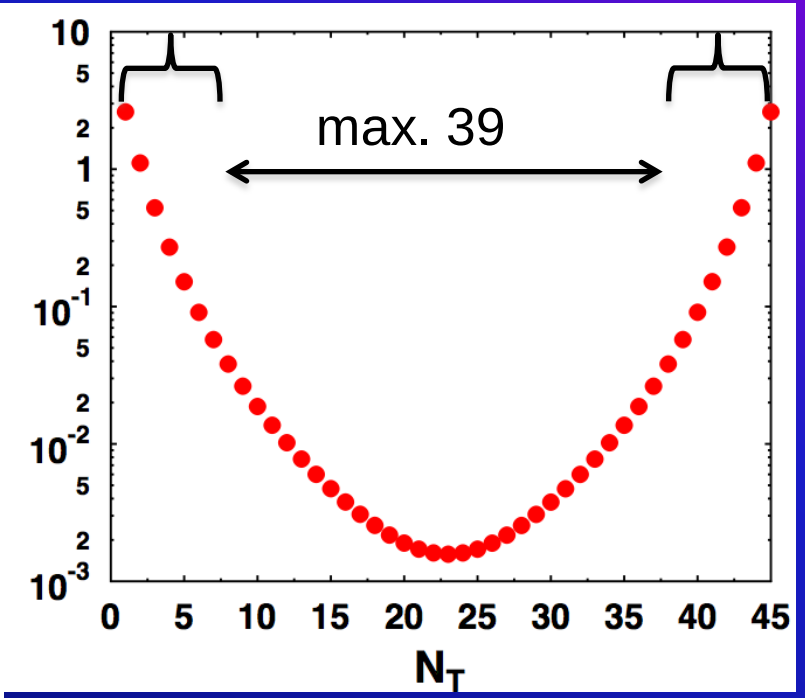


Data Point Number Dependence

not used in the analysis
note: anisotropy = 4

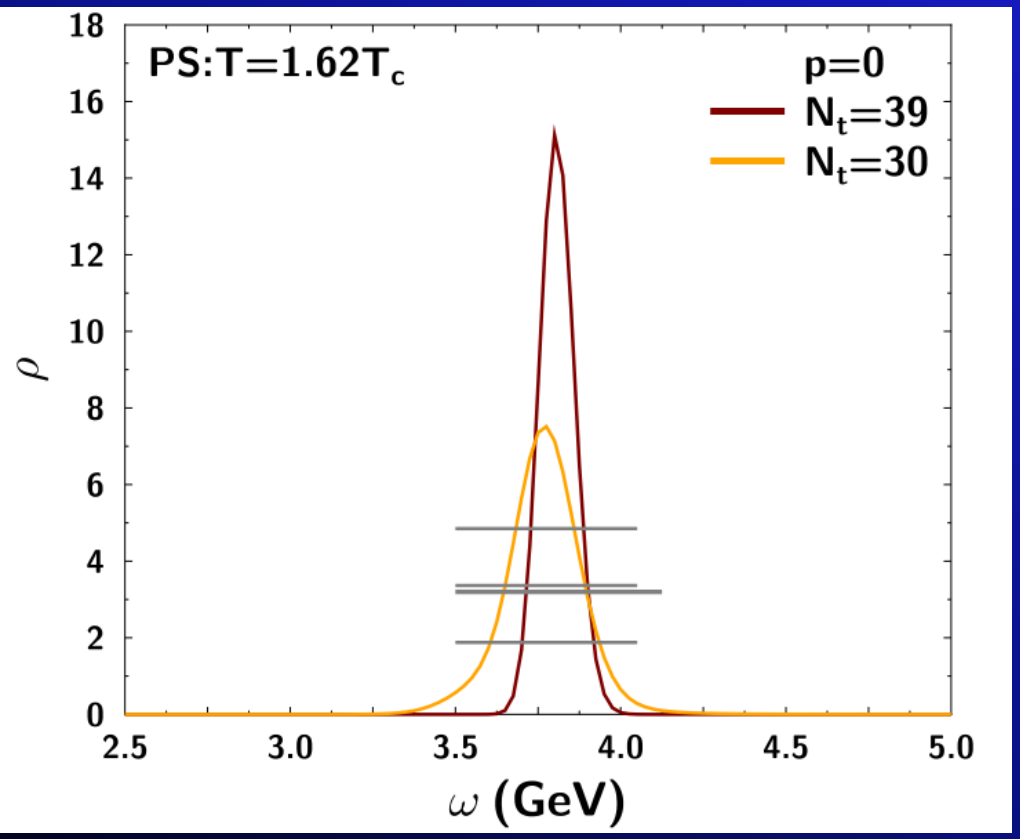
3 correlator 3





N_{data} Dependence of the Peak Shape

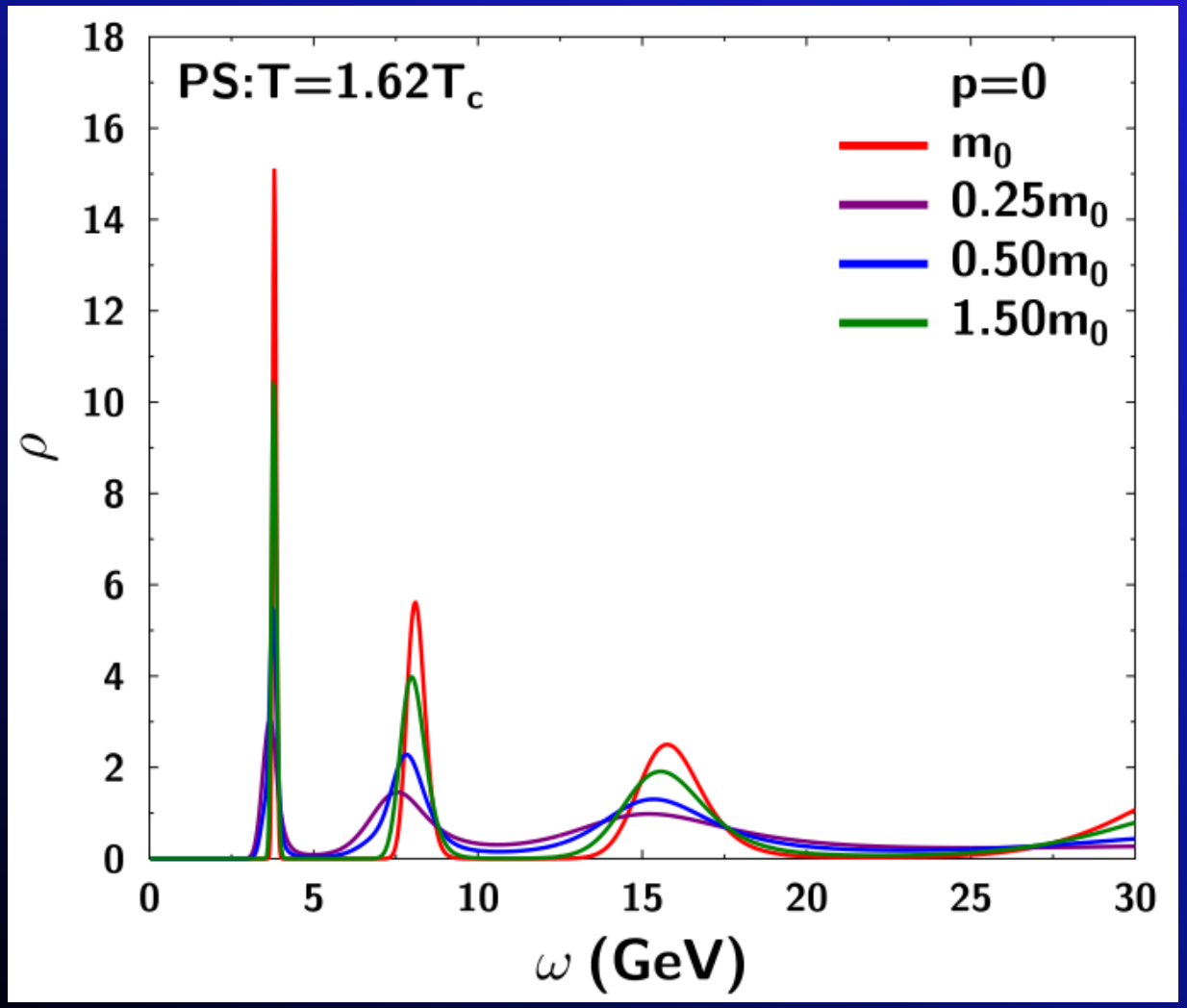
1st peak



- Larger N_{data}: sharper signal
- Average value of ρ is almost the same.
- Smaller N_{data}: larger error

It is necessary to have enough number of N_{data} in MEM analyses

Default Model Dependence



N_{data}=39

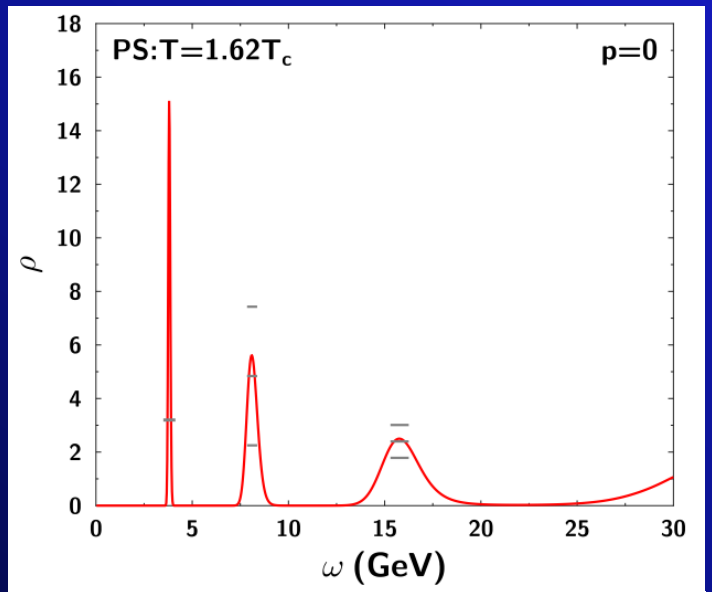
m_0=1.15

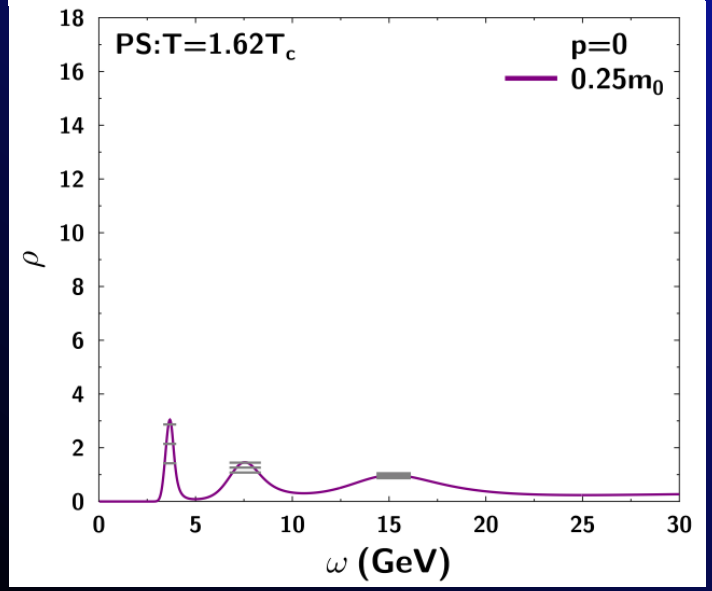
pQCD

+ Operator Renormalization

(as in Asakawa and Hatsuda (2004))

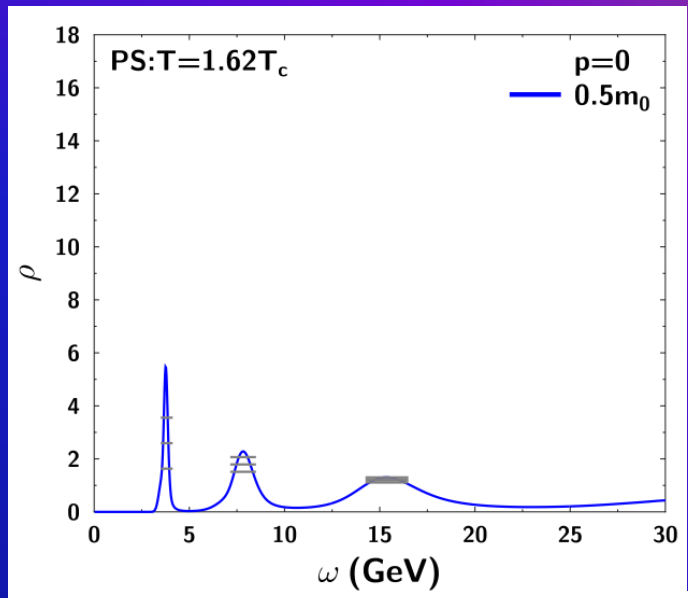
Error Analysis must also be done

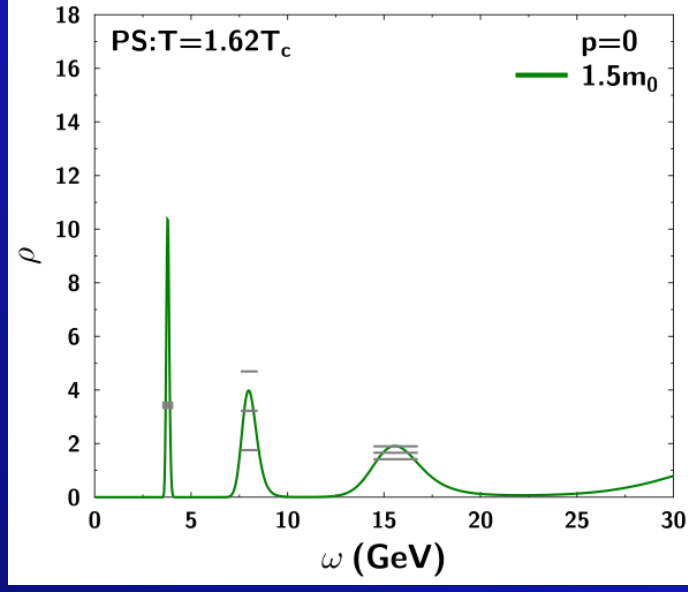




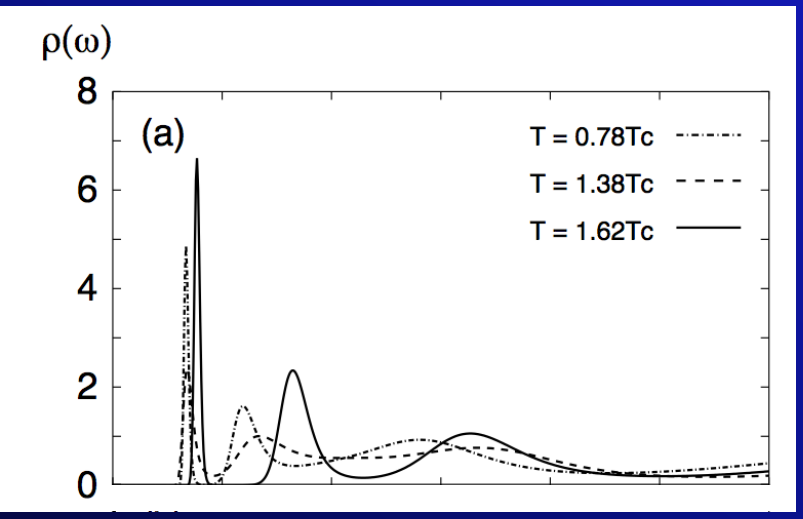
- Peak shape changes as default model changes
- But error size changes as well
- Error size takes minimum at m_0 among these four choices of default model

→ m_0 is best choice among these





Temperature Dependence of η_c "Mass"

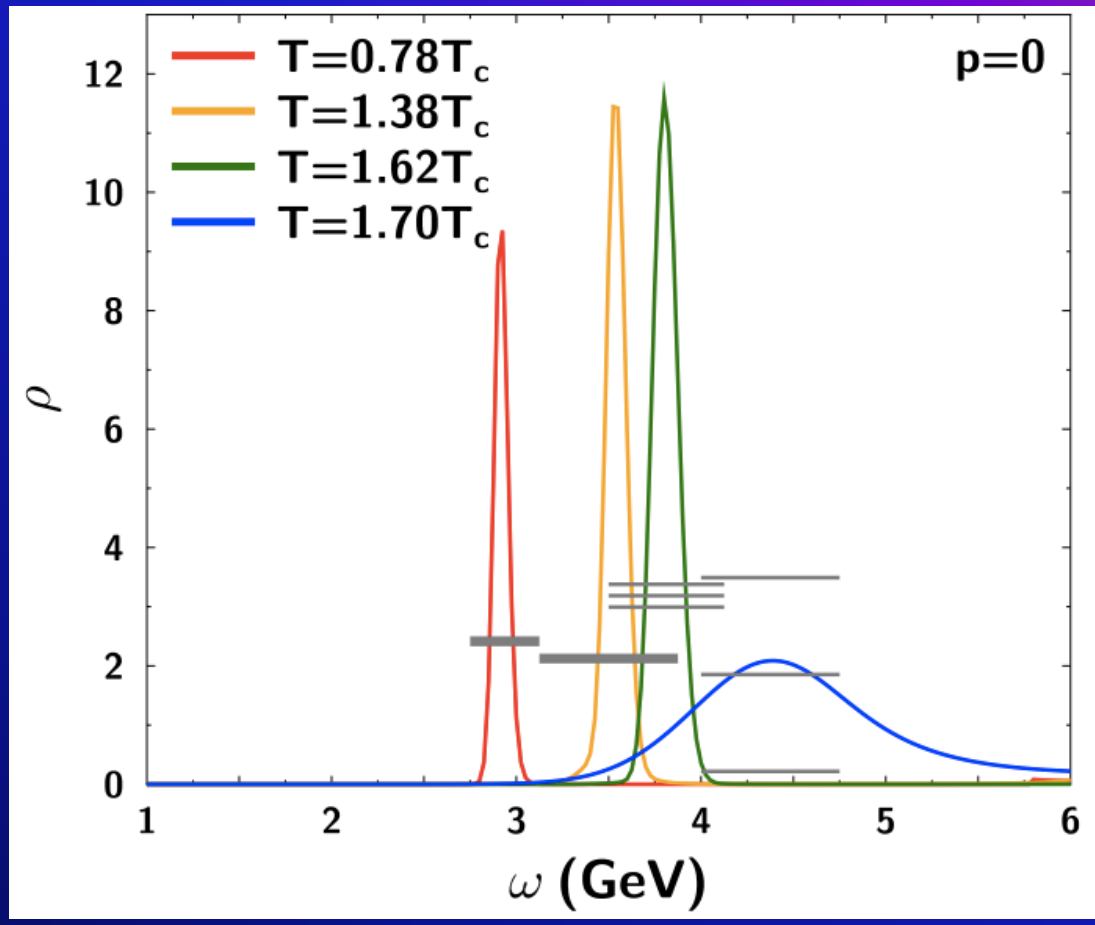


ω (GeV)

Asakawa and Hatsuda (2004)

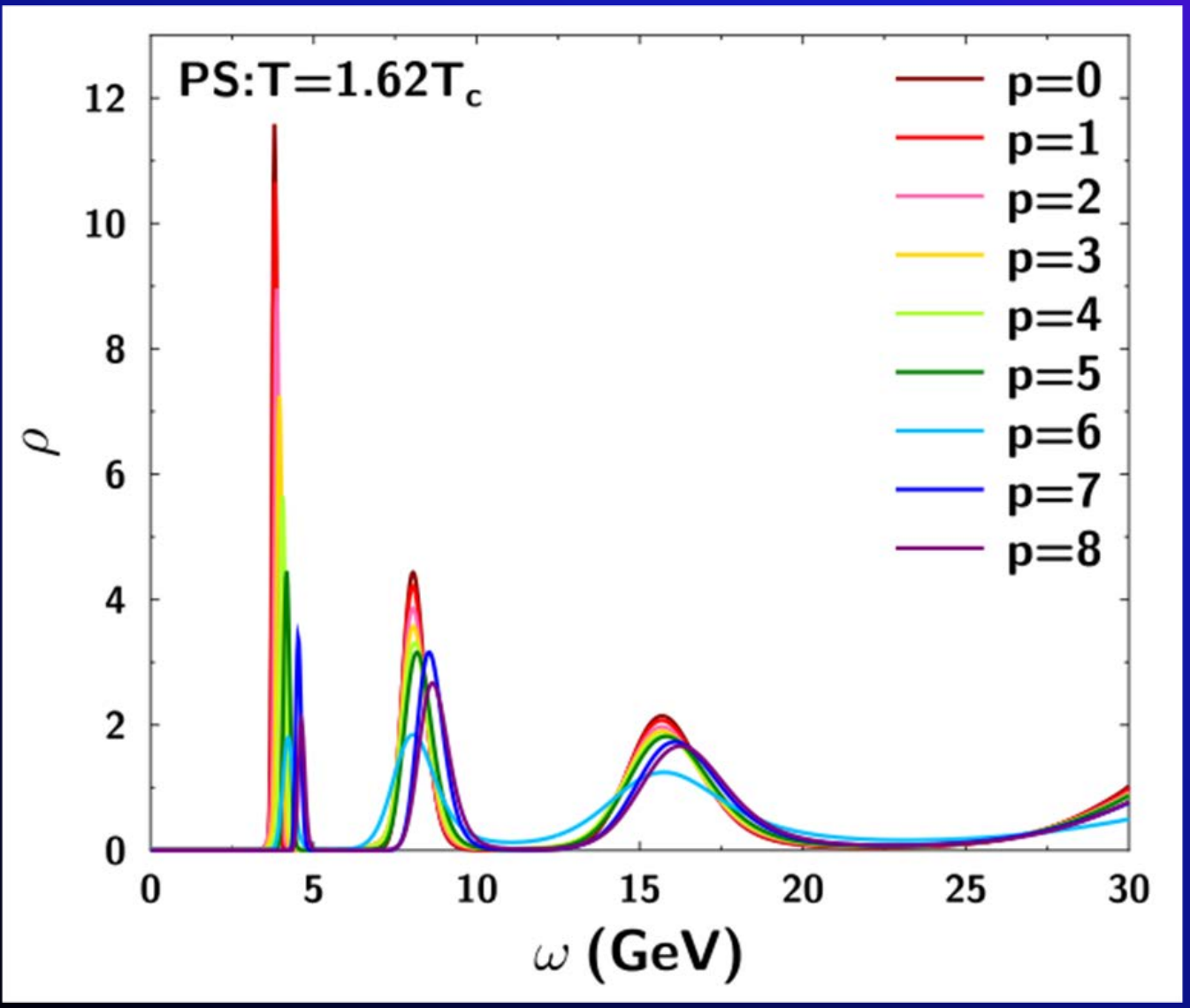
In the previous calculation, peak shift was not evident

low statistics, large p_{min}



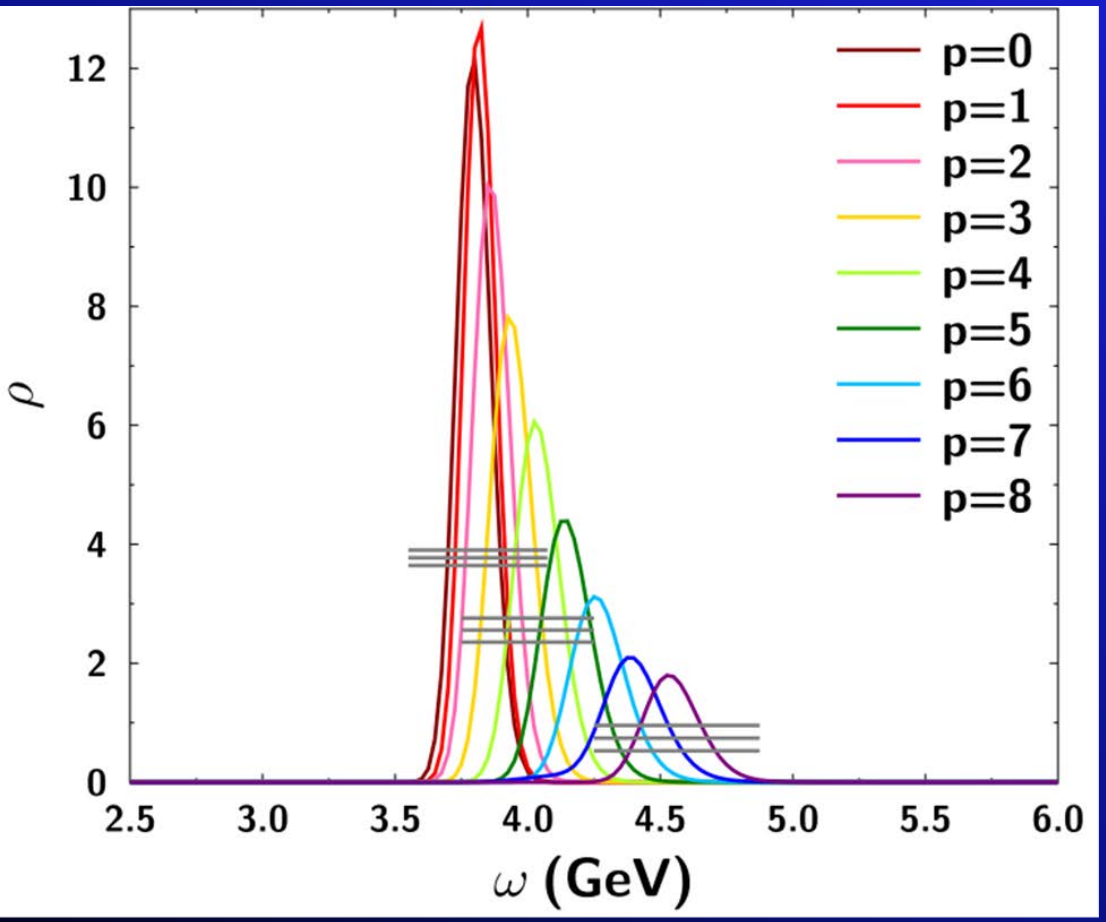
Now, peak shift is clear, it is upward

η_c at Non-zero Momenta



$P_{\min} \sim 0.5$ GeV

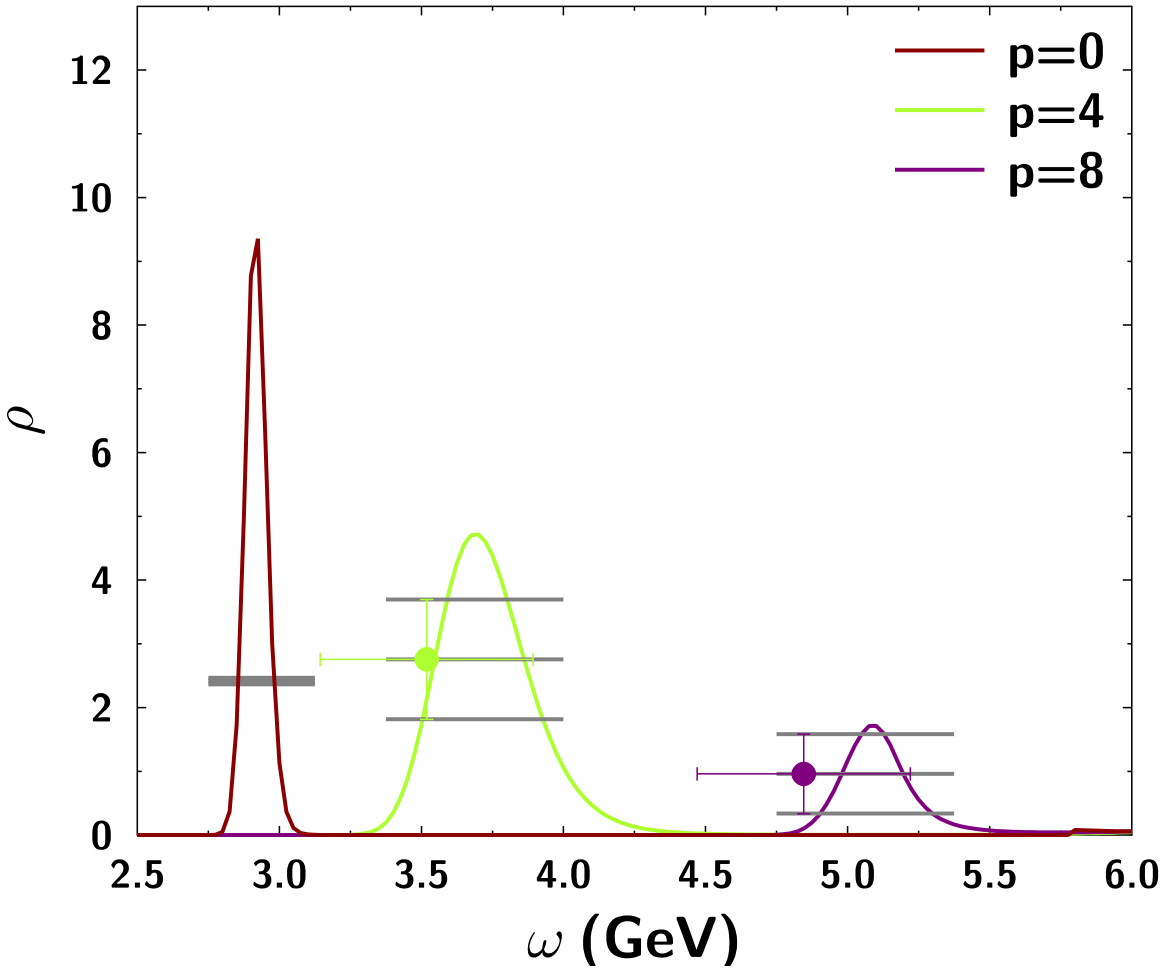
η_c at Non-zero Momenta



$T=1.62T_c$

- η_c exists at non-zero momentum (at least up to ~ 4 GeV)
- Apparently, the peak is lowered at finite momenta
- Default effect (residue decrease) + additional medium effect?

Dispersion Relation below T_c



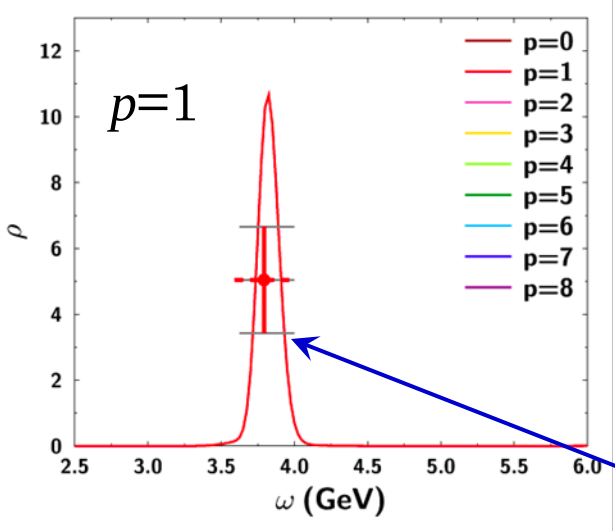
T=0.78T_c

$$\omega^2 = m|^2_{p=0} + \hat{p^2}_{\rm lattice}$$

$$\hat{p^2}_{\rm lattice} = \frac{2}{a_\sigma}\sin(pa_\sigma/2)$$

The dispersion relation at T=0.78T_c on the lattice is consistent with that in the vacuum

Despersion Relation in (Q)GP @ Finite T

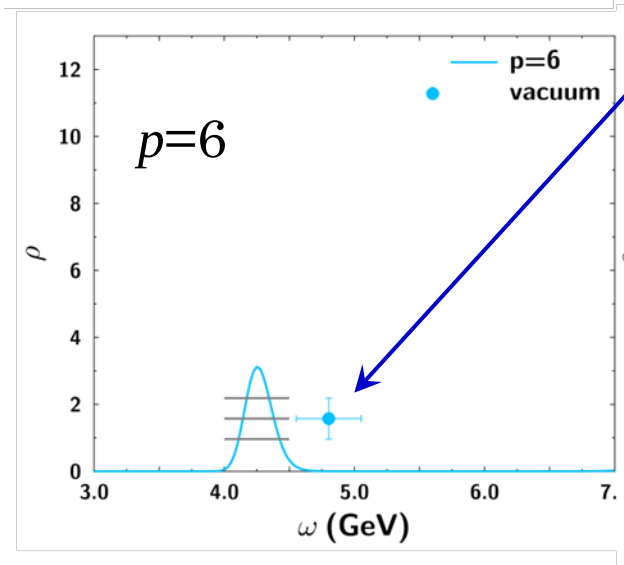


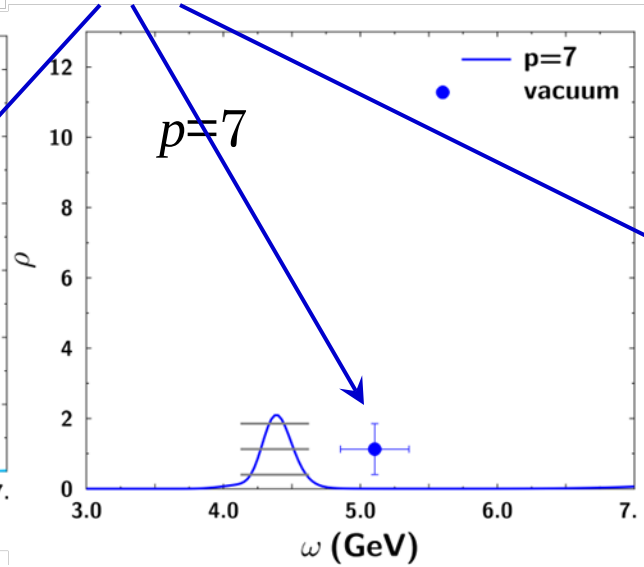
Above about p=3GeV, the modification of the dispersion relation becomes evident

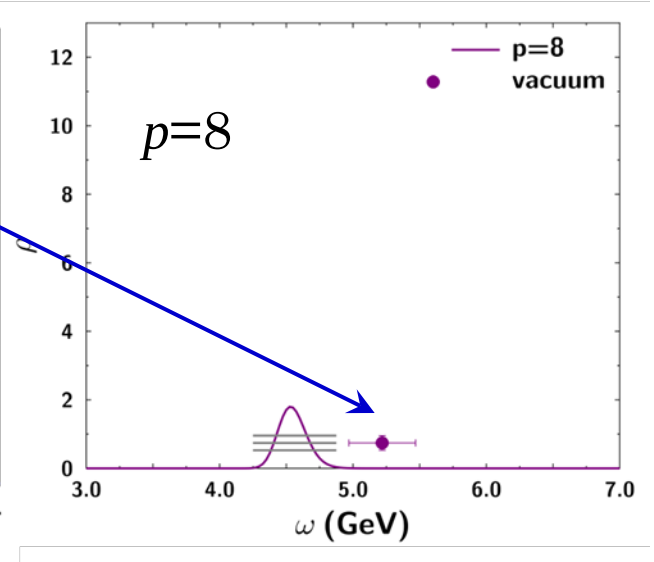
Clue to the structure of the resonance in medium?

$$\omega^2 = m|^2_{p=0} + \hat{p^2}_{\rm lattice}$$

$$\hat{p^2}_{\rm lattice} = \frac{2}{a_\sigma}\sin(pa_\sigma/2)$$





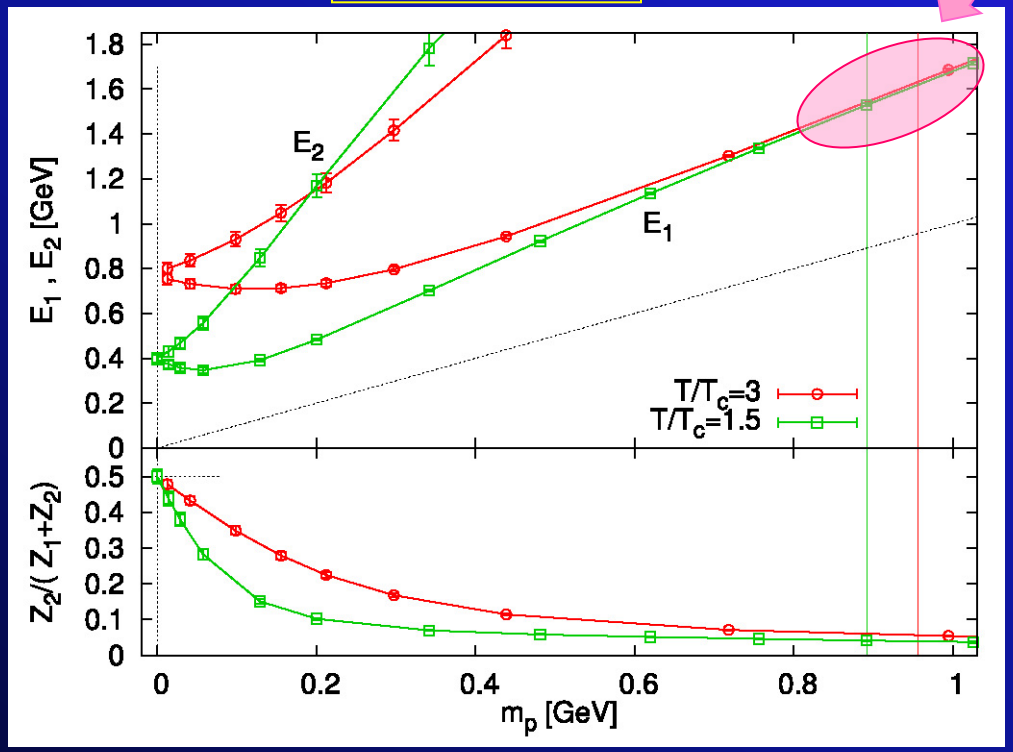


Threshold Enhancement?

Pole mass of charm barely changes as T increases

E_2: plasmino

E_1: normal mode



m_p = bare (free) quark mass

Karsch and Kitazawa, PRD (2009)

➡ Threshold enhancement cannot explain peak shift of η_c

Summary

- **Charmonia at Finite Temperature**
  - ✓ Spectral functions with large spatial volume (finite T effect)
    with many data points in τ (needed for MEM analysis)
  - ✓ At p=0
    - η_c survives up to $T=1.62T_c$ (confirms previous results)
    - Peak position ("mass") moves up as T increases
    - This cannot be explained by threshold enhancement
  - ✓ At $p\neq0$ ($T=0.78T_c$ and $1.62T_c$)
    - η_c exists at least up to p=4GeV
    - Peak height decreases as p increases
    - Dispersion relation is modified above T_c (validity of potential model?)
- **To be Finished Soon**
  - ✓ Separation of the two spectral functions in V channel
  - ✓ Calculation with smaller quark masses